ASSESSING ECONOMIC AND FISCAL REFORMS IN LEBANON.
A DYNAMIC CGE ANALYSIS WITH DEBT CONSTRAINTS

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Abstract
Since the early Nineties, Lebanon has been undertaking a number of economic reforms, covering in particular international trade and internal fiscal policy issues. Simultaneously, debt has been skyrocketing, partially justified by reconstruction needs after the end of the civil war. Fostering economic growth seems to be the only way out of the debt trap, but reforms intended to stimulate growth may well have adverse short run effects on public and external deficits. We construct a dynamic open economy CGE model with debt constraints in the sense that external debt requires physical capital as collateral. This model allows us to study the effects of a number of important economic policy issues (fiscal policy reform, WTO membership, FDI) in a multisectoral growth setting under the realistic assumption that debt constraints relax when the economy starts growing. In particular, this paper, reports results on scenarios of trade liberalization and political stabilization.

Key words: dynamic CGE, Lebanon, trade liberalization, FDI, political stability

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I Introduction

Lebanon has taken major steps towards integration in the world trading system. On the one hand, an Association Agreement\(^1\) with the European Union was signed June 17, 2002, in Luxembourg. On the other hand, an observer status at the WTO was granted since 1999. Both decisions constitute important achievements towards trade liberalization and further opening up to foreign capital flows. Besides, Lebanon is a member of the GAFTA\(^2\) since its creation in 1997 and joined the Agadir Agreement in 2004. A number of bilateral treaties with OECD as well as other world countries are intended to further promote the free exchange of goods, services and capital.

The preferential as well as the non-discriminatory trade liberalization process takes place in the immediate aftermath of the introduction of a number of internal fiscal policy reform measures. These have been undertaken with the primary goal of addressing fiscal imbalances, which have been steadily growing since the mid Nineties. Important modernization steps have involved the introduction of the VAT in 2002 and of a General Income Tax in 2003. This was preceded (in 2002) by the introduction of a new system for the deduction at source of the income tax on salaries and wages. Such measures, broadening and consolidating the tax base, are expected to counteract the likely negative fiscal impacts of trade liberalization due to the loss in public revenues. Even after the tariff system reform of 1993 which brought about a strong simplification of the tariff structure, the protection level in Lebanon is still somewhat high if compared to other developing countries (see Haddad (2004)). The high reliance of government revenues on tariff proceeds has been pointed out by Nashashibi (2002) and reinforced by Dessus and Ghaleb (2004); the latter authors report that even after the 2001 unilateral tariff cut tariff proceeds amount to about one quarter of government revenues. According to an older paper by Ghesquiere (1998), the preferential liberalization with the EU will account for a revenue loss equal to 4.2% of GDP over the agreement implementation period.

As to the expected benefits of trade liberalization, previous studies suggest only limited gains in terms of trade creation, lower import prices, higher consumer welfare or increased competition in the domestic economy. See, e. g., Martin (1996, 2000) who studies the impact of the implementation of the Agreement by means of a simple static CGE model. In his study, losses from trade diversion are found to exceed the gains from trade creation, resulting in a net welfare loss of 0.3% of GDP. This is due to the high share of Lebanon’s imports from the EU (around of 50%) and to the broadly poor export performance. Similar results are obtained by Dessus and Ghaleb (2004) under the assumption, however, of a non-discriminatory trade liberalization. They quantify the loss of public revenues at about 17% with only a negligible increase in GDP (0.1%). Somewhat larger benefits

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\(^1\) Henceforth “the Agreement”.

\(^2\) Greater Arab Free Trade Agreement.
(GDP growth of 0.4%) are achieved by introducing further reforms, in particular by removing regulations with anticompetitive effect.

The static structure of these models does not allow for the quantification of long-run welfare gains. It is one aim of this paper to close this gap and allow for productive investment and sectoral reallocation of capital, which may possibly induce additional GDP growth and positive welfare effects.

In order to account for investment, we also need to consider FDI, i.e. the import of resources from the world capital market. We model Lebanon’s integration in the global economy with limited capability of international borrowing. Specifically, we follow Barro, Mankiw and Sala-i-Martin (1995) in assuming that external debt requires collateral. Due to lack of data, however, we do not consider human capital as the limiting factor in capital demand, but rather adopt the approach of Cohen and Sachs (1986), who postulate that only a fraction $\nu$ of the actual capital stock may be used as collateral. Using this setup, which, to our knowledge, is novel in the CGE literature, we calibrate a dynamic multi-sectoral CGE model on the most recent National Accounts’ data (MOET\textsuperscript{3}, 2003).

Our simulations confirm that positive effects of trade liberalization on aggregate economic activity in Lebanon are tiny even in a long term perspective. On the other hand, positive developments in political stability raise the reliability of Lebanon as a debtor on the world capital market and thus lower the collateral requirements. We show that progress along these lines, for instance due to Syrian withdrawal from Lebanon or peaceful settlement of ethnic or religious disputes, may be economically more important than all efforts of trade liberalization. However, it should be borne in mind that trade liberalization and political stabilization are by no means rivalries. Quite to the contrary, they may positively influence each other and the EU’s Association Agreement is actually intended to make progress along both fronts.

The sequel of the paper is organized as follows. Section 2 provides an overview on main agreements on trade and investments involving Lebanon. Section 3 describes the structure of the CGE model that we use to quantify the effects. Section 4 briefly presents main calibration issues and the solution technique. Simulations of gradual non-discriminatory tariff reductions and economic benefits of political stability are carried out in Section 5. The main conclusions are summarized in the final section of this paper.

