A Wavelets Analysis of MENA stock markets

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Abstract

In this paper we apply the wavelets methodology to the analysis of the comovements of for some MENA countries from June 1997 until March 2005. We decompose weekly stock market returns into different time scale components using the *non-decimated discrete wavelet transform* and then analyze the relationships among these variables at the different time scales.

Keywords: Stock market returns, Wavelet correlation, Comovements
JEL classification: C22, E31

1 Introduction

According to the leading view, the process of globalization which followed the trade and financial liberalization of the nineties should enhance the degree of comovements among national economies. Such an effect is expected to raise over time both business cycles synchronization and stock markets comovements across countries. In such a context of global interdependence the equity markets of emerging countries may appear as an interesting opportunity for international investors due to the potential gains from international stock market diversification for constructing financial portfolios (Chen *et al.*, 2002, Bekaert and Harvey, 2002, 2003). Such benefits are further enhanced in the more recent period by the recent trend of the international stock market indices of the developed countries to become more and more integrated. In empirical studies concerning emerging stock markets Middle East and North African (MENA) countries have received less attention in comparison to, for example, Asian and Latin American regions.

Most of the empirical studies investigating the interdependence between international stock markets have been based on the estimation of a correlation matrix of stock market index returns and/or on multivariate analysis techniques, such as cointegration theory and principal component analysis.[1] These techniques, particularly cointegration analysis, analyze the interations between stock

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[1]Examples of application of these techniques to MENA and other emerging markets are Gunduz and Omran, 2002, Neaime, 2003 and Da Costa *et al.,* 2005.

market indices by examining either their short run or long run relationships, as the time series methodologies employed may separate out just two time scales in economic time series, *i.e. the short run and the long run.* But the stock market provides an example of a market in which the agents involved consist of heterogeneous investors making decisions over different time horizons (from minutes to years) and operating at each moment on different time scales (from speculative to investment activity). In this way, the nature of the relationship between stock returns may well vary across time scales according to the investment horizon of the traders, as the small time scales may be related to speculative activity and the coarsest scales to investment activity.

In such a context, where both the time horizons of economic decisions and the strenght and direction of economic relationships between variables may differ according to the time scale of the analysis (see Ramsey and Lampart, 1998a), a useful analytical tool may be represented by wavelet analysis. Wavelets are particular types of function $\omega(x)$ that are localized both in time and frequency domain and used to decompose a function $f(x)$, *i.e.* a signal, a surface, a series, etc.., in more elementary functions which include informations about the same $f(x)$. The main advantage of wavelet analysis is its ability to decompose macroeconomic time series, and data in general, into their time scale components. Moreover, because of the translation and scale properties, nonstationarity in the data is not a problem when using wavelets and prefiltering is not needed. Several applications of wavelet analysis in economics and finance have been recently provided by Ramsey and Lampart (1998a, 1999b), Ramsey (2002), In and Kim (2003), Kim and In (2003), and Lee (2004).

In this paper we revisit the issue of comovements among emerging stock markets applying the discrete wavelet decomposition analysis. In particular, we investigate the interrelationships among five major MENA equity markets over the various time horizons using weekly stock indices data starting from June 1997 until March 2005. The paper is organized as follows: section 2 investigates the comovements among MENA stock market returns using both unconditional and wavelet correlation coefficients while section 3 concludes the paper (the main properties of the wavelets and the method for calculating the wavelet correlation coefficient through wavelet variance and covariance are dealt with in the Appendix).

2 Comovements of MENA equity markets

The data used in this paper consist of the weekly stock market indices of Egypt, Israel, Jordan, Morocco and Turkey. Data are retrieved from Datastream for a time period from June 1997 until March 2005 and are expressed in local currencies. The use of weekly data is useful as they avoid nonsynchronous trading problerms arising from different operating hours and time zones.

The most commonly used measure to analyze comovements among international equity markets is unconditional correlation analysis. Cross-country correlations have been largely used to obtain a static estimate of the comovements of actual returns across countries (see, for example, Dumas *et al.*, 2000). In what follows we investigate the extent of comovements among some major emerging MENA stock markets: a) at the time series level using unconditional correlation analysis (sub-section 2.1), and b) at the various scales using wavelet correlation analysis (sub-section 2.2).

