Inflation and the real effects of monetary policy

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Abstract:
The relationship between money and real economic activity is still a very appealing subject in macroeconomics. A number of studies have sought to characterise this link with the assumption that money has symmetric effects on real variables. In this paper we reconsider the empirical evidence on the possible asymmetric effects on real activity of changes in short term nominal interest rate. We concentrate on the effects of positive and negative monetary shocks. In the literature, the latter often have real effects whereas the former are neutral. Models based on costly price adjustments demonstrate that a tight monetary policy has a larger absolute impact that an easy monetary policy. Ball and Mankiw (1994) have developed a model with inflation trend in this line of research. Here we test the assumption that inflation plays a key role in these asymmetric effects using the markov-switching model of Hamilton (1989). From the analysis of France and Italy we find strong evidence about the distinction between expansionist and restrictive shocks. Besides results support empirical predictions of Ball and Mankiw's model that negative shocks effects increase with inflation. These findings are particularly important in the European context in which single monetary policy is conducted by the European Central Bank. They imply that monetary authorities must take into account the behaviour of the inflation process. So we wonder what is the future for such asymmetries since European countries have converged through low inflation rates. Could the low inflation rate contribute to reduce asymmetries in Monetary Union?

Keywords: inflation rate, Markov switching model, monetary shocks.

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1 Introduction

The relationship between money and real economic activity is still a very appealing subject in macroeconomics. A number of studies have sought to characterise this link with the assumption that money has symmetric effects on real variables. Nevertheless recent studies on this issue are particularly relevant in light of new works which predict asymmetric effects.

From a theoretical point of view, microeconomic foundations have conducted to this renewed interest. Models based on sticky wages or costly price adjustments demonstrate that a tight monetary policy has a larger absolute impact than an easy monetary policy. Ball and Mankiw (1994) have developed a model with inflation trend in this line of research. It is based on costly price adjustments and implies a convex short-term aggregate supply curve. Ball and Mankiw’s model is able to generate asymmetric price adjustment behaviour on the part of firms. This model provides a theoretical rationale for the empirical finding of DeLong and Summers (1988), Cover (1992), Morgan (1993) and Karras (1996) who conclude that monetary shocks have asymmetric effects on real activity with contractionary monetary policy having a larger absolute impact than expansionary monetary policy. Besides this model shows a positive relation between the degree of asymmetry and the level of trend inflation: increases in trend inflation diminish the real effects of positive monetary shocks and increase the real effects of negative monetary shocks.

Having established the asymmetric properties of monetary shocks on output in previous research (Huchet (2003)), we investigate in this paper whether the observed asymmetric effects are related to the rate of inflation itself. We apply Cover’s (1992) methodology which is based on a two steps proceeding over the period 1970-1998. To investigate the dynamic implications of the Ball and Mankiw’s model, we allow for a time-varying relation between the degree of asymmetry and a measure of average inflation based on Hamilton's (1989) Markov regime-switching model. This Markov model provides a very relevant framework for analysing the differential real effects of monetary shocks across inflation regimes. In particular, this technique allows us to model and identify shifts in the mean of the inflation process. To determine whether the behaviour of inflation
is important for isolating asymmetric effects of monetary shocks as well as for explaining changes in the degree of asymmetry, we construct a measure of average inflation based on expected inflation series from a two-state Markov switching model. Hamilton's approach enables to specify first uncertainty about regime in date t and then the change from on regime to another.

The remainder of the paper is organised as follows. Section 2 is devoted to the Hamilton's approach. Section 3 provides an overview of the Ball and Mankiw’s model and describes the estimation strategy used to measure the real effects of monetary policy in France and Italy taking into account the inflation rate shifts. Section 4 presents the empirical results.

2 Hamilton's (1989) model

Hamilton (1989) establishes the method of modelling time series with changes in regime following a Markov process as one alternative of the non linear modelling methods. His approach is based on Goldfeld and Quandt (1973) ’s Markov switching regression to characterise the change in the parameters of the time series process.

2.1 Framework

Hamilton (1989) applies an univariate technique to US real GNP. He uses quarterly data running from 1951:2 to 1984:4 and fits an AR(4)-process. The model is written:

\[
y_t - \mu_{s,t} = \phi_1(y_{t-1} - \mu_{s,t-1}) + \phi_2(y_{t-2} - \mu_{s,t-2}) + \phi_3(y_{t-3} - \mu_{s,t-3}) + \phi_4(y_{t-4} - \mu_{s,t-4}) + \epsilon_t
\]

with  \( \epsilon_t \sim N(0,1) \)

\[
\mu_{s,t} = \alpha_0 + \alpha_1 S_t
\]

\( s_t \) is an unobservable discrete-valued state variable

\( s_t = 1 \) if high growth state, 0 otherwise

Nonlinearity of the model arises because the process is subject to discrete shifts in the mean, between high-growth and low-growth states. These discrete shifts have their own dynamics,
specified as a two-state first-order Markov process. Transition probabilities between regimes are then:

\[
\begin{align*}
P(s_t = 1|s_{t-1} = 1) &= p  \\
P(s_t = 0|s_{t-1} = 1) &= 1 - p  \\
P(s_t = 0|s_{t-1} = 0) &= q  \\
P(s_t = 1|s_{t-1} = 0) &= 1 - q
\end{align*}
\]

Where each of the transition probabilities are restricted to be non-negative and belong to the unit interval.

