Modelling of Structural Changes in Demand for Money

Cointegration Relations

Hannu Koskinen
University of Jyväskylä
School of Business and Economics

hakoskin@econ.jyu.fi

Abstract

In this paper the multivariate cointegration technique coupled with a smooth nonlinear trend of time is applied to model the demand for money. Unmodelled gradual structural changes in the cointegration parameters affect the specification of the cointegration relations so that the number of cointegrating vectors found by linear methodology is smaller than suggested by the economic theory. Here the demand for broad money in Finland during 1980 – 1996 is analysed. It turns out, that if the cointegrated VAR model is extended with a suitable nonlinear deterministic trend of time related to the intercept term, then the missing cointegration relation between broad money and the scale variable is found and the cointegration space can then be identified.

Keywords: gradual structural change, cointegration, nonlinear models
1. Introduction

This study examines how the potential nonlinearity due to structural change in the demand for money relation could be modelled and then re-evaluates the stability problem from the monetary policy point of view. When monetary policy involves strategies that in any sense rely on monetary targeting, then a stable demand for money relation is crucial (Lütkepohl and Wolters, 1999). Changes in regimes and technology as well as some exogenous shocks are examples of the kinds of change that can alter the behaviour of economic agents gradually and thereby the parameters and the relations between variables. Then the parameters of a linear model are not stable and the relations could not be correctly specified. However, it is possible to model such smooth structural changes by allowing for nonlinearities. Hence, even in the case of nonlinearities due to structural changes, we can find a stable cointegrating relation between the variables by using nonlinear modelling (see Ripatti and Saikkonen, 2001; Lin and Teräsvirta, 1994; and Lanne, 2002 and references therein), as an example in this paper will show.

Ripatti (1998) and Mannonen (2000), using Finnish data over the 80s and 90s, show that the parameters of interest for the demand for broad money are unstable. In particular, they did not find a stable relation between the scale variable and broad money. An explanation for this result could be the effect of institutional changes and business cycle fluctuations in 1990s. These kinds of fluctuations and changes can cause nonlinearities in macroeconomic relations (see Potter, 1999 p. 516) and distort the identification of these relations. In order to evaluate the adequacy of these parameters we first have to model the existing nonlinearities, as unmodelled nonlinearity will distort the parameters.

Nonlinear modelling of economic time series has aroused growing interest amongst econometricians (see, for example, Potter 1999 for a review). The modelling of smooth transition regression has mainly concentrated on short-run dynamics when the long-run equilibrium has been assumed to be stable (Teräsvirta, 1994). To widen the perspective in the case of nonlinearities one should also consider cases where the long-run equilibrium alters in multivariable cointegrating relations.
This paper focuses on cointegration analysis in a case when gradual change in the parameters of interest could accompany gradual change in the behaviour of economic agents. Recently, nonlinearities have been applied to cases of multivariate vector auto regression (VAR) e.g. by Gennari (1998), Ripatti and Saikkonen (2001), Johansen, Mosconi and Nielsen (2000) and He, Teräsvirta and González (2002), who derived a parameter constancy test. Whereas Gennari estimated the nonlinear deterministic part in a single equation framework and then transferred it to multivariate VAR procedure of Johansen (1991), this paper proceeds along the lines of Ripatti and Saikkonen by estimating the nonlinear part in a multivariate setting. Ripatti and Saikkonen focused on a model with a gradually changing constant term in the cointegration space, it is thus implicitly assumed that there are no (linear) trend components within the variables involved. Johansen et al. focused on structural breaks in the linear time trend in the cointegration space. This paper focuses on gradual and smooth change of intercept term (constant) with the nonlinear component in the cointegration space, but no constant or linear time trends constrained to it (for the motivation to this see e.g. Ripatti and Saikkonen 2001).

The theory for estimation and statistical inference for the models considered in this paper are those developed by Teräsvirta (1994) to univariate stationary models and subsequently extended by Saikkonen (2001, a, b) to multivariate cases where stationarity conditions are violated in the long-run.