2.1 Unconditional correlation analysis

In this sub-section we begin our analysis by presenting the descriptive statistics of the five MENA stock markets returns series, and then turn to the analysis of stock market returns series comovements. These stock market indices are transformed to compounded week-to-week stock market returns by calculating 100 times the natural logarithmic differences of the weekly stock prices, that is $100 * \ln\left(\frac{P_t}{P_{t-1}}\right)$.

Table 1: Summary statistics of weekly stock returns

	Egypt	Israel	Jordan	Morocco	Turkey
Mean	0.323	0.211	0.309	-0.045	0.642
Median	0.115	0.520	0.125	-0.059	0.814
Maximum	21.568	8.426	7.207	10.301	28.572
Minimum	-11.812	-11.053	-8.605	-6.615	-36.860
Std. Devn.	4.129	3.059	2.021	1.977	7.367
Skewness	0.351	-0.350	0.233	0.751	-0.154
Kurtosis	4.961	3.630	4.882	7.300	5.221

Table 1 reports the summary statistics of the percent weekly stock index returns series. We may note that with an average weekly return of 0.64% the Turkish market shows the highest performance across the five MENA stock markets together with the highest variability as mesured by the standard deviation of returns. Among the other stock markets, Egypt and Israel display average weekly returns values of about 0.30% and 0.20%, respectively, with higher average returns associated to higher volatility, according to the usual high return - high standard deviation characteristic of emerging markets. Jordan and Morocco stock markets presents features in contrast with the conventional wisdom of high return and high risk: indeed, both markets display less volatility than other MENA stock markets volatility, but while Jordan's average weekly returns are among the highest (about 0.30%), Morocco has negative returns.[2] As regards the distribution of the weekly stock market returns, the excess kurtosis and skewness measures are indicative of evidence against normal distribution in all cases.[3]

[2] Previous studies over the period between 1995 (or 1997) and 2000 (Gunduz and Omran, 2000, Hakim, 2000 and Hakim and Neaime, 2003) find lower (higher) average returns for Jordan and Turkey (Morocco) and similar values for Egypt and Israel. These differences indicate that these countries have experienced a large increase of their stock market index returns in the last five years.

[3] For a normal distribution the skewness and kurtosis measures should be 0 and 3 respectively. A formal test for normality of the returns distribution based on such measures is

Table 2: Cross-correlations among weekly stock index returns series

Time series	Egypt	Israel	Jordan	Morocco	Turkey
Egypt	1.00				
Israel	0,150*	1.00			
	(0,050)				
Jordan	0,126*	0,079	1.00		
	(0,060)	(0,043)			
Morocco	0,041	0,061	-0,055	1.00	
	(0,048)	(0,048)	(0,062)		
Turkey	0,011	0,265*	0,030	0,100	1.00
	(0,046)	(0,052)	(0,049)	(0,053)	

Note: HAC standard errors (in parenthesis) and * indicates significance at the 5% level

Table 2 reports cross-correlations among weekly stock index returns series of the five MENA stock markets. The results show that MENA stock markets tend to have a moderate stock return correlation between them, with an average correlation equal to 0.081 and a maximum correlation among all equity markets amounting to a just .265. Indeed, correlation coefficients do not significantly differ from zero only in three cases: Egypt and Israel, Egypt and Jordan, and Israel and Turkey. Thus, MENA stock markets do not appear to be correlated among themselves, with significant correlations probably influenced more by the degree of development of emerging stock markets than by their location.

2.2 Wavelet correlation analysis

In order to analyze if the pattern of comovements across MENA equity markets is time-scale dependent, we investigate the correlations among stock market returns at the different time scales using wavelet analysis as it enables us to separate a signal into multiresolution components. We decompose the weekly stock market index returns series of the five MENA countries into their time-scale components using the *non-decimated discrete wavelet transform* which is a non-orthogonal variant of the classical discrete wavelet transform. The *non-decimated discrete wavelet transform,* unlike the orthogonal discrete wavelet transform, is translation invariant, as shifts in the signal do not change the pattern of coefficients. Since we use weekly data, after the application of the translation invariant wavelet transform (MODWT) we obtain six crystals from the coarsest scale s5 to the finest scale d1. In particular, i) a set of coefficients $s_{J,k}$ representing the underlying smooth behaviour of the signal at the coarsest scale $s5$, namely sixty four weeks; ii) a second group of crystals, from $d5$ to $d1$, containing detailed coefficients, $d_{J,k}$, ..., $d_{1,k}$, representing progressively finer scale deviations from the smooth behaviour, and corresponding to 32-64,16-32, 8-16, 4-8 and 2-4 weeks period, respectively.

represented by the Jarque-Bera statistic. The null hypothesis of normal returns is rejected for all markets of our sample.