\textsuperscript{3} Ministry of Economy and Trade of the Republic of Lebanon.
Main agreements on trade and investments

Lebanon’s integration process in the global economy is based on several trade agreements and various treaties. The former include multilateral agreements (as in the case of WTO) and agreements at regional and bilateral level. Main regional agreements are the Euro-Mediterranean Agreement, enhancing North-South integration as well as the GAFTA and the Agadir Agreement promoting South-South integration. Bilateral agreements have been signed with, Syria (1993), Kuwait (1996), Egypt (1998) and the United Arab Emirates (2000). The Agreement for an Economic Free Trade Zone with Jordan (signed in 1992) is expected to be ratified before long. The latter involves a number of treaties on investments promotion and protection as well as on double taxation.

The Euro-Mediterranean Agreement is part of a wider programme, known as the Euro-Mediterranean Barcelona Process, involving a number of Mediterranean Countries. This follows the 1977 Cooperation Agreement (signed on 3rd May 1977) which grants Lebanese industrial exports duty-free access to EU markets (after satisfying strict rules of origins). The Agreement covers several areas of cooperation, including the progressive liberalization of trade in goods through a gradual phasing out of tariff and non-tariff barriers, with the ultimate goal of establishing a Mediterranean Free Trade Area. In addition, the Agreement seeks to liberalize trade in services and the rights of establishment, while widening the FDI potential in Lebanon.

The ratification by all parties being a compulsory condition for the Association Agreement to enter into force, Lebanon and the European Community signed a bilateral Interim Agreement. This reproduces the provisions contained in the Agreement on trade and trade-related matters. From the fifth year onward, customs duties on industrial imports from the EU will be gradually reduced by 12% per year until they reach zero in year 12. The phasing of tariff elimination will take into account the extent to which such imports compete with domestic production. Reductions in tariffs on agricultural products, fisheries and processed agricultural products are scheduled in a single shot in the fifth year. Customs duties on processed agricultural products are to fall by a maximum of 30% of the original duty, provided that imports currently subject to a 5% tariff will be fully liberalized. Lebanon's exports of industrial products to the EU will continue to be allowed free of customs duties (with the exception of certain listed sensitive agricultural and agro-industrial imports) as already granted in the Cooperation Agreement.

The GAFTA deals essentially with trade in goods. Tariffs are to be reduced by 10% per year over a period of 10 years, beginning in 1998. Lebanon (as well as each other partner) was allowed to set up a list of products to be excluded from the tariff reduction scheme for the first three

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4 Lebanon ratified the Agreement in December 2002.
5 March 2003.
years to allow industry restructuring. The trade liberalization process has been accelerated in 2002 and is expected to achieve full reduction by 2005.

The Agadir Agreement was signed in May 2001 by two Maghreb Countries (Morocco and Tunisia) and two Mashreq Countries (Egypt and Jordan) in order to establish a free trade area open to the other Maghreb\textsuperscript{6} and Mashreq\textsuperscript{7} Countries. According to Hamoudeh (2002) the process for achieving the free trade area should start in 2003 with a 65% reduction in tariffs, followed by a further 15% in 2004, and a 10% in 2005 and 2006. Agadir Member Countries are allowed to cumulate the Agadir rules of origin with those of the Euro-Mediterranean Agreement, while granting their exports an easier access to the EU markets.

In February 1999 Lebanon submitted its application for accession to the WTO and two months later (April 1999) was granted observer status. A National Committee on Accession to the WTO was established in May 1999 with the task of investigating the existing legal framework in order to ensure conformity of Lebanon’s foreign trade regime to the WTO requirements. Following the submission of its Memorandum on the Foreign Trade Regime in May 2001, the first round of negotiations was held in October 2002 and a second round in December 2003. The WTO membership will eventually contribute to a stable and conducive investment environment. The investors' perceived commercial risk in investing in Lebanon will decrease, thus attracting new FDI.

Further trade and investment promotion is sought through a number of treaties with Armenia, Australia, Austria, Azerbaijan, Belarus, Benelux, Bulgaria, Canada, Chile, China, Croatia, Cuba, Cyprus, Czech Republic, Egypt, Finland, France, Gabon, Germany, Greece, Hungary, Iran, Italy, Kuwait, Malaysia, Morocco, the OPEC Fund, Pakistan, Romania, Russia, Slovakia, Spain, Sweden, Switzerland, Syria, Tunisia, Ukraine, Uruguay, the United Arab Emirates, the United Kingdom and Yemen. Treaties for the avoidance of double taxation have already been ratified with 20 countries and signed with eight additional countries.

\section{A dynamic CGE Model}

The Lebanese economy is modeled as a dynamic small open economy (cf. Devarajan and Go (1998)), i.e. the domestic country is a price taker on international markets. International borrowing is constrained by the requirement of collateral, adopting ideas of Cohen and Sachs (1986) and Barro, Mankiw and Sala-i-Martin (1995). Specifically, external debt requires collateral and only a fraction $\nu$, $0 < \nu < 1$, of physical capital can serve this purpose.

\textsuperscript{6} Algeria, Libya, Mauritania.
\textsuperscript{7} Israel, Lebanon, Syria and the Gaza Strip.
The model is formulated in discrete time. In each period $t$ a population of $\Omega_t$ identical individuals grows at a constant exogenous rate of $\gamma^O$. The population cannot migrate. Labor, however, can move freely between domestic production activities.