The approach taken in this paper is to form an economic theory-based assumption about the number of the relations between the variables forming the VAR. The demand for money is then derived from the money-in-the-utility-function approach; this is done in chapter 2. The description of the theoretical statistical model for structural changes is done in chapter 3. In chapter 4 the Finnish monthly data for broad money demand is analysed by linear methods. An empirical application for the estimation of nonlinearities is performed in chapter 5. Finally, chapter 6 concludes. It examines the results from the statistical and economic theory point of view. Some suggestions for future research are also made.
2.1 Money demand

In this paper we incorporate money balances into the representative agent utility function. This money-in-the-utility-function (MIUF) model was originally developed by Sidrauski (1967), who employed a neoclassical growth framework to the study of monetary phenomena. In this context the analysis of monetary issues relevant to monetary policy implementation is straightforward. The parameterisation of the utility function in this paper is the one originally developed by Ripatti (1998). The advantage of this approach is that it gives the number and contents of theoretical relations.

A general form of the representative household’s utility function, when money is introduced into it consists of flow of services yielded by money and consumption in real terms. The utility function is assumed to be increasing in both arguments, strictly concave and continuously differentiable. In the case of rational economic agents, the flow of services yielded by money consists only of real money balances \( M/P \), where \( M \) is the nominal amount of money and \( P \) is price level.

The household maximises the discounted sum of the expected utility from consumption and from money under the budget constraints specified later:

\[
E_0 \sum_{t=0}^{\infty} \delta^t \left( u(C_t) + \xi_t v(M_t / P_t) \right),
\]

(2.1)

where \( \delta^t \) is the discount factor, \( \xi_t \) is a stochastic weight on the real money balances in the utility function and \( C_t \) is the real value of consumption.

In this paper I will focus on the long-run (preference) parameters and their constancy. This refers to a situation of equilibrium in the money market. Therefore we do not include the adjustment costs, which drive the dynamics of the system, in the equations. The steady state solution alone implies that money is neutral. But the dynamics continue to exist even if we do not model them here, and thus the real nature of money deviates from the neutrality assumption.
The household allocates its real income, $y$, and other earnings among consumption goods ($C_t$), bonds ($B_t$) and real money balances, $M_t / P_t$, which pay a gross return of $P_t / P_{t+1}$. A bond is here a commonly available asset (a financial market instrument), which pays a gross real return $1 + r_t$ (from time $t$ to time $t+1$) and (nominal) gross return $I_t \equiv 1 + i_t$. Money also yields a nominal return (own yield of money) of $O_t \equiv 1 + o_t$ for some definitions of it. The budget constraint that the household faces is

$$C_t + B_t + M_t / P_t < y + (O_{t-1} M_{t-1}) / P_t + (1 + r_{t-1}) B_{t-1}. \quad (2.2)$$

The first order conditions of the household's optimisation problem (2.1) subject to (2.2) can be written as

$$\frac{\bar{\xi}_t v'(M_t / P_t)}{u'(C_t)} = 1 - \frac{O_t}{I_t}. \quad (2.3)$$

After parameterisation of the utility (to constant relative risk aversion form) function the stationary equilibrium looks as follows;

$$1 - \frac{O_t}{I_t} = \bar{\zeta} C^{\rho} \left( \frac{M}{P} \right)^{-\omega}. \quad (2.4)$$

In the steady-state, the stochastic processes should have finite variance, which is not the case if any of the variables in the model are integrated of order $d$, $I(d)$, $d$ is a positive integer. However, it is possible that a linear combination of these $I(d)$ variables is stationary, and that they are cointegrated. In the (log of) stationary equilibrium the adjustment costs are zero and $M_t = M$, $C_t = C$, $I_t = I$ etc., It is assumed that the linearized version of the steady-state solution of the model should represent the stationary linear combination of the variables and we obtain the following log-linear equation for the level parameters:

$$(m - p - \frac{\rho}{\omega} c) + (i - o) = (m_t - p_t - \frac{\rho}{\omega} c_t) - (i_t - o_t) + (1 - O / I)(\zeta_t - \bar{\zeta}) \quad (2.5)$$
where the last term \((1-O/I)(\zeta_t - \zeta)\), is the deviation of the log-linearized velocity process from its steady-state value and lower cases denote the log-transformation of the variable, \(\log \xi_t = \zeta\), \(o\) and \(i\) are the own yield of money and return on the nominal bond in fractions. Moreover, \(\rho\) and \(\omega\) are risk aversion parameters; \(\rho\) for consumption and \(\omega\) for money.

