Regime Shifts and the Stability of Backward Looking Phillips Curves in Open Economies*

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Abstract

In this paper we assess the stability of open economy backward looking Phillips curves estimated over two different exchange rate regimes. The time-series we deal with come from the simulation of a New-Keynesian hybrid model suited for performing monetary policy analysis. Our results confirm Lindé (2001)’s finding on the low power of the Chow (1960) test in small samples. However, we do not find strong statistical support for the quantitative relevance of the Lucas critique when the ‘true’ model of the economy is featured by a positive but low degree of forwardness.

Keywords: Lucas Critique, forwardness, backward looking Phillips curves, exchange rates, Chow test.

JEL Classification: E17, E52, F41

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'Given that the structure of an econometric model consists of optimal decision rules of economic agents, and that optimal decision rules vary systematically with changes in the structure of the series relevant to the decision makers, it follows that any change in policy will systematically alter the structure of econometric models.' (Robert E. Lucas Jr., 1976, p.41)

'[The] question of whether a particular model is structural is an empirical, not theoretical, one.' (Robert E. Lucas Jr. and Thomas J. Sargent, 1981, pp. 302-303).

1 Introduction

Since the publication of the seminal paper by Lucas (1976), many researchers have undertaken efforts toward the microfoundation of economic models to be employed for performing policy analysis. Indeed, one of the fields that has been intensely affected by this push toward microfoundation is the monetary one (see e.g. Rotemberg and Woodford 1997; McCallum and Nelson, 1999a,b). Nevertheless, a different strand of this literature (e.g. Rudebusch and Svensson 1999,2002; Ball 1999,2000) has focussed on ad-hoc backward looking models, so following in spirit the VAR models popularized by Sims (1980). Indeed, backward looking models are quite appreciable from at least two different but important viewpoints: They tend to offer a quite good fit of the data, and their dynamics closely resemble those filtered with structural VARs, an issue that pure forward looking models have some troubles in dealing with (Estrella and Fuhrer, 2002).

Evidently enough, backward looking models are right the policy tools criticized by Lucas (1976). Indeed, if agents are rational, reduced-form coefficients (i.e. those of the backward looking models, if the 'true' model is featured by some degree of 'forwardness') should in principle be quite unstable over different policy regimes. But are they relevantly unstable over different regimes? After all, the relevance of the Lucas critique is fundamentally a quantitative issue (Lucas and Sargent, 1981). Then, a question naturally arises: Is the Lucas critique quantitatively important when backward looking monetary models are estimated and employed over different
monetary policy regimes?

Some recent studies based on U.S. data suggest a negative answer.\(^1\) Somewhat surprisingly, not that much attention has been posed yet to open-economies, despite their growing importance in terms of exchanges in goods and services and financial tradings all over the world (see e.g. Lane 2001; Sarno 2001). Then, in this paper we aim at proposing a first evaluation of the importance of the Lucas critique for open-economy policy models in the context of the modern monetary policy literature.

To investigate this issue, we employ a hybrid open-economy model (i.e. a convex combinations of forward and backward-looking schedules) to obtain simulated time-series for variables such as inflation, output gap, real exchange rate, and policy rates. In performing our simulations, we allow for a regime shift resembling what historically happened in several open-economies, i.e. the shift from a ‘controlled’ nominal exchange rate volatility to a floating exchange rate framework with the latter corresponding to a CPI inflation targeting. Then, we employ these simulated time-series to estimate a backward looking version of the possibly most important schedule in monetary policy, i.e. the Phillips curve. We do so because of the discussion that has been taking place for some years now on how to formalize the relationship between inflation and the real side of the economy. In fact, while on the one hand the expectations augmented New-Keynesian Phillips curve has become the workhorse model of modern research in monetary policy (e.g. Clarida, Gali, and Gertler 1999), on the other hand researchers such as Mankiw (2001) and Estrella and Fuhrer (2002) have shown that this model is just at odds with the facts. In particular, the failure of the New Keynesian Phillips curve refers to its predictions concerning i) disinflationary booms caused by fully credible disinflations, ii) a low-autocorrelated inflation rate, and iii) an immediate and one-shot reaction to a monetary policy shock. To fix this problem, Estrella and Fuhrer (2002) suggest to go for hybrid/backward looking mod-

\(^1\)See Rudebusch (2003) and Estrella and Fuhrer (2003). Notice that here we are referring to contributions that are very closely related to our object of investigation, i.e. the empirical relevance of the Lucas critique for backward looking monetary policy models. In general, the quantitative importance of the Lucas critique has been subject to wide attention since 1976. For surveys in this sense, see Favero and Hendry (1992) and Ericsson and Irons (1995).
els. Therefore, the attention we place on a backward looking Phillips curve is - we believe - justified.

Basically, the 'simulation-and-estimation' methodology we employ here is that proposed by Taylor (1989). Taylor points toward an assessment of the stability of several VAR-type schedules in an open-economy framework. By implementing the above described strategy with an estimated open-economy model, Taylor reaches the conclusion that the Lucas critique's importance is not quantitatively overwhelming.