The model is a nonlinear combination of discrete and continuous dynamics with nine parameters \((\alpha_0, \alpha_1, p, q, \phi_1, \phi_2, \phi_3, \phi_4, \sigma)\). The used variable is \(y_t = 100 \ln \left(\text{real GNP}_t/\text{real GNP}_{t-1}\right)\). It implies that \(\alpha_0\) is the mean quarterly percentage rate of growth in the low-growth state and \(\alpha_0 + \alpha_1\) is the mean quarterly percentage rate of growth in the high-growth state. Identification of the two states is an arbitrary choice which constitutes to fix the recession regime that one which has a negative intercept.

Since \(S_t\) is not observed, Hamilton presents an algorithm for drawing such probabilistic inference in the form of a nonlinear iterative filter\(^1\).

### 2.2 Results

Hamilton’s results can be summarised in table 1. It turns out that the estimated two means can be associated with the dynamics of the business cycle, one being negative (-0.35%) during state 0 (low-growth) and the other being positive (+1.17%) during state 1 (high-growth).

One interesting implication of the Markov framework is that we calculate from the maximum likelihood parameter estimates the expected duration of a typical recession and expansion. They are derived from the following result:

\(^1\) See details of the filter in appendix A1.
They imply a persistent about 4.1 quarters for a recession and 10.5 quarters for an expansion. They prove a high asymmetry during U.S. GNP cycle.

3 Application to the relationship between a monetary shock and the real activity

A former study of the effects of monetary policy on real activity realised under Monetary Union regime (Huchet (2003)) suggests the existence of asymmetries in France and Italy. Particularly, the behaviour of the inflation rate changes over the past thirty years. So we extend the Cover (1992)’s approach in order to test if monetary policy has different effects depending on inflation rate.

3.1 The theoretical contribution of Ball and Mankiw (1994)

Ball and Mankiw suppose that a continuum of firms in imperfect competition form the economy. Their specification combines elements of time- and state-contingent price adjustment. They look at a model in which firms can set prices every second period without cost, but subject to a menu-cost if they wish to change prices between periods. As with time-contingent pricing, the firm adjusts on a regular schedule (every two periods) but as with state-contingent pricing, firm can adjust whenever circumstances substantially change by paying a menu cost. The introduction of a positive inflation trend implies that monetary shocks have no more symmetric effects. With such a positive inflation trend, expansionist monetary shock means that the firm's desired relative price rises while its actual relative price is falling, creating a large gap between desired and actual prices. As a result, many firms prefer to pay a menu cost and to adjust their prices because of the existence of inflation that magnifies the change in demand. On the other hand, negative shock with positive trend inflation leads to a decrease in firm’s desired price. Because of the smaller discrepancy, less
firms pay the menu cost and adjust the prices because inflation automatically leads to a downward change in relative prices.

This demonstrates how the addition of positive inflation trend to costly prices adjustment model can generate asymmetric responses of output to monetary shocks. As positive shocks are more likely to induce price adjustment than negative shocks, aggregate demand increases have smaller absolute impact on output than equivalent decreases in aggregate demand. Lastly, the asymmetry degree will be accentuated with higher inflation rates. Ball and Mankiw’s model implies that the asymmetric nature of the effects of monetary shocks on output is intensified by increases in the rate of inflation. The logic of their theory is the following: along with price adjustment costs, positive trend inflation brings about the downward rigidity of prices, and this downward rigidity induces asymmetry in the effects of monetary shocks.

To better illustrate the extension we are proposing on Cover’s methodology, we begin by presenting an outline of his 1992 model.

### 3.2 Cover (1992)’s model extension

Following the work of Cover (1992), money-output system consists on two equations. We first estimate the reaction function of the national central bank and then, in a second stage, we estimate the output equation for each country.

The function of reaction can be written:

\[
i_t = \text{cst} + \alpha i_{t-1} + \beta \pi_{t-1} + \gamma y_{t-1} + \varphi Z_t + \varepsilon_t
\]  

where \( i, \pi, y \) stand respectively for interest rate level, inflation rate and output gap. \( Z \) corresponds to the external constraint represented by German interest rate. Potential GDP is approached by the Hodrick-Prescott filter and \( \varepsilon_t \) is an error term assumed to be uncorrelated with any available information.