In order for the equation (2.5) to have a stationary right-hand side, the levels of the variables on the same side, \(z_t = [m_t, p_t, c_t, i_t, o_t]\)', should be cointegrated. This particular parameterisation (2.5) suggests two cointegration vectors. The first is the net opportunity cost of money, \(i_t - o_t\), and the second is the ‘adjusted’ velocity, \(m_t - p_t - c_t \rho \omega^{-1}\). Interest rates are nominal, and so according to the Fisher parity, short-term interest rate depends on expected inflation. The expectations hypothesis of the term structure of interest rates predicts that the long-term interest rate is the average of the expected short-term interest rate for the entire time to maturity. Hence inflation is included in these nominal rates of interest. The Fisher parity and the expectations hypothesis of the term structure of interest rates are in many cases rejected where an empirical study relies on linear modelling of time series. If we notice the potential nonlinearities and asymmetric behaviour of the time series, then it becomes less easy to reject these hypothesis (for a discussion and examples, see e.g. Enders and Granger (1998)).

Other parameterisations with different numbers of cointegration vectors are also possible. When there are five integrated variables, in case of order one, the maximum number of cointegrating vectors will be four. Thus what is needed in any case, is to test for cointegration rank, after which one can apply the restrictions implied by the theoretical model to identify the cointegration vectors (in the case of single cointegration vector we can combine the terms of the two levels on the right-hand side of (2.5)). It can then be assumed that the variables in \(z_t\) are cointegrated and that \(\beta\) represents the cointegration vectors. Then we have

\[
\beta = \begin{bmatrix}
0 & 0 & 0 & 1 & -1 \\
-1 & 1 & \rho/\omega & 0 & 0
\end{bmatrix}
\]

and

\[
z_t = [m_t, p_t, c_t, i_t, o_t].
\]
The price homogeneity restriction imposed to the long-run relations hereafter, as in the case of earlier studies (e.g. Ripatti 1994, 1998), could not have been rejected for Finland for the period under analysis and we use the same data set as Ripatti in 1998. It is important to note that shorter periods could reveal deviations from that restriction. Moreover as the Fischer parity implies, we cannot include inflation twice in the relations as it is already included in the nominal rates of interest and the otherwise resulting multicollinearity would bias the parameters.

3. Statistical model

3.1 Linear Model

An $n$-dimensional time series generated by a VAR process of order $k$, which has an observable outcome $y_t, t = 1, \ldots, T$, can be written in the difference form

$$
\Delta y_t = d_t + \Gamma_1 \Delta y_{t-1} + \ldots + \Gamma_{k-1} \Delta y_{t-k+1} + \Pi y_{t-1} + \varepsilon_t,
$$

where $\Delta$ is a difference operator, $\Pi$ and $\Gamma_i$ ($i = 1, \ldots, k-1$) are $(n \times n)$ matrices of unknown parameters and $d_t$ is a deterministic $(n \times 1)$ sequence of level parameters. Furthermore, the initial values $y_{-k+1}, \ldots, y_0$ are observable and $\varepsilon_t \sim \text{NID}(0, \Omega)$ with $\Omega$ positive definite. It is also assumed that matrix $\Pi$ is of reduced rank $r$ ($0 < r < n$) so that we can write

$$
\Pi = \alpha \beta
$$

where $\alpha$ and $\beta$ are $n \times r$ matrices of full column rank. We also assume, that the parameters of the model satisfy the conditions of Johansen’s (1995, p. 49) version of Granger’s representation theorem because we are interested in time series cointegrated of order (1, 1). Thus we can conclude, that the ML estimator of the space spanned by $\beta$ and the rest of the parameter estimators can be found by Johansen’s algorithm (1991). So with suitably
specified initial values, both $\Delta y_t$ and $\beta' y_t$ are stationary around deterministic trends (see Johansen, 1995 p. 49).

