Complex Dynamics and Financial Fragility in an Agent Based Model

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Abstract

In this paper, we model an agent-based economy in which heterogeneous agents (firms and a bank) interact in the financial markets. The heterogeneity is due to the balance sheet conditions and to size. In our simulations, at the aggregate level, output displays changes in trend and volatility giving rise to complex dynamics. The average solvency and liquidity ratios peak during recessions as empirical analysis shows. At the firm level the model generates: i) firm sizes left-skewed distributed, ii) growth rates Laplace distributed. Furthermore, small idiosyncratic shocks can generate large aggregate fluctuations.

1 Introduction

The financial crises of the last decade have reawakened economists to the idea that aggregate activity is affected by financial factors. From the empirical point of view, financial fragility is either identified in term of flow measure, as insolvency (the ratio between debt commitments to profits), or as a stock variable, liquidity (the ratio between debt and capital). Both ratios have been quite a good leading indicator for recessions during the past 50 years. Every recession is forestalled by a sensible rise of the ratios and only when the peak is exceeded does the recovery begin. From the theoretical point of view, several models have been proposed that explicitly point out how financial factor may amplify and propagate business fluctuations (the so-called financial accelerator mechanism: Bernanke and Gertler [6, 7], Bernanke *et al.* [5]; Greenwald and Stiglitz [13, 14, 15]; Kiyotaki and Moore [16, 17]); Cooley and Quadrini [8].)

Besides some differences, the "financial accelerator" macro-models contain several common features.

- Some informational frictions on the markets (asymmetric information) which introduce a wedge (the premium for external funds) between the cost of internal and external funds.
- This premium is an endogenous variable, which depends inversely on the borrower's balance sheet.

- A positive relationship exists between the individual borrowers' balance sheets and aggregate activity.
- Fluctuations and growth are "supply driven".

The financial accelerator approach has several drawbacks. It assumes that agents are heterogeneous but: (i) the dynamics of the variance of their financial position is not analyzed, and (ii) there is no direct interaction among the agents themselves. This is particularly troublesome since heterogeneous agents' interaction is the ultimate cause of the several scaling laws literature has pointed out as a universal characteristic of firm size distribution, their growth rate as well as business cycles (Ormerod [19], Stanley *et al.* [20], Axtell [4], Gaffeo *et al.* [11, 12]).

In this paper, we model an agent-based economy (along the lines of Delli Gatti *et al.* [9] hereafter DGP) in which heterogeneous agents (firms and a bank) interact in the financial markets giving rise to complex dynamics (a quite sympathetic approach is the one os Aoki [1, 2, 3]). The heterogeneity is due to the balance sheet conditions ("hedge", "speculative" and "Ponzi" see Minsky [18]) and to their size (small, medium and large) as well.

The model is written at the level of individual analysis (it is a bottom-up model) and generates:

- firm sizes left-skewed distributed;
- growth rates Laplace distributed;
- moreover, it shows that small idiosyncratic shocks generate large aggregate fluctuations;

The paper is organized as follows. Section 2 illustrates the DGP model modified in order to include the banking system. Section 3 discusses the simulation of the model. Section 4 concludes.

2 The Model

The real (supply) side of the model is based upon the firm's behavior. Each firm chooses how much to produce (i.e. how much to invest) on the base of its balance sheet, the financial fragility of which is proxied by the equity ratio.

2.1 The Firm

The model is populated by a large, given[1], number of different firms. Each firm produces an homogeneous good by means of a constant return to scale technology. Capital is the only input. The production function is linear:

[1] For the sake of simplicity, we assume that when a firm goes bankrupt a new one enters the market. The entrant has the following characteristics: $K_{it} = 100$ and $A_{it} = 40$ and $L_{it} = 60$.

$$Y_{it} = \phi K_{it} \tag{1}$$

where Y_i and K_i are output and capital of the i-th firm and ϕ is capital productivity.

Firms sell their output at an uncertain price because of their limited knowledge of market conditions. The individual selling price, p_{it}, is a random variable with expected value $E(p_{it}) = P_t$ and finite variance where P_t is the market price; therefore p_{it} is an idiosyncratic shock. As a consequence, the relative price, $u_{it} = p_{it}/P_t$, is a positive random variable with expected value $E(u_{it}) = 1$ and finite variance. Real revenue from sale is $u_{it}Y_{it}$.

The balance sheet of the firm is:

$$K_{it} = L_{it} + A_{it} \tag{2}$$

where A_{it} is the equity base and L_{it} the debt of i-th firm at t. In the following we will refer to $a_{it} \equiv \frac{A_{it}}{K_{it}}$ as the "equity ratio" and $\beta_{it} \equiv \frac{L_{it}}{K_{it}}$ as the "debt ratio" (of course we have $\alpha + \beta = 1$, $\alpha > 0$, $\beta < 1$).