Similarly to Taylor (1989)'s contribution, we consider a shift from a 'controlled' exchange rate regime to a 'floating' one, shift whose importance is evident in the light of both historical evidence and the discussion on the choice of the optimal exchange rate regime (see e.g. Corden 2002). Contrarily to Taylor (1989), we do allow for imperfect exchange rate pass-through (so capturing the insights coming from Campa and Goldberg, 2002) and non-standard exchange rate expectations' formation (as suggested by Frankel and Froot, 1987), in order to handle a credible model from the dynamics viewpoint. More importantly, in our study we employ a statistical tool, i.e. the Chow (1960) breakpoint test, to assess the stability of the estimated coefficients of our reduced-form Phillips curve.\footnote{By contrast, Taylor (1989) just compares sub-sample estimates in a 'qualitative' manner.}

A note about the Chow test is needed. In fact, this is the test that most of the researchers employ when assessing coefficients' instability. About this point, in a recent contribution Lindé (2001) employs a monetary policy model calibrated with U.S. data, simulates a regime-shift, and show that the Chow (1960) test may indeed lead to wrong conclusions on the stability of estimated coefficients because of its low-power in small samples. Therefore, his claim is that several researchers have found the Lucas critique not to be important just because of their choice of relying on such a statistical tool. Aware of Lindé (2001)'s warning, we perform our 'simulation and estimation' exercise with three different sample-lengths, in order to not to be misguided by the indications coming from the Chow test.

Our results confirm Lindé (2001)'s finding on the low-power of the Chow test in small samples. However, some qualifications are needed. In fact, ac-
cording to the set-up at hand, the (in)stability of the estimated coefficients is connected to the ‘degree of forwardness’ of the ‘true’ model of the economy. In particular, if our regime shift occurs in an economy predominantly backward looking, then the impact on the coefficients of the reduced form Phillips curve turns out to be negligible also in large samples. This seems to be good news for the reliability of the monetary policy analyses performed with open economy backward looking models, e.g. the assessment of different policy rules (e.g. Ball 1999,2000), or the evaluation of the sacrifice ratio (e.g. Leitemo and Røste, 2003).

The paper is structured as follows. In Section 2 we present the small macro model we employ to produce the simulated time-series of interest. In Section 3 we offer a very simple example to explain why a regime shift might harm the stability of backward-looking models’ coefficients. Section 4 contains an explanation of the steps we implement to perform our econometric exercise. In Section 5 we present our findings, whose robustness is discussed in Section 6. Section 7 concludes, and References follow.

2 A simple open-economy macro-model

We present here the ‘true’ model we employ for performing our numerical simulations. In our open-economy framework, the Phillips curve and the IS schedule defining the paths of the domestic inflation rate and the output gap read as follows:

\begin{align}
\pi_{t+1} &= \mu_\pi E_t \pi_{t+2} + (1 - \mu_\pi) \pi_t + \alpha_y y_t + \alpha_q E_t q_{t+1} + u_{t+1} \\
y_{t+1} &= \mu_y E_t y_{t+2} + (1 - \mu_y) y_t - \beta_r (i_t - E_t \pi_{t+1}) + \beta_q q_t + \beta_y y^*_t + v_{t+1}
\end{align}

where \( \pi_t \) is the annualized quarterly inflation, \( y_t \) is the output gap (i.e. the log-difference between the real GDP and a measure of potential output), \( q_t \) is the real exchange rate, \( i_t \) is the short-term nominal interest rate controlled by the Central Bank, \( u_t \) and \( v_t \) are iid processes with zero mean and standard deviations \( \sigma_u \) and \( \sigma_v \), and \( y^*_t \) is the foreign output gap (as, in general, starred variables refer to foreign variables).
Equations (1) and (2) are fairly in line with those in Svensson (2000) and Leitemo and Söderström (2003). In particular, equation (1) determines the domestic inflation rate as a function of the expected inflation rate, the lagged one, and the lagged values of the output gap and the real exchange rate. This is an open economy version of a hybrid Phillips curve, in which the inflation rate is pre-determined one period, it is endogenously inertial (due to e.g. wage contracting as in Fuhrer and Moore, 1995, or to indexation of those prices that are not re-optimized in a given period, as in Christiano et al, 2003, and in Smets and Wouters, 2003), and it also takes into account the effect of expected costs of imported intermediate inputs. The 'cost push shock' $u_{t+1}$ is justified by the time-varying markup of monopolistically competitive firms, as in Smets and Wouters (2003). Equation (2) defines the path of the output gap, which is caused by expectations on future output gap’s realizations as well as past values (the latter finding its rationale in e.g. habit formation, as in Fuhrer 2000), the ex-ante real interest rate, the real exchange rate, which proxies the increased demand for domestic goods driven by exchange rate depreciation, and the foreign output gap, which captures the increased demand for domestic goods due to the booming foreign economy. The stochastic component of the aggregate demand curve, i.e. $v_{t+1}$, may be interpreted as a preference shock (Smets and Wouters, 2003).

Notice that equations (1) and (2) allows for explicit lags in the transmission mechanism; in fact, it is hard to derive these lags from microfoundations, but they are quite useful to match the apparent gradual response of inflation and output to monetary policy shocks. Indeed, the introduction of these lags exert a significant quantitative impact on the monetary policy transmission.
mechanism, as shown by Dennis and Söderström (2002).