We assume that central bank has an objective of inflation rate. To achieve this goal monetary authorities manipulate nominal interest rate in accordance with the state of the economy.
Besides, because of the European Monetary System creation, we take into consideration the external constraint. The money gap is not introduced because data on monetary aggregates are not available over the seventies. Lastly, we do not introduce here an inflation objective because of high inflation rates over the period. Residuals $\varepsilon_t$ issued from this estimate of the reaction function are employed as the proxy of monetary policy. They are interpreted as the monetary policy unanticipated shocks.

To test for the asymmetric effect of the positive and negative policy, two additional series are created:

$$\text{shock}^- = \max (\varepsilon, 0)$$  \hspace{1cm} (3)

$$\text{shock}^+ = \min (\varepsilon, 0)$$  \hspace{1cm} (4)

A positive not anticipated shock "shock\(^+\)" corresponds to expansionist monetary policy. So it equals the policy proxy $\varepsilon_t$ when the proxy is negative since the interest rate is the endogenous variable. A negative unanticipated shock "shock\(^-\)" corresponds to a restrictive monetary policy that is to say to a positive proxy.

These new series are then used as explanatory variables in output process to evaluate their impacts on the activity:

$$\Delta y_t = \text{constant} + a \Delta y_{t-1} + \sum_{i=1}^{4} b_{t-i}^+ \text{shock}^+ + \sum_{i=1}^{4} b_{t-i}^- \text{shock}^- + v_t$$  \hspace{1cm} (5)

The significance of the sum of coefficients $b_{t-i}^+$ and $b_{t-i}^-$ implies that the expansionary and contractionary policies influence output, respectively.

Up to now, the system simply allows asymmetric effects of monetary shocks but this asymmetry is supposed constant over time. The question of knowing whether inflation determines significantly the degree of asymmetry can be studied while making depend the monetary shocks on the average inflation.
Indeed, drawing on the work of Ball and Mankiw, the degree of asymmetry is expected to be positively associated with changes in inflation, suggesting a time-varying degree of asymmetry. Because the influence of monetary policy varies with the inflation rate, the asymmetry is specified as a function of inflation. We use, as Rhee and Rich (1995), an expected inflation series based on Hamilton's (1989) Markov regime-switching model as a proxy for the average level of inflation.

The 2-state Markov switching model for inflation is given by:

\[
\pi_t - \mu(S_t) = \sum_{i=1}^{r} \phi_i [\pi_{t-i} - \mu(S_{t-i})] + \nu_t
\]  \hspace{1cm} (6)

where \( \pi \) is the inflation rate, \( \nu_t \sim \text{i.i.d. } N(0, \sigma^2) \). \( \mu(S_t) \) stands for the mean of the inflation process and it is assumed to vary across unobserved regimes. The number of autoregressive lags \( r \) is equal to four.

To make this model tractable, the econometrician must specify a stochastic process for the variable \( S_t \). We suppose a first-order Markov process with transition probabilities supposed constant over time:

\[
p_{ij} = P[S_t = j \mid S_{t-1} = i] ; i, j = 1, 2
\]  \hspace{1cm} (7)

with \( \sum_{j=1}^{2} p_{ij} = 1 ; i = 1, 2 \) \hspace{1cm} (8)

Where \( S_t \) is an index of the regime and is a discrete-valued random variable which can take on two possible values. The corresponding transition probability matrix is given as:

\[
P = \begin{bmatrix}
  p_{11} & p_{12} \\
  p_{21} & p_{22}
\end{bmatrix}
\]  \hspace{1cm} (9)

The state-dependent means of the inflation process are specified linearly as:

\[
\mu(S_t) = \alpha_1 + \alpha_2 S_t
\]  \hspace{1cm} (10)

If the sequence of states was known, it would be possible to write the joint conditional log likelihood function of the sequence \( \{\pi_t\} \). Since we do not observe \( S_t \), but only \( \pi_t \) from time 0 to T,
Hamilton (1989) provides details on a nonlinear filter algorithm that permits Maximum Likelihood estimation of the unknown parameters of the Markov switching model. It allows to draw an inference about the unobserved regime at time $t$ based on the observed sequence of data $\{\pi_t\}$. The resulting filter probabilities $P[S_t / \{\pi_t\}]$ give information on the regime in which the series will probably be. We can then build the forecast of the inflation rate according to the following formula:

$$\Pi_t^{\text{AVG}} = E[\pi_t | \{\pi_{t-j}\}]$$

The expected inflation series obtained provides a proxy for the level of average inflation:

$$\Pi_t^{\text{AVG}} = E[\pi_t | \{\pi_{t-j}\}]$$

To account for the influence of this inflation series on the asymmetry in price adjustment of firms, we consider the following linear specifications for the time-varying coefficients on the positive and negative monetary shocks:

$$b^+_{t-i} = \beta + \beta^+_{t-i} \Pi_t^{\text{AVG}}$$

$$b^-_{t-i} = \beta + \beta^-_{t-i} \Pi_t^{\text{AVG}}$$

Where $\beta$, $\beta^+_{t-i}$, $\beta^-_{t-i}$ are unknown parameters. The common constant term $\beta$ incorporates the implication of the Ball and Mankiw model that monetary shocks have symmetric effects under price stability. The coefficients $\beta^+_{t-i}$ and $\beta^-_{t-i}$ denote the influence of interacting variables of the products of the monetary policy and the average inflation rate. They are coefficients that capture the extent to which the effects of the monetary shocks depend on inflation.