Because of asymmetric information in the capital market we assume equity rationing.

Each firm incurs financing costs, CF_{it} [2].

$$CF_{it} = r_{it}L_{it} + r_{it}A_{it} = r_{it}K_{it} \tag{3}$$

where r_{it} is the rate of interest.

Profit (π_{it}) is the difference between revenues and costs:

$$\pi_{it} = u_{it}Y_{it} - r_{it}K_{it} \tag{4}$$

Bankruptcy occurs if net worth (A) becomes negative. Net worth is equal to net worth in previous period plus (minus) retained profits (losses).

$$A_{it} = A_{it-1} + \pi_{it} < 0 \tag{5}$$

As pointed out by the literature on asymmetric information, firms' objective function (4) has to include the expected bankruptcy cost ($E(C^B)$):

$$E(C^B) = Pr^B C^B \tag{6}$$

where Pr^B is the bankruptcy probability. Each firm maximizes

$$\Gamma_{it} = E(\pi_{it}) - E(C^B) \tag{7}$$

We assume a simple quadratic functional form for bankruptcy costs:

$$C^B_{it} = cY^2_{it} \tag{8}$$

[2] Indeed we should write $CF_{it} = r_{it}L_{it} + r^A_{it}A_{it}$, where r^A_{it} is the return on equity base. For the sake of simplicity, we assume that the return on equity base equals the interest rate: $r^A_{it} = r_{it}$.

where c is a constant.

Substituting for the real profit in (5), bankruptcy occurs if

$$u_{it} < \frac{r_{it}K_{it}}{Y_{it}} - \frac{A_{it-1}}{Y_{it}} \tag{9}$$

Assuming, for the sake of simplicity, that u_{it} has an uniform distribution with support $(0, 2)$

$$Pr^B = \frac{r_{it}K_{it}}{2Y_{it}} - \frac{A_{it-1}}{2Y_{it}} \tag{10}$$

expected bankruptcy cost (6) can be rewritten as:

$$E(C^B) = \frac{c}{2}Y_{it}(r_{it}K_{it} - A_{it-1}) = \frac{\phi c}{2}K_{it}(r_{it}K_{it} - A_{it-1}) \tag{11}$$

and the objective function becomes:

$$\Gamma_{it} = \phi K_{it} - r_{it}K_{it} - \frac{\phi c}{2}(r_{it}K_{it}^2 - K_{it}A_{it-1}) \tag{12}$$

Maximizing (12) with respect to K_{it}, one gets

$$K_{it}^* = \frac{1}{cr_{it}} - \frac{1}{c\phi} + \frac{A_{it-1}}{2r_{it}} \tag{13}$$

Investment is:

$$I_{it} = K_{it}^* - K_{it-1} \tag{14}$$

and therefore

$$I_{it} = \frac{1}{cr_{it}} - \frac{1}{c\phi} + \frac{A_{it-1}}{2r_{it}} - K_{it-1} \tag{15}$$

while the capital accumulation rate is,

$$\tau_{it} = \frac{1}{c\phi r_{it}K_{it-1}}(\phi - r_{it}) + \frac{1}{2r_{it}}a_{it-1} - 1 \tag{16}$$

It depends positively on the firm's equity ratio and negatively on the interest rate: the higher the financial fragility, the lower τ is. Furthermore according to this equation the firm's dimension (proxied by K_{it-1}) is crucial in determining the accumulation rate. The first term in the r.h.s. decreases with the size of the firm.

The demand for credit, L_{it}^d is:

$$L_{it}^d = L_{it-1} - \pi_{it-1} + I_{it} \tag{17}$$

where L_{it-1} is the stock of credit in previous period. Substituting (15) into (17) one gets

$$L_{it}^d = \frac{1}{cr_{it}\phi}(\phi - r_{it}) - \pi_{it-1} + \left(\frac{1 - 2r_{it}}{2r_{it}}\right) A_{it-1} \tag{18}$$

Some observations are in order:

- For reasonable values of the rate of interest ($r < 0.5$) the demand for credit is an increasing function of the equity base;
- Losses (profits) in the current period (t) will increase (decrease) the demand for credit in $t+1$;
- The lower the cost of investment (r_{it}) with respect to its revenue (ϕ), i.e. the higher is ($\phi - r_{it}$), the higher is the demand for credit.

2.2 The bank

In this section we modify the DGP model by introducing the credit market. For the sake of simplicity we model the credit relation of the firms' sector with one monopolistic bank. The bank's supply of credit is proportional to its current equity base which rises (lowers) if past profits were positive (negative) and it is independent on the rate of interest (the supply of credit is vertical). The bank accommodates the demand for credit of each single firm, but, because of asymmetric information, the contractual rate of interest depends on the size of the loans and the collaterals as well. There exists also a quantitative channel between the firms' and the bank's equity bases which may generate a domino effect. In fact, if a firm goes bankrupt, it is not able to fulfil the debt commitments and, as a consequence, bank's profit and equity base shrink. As a consequence, the supply of credit shrinks and the interest rate rises, depressing the demand for investment and output. If the bankruptcies are spread out, the financial market may collapse and the real market comes to a halt.