The nominal exchange rate $s_t$ is a key-element in our analysis. In our model, its temporal evolution is driven by the following hybrid stochastic version of the uncovered interest parity (UIP) condition:

$$i_t = i_t^* + \mu_s E_t s_{t+1} + (1 - \mu_s) s_{t-1} - s_t + \varphi_t$$  \(3\)

where the risk-premium $\varphi_t$ follows an AR(1) process with root $\rho_\psi$ and a zero-mean stochastic error $\psi_t$ whose standard deviation is equal to $\sigma_\psi$. Clearly, when $\mu_s = 1$, eq. (3) is a standard stochastic UIP condition. However, there is a certain evidence of backward-lookingness in the exchange rate expectations' formation (e.g. Frankel and Froot, 1987). We model this possibility by allowing for the lagged nominal exchange rate $s_{t-1}$ to play an active role in the exchange rate determination; this happens when $0 \leq \mu_s < 1$. Notice that when $\mu_s = 0$ the UIP equation (3) assumes the backward looking flavor that Debelle and Wilkinson (2002) attribute to it.

As indicated above, one of the arguments (potentially) of interest for the central banker is the CPI inflation rate $\pi_{t}^{CPI}$, which is defined as

$$\pi_{t}^{CPI} = (1 - \chi)\pi_t + \chi \pi_{t}^{M}$$  \(4\)

where $\chi$ is the weight of imported goods in the aggregate consumption basket, and $\pi_{t}^{M}$ stands for imported inflation. Following Leitemo and Söderström (2003), we define the imported price level $p_{t}^{M}$ as follows:

$$p_{t}^{M} = (1 - \theta)p_{t-1}^{M} + \theta(p_{t}^* + s_t)$$  \(5\)

Importantly, the parameter $\theta$ allows for the possibility of deviating from the law of one price in the short-run. In fact, if $0 \leq \theta < 1$, then the imported price level does not immediately fully adjust after that a shock has hit the foreign inflation rate or the nominal exchange rate. This price stickiness

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6We shape the stochastic component $\varphi_t$ as an AR(1) process mainly to capture the commonly observed persistence of the risk-premium, as in McCallum and Nelson (1999b), Svensson (2000), and Leitemo and Söderström (2003).

7Frankel and Froot (1987) concentrate on three departures from rational expectations, i.e. distributed lags, adaptive expectations, and regressive expectations; in their paper, all these three models turn out to be supported by the data.
is intended to capture the imperfection of the exchange rate pass-through observed in the real world, imperfection that tend to be much less important in the long run, as shown in Campa and Goldberg (2002). The strategy of modelling price stickiness and not pricing-to-market is also followed by Smets and Wouters (2002) and Lindé et al (2003).

Since the real exchange rate \( q_t \) is defined as

\[
q_t = s_t + p_t^* - p_t
\]  

equations (4), (5), and (6) suggest the following link between real exchange rate and CPI inflation:

\[
\pi_{CPI}^t = (1 - \chi)\pi_t + \chi[(1 - \theta)\pi_{M}^{t-1} + \theta(\pi_t + \Delta q_t)]
\]  

which makes it clear that (the change of) the real exchange rate exerts an impact over CPI inflation. As far as the Rest-Of-the-World (ROW henceforth) is concerned, our formalization follows the one proposed by Svensson (2000). In particular, we assume that ROW follows a Taylor rule, i.e.

\[
i_t^* = (1 - \rho_i^*) (f^*_{\pi} \pi_t^* + f^*_{y} y_t^*) + \rho_i^* i_{t-1}^* + \zeta_t^*
\]  

where \( f^*_{\pi} \) and \( f^*_{y} \) are the coefficients respectively associated to foreign inflation and foreign output gap, \( \rho_i^* \) is the interest rate smoothing coefficient, while \( \zeta_t^* \) is a zero-mean white noise process with variance \( \sigma_{\zeta}^* \). To catch the persistence typically observed in macro data, \( \pi_t^* \) and \( y_t^* \) are defined as AR(1) processes, i.e.

\[
\pi_{t+1}^* = \rho_{\pi} \pi_t^* + u_{t+1}^*
\]  

\[
y_{t+1}^* = \rho_{y} y_t^* + v_{t+1}^*
\]  

with \( u_{t+1}^* \) and \( v_{t+1}^* \) being i.i.d. processes whose variances are respectively \( \sigma_u^* \) and \( \sigma_v^* \).

\(^8\)Campa and Goldberg (2002) investigate exchange rate pass-through short-run and long-run elasticities on a sample of 25 OECD countries for the period 1975-1999. It turns out that in the short-run the law of one price is rejected in 22 out of 25 cases, but in the long-run 16 cases out of 25 support an elasticity statistically equivalent to one.
2.1 Optimal monetary policy

The monetary authorities’ behavior closes the model. In our framework, the Central Banker controls the short-term nominal interest rate $i_t$, and aims at minimizing the volatility of the arguments belonging to his loss function. Of course, different monetary policy regimes go hand-in-hand with different penalty function. The following generic loss function captures the two different regimes we will analyze in our exercise:\footnote{In fact, the CB solves an intertemporal problem featured by the following loss function:

$$E_t \sum_{\tau=0}^{\infty} \delta^\tau \left( \sum_{i=1}^{n} x_i^2 i_{t+\tau} \right),$$

where $x_i$ is one of the $n$ arguments targeted by the monetary authorities. As shown by Rudebusch and Svensson (1999), the conditional expectation presented here tends to the unconditional expectation discussed in the text for $\delta \rightarrow 1$. In this study, we fix the discount factor $\delta$ to be equal to .99, a standard choice given the quarterly frequency assumed for our model.}

$$E(L_t) = \lambda_{s} \text{Var}(\Delta s_t) + \lambda_{\pi CPI} \text{Var}(\pi_t^{CPI})$$
$$+ \lambda_{y} \text{Var}(y_t) + \lambda_{\Delta i} \text{Var}(\Delta i_t) \quad (11)$$