### 3.2 Nonlinear Model

In its deterministic sequence $d_t$ the standard model described above may contain an intercept term, several dummies and possibly a linear time trend. The standard model may fail if the mean of the error correction term $\beta' y_t$ changes in a nonlinear fashion. As a consequence the stationarity conditions are violated and the parameters of the model are not properly estimated. In this study the nonlinear deterministic trend of time is absorbed into the cointegrating relations but, however, the intercept term cannot be restricted into the cointegration space here. Then the sequence $d_t$ is of the form

$$d_t = \chi_t - \alpha g_t(\mu)$$

(3.2)

where $\chi_t$ is a $n \times 1$ column vector of those deterministic components that are not included in the cointegrating relations and $g_t(\mu)$ is a generally nonlinear deterministic function of time with parameter vector $\mu$. Thus we can proceed as Ripatti and Saikkonen (2001) and rewrite the equation (3.1) as

$$\Delta y_t = \chi_t + \alpha (\beta' y_{t-1} - g_t(\mu)) + \Gamma_1 \Delta y_{t-1} + \ldots + \Gamma_k \Delta y_{t-k+1} + \epsilon_t.$$  

(3.3)

A convenient way to specify the nonlinear sequence $g_t(\mu)$ is to assume that it depends on $t/T$ such that $g_{t/T; \mu} = [0, 1]$. The function $g_{t/T; \mu}$ may be specified as

$$g_{t/T; \mu} = [1 + e^{-g(t/T - \tau)}]^{-1} \delta$$

(3.4)

or

$$g_{t/T; \mu} = [1 - e^{-g(t/T - \tau)^2}] \delta$$

(3.5a)

or
where \( \mu = [\delta', \gamma, \tau]' \) with \( \delta \) unknown \( n \times 1 \) parameter vector while \( \gamma \) and \( \tau \) are scalar parameters with \( \tau \) and \( \gamma > 0 \). These functions model a smooth or continuous structural change in the coefficients of a dynamic regression model (see Teräsvirta, 1998, for a review). In (3.4) the smooth change is modeled by a logistic function and the idea is to add a new level to the equilibrium described by \( \beta'y_t \). In (3.5a) and (3.5b) the density function of the normal distribution is utilised and the change about parameter \( \tau \), which determines the average location of the change, is symmetric. Parameter \( \gamma \) is a slope parameter which indicates how rapid the change is. The smaller the value of \( \gamma \), the longer it takes for the term in to reach its new level. If in (3.4) the value of \( \gamma \) is ‘large’, we are close to the case of a sudden structural break i.e. step dummy (for details see, for example, Ripatti and Saikkonen, 2001). Finally, there can be more than a single smooth transition in the term to which the transition applies.

The parameters of the nonlinear model in (3.3) can be estimated by ML, see Saikkonen (2001 a,b). Conventional dummy variables may also be included. The most important cases to be excluded are structural breaks with unknown break dates (conventional regime switching models) or dummy variables with jump dates which depend on unknown parameters. This maximisation problem is naturally nonlinear. Saikkonen (2001a) shows that, under suitable regularity conditions, the ML estimators are consistent and standard inference can be applied except for the linearity hypothesis because of identification problem of the slope parameter, see Lin and Teräsvirta,1994.
4. Empirical Study

4.1 Data and history

The data are from the Bank of Finland and Statistics Finland databases and consist of seasonally unadjusted monthly data on harmonised broad money (M3H) defined in February 2002, a monthly volume indicator (a combined index of various indicators such as industrial production, retail sales, consumption of electricity, etc.) for real gross domestic product (GDP) as a scale variable (a proxy for consumption), the consumer price index (1990 = 100), the three-month money market interest rate as an opportunity cost for money and the own yield on money from the years 1980:1 – 1996:2. Graphs of the variables and specific definitions are shown in Figure A1 in Appendixes.

During the time period considered in this study the financial institutions in Finland changed significantly; see e.g. Vihriälä (1990) and Koskenkylä (ed.) (2003). In the 1980s deregulation of the financial market took place in Finland as in other OECD countries; deposit and loan rates were liberalised, credit rationing was abolished, money markets were created and capital movements were deregulated. As a result of cross-border trade in financial services (tighter competition), technological advancement and banking crisis of the early 1990s the Finnish banking sector changed substantially. Recession of the early 1990s and some severe crisis in the development of international economy resulted to floating of Finnish markka in 1992. Then the Finnish markka devaluated substantially. In 1996 Finland joined to the Exchange Rate Mechanism (ERM) system and the course of Finnish markka was fixed against the other euro currencies. These changes could be seen as reasons for economic agents to alter their behaviour in a nonlinear manner (in some cases also in advance when the agents know the forthcoming mandatory change).
4.2 Linear model for Finnish money demand

Before we consider including any nonlinear component in the cointegrating relations, we proceed as Ripatti (1998) (in fact we analyse the same data set and time period) and form a linear empirical model from the equilibrium relations between the variables. These parameters then reflect the parameters of the utility function (Ripatti 1998, p. 88).