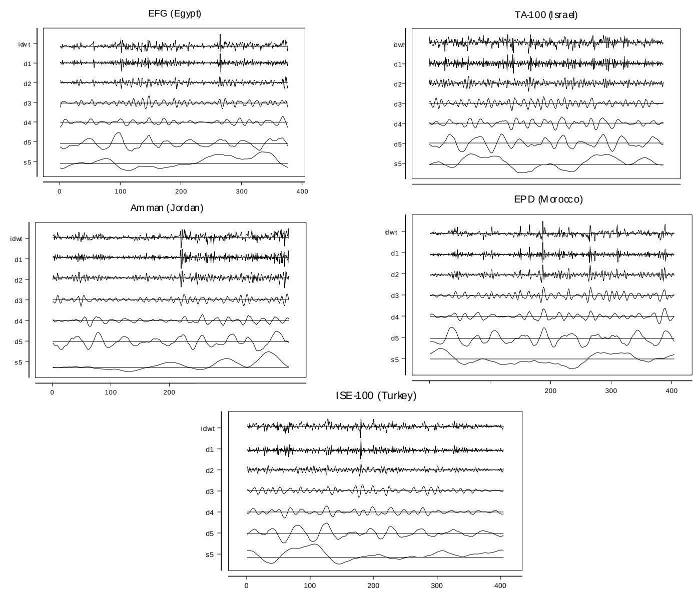

Figure 1: Non-decimated discrete wavelet transform of the stock market returns signals of the MENA countries using "s8" symmlet wavelet

In figure 1 we report the non-decimated discrete wavelet transform of the weekly stock index returns for the five MENA countries accomplished using the wavelet symmlet "s8". The first line in each chart shows the plot of the original series, the last the smooth signal, i.e. the overall trend of the series, and between them the detail signals from the high-frequency d1 to the low frequency d5 crystals.

In tables 3a to 3c we report the wavelet correlations coefficients among the stock index returns of the five MENA countries over the different time scales. The results from the wavelet correlation coefficients analysis show that at the longest scales, *i.e.* from s5 to d5, the relationships between the MENA countries are generally positive[4] and mostly significant, with the number of significant relationship increasing and the magnitude of this relationships decreasing as the wavelet time scale decreases. At the other wavelet scales there are a limited number of significant relationships between Israel and i) Morocco (at the d4 and d3 wavelet scales), ii) Egypt (at the d3 and d2 scales), and iii) Turkey (at the finest, d2 and d1, scales). In particular, looking at individual countries we observe that:

– Egypt has a positive significant relationship with Israel at almost all wavelet scales, and with the other MENA countries only at the longest scales;

[4] The only exception is the negative significant correlation between Egypt and Morocco at the D5 scale.

– Israel a positive significant relationship with Egypt and Turkey at almost all wavelet scales, and with the other MENA countries only at the longest scales;

– Jordan has no significant relationship at the longest wavelet scales (except Egypt), but positive significant relationship at the d5 scale with the other MENA countries (except Morocco);

– Morocco has no significant relationship with Jordan and Turkey at any wavelet scale, and significant relationship (sometimes negative) with Egypt and Israel starting from the intermediate, d3, and longest, d5, scales, respectively;

– Turkey has a positive significant relationship with Israel at almost all wavelet scales and with Egypt at the longest scales, but no significant relationship with Morocco at all.