In particular, the weights $\{\lambda_{s}, \lambda_{\pi CPI}, \lambda_{y}\}$ are structural parameters of our framework, i.e. the preferences of the Central Banker over the targeted arguments (Svensson, 1999), and identify the regimes we will work with; by contrast, the interest rate smoothing argument $\text{Var}(\Delta i_t)$ is mainly introduced to avoid counterfactual extreme fluctuations of the short-term nominal interest rate.\footnote{In performing our simulations, we will consider for the interest rate smoothing argument a relative weight $= 0.2$, as in Rudebusch and Svensson (2002).}

The loss function (11) deserves some explanations. As already mentioned, we aim at mimicking a shift from a 'limited flexibility' nominal exchange rate regime to a flexible one characterized by CPI inflation targeting, a shift that concerned a certain number of countries in the past decades (Reinhart and Rogoff, 2002); a few examples are collected in Table 1.\footnote{Notice that in most of the selected cases Reinhart and Rogoff (2002) do not find evidence of a fixed exchange rate regime; this is why we analyze a form of 'controlled' exchange rate volatility.}

\[\text{Table 1 about here}\]
Then, our strategy is that of simulating the ‘controlled’ nominal exchange rate regime by implementing the following targeting strategy:\textsuperscript{12}

\[ E(L_t) = \lambda_{\Delta s} Var(\Delta s_t) + \lambda_{\Delta i} Var(\Delta i_t) \]  

(12)

This is a targeting of the stationary difference existing between today’s and yesterday’s nominal exchange rate. Given the equation involving the real exchange rate (6) and the UIP (3), this calls for an optimal policy rule that takes into account both domestic elements and, above all, foreign variables such as the foreign policy rate and the risk-premium. In this sense, we believe our approximation of a controlled exchange rate regime may be considered as being fairly satisfactory.\textsuperscript{13} By contrast, after the regime shift the monetary authorities will follow a CPI inflation targeting strategy, identified by the following penalty function:\textsuperscript{14}

\[ E(L_t) = \lambda_{\pi CPI} Var(\pi_{CPI}) + \lambda_y Var(y_t) + \lambda_{\Delta i} Var(\Delta i_t) \]  

(13)

### 2.2 A note on the solution of the Central Banker’s problem

In computing the solution of the Central Banker’s problem, we assume that the central banker is conducting monetary policy under discretion. Indeed,\textsuperscript{12}

To mimic the first regime, one might think of setting a policy rule like \( i_t = i_1 + \varphi_t \), in order to ‘exploit’ the UIP and stabilize the nominal exchange rate. However, this strategy would lead to ex-post instability of the exchange rate as well as indeterminacy, as shown by Benigno, Benigno, and Ghironi (2003). Anyhow, the optimally computed policy rule under the ‘controlled’ nominal exchange rate regime attributes a quite important role to \( i_1 \) and \( \varphi_t \), as shown in Table 3; moreover, our results are robust to an alternative formalization of such a regime, i.e. when a loss function such as \( Var(i_t - i_1 - \varphi_t) + \lambda_{\Delta i} Var(\Delta i_t) \) is considered.

\textsuperscript{13}A discussion on possible alternative ways of shaping a ‘controlled’ exchange rate regime is offered in the Appendix of this paper available upon request.

\textsuperscript{14}Notice that in this work we are considering monetary policy shifts that are intimately related to the open economy dimension of the model. In other words, in a closed economy set up the targeting schemes we discuss in the text are just not replicable. Then, we are not claiming that a regime shift having no consequences on the estimated reduced-form coefficients in a closed economy set-up might indeed have an impact when we open up the economy. Indeed, this might be an interesting issue for further research.
this assumption seems to be quite reasonable for describing the inflation targeting framework widely adopted all over the world by many central banks for some time now (Bernanke and Mishkin, 1997). It is possible to show that the solution of the monetary authorities’ problem in this context is given by the following feedback rule:\footnote{See Söderlind (1999) for details, also present in the Appendix of the paper available upon request.}

\begin{equation}
\dot{i}_t = f x_{1t}
\end{equation}

where \( f \) is a (1x9) vector whose elements are complicated convolutions of the policymakers’ preferences and the parameters of the economy, while \( x_{1t} \) is the (9x1) vector of the pre-determined variables of the problem, i.e. \( x_{1t} = \begin{bmatrix} \pi_t & y_t & \varphi_t & \pi_{t-1}^M & \ddot{i}_t^* & \dddot{i}_t^* & \pi_t^* & y_t^* & \ddot{i}_{t-1} \end{bmatrix}' \). This endogenous targeting rule closes the model.