The system of equations (13) is then introduced in equation (5). The role of inflation in monetary shocks effects on activity fluctuations can then be studied. The reaction function and the output equation are then jointly estimated by a nonlinear iterative procedure. The system is estimated by nonlinear generalised least squares.
4 Empirical results

Details on French and Italian data are presented in appendix A2. The evolution of inflation rates is represented in figure A2-1. It shows that Italian inflation rate is always above the French one (dotted line) on whole sample period. Moreover both inflation rates vary over the period. It suggests that the assumption of fixed asymmetry is not the most relevant. So it is interesting to study Italian and French real economies reactions with the assumption of time varying asymmetry. Because the focus of this study concerns the existence and dynamics of the asymmetric effects of monetary shocks, we do not present the estimated coefficients for the reaction function and only report the estimated coefficients for the output equation.

4.1 Two states Markov switching model for inflation rate

To assess the role of average inflation for characterising the effects of monetary shocks on output fluctuations, we build a characteristic series of average inflation based on expected inflation series from markovian models with two regimes and four autoregressive parameters.

Table 2 reports the Maximum Likelihood estimates of the average inflation based on the two-states Markov switching model over the period 1970:1 – 1998:4. We define low state as the state with the lower average inflation rate. Besides, we analyse these two countries independently one of the other.

The results in table 2 document that the rate of inflation undergoes statistically significant shifts on average according to states whatever the studied country. The findings indicate that the inflation rate is characterised by dichotomous shifts between a low and high-mean state. The means of two states, ranging from high to low inflation are then 11,74 (\(\alpha_1 + \alpha_2\)) and 5,53 (\(\alpha_2\)) percent, respectively in France. They are about 15,77 (\(\alpha_1 + \alpha_2\)) and 7,14 (\(\alpha_2\)) percent in Italy. The transition probability \(p = 0,59\) in France demonstrates that once the inflation rate is in the high state, it tends

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3 We also have tried to estimate models with data from Netherlands. Results suggest that there is no change of regimes. This can be explained by the fact that this country has low inflation rates in the last decades.
to shift to the low state. The same remark can be made in Italy. The second regime displays a high
degree of persistence, with the transition probabilities being equal to 0.85 in France and 0.94 in
Italy. This remark is not so surprising when we inspect the figure A2-1.

Finally, three specification tests and the Hansen (1992) test seem particularly interesting in
order to make sure that the selected specification presents the good properties. Results are reported
in table 3.3.

The first three rows report the results to the tests for autocorrelation, ARCH effects and for
validity of Markov assumption. The interest is to test the linear (autocorrelation) or quadratic
dependence (ARCH structure) of transition probabilities compared to past. The test of validity of
the assumption of Markov rests on the definition of one order Markov chain. The bottom row
concerns the Hansen test. It represents standardised likelihood ratio statistics for the model of each
country. The asymptotic p-values are calculated according to the Hansen (1992)’s method.

According to table 3, the first tests indicate that the selected specification for each country
does not comprise a phenomenon of autocorrelation nor of ARCH phenomenon. Besides results of
the assumption of one order Markov chain are in favour of the nonlinear specification in both
countries. These calculations enable us to conclude that the estimate in each studied country is not
skewed.

Let us look at now the Hansen test. The null hypothesis in France of a single regime for the
inflation dynamic is not rejected. Indeed, the p-value is not statistically significant since the critical
value at 5 % significance level is approximately 3. A contrario, the results obtained for Italy are in
favour of the existence of a markovian model.

Nonetheless, these last results must be considered with cautious. Indeed, according to
literature, there is no good test which could permit to rule for a model. The selection of the number
of regimes is often done arbitrarily. It is however recommended to carry out the multiple tests of
specifications suggested by Hamilton (1996). As our specification tests are satisfied for the two

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3 Estimates are realised with the program of Hamilton (1989).
studied countries and to ensure comparability, we maintain the assumption that Italian and French inflation rates follow a two-states markov switching model. The aim of this paper is to measure real effects of monetary policy.

Graphs 1 and 2 plot the implied probabilities that the inflation rate is in high state at time t for both countries ($S_t =1$). The filter probabilities depicted in graph 1 associate the inflation rate with the low-mean state most of the period since 1986 with the exception of 1989 – 1990 (German reunification). During the intermediary period there are recurrent shifts between the low and high-mean states. Low state in France seems correspond to the policy of competitive desinflation and the period towards EMU. According to graph 2 the high state in Italy seems correspond to the sub-period 1974-1980. This period is affected by oil shocks. Moreover, this graph shows that the high state also appears but less significantly over the sub-period 1989-1991 (German reunification).