Table 3 - Wavelet correlation coefficients at different scales

d5-s5	Egypt	Israel	Jordan	Morocco	Turkey
Egypt	1.00	0,605*	0,650*	0,522*	0,427*
Israel	0,296*	1.00	0,286*	0,455*	0,735*
Jordan	0,385*	0,421*	1.00	0,374*	0,113
Morocco	-0,302*	0,184	0,049	1.00	0,136
Turkey	0,389*	0,605*	0,427*	0,060	1.00
d3-d4	Egypt	Israel	Jordan	Morocco	Turkey
Egypt	1.00	0,053	-0,030	0,006	0,135
Israel	0,271*	1.00	0,242*	-0,109	0,307*
Jordan	0,130	0,091	1.00	-0,056	0,078
Morocco	0,023	0,224*	-0,029	1.00	-0,057
Turkey	0,127	0,138	0,069	-0,032	1.00
d1-d2	Egypt	Israel	Jordan	Morocco	Turkey
Egypt	1.00	0,240*	0,044	0,089	0,017
Israel	0,021	1.00	0,128	0,017	0,356*
Jordan	-0,025	-0,129	1.00	-0,055	-0,058
Morocco	0,088	0,053	0,077	1.00	0,046
Turkey	0,148	0,191*	-0,105	-0,007	1.00

Note: Wavelet correlations at the s5, d4 and d2 scales in the upper triangle and at the d5, d3 and d1 scales in the lower triangle. * indicates significance at the 5% level

3 Conclusion

In this paper we have analyzed stock market returns comovements among some MENA countries using wavelet multi-scale correlations on a scale by scale basis through the application of a non-decimated discrete wavelet transform. The results based on unconditional and wavelet correlation coefficients indicate that:

- at the longest wavelet scales, the wavelet correlations coefficients exceeds substantially unconditional correlations;

- the shorter the time scale (high frequencies) the smallest the number of significant comovements of MENA stock market returns are; and

- the magnitude of the comovements increases as the wavelet time scale increases.

Therefore, our results confirm that the decomposition of the relationships over different time scales has important implications for the analysis of stock markets correlations as wavelet correlation analysis, in contrast to conventional unconditional correlation analysis, allows us to show that the comovements among MENA stock market returns, especially between some major MENA countries, tends to become stronger as the time horizon increases.

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A Wavelet analysis

In this appendix we present first the basic concepts of wavelet analysis, then present the method for calculating the wavelet variance and covariance from the data decomposed by the *non-decimated discrete wavelet transform.* Wavelet analysis, roughly speaking, decomposes a given series in orthogonal components, as in the Fourier approach, but according to scale (time components) instead of frequencies. The comparison with the Fourier analysis is useful first because wavelets use a similar strategy: find some orthogonal objects (wavelets functions instead of sines and cosines) and use them to decompose the series. Second, since Fourier analysis is a common tool in economics, it may be useful in understanding the methodology and also in the interpretation of results. Saying that, we have to stress the main difference between the two tools. Wavelet analysis does not need stationary assumption in order to decompose the series. This is because Fourier approach decomposes in frequency space that may be interpreted as events of time-period T (where T is the number of observations). Put differently, spectral decomposition methods perform a global analysis whereas, on the other hand, wavelets methods act locally in time and so do not need stationary cyclical components. Recently, to relax the stationary frequencies assumption it has been developed a windowing Fourier decomposition that essentially use, for frequencies estimation, a time-period M (the window) event less than the number of observations T. The problem with this approach is the right choice of the window and, more important, its constancy over time.

Many economic and financial time series are nonstationary and, moreover, exhibits changing frequencies over time. Much of the usefulness of wavelet analysis has to do with its flexibility in handling a variety of nonstationary signals. Indeed, as wavelets are constructed over finite intervals of time and are not necessarily homogeneous over time, they are localized in both time and scale. Thus, two interesting features of wavelet time scale decomposition for economic variables will be that, i) since the base scale includes any non-stationary components, the data need not be detrended or differenced, and ii) the nonparametric nature of wavelets takes care of potential nonlinear relationships without losing detail (Schleicher, 2002).