2.2.1 The supply of credit and credit market equilibrium.

Bank's balance sheet is:

$$L_t = D_t + E_t \tag{19}$$

where L_t is the aggregate supply of credit ($L_t = \sum_i L_{it}$), D_t deposits and E_t the equity base of the bank.

The problem of the bank is more complex than the firms' one, since it has to determine the aggregate level of credit and to allocate it among the firms.

To determine the aggregate level of credit supply we assume that there is a risk coefficient the bank has to respect (see Estrella [10])

$$L_t \leq \frac{E_{t-1}}{\alpha} \tag{20}$$

where α is a constant.

In the following we assume that the aggregate supply of credit is given by:

$$L_t = \frac{E_{t-1}}{\alpha} \tag{21}$$

The problem of determining the supply of credit for each firm would be a trivial goal if information were complete and firms were homogeneous (the same financial soundness and dimension).

Because of asymmetric information the bank does not know the "true" equity ratio of the borrowers, but can observe its size, which is used as a collateral. We assume that the individual supply of credit is a function of the "relative" collaterals, i.e. the capital stock of the i-th firm relative to the aggregate capital stock, times $L_t = \frac{E_{t-1}}{\alpha}$:

$$L_{it}^s = f(K_{it-1}) = L_t \frac{K_{it-1}}{K_{t-1}} \tag{22}$$

where $K_{t-1} = \sum_i K_{it-1}$.

Therefore, highly collateralized borrowers have a higher credit supply.

Credit market is in equilibrium when (18) equals (22). The rate of interest is endogenously determined by solving the equation.

$$r_{it}^* = \frac{2 + cA_{it-1}}{2c\left\{\frac{1}{c\phi} + \frac{L_t K_{it-1}}{K_{t-1}} + \pi_{it-1} + A_{it-1}\right\}} \tag{23}$$

Note that the equilibrium level of credit is equal to the credit supply (since this last is independent from the interest rate) and that a rise (decrease) of profits and the "relative" collaterals (of the equity base) decreases the rate of interest.

2.2.2 The level of bank's equity ratio.

In dealing with the dynamics of the bank's equity level and ratio, it is interesting to note what happens when a firm goes bankrupt.

According to our assumption, a firm goes bankrupt when its equity base becomes negative. In such a case $K_{it} < L_{it}$ and the firm cannot refund its own loan. Following the Japanese jargon we call the difference $L_{it} - K_{it}$ as "bad debt". Bad debt for the bank is:

$$B_{it} = \begin{cases} -A_{it} & if \quad A_{it} < 0 \\ 0 & if \quad A_{it} \geq 0 \end{cases} \tag{24}$$

The bank's equity stock is

$$E_t = \Pi_t^b + E_{t-1} - \sum_i B_{it-1} \tag{25}$$

where bank's profit is:

$$\Pi_t^b = \sum_i r_{it} L_{it} - r_t^D D_{t-1} - \bar{r}_t E_{t-1} = \sum_i r_{it} L_{it} - \bar{r}_t (1-\omega) D_{t-1} - \bar{r}_t E_{t-1} \tag{26}$$

where r_t^D is the rate of interest on deposit, $\bar{r}_t$ is the average rate of interest and ω is the profit mark up for the bank. For the sake of simplicity we assume that $r_t^D = (1-\omega)\bar{r}_t$.

Debt is a source of indirect interaction. Bankruptcies shrink bank's equity base and tightens credit market conditions for every firm because the aggregate supply of credit shifts to the left:

$$\frac{E_1}{\alpha} < \frac{E_0}{\alpha} \Rightarrow L_1^s < L_0^s \tag{27}$$

As a consequence, the individual supply of credit shrinks and the interest rate increases deteriorating firms' balance sheets. If there exists a large enough critical mass of bankrupted firms a domino effect may set up. The next section shows some cases in point.

3 Simulations

We simulate, by using SWARM, a setting with 100 firms (there is a 1 to 1 replacement if a firm goes bankrupt, so that the total number of firms is constant) which have credit relations with one bank. The parameters of the simulation are the following: $\phi = 0.1$, $c = 1$, $\alpha = 0.08$, $\omega = 0.002$ (results are quite robust with very different parameters value and number of firms). The (complex) behavior of the aggregate output shows that phases of smooth growth follow periods of large output variability, different slopes and sudden drifts appear from time to time. While growth and fluctuations can be attributed to changes in the equity ratio, shifts, drifts and volatility changes can be attributed to the micro-interaction of the single units (firms and bank) and to the small idiosyncratic shocks.