Graphs 3 and 4 plot the level of average inflation implied by the expected inflation series from the Markov switching model in France and Italy respectively. These two figures reflect the same states highlighted starting from the filter probabilities. According to the graph 4, the variability would seem less important in Italy. Nevertheless, the mean is greater and the observed variability over the second half of the 70s is also more important.

### 4.2 Asymmetric effects of monetary policy

Our primary interest is to test the asymmetry hypotheses. The first thing we look at is whether there is an asymmetry between positive and negative monetary policy shocks. To do this we introduce these two kinds of monetary shocks depending on inflation rate as explained in section 3.2 into the production equation (5). We finally estimate jointly the reaction function and this output equation. Table 4 summarises the empirical findings while assuming time-varying asymmetries in equation (5). The significance of coefficients $\beta_t^{+}$ and $\beta_t^{-}$ implies that the expansionary and

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*Details are in appendix A3.*
contractionary policies influence output, respectively. Nevertheless the size of the effects is not similar with positive or negative shocks.

While the estimates of $\beta^+_{t-i}$ are statistically less significant in France at conventional significance levels, the estimates of $\beta^-_{t-i}$ are typically statistically significant at the 5% level and offer strong evidence that the size of the effects of negative monetary shocks is exacerbated by increases in average inflation. Specifically, the estimates of $\beta^-_{t-i}$ are much larger than the corresponding estimates of $\beta^+_{t-i}$, while the evidence from Wald tests examining the joint statistical significance of the coefficients on the monetary shocks documents that output displays a highly statistically significant association with negative shocks. Last the results in France provide support for the proposition that the economy responds more to negative monetary shocks than to positive shocks.

Let us now examine the Italian case. The measures of real effects are less clear. Indeed, the estimates of $\beta^-_{t-i}$ and $\beta^+_{t-i}$ seem statistically significant. However, the estimates of $\beta^-_{t-i}$ are strongly statistically significant at 5% level whereas those of $\beta^+_{t-i}$ are statistically significant only at 10% level according to the first Wald tests.

According to these first reports, France and Italy seem to react more with negative monetary shocks. A closer examination of table 4 offers support for the view that monetary shocks have asymmetric effects on output. More specifically, of particular interest here is the null hypothesis in which the coefficients on the easy policy are jointly equal to those on the tight policy. The first Wald tests confirm that France and Italy are more sensitive to negative monetary policy. Besides, a Wald test of the null hypothesis that $\beta^+_{t-i} = \beta^-_{t-i}$ rejects the hypothesis at the 1% level in France and 5% in Italy. In a same manner the test for the null assumption $\Sigma \beta^+_{t-i} = \Sigma \beta^-_{t-i}$ is rejected in France at 1% significance level and in Italy at 5%. The Wald tests typically reject the null hypothesis that the coefficients on the average inflation variable and on negative shocks are jointly equal. This last
result is very important here because it proves that inflation rate plays a key role and that negative monetary shocks effects increase with inflation.

The finding that positive monetary shocks display a smaller impact on output is consistent with the implications of the Ball and Mankiw model without any doubts in France and can be accepted in Italy. As we have already specified, the findings also provide general support for the implications of the Ball and Mankiw’s model concerning the dynamic relation between the degree of asymmetry and average inflation.

Graphs 5 and 6 depict the estimated negative monetary shock coefficients (sommeg) for both countries, implied by the regression results in table 4, and the inflation rate (frti for France and ti for Italy). These figures demonstrate the changing influences of restrictive monetary policy, that is, the time varying of $\left( \sum b_{t-i}^{-}\text{shock}_{i-t}^{-} \right)$.

Plots 5 and 6 suggest that negative monetary shocks decrease output during accelerating inflation periods. Moreover, the inspection of positive shocks shows that expansionist policy increases output during the low inflation state specified previously. In conclusion, these last considerations support the implications of the Ball and Mankiw’s model with negative shocks being larger with positive trend inflation. We notice also that the magnitude of negative shocks effects is more important up to the half of the eighties which is a period characterised by higher inflation rate. Lastly, our results show that the variability of negative shocks effects on output growth is not the same according to the level of inflation rate.

5 Conclusion

This article examines French and Italian data to determine whether or not output asymmetrically responds to monetary policy shocks. Ball and Mankiw (1994) have provided microeconomic foundations for asymmetric effects of monetary shocks. They focus on asymmetric price adjustment by incorporating positive trend inflation in a menu cost model. Their theoretical framework not only implies that monetary shocks have asymmetric effects on output but also
implies that the degree of asymmetry is positively related to movements in average inflation. The contribution of this paper is to test formally the empirical predictions of Ball and Mankiw’s model for the effects of monetary disturbances on real activity in France and Italy.