The economy consists of $N$ mono-product industries (activities), each one producing a specific commodity indexed $m = 1, 2, \ldots, M$ \(^8\). Since the number of primary factors is lower than $M$ (number of tradable commodities), an “overspecialization” problem arises from the assumption of constant returns to scale technologies (see Samuelson 1953). This is solved through the adoption of the familiar Armington approach. Under this assumption, domestic actors consider commodities with identical statistical classification but different country of origin as imperfect substitutes. Imported and domestically produced commodities are used to “produce” an aggregate commodity, the Armington good, which is defined by a conventional CES function, unambiguously identified by the scale parameter, $\theta^m$, the elasticity of substitution, $\mu$ and the share parameter, $\phi^m$. Armington goods can be used either for consumption, which may be public or private, or for investment or as intermediate for production. The specifications of the CES function for each of the four types of Armington aggregates (private and public consumption, intermediates and investment) are to be found in the appendix (equations (A.11), (A.18), (A.25), (A.32)). On the export side, an analogous approach is based on constant-elasticity of transformation (CET) functions, as shown below.

Aggregate quantities (e.g. for consumption or investment) are also obtained from CES aggregators defined by the scale parameter, $\zeta$, the elasticity of substitution, $\kappa$ and the share parameter ($\psi^{C,m}$ for private consumption, $\psi^{I,m}$ for investment and $\psi^{G,m}$ for government consumption). The three CES aggregators (for private and public consumption as well as for investment) are reported in the appendix (equations (A.21), (A.28), (A.35)).

Perfect competition is assumed on all markets. Production factors are fully homogeneous and mobile across sectors.

### 3.1 Production

Sectoral production $Q^n_t$ employs value added $V^n_t$ and $M$ intermediate inputs according to the following Leontief fixed coefficients’ production function:

$$Q^n_t = \min \left\{ \frac{V^n_t}{a_{1,n}}, \frac{x^n_{1,n}}{a_{1,n}}, \frac{x^n_{2,n}}{a_{2,n}}, \ldots, \frac{x^n_{M,n}}{a_{M,n}} \right\}, \quad n = 1, 2, \ldots, N, \quad (1)$$

\(^8\) Under this assumption, the total number of produced commodities $M$ equals the total number of industries $N$.
where $x_{m,n}^t$ indicates the intermediate input $m$ used for production of commodity $n$ and $a_{m,n}^t$ is the corresponding fixed input requirement. Each intermediate commodity $m$ is an Armington aggregate of domestic origin $D_{F,m,n}^t$ and imported origin $M_{F,m,n}^t$ (see equation (A.11)). Value added, $V_n^t$ is produced with three primary inputs: physical capital $K_n^t$, labor services $u_{n}^t\Omega_{t}$ (being $u_{n}^t$ the number of per-capita worked hours in sector $n$ during period $t$) and land $L_a^t$. Value added of firm $n$ is generated under a Cobb-Douglas technology specified as follows:

$$V_n^t = A_n^t \left[ \left( K_n^t \right)^{bk_n'} \cdot \left( u_{n}^t\Omega_{t} \right)^{bh_n'} \cdot \left( L_a^t \right)^{bla_n'} \right],$$

(2)

where $A_n^t$ is the total factor productivity in sector $n$. The assumption of constant returns to scale implies:

$$bk_n' + bh_n' + bla_n' = 1. \tag{3}$$

Since firms operate in a fully competitive environment, the production elasticities are equal to the respective factor income shares.

Since the cost of intermediates is constant for a given amount of output, the cost minimization problem can be reduced to the minimization of:

$$C_{n}^{V,n} = P_{n}^{r} \cdot K_{n}^{t} + w_{t} \cdot u_{n}^{t}\Omega_{t} + P_{t}^{La} \cdot L_{n}^{a}, \tag{4}$$

under the value added production function constraint. The minimal value added cost is:

$$P_{t}^{V,n}Q_{t}^{n} = \frac{1}{A_{n}^t} \left[ \left( \frac{P_{t}^{r}}{bk_n'} \right)^{bk_n'} \left( \frac{w_{t}}{bh_n'} \right)^{bh_n'} \left( \frac{P_{t}^{La}}{bla_n'} \right)^{bla_n'} \right] Q_{t}^{n}, \tag{5}$$

where $P_{t}^{V,n}$ is the price of value added. The minimization of the value added costs gives the optimal demand of primary factors as well:

$$K_{n}^{t} = \frac{bk_n'}{P_{t}^{r}} P_{t}^{V,n} \cdot Q_{t}^{n}, \tag{6}$$

$$u_{n}^{t}\Omega_{t} = \frac{bh_n'}{w_{t}} P_{t}^{V,n} \cdot Q_{t}^{n}, \tag{7}$$

$$L_{n}^{a} = \frac{bla_n'}{P_{t}^{La}} P_{t}^{V,n} \cdot Q_{t}^{n}. \tag{8}$$
Due to the mono-product nature of each activity $n$, activity output $Q^n$ also equals the commodity supply $Y^m$ of each industry (see (A.13)). Domestic production $Y^m$ ($m = 1, 2, ..., M$) satisfies domestic demand, $D^m$, and foreign demand $E^m$. We denote with $P_{t,D,m}^m$ the price of the domestic commodity $m$ on the domestic markets and by $P_{t,E,m}^m$ the domestic (producer) price of exports of the same commodity $m$. For a given production $Y^m$, firms maximize the value of total sales (given by equation (A.14) in the appendix) under the CET restriction (equation (A.15)) as constraint. Solving this problem determines the optimal amount of domestically sold and exported goods (see the first order condition (A.16) in the appendix) as well as the producer price of the composite good (see equation (A.9)).