First we formed an unrestricted VAR for real money, real gdp, the short term interest rate and the own yield of money included in the system. Lag length \( k = 4 \) produced the best results to eliminate the autocorrelation of the residuals, but the normality conditions were violated due to excess kurtosis. Centred seasonal dummies were included in the model because of the strong seasonal pattern of the scale variable (GDP). Those dummies do not affect the asymptotic distributions of the test statistics. We added a dummy to the cointegration space to capture the possible effects of the financial liberalisation of the year 1987 for April 1987 onward (D87:4) to interest rate relation (for the motivation for this, see Ripatti and Saikkonen, 2001; there was a deregulation of issuing certificates of deposits in 1987). Therefore the statistics are calculated with Disco program by Nielsen (1993).

Table 1. Trace test of cointegration rank, Johansen & Nielsen (1993).

<table>
<thead>
<tr>
<th>( H_0 )</th>
<th>( \lambda_{trace} )</th>
<th>95%</th>
<th>97.5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r = 0 )</td>
<td>66.31</td>
<td>51.25</td>
<td>54.36</td>
</tr>
<tr>
<td>( r \leq 1 )</td>
<td>33.51</td>
<td>32.67</td>
<td>35.11</td>
</tr>
<tr>
<td>( r \leq 2 )</td>
<td>14.19</td>
<td>17.94</td>
<td>20.30</td>
</tr>
<tr>
<td>( r \leq 3 )</td>
<td>3.50</td>
<td>5.96</td>
<td>7.20</td>
</tr>
</tbody>
</table>

This (conventional) test for the rank suggests one cointegrating vector in the parameter space instead of the two expected according to the theory. The recursive test (Figure A2 in Appendixes) of Hansen-Johansen (1993) show that the cointegration rank is one, which is lower as expected. The results are parallel to those of Ripatti (1998) for broad
money, as in the main the same data was used (the only exception is for broad money, the definition of which was renewed). One reason for this could be that some of the parameters are integrated of order two, I(2). Another reason may be the presence of nonlinearity. As the stationarity conditions could be violated by this I(2):ness, we have to test it. The test is carried out according to the two-step procedure proposed by Johansen (1995). In this case the possible I(2):ness is clearly rejected by the test (results available upon request).

We then restricted the relations according to the theory presented in section 2.1 and the exact forms of the restrictions are

$$\beta^* = \begin{bmatrix} 0 & 0 & 0 & 1 & -1 \\ -1 & 1 & \rho/\omega & 0 & 0 \end{bmatrix} \quad \text{and} \quad z'_t = [m_t, p_t, c_t, i_t, o_t] .$$

In the restricted relations the parameters of the semi elasticity of the own yield of money, scale elasticity and step dummy 87:4 were estimated freely. The results are presented in table 2.

<table>
<thead>
<tr>
<th>Table 2. Restricted cointegration relations</th>
</tr>
</thead>
<tbody>
<tr>
<td>real broad money</td>
</tr>
<tr>
<td>------------------</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

The restrictions cannot be rejected by the LR-test, $$\chi^2(2) = 6.09 \ [0.11]$$. The potential presence of nonlinearity could affect the order of rank as the discussion in Ripatti and Saikkonen (1998) shows. Moreover the parameters are unstable as recursive estimates shown in appendixes reveals.
Figure 1 shows the cointegration relations restricted according to the theory derived in section 2.1 implies. The first vector that is not stationary in any sense (possibly due to the recession on account of the timing of the nonstationarity) is between money aggregate M3H and the scale variable (GDP as a proxy for consumption); these variables are in logarithmic form. The second vector is between interest rates which, instead, are not in logarithmic form. Because of very strong seasonal pattern the seasonality could be seen from the figures although seasonal dummies are included.

An explanation for the fact, that we did not find the expected cointegration with the scale variable and aggregate money, lie in the fluctuations of the business cycle during the 1990s. Fluctuations of this kind could induce nonlinearities into the macro economic relations (see Potter, 1999 p. 516). These unmodelled changes in the model could possibly be fixed by nonlinear modelling. As discussed earlier, the nonstationarity in the present case could potentially be modelled by a smooth deterministic function of time that describes the gradual change in the preferences of economic agents. Results of the stability test of Hansen-Johansen (1993) are shown in Figures A2 and B2 in the
Appendices and recursive estimates for the scale variable are shown in Figure A3 and B3 in the Appendices.