To sum up, the data support the hypothesis that monetary shocks display asymmetric effects on real activity in both countries. We find also strong support for the empirical predictions of Ball and Mankiw’s model after accounting for the behaviour of average inflation. Specifically, the evidence indicates that output responds less to positive monetary shocks than to negative shocks and these effects are more important with increases in average inflation. To conclude we find asymmetry on two levels. First strong asymmetry exists between positive and negative shocks. Second negative shocks effects increase with higher inflation rates. Lastly we note a variability of the impacts of restrictive shocks.

These findings are particularly important in the European context in which single monetary policy is conducted by the ECB. They imply that monetary authorities must take into account not only the behaviour of the inflation process but also the fact that all European countries can not react in the same way to positive and negative shocks. To conclude we wonder what is the future for such asymmetries. Indeed, these results are realised with past series when inflation rates were high. Today, European countries have converged towards low inflation rates. So we can wonder if the inflation rate could change the nature of the asymmetries. Could the low inflation rate contribute to reduce asymmetries in Monetary Union?

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References


Appendix

Appendix A1: the filter

Since \( S_t \) is unobserved but \( Y_t \) does, the idea of the nonlinear filter of Hamilton (1989) is to build an optimal inference on current state based on history of the actual values of \( Y_t \). The objective is to calculate the sequence of probabilities \( P\{S_t = 0 / Y_t, ..., Y_0 ; \Theta, P(S_{t-1} = 1)\} \) and the sequence of observations \( Y_t \) being given the parameters \( \Theta = \{\alpha_0, \alpha_1, \sigma^2, p, q\} \) for \( t = 0, ..., T \). We note thereafter \( \{P(S_t = 0 / Y_t, ..., Y_0 ; \Theta), P(S_t = 1 / Y_t, ..., Y_0 ; \Theta)\} = P (S_t / Y_{t-1}). \)

The filter involves the following five steps. Initial conditions are \( P(S_{t-1} / Y_{t-1}) \).

**Step 1**: calculate \( P\{S_t, S_{t-1} / Y_{t-1}\} \) with Bayes law:

\[
P(S_t = j , S_{t-1} = i / Y_{t-1}) = P \left( S_t = j / S_{t-1} = i \right) \ast P(S_{t-1} = i / Y_{t-1})
\]

**Step 2**: calculate the joint conditional density of \( Y_t \) and \( (S_t, S_{t-1}) \):

\[
f \left( Y_t, S_t = j, S_{t-1} = i / Y_{t-1} \right) = f \left( Y_t / S_t = j, S_{t-1} = i, Y_{t-1} \right) \ast P(S_t = j, S_{t-1} = i / Y_{t-1})
\]

**Step 3**: Marginalize the previous joint densities with respect to the states which gives the conditional density from which the (conditional) likelihood function is calculated:

\[
f \left( Y_t / Y_{t-1} \right) = \sum_{j=0}^{1} \sum_{i=0}^{1} f \left( Y_t, S_t = j, S_{t-1} = i / Y_{t-1} \right)
\]

**Step 4**: Combining the results from steps 2 and 3, calculate the joint density of the state conditional on the observed current and past realizations of \( Y \):

\[
P(S_t = j, S_{t-1} = i / Y_t) = \frac{f \left( Y_t, S_t = j, S_{t-1} = i / Y_{t-1} \right)}{f \left( Y_t / Y_{t-1} \right)}
\]

**Step 5**: The desired output is then obtained:

\[
P(S_t = j, S_{t-1} = i / Y_t) = \sum_{i=0}^{1} P(S_t = j, S_{t-1} = i / Y_{t-1})
\]

Nevertheless, initial value \( P(S_0 / Y_0, \Theta_0) \) is necessary. The best solution consists in taking as initial value the nonconditional or ergodic probabilities obtained starting from the transition matrix.
Appendix A2 : data

We use quarterly Italian and French series over the period 1970-1998 to examine the role of inflation regime in monetary policy real effects. Inflation rates are calculated as annual variations of the log of consumer price index. Consumer price index and real GDP series are issued from OECD publications. The short term interest rate comes from International Financial Statistics published by IMF.

First, it is very appealing to inspect inflation evolution in France and Italy. Indeed, it clearly appears on the below graph that the stochastic dynamic of inflation is not remained constant on the whole of the period and that, consequently, the price adjustment behaviour of firms is not remained invariant over the sample period.