### 3.2 Capital accumulation

Capital dynamics follow the standard neoclassical capital accumulation equation:

$$K_{t+1} = I_t + (1 - \delta)K_t,$$

where $I_t$ are aggregate investments (see Section 3.3) and $\delta$ is the capital depreciation rate. The investment good $I_t$ has a structure similar to the consumption composite: It is a CES composite (see equation (A.28) in the appendix) of $m$ Armington aggregates (see equation (A.25) in the appendix). This CES composite has nominal investment price $P^{I_t}_t$. Investments are financed through households’ savings, $S^{C}_t$, capital depreciation $\delta K_t$, and net foreign direct investment, $D_{t+1} - D_t$, which is subject to collateral requirements (see Section 3.5):

$$P^{I_t}_t I_t = S^{C}_t + \delta \cdot P^{I_t}_t K_t + (D_{t+1} - D_t),$$

where $S^{C}_t$ and $D_t$ are nominal variables.

### 3.3 Consumption and leisure

Generalizing Devarajan and Go (1998), infinitely-lived households can choose between consumption and leisure. In each period $t$ they are endowed with one unit of time, part of which is supplied on the labor market. The remaining time is devoted to leisure activities. The representative agent is represented by the following instantaneous utility function:

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9 The producer price of exports may entail export subsidies.
\[ U_t(c_t, u_t) = \left[ (1-\eta)\frac{(c_t)^{1-\alpha}}{1-\alpha} - 1 + \eta \frac{(1-u_t)^{1-\beta}}{1-\beta} - 1 \right] \cdot \Omega_t, \quad (11) \]

where \( c_t \) is the per-capita composite consumption at time \( t \) and \( u_t \) is the fraction of time spent working. Parameter \( \alpha (\beta) \) is the absolute value of the inverse of the elasticity of substitution between consumption (leisure) at any two points of time and \( \eta \) is the leisure share parameter.

At the beginning of her life \( t = 0 \), the representative agent maximizes her lifetime utility:

\[ U_0 = \sum_{t=0}^{\infty} \left( \frac{1}{1+\rho} \right)^t U_t, \quad (12) \]

with being \( \rho \) the consumer’s positive and constant rate of time preference.

Households’ assets are ownership claims on two types of durables, productive capital and land. We denote the aggregate household’s productive capital assets by \( K_t \) and land assets by \( L_a_t \). Households behave competitively, taking as given the real domestic interest rate \( r_t \), the price of capital goods \( P_t^K \) and the wage rate \( w_t \), paid per unit of labor services. The total income of the aggregate of households encompasses labor income, \( w_t \cdot u_t \Omega_t \), and asset income. This is the sum of capital rents, \( P_t^K r_t \cdot K_t \), and land rents, \( P_t^{La} \cdot L_a_t \). In addition, households receive a net lump transfer \( T_t^G \) from the government and foreign remittances \( T_t^W \). These two latter variables grow at the exogenous rate \( \gamma^\Omega \) in steady-state. Total disposable income is given by:

\[ YD_t = (1-\tau) \left[ (1-\tau^L) w_t u_t \Omega_t + (1-\tau^K) P_t^K r_t K_t + (1-\tau^{La}) P_t^{La} L_a_t \right] + T_t^G + T_t^W - \tau \cdot D_t, \quad (13) \]

where \( \tau \) is a general income tax rate and \( \tau^L, \tau^K, \tau^{La} \) are factor income tax rates on labor, capital, land income respectively.

Net income is allocated to consumption and to savings. In each period \( t \), households choose among a variety of domestic and imported goods. As illustrated in Section 3.1, each consumption good \( c_t^m \) is an Armington aggregate of domestic goods \( D_t^{C,m} \) and imports \( M_t^{C,m} \) (see equation (A.18)). The final composite good is the result of a CES-aggregation (see equation (A.21)) of each of the \( m \) commodities. Its market price is \( P_t^{C} \). Savings \( S_t^{C} \) are used to accumulate productive capital or may be rented out on the international capital market. The budget constraint of the household is given by:
\[ \Omega_t \cdot P_t^C c_t + S_t^C = YD_t. \]  

(14)

The household optimization problem is to maximize the lifetime utility \( U_0 \) in equation (12), subject to the budget constraint in equation (14), the stock of initial assets (capital, \( K_0 \), land, \( L_{a_0} \), and nominal debts, \( D_0 \)) and the borrowing constraint in equation (19). The solution of the optimization problem gives the optimal demand for leisure:

\[
(1-u_t)^\beta = (f_t)^\beta = \frac{1-\eta}{\eta}(1-\tau)(1-\tau^t)w_t, \tag{15}
\]

being the Euler condition given by:

\[
\left( \frac{c_t}{c_{t+1}} \right)^{\alpha} \frac{P_{t+1}^C \cdot u_t - P_t^C}{P_t^C \cdot u_{t+1}} = \frac{1+\rho}{1+\rho} \frac{1+(1-\tau)(1-\tau^t)\nu_t - \delta}{(1+\rho)\nu}. \tag{16}
\]

### 3.4 Government behavior

Public revenues \( R_t \) include general income taxes and single factor taxes on labor, capital and land income. Additionally the government raises import taxes on imported quantities and collects indirect taxes.