Our model (1)-(10) and (14) is quite flexible, and it can be easily forced to assume a pure forward looking fashion (i.e. \( \mu_x = \mu_y = \mu_s = 1 \)) or a pure backward looking structure (i.e. \( \mu_x = \mu_y = \mu_s = 0 \)). We think of it as representing a fair compromise between highly stylized formalizations of the open economy framework (e.g. Ball, 1999,2000) and more complex, fully-fledged ones (e.g. Smets and Wouters, 2002). Although not fully in line with representations whose beauty is due to theoretical coherence coming from microfoundation (e.g. Gali and Monacelli, 2003), this model is actually quite close to one of them, i.e. Lindé, Nessén, and Söderström (2003).

### 2.3 Model parametrization

The benchmark parametrization used in our exercise is largely borrowed from the existing literature dealing with dynamic stochastic monetary modeling. In fact, we make no attempt of estimating the model; instead, we select plausible parameters values in order to provide a first assessment on what it might be the relevance of the Lucas critique in an open-economy environment. In particular, the domestic economy is almost fully parametrized on the basis on Leitemo and Söderström (2003)’s paper.\footnote{Since Leitemo and Söderström (2003) work with a quarterly inflation rate, while we work with a four-quarter inflation rate, we re-scaled their Phillips curve coefficients by} As far as the degree of
forwardness of the UIP condition is concerned, we set $\mu_s = .7$, i.e. slightly larger than the degree of forwardness of the Phillips curve $\mu_\pi = .5$ and that of the IS equation $\mu_y = .3$; we make this choice to recognize to the nominal exchange rate its feature of 'forward looking determined asset price' (Svensson, 2000). Moreover, we set the exchange rate pass-through coefficient $\theta = .5$, in line with many of the point estimates in Campa and Goldberg (2002). The foreign economy is parametrized as in Svensson (2000); as a sole exception, we enriched the ROW Taylor rule with the interest rate smoothing parameter $\rho_{is} = .75$, pretty much in line with the value estimated by Clarida, Gali, and Gertler (2000) for the US. All the parameters of the benchmark model are collected in Table 2. Finally, the ‘controlled nominal exchange rate volatility’ regime (Loss function [12]) is featured by $\lambda_{\Delta s} = 1, \lambda_{\Delta i} = .2$; by contrast, the ‘CPI Quasi-Strict Inflation Targeting’ regime (Loss function [13]) is characterized by $\lambda_{\pi CPI} = 1, \lambda_y = .5, \lambda_{\Delta i} = .2$.

The models is thought for ‘replicating’ quarterly dynamics. All the variables are in log-deviations with respect to their steady states, which are normalized to zero. The timing of the model goes as follows: at the beginning of the $t^{th}$-period, shocks strike the economy; then, private agents form their expectations; finally, CB sets the policy rate. As testified by its impulse response functions, the model is quite appealing from an empirical viewpoint, i.e. it is a tool capable to produce sensible dynamics. Given the similarity between our impulse response functions and those reported in Svensson (2000) and Leitemo and Söderström (2003), we refer to those contributions for a detailed comment, just adding that - as expected - the reaction of the real exchange rate to all the shocks considered in our simulations is milder under the ‘controlled’ nominal exchange rate regime.

[Figures 1-2 about here]

Table 3 collects the coefficients of the optimal rules (14) conditional to the regime shift described above and the model parameters as in Table 2. As expected, huge differences exist between optimal rules associated to different targeting schemes. In particular, the Central Banker attributes a large importance to the elements entering the UIP condition in the Pre-Shift phase, multiplying them by 4.
and mainly to domestic elements in the Post-Shift period. Notably, the shift also leads to a higher optimal interest rate smoothing, given the higher concern toward inflation stabilization under discretion (Woodford, 2003).

[Tables 2-3 about here]

3 Why may the Lucas critique affect backward looking models?

Before moving to the description of our exercise, we present a simple example to explain why the Lucas critique may affect reduced-form backward-looking models. Consider the following framework:

\[ x_t = \theta E_t \sum_{j=0}^{\infty} y_{t+j} + \varepsilon_t \quad (15) \]

\[ y_t = \phi y_{t-1} + v_t \quad (16) \]

where \( x_t \) is the variable targeted by the policy makers, \( y_t \) is the policy variable, \( \varepsilon_t \) and \( v_t \) are white noise exogenous shocks, and \( 0 < \phi < 1 \). In this model, agents form expectations on the future path of the policy variable \( y_t \); the equilibrium value of the target-variable \( x_t \) is right a function of these expectations. By plugging (16) into (15) and imposing rational expectations, we obtain the following equation:

\[ x_t = \gamma y_t + \varepsilon_t \quad (17) \]

where \( \gamma = \frac{\theta}{1-\phi} \). Then, if the econometrician estimates \( \gamma \) in (17) and use this model to perform policy simulations based on alternative policies \( \{y_{t+j}\}_{j=0}^{\infty} \) (which is to say, based on alternative values of \( \phi \)), he will miss the link existing between new policies \( \{y_{t+j}\}_{j=0}^{\infty} \) and the corresponding new values for \( \gamma \).