Figure A2-1 : Evolution of inflation rate in France and Italy
Appendix A3: Specification tests

Specification tests are based on likelihood function and scores (Hamilton (1996)). The score is defined as the derivative of the log of the conditional likelihood with respect to the true parameter values \( \theta \):

\[
\int_{\theta} \frac{\partial \ln(f(Y_t/Y_{t-1}))}{\partial \theta} \equiv \int \frac{\partial \ln(f(Y_t/Y_{t-1}))}{\partial \theta} 
\]

(A1)

If the model is correctly specified, the score should be impossible to forecast on the basis of any information available at date \( t-1 \):

\[
E_{t-1}(\int_1) = 0
\]

(A2)

The score permits construction of the statistic of Lagrange Multiplier test (Hamilton(1996)). The intuition appears by using the following model:

\[
\int_j(t) = \gamma_0 + \sum_{j=1} \gamma_j \int_j(t-1) + \varepsilon_t
\]

(A3)

for a certain combination of \( i \) and \( j \). The above property (A2) implies, for a well specified model and convergent estimates, the following condition:

\[
\gamma_j = 0, \forall j
\]

(A4)

It permits to test several assumptions like residual non autocorrelation, ARCH effects and the validity of Markov assumption.

To test for autocorrelation:

\[
\int_{\alpha_1}(t) = \gamma_1 \int_{\alpha_1}(t-1) + \mu_t
\]

We wish to test the null hypothesis of non autocorrelation \( (\gamma_1 = 0) \). The LM test is asymptotically \( \chi^2(1) \).

To test for ARCH structure:

\[
\int_{\sigma^2_t}(t) = \gamma_2 \int_{\sigma^2_t}(t-1) + \mu_t
\]
It determines a ARCH structure in residuals in state one through correlation between squared residuals at time t and at time t-1.

**Test for validity of Markov assumption:**

The test enables to verify the hypothesis of a one order Markov chain. The property of a Markov chain of order one is:

\[ P(S_t = j|S_{t-1} = i, \Theta_{t-1}) = P(S_t = j|S_{t-1} = i, S_{t-2} = k, ..., \Theta_{t-1}) \]

Then we can test the assumption:

\[ P(S_t = 1|S_{t-1} = 1, \Theta_{t-1}) = P(S_t = 1|S_{t-1} = 1, S_{t-2} = 1, ..., \Theta_{t-1}) \]

with the following regression:

\[ \int_{p11} (t) = \gamma \int_{p11} (t-1) + \mu_1 \]

If parameters associated with the regressor are not null, the hypothesis of one order Markov chain is violated.

Finally, the question is whether our markov switching model fits well data in both countries. The hypothesis test is one that tests the hypothesis of linearity against non linearity of the model. Put differently, it tests for the null of one against two regimes in our markov switching model.

\[ H_0 : \alpha_2 = 0 \text{ (only one regime)} \]
\[ H_1 : \alpha_2 \neq 0 \text{ (two regimes)} \]

It would be tempting to apply a conventional t test or likelihood ratio test for the significance of \( \alpha_i \) but this test statistic does not have standard null distributions because, under the null hypothesis, the transition probabilities are not identified. Instead, Hansen views the likelihood function as an empirical process of the unknown parameters and uses empirical process theory to derive a bound for the asymptotic distribution of a standardized likelihood ratio statistic.
Table 1: Maximum likelihood estimates of parameters and asymptotic standard errors based on data for US real GNP

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1$</td>
<td>1.522</td>
<td>0.263</td>
</tr>
<tr>
<td>$\alpha_0$</td>
<td>-0.357</td>
<td>0.265</td>
</tr>
<tr>
<td>$p$</td>
<td>0.904</td>
<td>0.037</td>
</tr>
<tr>
<td>$q$</td>
<td>0.755</td>
<td>0.096</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.769</td>
<td>0.066</td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>0.014</td>
<td>0.120</td>
</tr>
<tr>
<td>$\phi_2$</td>
<td>-0.058</td>
<td>0.137</td>
</tr>
<tr>
<td>$\phi_3$</td>
<td>-0.247</td>
<td>0.107</td>
</tr>
<tr>
<td>$\phi_4$</td>
<td>-0.213</td>
<td>0.110</td>
</tr>
</tbody>
</table>

Source: Hamilton (1989)

Table 2: Maximum likelihood estimates of equation (6)

<table>
<thead>
<tr>
<th></th>
<th>France</th>
<th>Italy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1$</td>
<td>6.217***</td>
<td>8.628***</td>
</tr>
<tr>
<td></td>
<td>(2.71)</td>
<td>(2.68)</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>5.537***</td>
<td>7.145***</td>
</tr>
<tr>
<td></td>
<td>(2.71)</td>
<td>(2.67)</td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>1.919***</td>
<td>1.321***</td>
</tr>
<tr>
<td></td>
<td>(0.16)</td>
<td>(0.12)</td>
</tr>
<tr>
<td>$\phi_2$</td>
<td>-1.181***</td>
<td>-0.152</td>
</tr>
<tr>
<td></td>
<td>(0.36)</td>
<td>(0.19)</td>
</tr>
<tr>
<td>$\phi_3$</td>
<td>0.158</td>
<td>-0.322</td>
</tr>
<tr>
<td></td>
<td>(0.30)</td>
<td>(0.21)</td>
</tr>
<tr>
<td>$\phi_4$</td>
<td>0.091</td>
<td>0.131</td>
</tr>
<tr>
<td></td>
<td>(0.11)</td>
<td>(0.14)</td>
</tr>
<tr>
<td>$\sigma_v$</td>
<td>0.143***</td>
<td>0.513***</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.10)</td>
</tr>
</tbody>
</table>