5. Nonlinear model for Finnish money demand

We continued modelling the demand for money in Finland by adding a nonlinear part into the cointegration space to model the change in the intercept term (but, of course, the change could happen in any of the parameters). This enables us to judge whether the nonlinear extension gives a model with better stability properties.

We continue with a model that was estimated by linear methods in previous chapter and is derived and constrained according to the theory outlined in section 2.1. As in the linear case, we added a step dummy, d87: 4, to approximate a change in the intercept term in the interest rate relation. Ripatti and Saikkonen (2001) modeled the same change in the interest rate relation with a gradually changing smooth deterministic trend of time that is additive to the intercept term. As we want to limit the number of transition variables to be estimated we do not include a separate deterministic trend to this interest rate relation. From the graphs of the restricted cointegration relation shown in figure 1 we can get an idea of the possible timing and shape of the gradual change. Therefore we try different specifications of the shape of underlying nonlinearity (functions 3.4 – 3.5b). The highest p-value is for a model that is specified as function 3.5b, in which case the transmission has been a symmetrical ‘bell shape’ change. The timing of the nonlinear trend is found in its natural place in the deepest years of the recession of 1990s.
Figure 2. The estimated nonlinear deterministic time trend.

In order to justify the inclusion of the specified nonlinear trend in the cointegrating relation, we have to test it. Following Lin and Teräsvirta (1994) and Ripatti and Saikkonen (2001) we assume that the function $g(x; \mu)$ can be approximated by a Taylor series expansion, i.e. we write model (3.3) as

$$
\Delta y_t = \chi_t + \alpha(\beta y_{t-1} + \Psi w_t)) + \Gamma q_t + e_t
$$

(5.1)

where $\Psi = [\Psi_1, \Psi_q]$, $w_t = [(t/T)^{n1} \ldots [(t/T)^{nq}]'$ and $e_t$ is an error term which equals the true error term $\varepsilon_t$ when the linearity hypothesis holds. The form of $w_t$ determines the form of the deterministic term. We proceed by testing the restrictions implied by the theory presented in section 2.1 in models with different specification for the deterministic term. The results for applied LR type test to the auxiliary model (5.1) (not reported here, available upon request) for the different specification of the underlying model clearly support the specification of the model as in (5.2) with the parameters reported later, the p-value for accepting the restrictions is 0.31. Visual inspection of Figure B3 in Appendices gives support to this conclusion.
The preferred restricted model is

$$\Delta y_t = \chi - \alpha (y_{1,t-1} - A y_{2,t-1} - d - \phi f((l/T) - \tau)) - \Gamma q_t + \epsilon_t$$ (5.2)

where $\chi$ is a $4 \times 1$ parameter vector of the intercept terms, $y_t = [y_{1,t}', y_{2,t}']'$ with $y_{1,t} = [\text{Lirm3}_t, \text{ii}_t]'$ and $y_{2,t} = [\text{yy}_t, \text{iownt}_t]'$, where $\text{Lirm3}_t$ is the logarithmic index of the real money balances, $\text{ii}_t$ is the nominal 3-month interest rate, $\text{yy}_t$ is a logarithmic index of the scale variable (GDP volume indicator, 1990=100) and $\text{iownt}_t$ is the nominal own yield of money. Moreover, $A$ is a $2 \times 2$ matrix and $\phi$ is a $2 \times 1$ vector of parameters and $f(\gamma((l/T) - \tau)) = \exp\{\gamma ((l/T) - \tau)^2\}$. It is then assumed that the form of nonlinearity is the same in each cointegrating relation. The step dummy $d87:4$ is included in the $2 \times 1$ vector $d$. 