\[
R_t = \tau \left[ (1-\tau^t)w_t \cdot u_t \Omega_t + (1-\tau^K)P_t^l r_t \cdot K_t + (1-\tau^{L_a})P_t^{L_a} \cdot L_{a_t} \right] + \\
+ \tau^t \left[ w_t \cdot u_t \Omega_t + \tau^K \left( P_t^l r_t \cdot K_t \right) + \tau^{L_a} \left( P_t^{L_a} \cdot L_{a_t} \right) \right] + \\
+ \sum_{n=1}^{N} \nu_t^P P_t^{Q_x} Q_t^x + \sum_{m=1}^{M} \left[ \phi_t^m \left( 1 + \theta_t^m \right) + \theta_t^m \right] P_t^{M,w,m} M_{t,m}^w,
\]

with \( M_{t,m}^w \equiv M_{t,m}^F + M_{t,m}^C + M_{t,m}^I + M_{t,m}^G. \)

Government outlays \( O_t \) consist of purchases of consumption goods and services in the aggregate quantity \( G_t \), and payments abroad, \( B_t^G \). Direct lump transfers \( T_t^G \), in the amount of revenues exceeding expenditures\( P_t^G G_t + B_t^G \), are paid to consumers.

\[
O_t = P_t^G G_t + B_t^G + T_t^G. \tag{18}
\]

Government consumption of commodity \( m \), \( G_t^m \) is an Armington aggregate of domestic good \( D_t^{G,m} \) and imported commodity \( M_t^{G,m} \). The aggregate government consumption \( G_t \) is a CES-composite of all commodities \( G_t^m \). The variable payments abroad, \( B_t^G \) grows in the steady-state at the exogenous rate \( \gamma^G \).
3.5 **Foreign trade and international borrowing**

International linkages of the domestic economy encompass trade as well as financial flows. Due to data limitations, the model currently allows for just a single trading partner which is also the only foreign direct investor. Trade relations are modeled taking into account import tariffs and indirect taxes raised on imported goods. These accrue to the government. Export activity is neither subsidized nor subject to tariffs or taxes (see (A.6)). Quotas are excluded. Non-tariff barriers are not considered due to lack of reliable data.

As mentioned in Section 3.3, households are allowed to borrow from abroad while financing a share of private capital accumulation. The so called collateral rule (as introduced in Barro, Mankiw, Sala-i-Martin (1995) and further developed in Penalver (2000)) requires that households’ external borrowing is constrained to a fraction of the existing physical capital which is used as collateral for foreign debt:

\[
D_t \leq \nu \cdot P_t^iK_t \quad \text{with} \quad 0 < \nu < 1. \tag{19}
\]

3.6 **Market clearing**

Factor market clearing requires:

\[
K_t = \sum_{n=1}^{N} K^n_t, \tag{20}
\]

\[
u = \sum_{n=1}^{N} \nu^n_t, \tag{21}
\]

\[
La_t = \sum_{n=1}^{N} La^n_t. \tag{22}
\]

On the domestic goods markets, equilibrium is given by:

\[
D_t^m = D_t^{F,m} + D_t^{C,m} + D_t^{I,m} + D_t^{G,m}. \tag{23}
\]

The equilibrium of the balance of payments requires that financial inflows, due to exports and transfer payments from the rest of the world as well as from new foreign debt, equal financial outflows due to imports, government payments abroad, foreign debt reimbursement and interest payments on debt:

\[
\sum_{m=1}^{M} P_t^{E,W,m} \cdot E_t^m + T_t^W + D_t^{r+1} = \sum_{m=1}^{M} P_t^{M,W,m} \cdot M_t^m + B_t^G + (1 + \varpi) D_t. \tag{24}
\]
Model calibration and solution technique

The model is calibrated so that its steady-state solution reproduces the data assembled in a social accounting matrix (SAM) for the base year 1997. The underlying idea is that this matrix measures Lebanon’s economic performance in 1997, i.e. basically prior to its recent efforts of integrating into the world economy, cf. Section 2. Unless otherwise specified, data are taken from the National Accounts’ (MOET, 2003) – actually 1997 data are the most recent National Accounts data currently available. The social accounting matrix (SAM) is based on these data and is the result of the authors’ own calculations. The SAM distinguishes 8 production sectors and 3 production factors.

Calibration of model parameters is based on the SAM and some additional assumptions. In particular, the population growth rate is assumed to equal to 1.4% per annum according to World Bank (2004) and the world real interest rate \( r \) is exogenously set at 4%. Substitution elasticities are obtained from the existing literature wherever possible.

On the demand side, the elasticity of substitution between different commodities is set at 0.9, while the Armington elasticity of substitution between domestic and imported good is set at 0.5 (see Devarajan, Go, Li, 1999). On the supply side the elasticity of substitution between domestic and exports is -2.3. In the utility function, the inverse of the consumer’s elasticity of intertemporal substitution and the elasticity of leisure are set equal to 1.

Once elasticity values have been fixed, model share and scale parameters are calibrated from the SAM. To save space, only few aspects of the calibration shall be discussed. In particular, the parameter \( \nu \), denoting the fraction of foreign debt to domestic capital, is not a free parameter but directly calibrated from data assembled in the SAM. Subtracting consumption and investment expenditures from households’ net factor income (including transfers from the government as well as from abroad) and relating this figure to gross capital income we obtain \( \nu = 0.393 \).

The model is programmed in Gauss and solved with the method of backward integration, cf. Brunner and Strulik (2002). In this method, the algorithm sets off in an arbitrarily small neighborhood of the post-shock steady state and iterates backwards on the saddle path. Since time is reversed in this method, all instable trajectories become stable in the sense that they converge to the true saddle path. Hence, choosing a starting value arbitrarily close to the post-shock steady state gives excellent approximations to the saddle path.