Transition probabilities matrix:

\[
\begin{bmatrix}
0.5908 & 0.1506 \\
0.4091 & 0.8493 \\
\end{bmatrix}
\begin{bmatrix}
0.5500 & 0.0580 \\
0.4500 & 0.9420 \\
\end{bmatrix}
\]

(…) Standard errors; *, ***: significant at the 1% level.
Table 3: Tests

<table>
<thead>
<tr>
<th></th>
<th>France</th>
<th>Italy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Autocorrelation</td>
<td>1.29 (0.27)</td>
<td>0.98 (0.41)</td>
</tr>
<tr>
<td>ARCH</td>
<td>2.26 (0.135)</td>
<td>3.59 (0.16)</td>
</tr>
<tr>
<td>Markov</td>
<td>1.53 (0.197)</td>
<td>0.81 (0.51)</td>
</tr>
<tr>
<td>LR*</td>
<td>1.30 (0.73)</td>
<td>3.27 (0.02)</td>
</tr>
</tbody>
</table>

(... P-value)

Table 4: Non linear estimates

<table>
<thead>
<tr>
<th></th>
<th>France</th>
<th>Italy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.007 (7.34)***</td>
<td>0.0035 (2.58)***</td>
</tr>
<tr>
<td>a</td>
<td>0.008 (0.088)</td>
<td>0.38 (4.03)***</td>
</tr>
<tr>
<td>$\beta_{t-1}$</td>
<td>-0.18 (-1.99)***</td>
<td>0.0199 (0.77)</td>
</tr>
<tr>
<td>$\beta_{t-2}$</td>
<td>-0.20 (-2.22)***</td>
<td>-0.048 (-1.72)*</td>
</tr>
<tr>
<td>$\beta_{t-3}$</td>
<td>-0.28 (-2.86)***</td>
<td>-0.053 (-1.80)*</td>
</tr>
<tr>
<td>$\beta_{t-4}$</td>
<td>-0.13 (-1.49)</td>
<td>-0.08 (-1.66)*</td>
</tr>
<tr>
<td>$\beta^t_{t-1}$</td>
<td>-0.008 (-0.66)</td>
<td>-0.063 (-2.08)*</td>
</tr>
<tr>
<td>$\beta^t_{t-2}$</td>
<td>-0.007 (-0.60)</td>
<td>-0.114 (-2.55)*</td>
</tr>
<tr>
<td>$\beta^t_{t-3}$</td>
<td>-0.006 (-0.55)</td>
<td>-0.042 (-1.04)</td>
</tr>
<tr>
<td>$\beta^t_{t-4}$</td>
<td>-0.007 (-0.64)</td>
<td>-0.025 (-0.063)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.017 (0.58)</td>
<td>0.036 (0.35)</td>
</tr>
</tbody>
</table>

Tests of Wald

<table>
<thead>
<tr>
<th></th>
<th>France</th>
<th>Italy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{Ho : } \beta = 0$</td>
<td>13.6***</td>
<td>11.2**</td>
</tr>
<tr>
<td>$\text{Ho : } \beta^* = 0$</td>
<td>6.1</td>
<td>8.3*</td>
</tr>
<tr>
<td>$\text{Ho : } \sum \beta^- = 0$</td>
<td>9.9***</td>
<td>2.4*</td>
</tr>
<tr>
<td>$\text{Ho : } \sum \beta^+ = 0$</td>
<td>4.6**</td>
<td>2.5</td>
</tr>
<tr>
<td>$\text{Ho : } \beta^* = \beta^-$</td>
<td>14.5***</td>
<td>11.9**</td>
</tr>
<tr>
<td>$\text{Ho : } \sum \beta^+ = \sum \beta^-$</td>
<td>50.9***</td>
<td>7.8**</td>
</tr>
<tr>
<td>$\text{Ho : } \beta = \beta^-$</td>
<td>10.7**</td>
<td>10.5**</td>
</tr>
<tr>
<td>$\text{Ho : } \beta^* = \beta$</td>
<td>6.2</td>
<td>7.7</td>
</tr>
</tbody>
</table>

(... t-stat; *, **, *** significant at 10, 5 and 1%.)
Graph 1: France – High state St=1-Implied probabilities

Graph 2: Italy – High state – St=1- Implied probabilities

Graph 3: France – Average inflation series from the two states Markov switching model
Graph 4: Italy – Average inflation series from the two-states Markov switching model

Graph 5: Restrictive monetary policy in France

Graph 6: Restrictive monetary policy in Italy