Figure 3. Cointegration relation with estimated nonlinear time trend.
Visual inspection of figure 3 indicates, that the cointegrating vector between the money aggregate and the scale variable is stationary. Thus there is evidence in favour of a cointegration relation if the changes in behaviour of economic agents occur in the nonlinear fashion given by model (5.2) form. The ML estimates of the long-run parameters of the model are as follows (standard errors of the level parameters are given in parenthesis):

$$\Delta \begin{bmatrix} Lirm3 \\ yy \\ ii \\ iownt \end{bmatrix} = \begin{bmatrix} -0.6776 & -0.1647 \\ -0.0097 & 0.0769 \\ 0.2153 & -0.0512 \\ -0.2709 & -0.0133 \end{bmatrix} \begin{bmatrix} Lirm3 \\ iy \\ i_{t-1} \end{bmatrix} - \begin{bmatrix} 2.4796 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} w_{t-1} \\ ywnt \end{bmatrix} + \Delta \begin{bmatrix} \chi \\ \Gamma q_t \end{bmatrix} + \epsilon_t$$

The parameters for the scale elasticity and semielasticity of the own-yield of money are quite reasonable in the model that contains the nonlinear time trend. Nonetheless the magnitude of these parameters is greater than expected. A theoretically and empirically congruent explanation could, however, be found: the contents of the components belonging to the money aggregate M3H were formed at the beginning of the period under study and no such components existed before the deregulation of the money market. Moreover, as Fase and Winder (1997) show, the strong increase in broad money (in Europe) should be attributed to portfolio investment considerations rather than to an expansionary monetary policy. The same market formation is naturally also behind the semi elasticity parameter of the own-yield of money.

The nonlinear deterministic trend of time is restricted to appear only in the relation between money and its scale variable, because in the other relation for the interest rates it is not accepted by the LR-test and only distorts the relation in an unstationary direction.
All the parameters have been estimated as conditional on the location and slope parameters of the transition process (τ- and γ-parameters which are found after several iterations of estimation conditional on the other parameters).

6. Conclusions

The purpose of this paper was to examine how the potential nonlinearity due to structural change in the demand for money relation could be modelled and then re-evaluate the stability problem from the monetary policy point of view. The proposed approach is demanding because of the statistical problems related to the ML estimation of the parameters in the case of a nonlinear time trend. The results show that a stable demand for money relation can be found by utilizing nonlinear methods. It is the level of money holdings with respect to the scale variable which appear to have been affected by the recession because nonlinearity seems to be related to the intercept term. By taking into account the potential nonlinearities we can thus receive a reasonable solution from the stability point of view. But from the standpoint of monetary policy some considerations remain. First, where do these nonlinearities due to structural changes come from? Then, what effect do these changes have on the attainment of the objectives of a given monetary policy?

It could be that the preferences of economic agents have changed during period under study or that the policy regime has changed thereby inducing changes in the behaviour of agents (Lucas, 1976) or that the technologies have changed. It is reasonable to assume that the recession would have affected the level of financial assets held by agents but not their preferences because the smooth transition is related to the intercept term. Hence the demand for money is not unstable if we allow for nonlinearities in the model. If the only reason for rejecting money growth as an intermediate target has been instability, then it has to be re-evaluated. In the linear world we thus can draw the wrong conclusions and inferences. Evaluating the second question is more problematic. Recent research has shown that the connections between the monetary transmission mechanism and the
objectives of monetary policy depend to a large extent on the structure of the economy (Juselius, 1999 in Lütkepohl & Wolters (ed)). Therefore we have to take into account the sources of nominal rigidities (Walsh, 2002) and the framework (monetary – or inflation targeting or a mixture of both) under which monetary policy is executed. Before we can make any conclusions about the usefulness of money growth as an intermediate target in monetary policy implementation, we have to step away from a linear world. This calls for a proper theoretical framework within which to analyse these monetary issues.
References


Appendices

A 1. Variables graphs

Variables are:

- **Lirm3**, log index of harmonised real money balances (1990 = 100, mm-pp)
- **pp**, consumer price index (1990 = 100)
- **yy**, log index of monthly volume indicator for real gross domestic production (1990 = 100) as a proxy for consumption
- **ii**, 3-month money market rate
- **iownt**, own yield of broad money after taxes
**A 2** Stability test for cointegration rank, Hansen-Johansen 1993, recursively estimated parameters.

A value larger than one means that the hypothesis is rejected, the uppermost line represents the test for the hypothesis \( r = 0 \), the second for \( r \geq 1 \) and so on.

**B 2** Stability test for cointegration space, Hansen-Johansen 1993

The cointegration space has not been stable for its parameters, as the hypothesis of stability is rejected in every case because of a test value larger than one.
A 3. Recursive estimate of the scale elasticity parameter without deterministic trends.

B 3. Recursive estimate of the scale elasticity parameter after adding the chosen deterministic trend of time.