As stressed by Lucas and Sargent (1981), while the theoretical point raised by Lucas (1974) is out of discussion, its empirical importance must be quantitatively evaluated; indeed, this assessment is the goal of our exercise.
The object of our test is the following open-economy version of the Phillips curve:

\[ \pi_t = \sum_{i=1}^{4} \left( \gamma_{\pi_i} \pi_{t-i} + \gamma_{y_t} y_{t-i} + \gamma_{\psi_t} q_{t-i} \right) + \xi_t \]  

Eq. (18) embeds all and no more than the variables present in the 'structural' Phillips curve (1), and it is intended to capture it in a backward looking fashion.\(^{17}\) In fact, it is nothing but an open-economy version of the one proposed by Rudebusch and Svensson (1999, 2002) for the US. Notably, with adequate restrictions on the coefficients \(\gamma_s\), this reduced-form equation collapses to the one by Ball (1999, 2000).\(^{18}\)

Then, is eq. (18) stable across the two different regimes (12) and (13)? In the next Section we investigate this issue.\(^{19}\)

4 Steps for assessing the statistical relevance of the Critique

Our algorithm to assess the importance of the Lucas critique goes as follows:

1. We simulate the Data Generating Process of the economy for \(I + T\) periods \textit{under the null of absence of regime shifts}. In doing so, we draw \(I + T\) times from a zero-mean normal distribution per each shock hitting the economy, i.e. \(u_t, v_t, \psi_t, u_t^*, v_t^*, \zeta_t^*\), shocks whose variances are indicated in Table 2. Notice that the first \(I = 100\) periods are simulated in order to obtain a stochastic vector of initial values for the model, and are just discarded before implementing Step 2.

\(^{17}\)Of course, it would be interesting to write (and estimate) the exact reduced form of the structural inflation equation (1). Unfortunately, given the somewhat complicated structure of the economic model at hand, this is far from being an easy task. Moreover, that of estimating a reduced form Phillips curve whose coefficients are complicated (and unknown) convolutions of the structural parameters of the economy is nothing but what an econometrician working with backward looking models typically does.

\(^{18}\)Ball (1999, 2000)'s Phillips curve reads as follows: \(\pi_t = \pi_{t-1} + \alpha y_{t-1} - \gamma(q_{t-1} - q_{t-2}) + \eta\). To be precise, in those papers \(y_t\) stands for the log of real output.

\(^{19}\)Notice that, given the flexibility of the model at hand, it would be quite easy to simulate several other regime shifts, e.g. CPI vs. domestic inflation targeting, strict vs. flexible inflation targeting, and so on. We leave these exercises for future research.
2. With the whole sample of simulated data (sample whose size is equal to \( T \)), we OLS estimate the 'reduced form' coefficients of the backward looking Phillips curve (18). Then, we compute the F-statistical value related to the Chow (1960) breakpoint test. To do that, we consider subsamples of equal size \( T_1 = T_2 = \frac{T}{2} \). Notice that here we are applying the Chow test to detect the break occurring at a known date, given that we perfectly know the date of the break.\(^{20}\) We perform our exercise with samples features by different sizes: a 'small' one (\( T = 200 \)), an 'intermediate' one (\( T = 500 \)), and a 'large' one (\( T = 1,000 \)).\(^{22}\) We do this in order not to obtain misleading indications from the Chow-test, whose power is low in small samples (Lindé, 2001);

3. We repeat Steps 1-2 \( N = 5,000 \) times. Once done so, we compute the F-critical value for the Chow test, so obtaining the corrected-per-sample-size critical value of the test (we go for a 5% statistical confidence);

4. We implement Step 1 allowing for the above described regime-shift at \( t = \frac{T}{2} \).\(^{23}\)

5. We implement Step 2. Once done so, we compare the F-statistical value with the F-critical value computed in Step 3. Notice that if the statistical value is larger/smaller than the critical one, the null of stability is rejected/non rejected;

6. We repeat Steps 4-6 \( N = 5,000 \) times. Then, we count how many times we rejected the null of stability (in Step 5). This rejection rate

\(^{20}\)To compute the F-statistic, we adopt the following formula (k stands for the number of estimated coefficients):\( \frac{(\hat{\sigma}_T^2 - \hat{\sigma}_{T_1}^2 - \hat{\sigma}_{T_2}^2)/k}{(\hat{\sigma}_{T_1}^2 + \hat{\sigma}_{T_2}^2)/(T - 2k)} \sim F(k, T - 2k) \) under the null of stability.

\(^{21}\)For a note on the Chow test vs. alternative ones when the break-date is unknown see Hansen (2001).

\(^{22}\)Of course, our labelling is not to be taken too seriously, given that a sample of 200 quarterly observations is frankly quite large in the real world.

\(^{23}\)Notice that in moving from the first regime to the second one we are assuming that agents are not concerned with any learning issue; this is a limitation of our approach, and probably renders our ‘in-lab’ exercise less close to reality than a study performed on actual data. On the other hand, this approach amplify the power of the Chow test, so rendering its suggestions (above all those coming from large samples) more reliable.
is a 'p-value' indicating the probability of rejecting the null of stability of the estimated backward looking Phillips curve at a 5% level of statistical confidence. If this rejection-rate is larger/smaller than .05, then the estimated backward looking schedules should be judged as being potentially unstable/stable, and the Lucas critique turns out to be empirically relevant/non-relevant. This is so because, if the Null holds true, just a 5% should show up (due to the level of confidence we selected). Then, a higher value would signal that the Null is statistically rejected.24

In performing our exercise we consider different parametrizations of our 'true' model of the economy. In particular, we take into account all the combinations of these batteries of forward looking degrees: \((\mu_\pi, \mu_y) = \{(0.3,0.1), (0.5,0.3), (0.8,0.8)\}\), \(\mu_s = \{0.4,0.7,0.9\}\). The first battery is that employed by Rudebusch (2003) in his study on the Lucas critique, while the second one is intended to explore the consequence of having different degrees of forward lookingness in the UIP condition (3). We now turn to the analysis of our results.