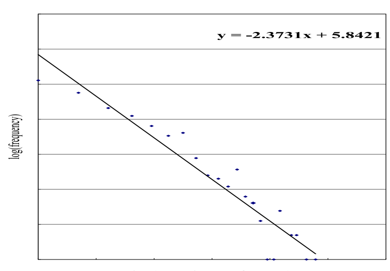

Figure 1: Power law in recession: log(growth rate of output) vs log(frequency)

Despite these interesting properties, we want to concentrate our attention on the ability of the model to replicate some recent empirical findings on industrial dynamics. Indeed, the simulation shows some stylized facts emphasized by the literature. In particular, as shown in figure 1, recession rates and length follow a power law (Gaffeo *et al.* [11]) while expansions are log-normal distributed. It has been suggested that, according to this evidence, recessions and expansions likely

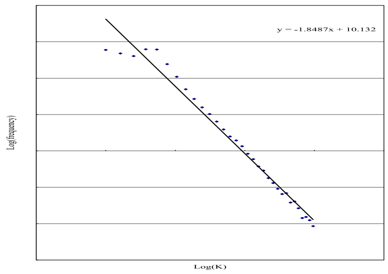

Figure 2: Distribution of the firm size: log(K) vs log(frequency).

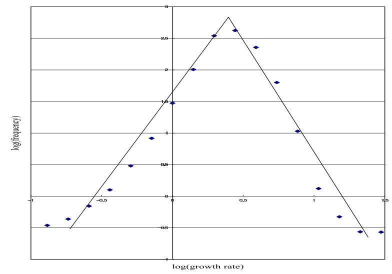

Figure 3: Distribution of the firm's growth rate: log(growth rate) vs log(frequency).

follow different statistical laws. This result could have profound implications for business cycle modeling. In this model the asymmetry between expansions and recessions is due to the fact that firms react differently to a (positive) negative shock, since the endogenous relation between capital accumulation and the equity base is concave.

Firm size distribution is skewed (Axtell [4]; Gaffeo *et al.* [12]) and distributed according to a power law (figure 2).[3] If we condition this distribution to business cycle phases, we have firms that are more equally distributed during recessions than expansions. Moreover, if firm distribution has fat tails, small idiosyncratic shock may lead a system populated by many heterogeneous agents to experience aggregate fluctuations. There is a persistent heterogeneity among firms regarding both dimension and financial position. The distribution with respect to dimension is asymmetric (Sutton [21]). The moments of distribution by financial position varies with the business cycle. Moreover, the simulations show that there is a tendency of the series to converge to a Power Law distribution. Both the distribution by dimension and by financial position keeps a positive dispersion. There is however a modification of the distribution with the different phases of the cycle.

As regards growth rate distribution of firms, literature was used to appeal to the law of proportional growth, or Gibrat's law, according to which the right skewed distribution of the firms' size may be attributed to the fact that growth rates are independent of the size of a firm. We find this distribution is better approximated by a tent-shaped curve (figure 3), i.e. by a Laplace distribution, rather a log-normal distribution with fat tails. Stanley *et al.* [20] , have shown that this stylized fact resembles better the empirical evidence on US firms.[4]

[3] There is some simulative evidence that, since firms' size distribution is power law, there is a link between the business cycle and the scaling behavior (see also Gabaix [22]). The average and the variance of the firm's distribution changes through the cycle according to the financial fragility process envisaged in the paper either with movements along the distribution curve and of the curve itself (conditioned to expansions-contractions).

[4] Moreover, we point out that, in agreement with the evidence in Stanley *et al* [20], the standard deviation of log growth rates decreases with size according to a power law.

4 Conclusive remarks

In this paper we presented an interacting heterogeneous agent based model which fits several empirical facts on firm size distribution and growth, and on the business cycle and growth as well. It has pointed out the vacuum of the standard (i.e. based upon the representative agent approach) microfoundation literature which is unable to provide sensitive answers to understand a world where, because of less than complete information, agents have to interact. All in all, we may say that macroeconomics (macroeconometrics) still lack sound microfondations. Economics is still adopting the classical mechanics approach, of XIX century physics, based upon the reductionist principle: since the aggregate is nothing but the sum of its components, in order to understand the working of the aggregate it is sufficient to understand the working of each single element. The quantum revolution of the last century, radically changed this perspective: according to the holistic approach it adopts, the aggregate is different from the sum of its component because of interaction. This is precisely the approach the agent based modeling strategy adopts. To analyse the aggregate dynamics as if it were the dynamics of a representative agent can not produce such false answers (and erroneous policies). The lack of analytical tools able to cope with heterogeneous interacting agents and their aggregate dynamics is one of the goals of future economic research.

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