Coming back to wavelets and going into some mathematical detail we may note that there are two basic wavelet functions: the father and the mother wavelets, $\phi(t)$ and $\psi(t)$, respectively. The formal definition of the father wavelets is the function

$$\Phi_{J,k} = 2^{-\frac{J}{2}} \Phi\left(\frac{t - 2^J k}{2^J}\right) \tag{1}$$

defined as non-zero over a finite time length support that corresponds to given mother wavelets

$$\Psi_{J,k} = 2^{-\frac{J}{2}} \Psi\left(\frac{t - 2^J k}{2^J}\right) \tag{2}$$

with $j = 1, \ldots, J$ in a J-level wavelets decomposition. The former integrates to 1 and reconstructs the longest time-scale component of the series (trend), while the latter integrates to 0 (similarly to sine and cosine) and is used to describe all deviations from trend. The mother wavelets, as said above, play a role similar to sins and cosines in the Fourier decomposition. They are compressed or dilated, in time domain, to generate cycles fitting actual data.

To compute the decomposition we need to calculate wavelet coefficients at all scales representing the projections of the time series onto the basis generated by the chosen family of wavelets, that is

$$\begin{aligned} d_{j,k} &= \int f(t)\Psi_{j,k} \\ s_{J,k} &= \int f(t)\Phi_{J,k} \end{aligned}$$

where the coefficients d_{jk} and s_{Jk} are the wavelet transform coefficients representing, respectively, the projection onto mother and father wavelets.

The orthogonal wavelet series approximation to a signal or function $f(t)$ in $L^2(R)$ is given by

$$f(t) = \sum_k s_{J,k}\phi_{J,k}(t) + \sum_k d_{J,k}\psi_{J,k}(t) + ... + \sum_k d_{j,k}\psi_{j,k}(t) + ... + \sum_k d_{1,k}\psi_{1,k}(t) \tag{3}$$

where J is the number of multiresolution components or scales, and k ranges from 1 to the number of coefficients in the specified components. The multiresolution decomposition of the original signal $f(t)$ is given by the sum of the smooth signal S_J and the detail signals $D_J, D_{J-1}, ..., D_1$,

$$f(t) = S_J + D_J + D_{J-1} + ... + D_j + ... + D_1 \tag{4}$$

where $S_J = \sum_k s_{J,k}\Phi_{J,k}(t)$ and $D_j = \sum_k d_{J,k}\Psi_{J,k}(t)$ with $j = 1, \ldots, J$ .The sequence of terms $S_J, D_J, ..D_j, ..., D_1$ in (4) represent a set of signals components that provide representations of the signal at the different resolution levels 1 to J, and the detail signals D_j provide the increments at each individual scale, or resolution, level.

In addition to the features stated above wavelet transform may decompose the variance of a stochastic process, wavelet variance, and the covariance between two stochastic processes, wavelet covariance. The wavelet variance is estimated using the wavelet series coefficients for scale 2^{j-1} through

$$\widehat{v}^2_{f(t)}\left(2^{j-1}\right) = \frac{1}{\widehat{N}_j}\sum_t \left[\mathbf{w}^{f(t)}_{j,t}\right]^2 \tag{5}$$

where the vector $\mathbf{w}$ are *n-dimension* vectors containing the coefficients s_J, d_J,......, d_1 of the wavelet series approximations, and $\widehat{N}_j = \frac{N}{2^j - L_j}$ with $L_j = \left[(L-2)\left(1-2^j\right)\right]$, and thus level j wavelet variance is simply the variance of the wavelet coefficients at that level (Gencay *et al.,* 2002). Similarly, the covariance is defined as:

$$\widehat{Cov}_{f(t)g(t)}\left(2^{j-1}\right) = \frac{1}{\widehat{N}_j}\sum_t \left[\mathbf{w}^{f(t)}_{j,t}\mathbf{w}^{g(t)}_{j,t}\right]^2 \tag{6}$$

Wavelet based correlation coefficients may then be obtained making use of the wavelet covariance $\widehat{Cov}_{f(t)g(t)}$ and the wavelet variances $\widehat{v}^2_{f(t)}$ and $\widehat{v}^2_{g(t)}$ as follows:

$$\widehat{\rho}_{f(t)g(t)}\left(2^{j-1}\right) = \frac{\widehat{Cov}_{f(t)g(t)}\left(2^{j-1}\right)}{\widehat{v}^2_{f(t)}\left(2^{j-1}\right)\widehat{v}^2_{g(t)}\left(2^{j-1}\right)} \tag{7}$$