5 Findings

Our results are collected in Table 4.25 This Table displays the 'p-values' (i.e. rejection-rates) computed as explained in the previous section.26 Some interesting results seem to come out. First, as long as the 'true' model of the economy is featured by low degrees of forwardness of the AD-AS schedules, the estimated Phillips curve shows a quite appreciable stability. In particular,

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24Since we are working with an empirical F-distribution, standard deviations for the computed rejection-rates should also be taken into account. However, the standard deviation is equal to \(\sqrt{\frac{\text{rej.rate}(1-\text{rej.rate})}{N}}\), i.e. the maximum value it can assume is .003536 (when the rejection-rate = .5). In claiming that an estimated equation is non-stable, we will consider large rejection-rates so to take into account the uncertainty surrounding them.

25In order to save space, we do not present in the paper the volatilities of the main economic variables computed under the various regimes and models considered in our exercise. The Matlab codes to compute these figures are available upon request.

26The \(\hat{R}^2\) of the estimated equations ranges from a maximum of .774 (registered in the sample \(T = 1,000\) for the model \((\mu_\pi, \mu_y, \mu_s) = (0.3,0.1,0.9)\)) to a minimum of .155 (\(T = 200, (\mu_\pi, \mu_y, \mu_s) = (0.8,0.8,0.9)\)).
when we consider the pair \((\mu_\pi, \mu_y) = (.3, .1)\) it is hard to detect a strong evidence of instability. In fact, the highest 'rejection rate' in this case is .151 (for \(T = 1,000\)), i.e. a figure far from being overwhelmingly in favour of a rejection of the null of stability. Things change in favour of a more significant rejection of the hypothesis of coefficients’ stability when we move toward more forward looking models, even if for the triple \((\mu_\pi, \mu_y, \mu_s) = (.5, .3, .7)\) we still get a quite low rejection rate, i.e. .128 (for \(T = 1,000\)). In fact, the estimated curve (18) turns out to be clearly unstable only when pretty large degrees of forwardness are taken into account. Interestingly, those high values are not necessarily the most interesting ones, at least given what some contributions on the US teach us about those parameters.\(^{27}\)

The exercise we run allows us to reach also another conclusion regarding the power of the test we employed. When we consider low degrees of forwardness, the Chow test suggests rejection of the null of stability neither when the small sample is employed nor when the intermediate one is taken into account, and it signals instability only in one case out of three when the large sample is considered. In fact, it is pretty hard to detect any difference in the computed 'rejection rates'. For intermediate values of \(\mu_\pi\) and \(\mu_y\), the difference is much more evident, above all when moving from the intermediate sample to the large one. Still, it seems to us that the Chow test is not really suggesting a definitive rejection, at least in one case out of three. This is interesting, given the marked emphasis that hybrid and backward looking models have recently been given by leading researchers in this field (e.g. Estrella and Fuhrer 2002; Rudebusch 2003; Fuhrer and Rudebusch 2003).

Finally, the Chow test robustly rejects the null of stability just when the 'structural' model of the economy is prominently forward looking. Wrapping up, if we think of \(T = 1,000\) observations as forming a large sample, our results line up with Lindë (2001)’s on the unreliability of the Chow test in small samples, but do not offer a robust support to the statistical relevance of the Lucas critique when backward looking models are employed for performing monetary policy analysis as long as the 'true' model of the economy

\(^{27}\) See e.g. Fuhrer (1997) for the Phillips curve, and Fuhrer and Rudebusch (2003) for the IS equation. However, there is a hot debate about the value of the 'degree of forwardness' of the Phillips curve. For a small survey, see Rudebusch (2002b).
is characterized by a large degree of endogenous persistence.

[Table 4 about here]

6 Robustness checks

Of course, our qualitative findings may be affected by some of the choices we made when setting up the 'true' model of the economy, when deciding which reduced form to estimate, and so on. Accordingly, we performed some checks to verify the robustness of our results. First, we replicated all the simulations/estimations previously presented with a lower weight for the 'CPI Quasi-Strict Inflation Targeting' regime, i.e. we took into account a value for $\lambda_y = .2$. Another check we performed was that of estimating a richer version of equation (18). In particular, we added four lags of both the domestic and the foreign short-term interest rate. Finally, we implemented a grid-check having as protagonists key-parameters, such as $\alpha_y$ (linking the output gap to the inflation rate in the Phillips curve 1), $\beta_r$ (interest rate sensitivity of the economy), and $\theta$ (indicating the degree 'imperfection' of the exchange rate pass-through); all in all, the results commented above turn out to be fairly robust to these perturbations.28