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10 We thank Salam Said for very helpful research assistance.
Simulation scenarios and results

As illustrated in Section 2, Lebanon implements trade liberalization mainly through bilateral and regional agreements. However, since previous studies demonstrated that the preferential (see Martin, (1996, 2000)) as well as the non-discriminatory (see Dessus and Ghaleb (2004)) trade liberalization have only minor static effects, we want to assess the maximum possible gain by allowing for dynamic adjustments and (counterfactually) the most generous form of tariff reduction (MFN). Even in this setting, however, it turns out that the economic impact of advances in political stability are larger than any kind of trade liberalization.

Our analysis begins at the time when the Interim Agreement was signed, so that time $t = 0$ corresponds approximately to 2003 in real time, and there is a 5-year transition period granted to Lebanon by the Interim Agreement for further structural reforms. We simulate a gradual reduction of tariff rates as specified in the Agreement, but counterfactually assume that this reduction applies to all trading partners rather than to just the EU countries. In order to compare the effects of the tariff liberalization with the economic benefits of political stability, we assume that the improvement in the political environment is managed progressively over a period of comparable length as in the trade liberalization scenario.

5.1 Gradual non-discriminatory tariff reductions

Title II of the Interim Agreement (see Table 1) is devoted to the free movement of goods and states that the free trade area will be established “over a transitional period not exceeding 12 years […].” Goods are divided into two categories, industrial products (chapter 1), and agricultural, fisheries and processed agricultural products (chapter 2). The second category is further divided into two classes, one including agricultural products and the other the processed agricultural products\(^{11}\). Exceptions to the provisions of chapter 1 are allowed for products listed in Annex 1 of the Agreement. These are agricultural and processed agricultural products as oil derivates or silk, cotton and wool.

Table 1 - Title II of the Interim Agreement (schematic structure)

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<th>Chapter 1 Industrial products</th>
<th>Chapter 2 Agricultural + Processed agricultural products.</th>
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<td>Annex 1 (some exceptions to Chapter 2)</td>
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<td>Protocol 2 Agricultural IMP</td>
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<td>Protocol 3 Processed agricultural products</td>
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<td></td>
<td>↓</td>
</tr>
<tr>
<td></td>
<td>Annex 1 EX</td>
</tr>
</tbody>
</table>

\(^{11}\) The classification at hand is based on the Combined Nomenclature 2002 (CN 2002) and of the Lebanese Custom Code (LCC).
Customs’ duties and charges with equivalent effect on imports of industrial products into Lebanon are to be abolished according to the schedule presented in table 2. A five-year transition period is allowed for appropriate structural reforms aimed at improving the competitiveness of the economy. By the fifth year the progressive reduction of tariffs’ will start according to the schedule indicated in the right column of the table (Interim Agreement, Art. 5).

**Table 2 - Tariff’s abolishment schedule according to the Interim Agreement**

<table>
<thead>
<tr>
<th>Time Frame</th>
<th>Rate (with reference to the basic rate)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5\textsuperscript{th} year</td>
<td>88%</td>
</tr>
<tr>
<td>6\textsuperscript{th} year</td>
<td>76%</td>
</tr>
<tr>
<td>7\textsuperscript{th} year</td>
<td>64%</td>
</tr>
<tr>
<td>8\textsuperscript{th} year</td>
<td>52%</td>
</tr>
<tr>
<td>9\textsuperscript{th} year</td>
<td>40%</td>
</tr>
<tr>
<td>10\textsuperscript{th} year</td>
<td>28%</td>
</tr>
<tr>
<td>11\textsuperscript{th} year</td>
<td>16%</td>
</tr>
<tr>
<td>12\textsuperscript{th} year</td>
<td>0%</td>
</tr>
</tbody>
</table>

Mapped onto the multi-sectoral structure of the model, this tariff reduction schedule will affect sector 2 (Energy & Water) and sector 3 (Manufacturing), which are the non-agricultural sectors of the economy.

Agricultural and fisheries’ imports (category 2, class 1) are addressed in Protocol 2. Tariff reduction for these products will occur in a single shot in the fifth Agreement year. Since the reduction will not affect all goods categories equally, we will adopt an aggregate tariff rate, as calculated on the base of Protocol 2. The depicted tariffs’ abatement framework will be applied to sector 1 (Agriculture). Processed agricultural products (category 2, class 2), including those listed in Annex 1, will be granted a less binding treatment according to the general statement of Art. 9, to be interpreted alongside Protocol 3. These product categories are however aggregated to the manufacturing sector (sector 3) in our model and will therefore be treated according to the provisions of chapter 1.

The provisions of the Interim Agreement are implemented in the model through an unexpected exogenous variation of the import tariff rate $\theta''$. Table 3 illustrates the (calibrated) benchmark value of $\theta''$ and its subsequent variations for each import sector.