7 Conclusions

We set up a small scale open economy dynamic stochastic model that allows for imperfect exchange rate pass-through and endogenous persistence in inflation, output gap, and nominal exchange rate. With this model, we simulated a regime shift, i.e. from 'controlled' to floating exchange rate. With our simulated data, we estimated a reduced-form equation, i.e. a backward looking Phillips curve, in order to evaluate its stability under such a regime shift. With this 'simulation-and-estimation' approach, we obtained some interesting results. i) Overall, our results do not support the statistical importance

\[28\text{Values investigated (single departures with respect to the benchmark model): } \alpha_y \in \{.15,.2,.25\}, \beta_r \in \{.1,.15,.2\}, \theta \in \{0,.2,.4,.6,.8,1\}. \text{ For sake of brevity, we do not display here the results of our robustness checks. However, the Matlab codes for replicating them are available upon request.}\]
of the Lucas critique for this formulation of the Phillips curve, at least when we do not assume large ‘degrees of forwardness’ for our ‘true’ model of the economy. This is good-news for policy analysis based on backward-looking models, above all when ‘milder’ policy shifts - like fairly small variations of a Taylor rule’s coefficients - are taken into account (as in Ball 1999,2000), or for assessments such that of the sacrifice ratio (as in e.g. Leiteno and Røste, 2003). ii) The more forward the ‘true’ model of the economy is, the higher is the probability of rejecting the null of stability of the estimated Phillips curve. This finding calls for a serious quantification of these key-parameters. iii) Our analysis confirms Lindé (2001)’s evidence against the power of the Chow-breakpoint test in small samples. iv) The impact of different processes for the nominal exchange rate formation is not clear, and deserves further investigation.

Of course, the flip-coin of our findings leads us to state that if the ‘true’ model of the economy is prevalently characterized by forward-looking agents, then backward-looking models might turn out to be severely unstable. Unfortunately, nothing guarantees that forward looking models would show a superior stability, as shown by Estrella and Fuhrer (2003). Therefore, we interpret the evidence in this and other papers on the Lucas critique as a call for monetary policy analyses based on a large variety of different models, an exercise already undertaken in some occasions, e.g. the NBER conference on Taylor rules whose contributions are collected in Taylor (1999).

Notice that the results reached in our study are strictly related to the magnitude of the shifts we considered. In fact, tougher shifts might lead to different conclusions on the stability of the Phillips curve. We think of this as being a possible extension of this work. Other extensions also come as natural. First, the Phillips curve is not the only protagonist of the monetary policy transmission mechanism; then, the stability of other schedules is equally important for a complete assessment of backward looking models. Then, an analysis country-by-country on the importance of the Lucas critique should be performed. This would imply the estimation of the structural model e.g. via ML/Bayesian methods, as in Ireland (2001,2004) or Smets and Wouters (2002), and would allow us to draw idiosyncratic conclusions concerning the stability of the estimated reduced form equations in a given country. Finally,
the stability of more flexible models having time-varying parameters could be assessed. This line of research has already been followed by Cogley and Sargent (2003) and Primiceri (2003), and promises to be quite fruitful.
References


<table>
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<th>Country</th>
<th>Pre-Shift classification</th>
<th>Post-Shift classification</th>
</tr>
</thead>
</table>

Table 1: **HISTORICAL EXCHANGE RATE REGIMES SHIFTS.**
Source: Reinhart and Rogoff (2002). The labels for the exchange rate regimes are those referring to Reinhart and Rogoff (2002)’s coarse grid (Table 4 in their paper), and go (from ‘fixed’ to ‘floating’) approximately like this: ‘peg’, ‘crawling peg’, ‘moving band’, ‘managed floating’, ‘freely floating’. Our cut-off for shifting from ‘controlled’ to ‘floating’ exchange rate regime is located between ‘moving band’ and ‘managed floating’.

25
Figure 1: IMPULSE RESPONSE FUNCTIONS UNDER CONTROLLED EXCHANGE RATE VOLATILITY. Strictly positive weights in the Loss function: $\lambda_{\Delta s} = 1$, $\lambda_{\Delta i} = .2$. 
Figure 2: **IMPULSE RESPONSE FUNCTIONS UNDER QUASI-\textit{STRICT} CPI INFLATION TARGETING.** Strictly positive weights in the Loss function: $\lambda_{\pi} = 1, \lambda_y = .5, \lambda_{\Delta i} = .2$. 
<table>
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Table 2: **BENCHMARK PARAMETRIZATION.** Sources of the parameters indicated in the text.

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<th>$\varphi_t$</th>
<th>$q_{t-1}$</th>
<th>$\pi_t^{M}$</th>
<th>$i_t^*$</th>
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<td>.11</td>
<td>.34</td>
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Table 3: **OPTIMAL REACTION FUNCTIONS UNDER ALTERNATIVE REGIMES.** Model parameter as in the benchmark case, see Table 2.
Table 4: **Estimated Backward-Looking Phillips Curve in Presence of a Regime Shift.** Note: ‘rej. rate’ indicates the probability of rejecting the Null of stability of the estimated equation at the 5-percent significance level on the basis of the Chow-breakpoint test. The Chow test 5-percent critical values were computed with a Monte-carlo experiment under the Null of absence of structural break. Number of lags = 4, interest rate smoothing weight = .2, number of sample draws N = 5,000, simulations run with the parameter values indicated in Table 2.

<table>
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<tr>
<th>'True' Model</th>
<th>Estimated Phillips curve (eq. 21)</th>
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