**Table 3 - Import tariff rate subsequent variations for each import sector**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark</td>
<td>0,076</td>
<td>0,046</td>
<td>0,036</td>
</tr>
<tr>
<td>5\textsuperscript{th} year</td>
<td>0,053</td>
<td>0,041</td>
<td>0,031</td>
</tr>
<tr>
<td>6\textsuperscript{th} year</td>
<td>0,053</td>
<td>0,035</td>
<td>0,027</td>
</tr>
<tr>
<td>7\textsuperscript{th} year</td>
<td>0,053</td>
<td>0,030</td>
<td>0,023</td>
</tr>
<tr>
<td>8\textsuperscript{th} year</td>
<td>0,053</td>
<td>0,024</td>
<td>0,018</td>
</tr>
</tbody>
</table>
In order to quantify the economic benefits of political stability, we take Moody’s rating system as indicator. This is appropriate since Moody’s ratings not only reflect changes in economic conditions, but also political concerns. For instance, the assassination of former Prime Minister Rafiq al Hariri prompted Moody’s to downgrade Lebanon by one notch (from B2 to B3). A statement by Moody’s issued March 24, 2005, made clear that the assassination was the only motive for downgrading Lebanon.\textsuperscript{12}

We first simulate an improvement in political stability, assuming an upgrade in Moody’s rating by 3 notches over a 20-year-period (see table 4). This corresponds to assuming that Lebanon manages to improve its creditworthiness to levels still below the rating currently held by Egypt and is hence a moderate aim for a twenty-year period. Secondly, we calculate the economic effects of a sudden deterioration in political environment as exemplified by the assassination of Mr Hariri. The related fall in creditworthiness is reflected by a minor but instantaneous downgrade (by one notch).

\textbf{Table 4 - Gradual improvement in political stability}

<table>
<thead>
<tr>
<th>Time Frame</th>
<th>Moody’s rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>1\textsuperscript{st} - 6\textsuperscript{th} year</td>
<td>B1 = Lebanon (1997)</td>
</tr>
<tr>
<td>7\textsuperscript{th} - 13\textsuperscript{th} year</td>
<td>Ba3</td>
</tr>
<tr>
<td>14\textsuperscript{th} - 19\textsuperscript{th} year</td>
<td>Ba2</td>
</tr>
<tr>
<td>20\textsuperscript{th} year</td>
<td>Ba1 = Egypt (2005)</td>
</tr>
</tbody>
</table>

The variations in Moody’s rating are depicted through an exogenous change in the fraction $\nu$ of foreign debt to productive capital. The change is set to be equal to +36% in the first and -12% in the second (Hariri) case. Since the initial fraction of debt to capital was calibrated as slightly less than 40%, this means a final $\nu$ value of around 53% or 35%, respectively.

\textbf{5.3 Simulation results}

The first scenario implies a reduction of import tariff rates by the fifth year of the agreement. This induces an immediate decrease in the domestic price of each imported commodity $m$:

\textsuperscript{12} See Moody’s Rating Action of March 24, 2005.
\[ P_{t}^{M,C,m} = \left(1 + \varphi_{m}\right) \left[ \left(1 + \vartheta_{m}\right) P_{t}^{M,W,m} \right] \]

which instantaneously affects the demand price \( P_{t}^{L,m} \) of the Armington aggregates \( (x_{i}^{m,n}, c_{i}^{m}, I_{i}^{m}, G_{i}^{m}) \). These price variations affect the overall price levels \( P_{C}^{I}, P_{I}^{C} \) and \( P_{G}^{I} \) of the three composite commodities.

The immediate effect of lower domestic price of imports is an increase in the demand for foreign goods by 1.6%. The general price reduction following from the abolishment of tariffs stimulates long-run demand for private consumption (which increases by 1.2%) and investment (which increases by 3%). Government consumption remains constant, being exogenously given. Figure 1 shows the dynamics of aggregate private consumption and aggregate investment.

The overall benefits of trade liberalization give rise to an increase in GDP by 2.7%. These results are in line with the findings of other studies (see for instance Martin (1996, 2000)). As demonstrated by Dessus and Ghaleb (2004), public revenues suffer a relevant fall, which in our case is around 9.3%.

A progressive increase in external borrowing possibilities, as induced by increased political stability, enlarges households’ savings. The first consequence is an increase in the demand for investments, which grows in the long run by 7.6%, according to the non-monotonic transition path showed in Figure 2. After a strong increase during the first periods, the demand for investment reaches its highest value at around the last period of exogenous increase in \( \nu \) and then descends to its steady-state value. The
initial short term boost in investment demand influences private consumption demand, which shows an opposite non-monotonic transition path (see Figure 3) which ends up with an increase of 0.9%. A further increase on the demand side is represented by exports, which grow steadily from the first transition period as far as achieving a 20.9% increase in the new steady state. On the supply side, consequences of the improvement in the political environment include a growth of the productive capital by 7.6%, which is supported by an FDI boost (in value) of 41.4% relative to the old steady-state. The overall demand increase brings about a long run GDP growth of 3.4%.

By contrast, a rapid (though limited) deterioration in the political environment due to a destabilizing incident like the assassination of the former Prime Minister has - not unexpectedly - generally negative effects. The underlying economic reasons are conceptually the same as before. Due to the fall in creditworthiness on global markets, the economy’s foreign borrowing constraint becomes stricter, thus limiting productive investment possibilities. Figure 4 shows the transition path of domestic consumption and investment. The second falls rapidly since the negative shock, leaving
productive capital exceeding demand requirements. On the other hand, due to stronger constraints on international borrowing, households modify their consumption and savings’ path and more income is allocated to consumption than to investment. These two effects together bring about a short term rise in consumption. Moreover, the fall in creditworthiness also implies deteriorating exports: In the new steady-state exports shrink by 6.1% relative to the benchmark. The overall negative effect is quantified by a GDP fall by 1% in the long term.

A comparison of the three scenarios shows the quantitative relevance of political stability in comparison to trade liberalization. Figure 5 depicts the long term GDP effects in the three scenarios. Leaving the positive effect of an increase in creditworthiness aside, evidence suggests how single destabilizing incidents as the one quantified here may undermine the positive effects of trade liberalization.
6 Conclusions

The main aim of this paper is to study the effects of Lebanon’s integration into the world economy with a dynamic CGE model. Unlike previous studies, the intertemporal structure of the model allows a dynamic quantification of the effects, including also the very long-run effects. Additionally, the analysis takes into consideration the role of FDI and its immediate dependence on Lebanon’s creditworthiness on the world capital market.

We show that even a non-discriminatory trade liberalization brings about not more than moderate effects with respect to GDP and in any case heavily affects public revenues. Political stabilization, by contrast, can easily produce larger gains than broad trade liberalization by attracting FDI into the economy. We hasten to add that trade liberalization and political stabilization are, of course, by no means exclusive, but that rather the two might complement each other.
References


Challenges for Arab Countries”. The University of Michigan Press. Studies in International Economics.


Appendixes

Appendix A. Glossary

\[a^{m,n}\] fixed input (commodity \(m\)) requirement for production (activity \(n\))

\(A^n\) total factor productivity (activity \(n\))

\(B^G\) government payments abroad

\(b^n_h\) production elasticity of labor (activity \(n\))

\(b^n_k\) production elasticity of capital (activity \(n\))

\(b^n_{la}\) production elasticity of land (activity \(n\))

\(c_i\) per-capita composite consumption demand

\(c^n_m\) per-capita consumption demand of Armington aggregate \(m\)

\(C^{v,n}_i\) value added cost

\(D_i\) stock of foreign debt

\(D^{C,m}_i\) consumption demand of domestic commodity \(m\)

\(D^{F,m}_i\) intermediates’ demand of domestic commodity \(m\)

\(D^{F,m,n}_i\) intermediates’ demand of domestic commodity \(m\) (activity \(n\))

\(D^{G,m}_i\) government consumption demand of domestic commodity \(m\)

\(D^{I,m}_i\) investment demand of domestic commodity \(m\)

\(D^n_i\) domestic demand of domestic commodity \(m\)
\( E_t^m \) export demand of commodity \( m \)
\( f_t \) per-capita amount of free time
\( G_t \) government composite consumption demand
\( G_t^m \) government consumption demand of Armington aggregate \( m \)
\( I_t \) composite investment demand
\( I_t^m \) investment demand of Armington aggregate \( m \)
\( K_t \) physical capital (total amount)
\( K_t^n \) physical capital (activity \( n \))
\( L_t \) land assets (total amount)
\( L_t^n \) land assets (activity \( n \))
\( m \) generic commodity
\( M \) total number of commodities
\( M_t^{C,m} \) consumption demand of imports (commodity \( m \))
\( M_t^{F,m} \) intermediates’ demand of imports (commodity \( m \))
\( M_t^{F,m,n} \) intermediates’ demand of imports (commodity \( m \)) by activity \( n \)
\( M_t^{G,m} \) government consumption demand of imports (commodity \( m \))
\( M_t^{I,m} \) investment demand of imports (commodity \( m \))
\( M_t^m \) domestic demand of imports (commodity \( m \))
\( n \) generic mono-product industry (activity)
\( N \) total number of mono-product industries (activities)
\( O_t \) government outlays
\( P_t^{A,m} \) demand price of Armington aggregate \( m \)
\( P_t^C \) optimal price of consumption (aggregate)
\( P_t^{D,m} \) price of the domestic commodity \( m \)
\( P_t^{E,m} \) domestic (producer) price of exports (commodity \( m \))
\( P_t^{E,W,m} \) world price of exports (commodity \( m \))
\( P_t^{G} \) optimal price of government consumption (aggregate)
\( P_t^{I} \) optimal price of investment (aggregate) commodity
\( P_t^{L} \) price of land
\( P_t^{M,m} \) domestic price of imports (commodity \( m \))
\( P_t^{M,W,m} \) world price of imports (commodity \( m \))
\( P_t^{P,m} \) price index of commodity \( m \)
\( P_t^{Q,n} \) market price of activity \( n \)
\( P_t^{V,n} \) minimal price of value added
\( Q_t^n \) domestic production (activity \( n \))
\( r_t \) real domestic interest rate
\( \bar{r} \) real world interest rate
\( R_t \) government revenues
\(S_C\) households’ savings
\(T_i^G\) net lump transfer from the government
\(T_i^W\) foreign remittances
\(u_t\) per-capita total amount of working time
\(u_t^a\) per-capita amount of working time (activity \(n\))
\(U_0\) lifetime utility
\(U_t\) instantaneous utility
\(V_t^n\) value added (activity \(n\))
\(w_t\) wage rate
\(x_{i,m,n}^m\) intermediate input demand of Armington aggregate \(m\) (activity \(n\))
\(Y_t^m\) domestic production (commodity \(m\))
\(YD_t\) disposable income

\(\alpha\) absolute value of the inverse of the constant elasticity of substitution in the instantaneous utility function (consumption)
\(\beta\) absolute value of inverse of the constant elasticity of substitution in the instantaneous utility function (leisure)
\(\chi^m\) scale parameter in the CET function (commodity \(m\))
\(\delta\) capital depreciation rate
\(\epsilon\) elasticity of transformation in the CET function
\(\phi^n\) domestic sales’ share parameter in the CET function (commodity \(m\))
\(\varphi^n\) share parameter in the CES function (Armington composite \(m\))
\(\gamma^{\alpha}\) population growth rate
\(\eta\) leisure share parameter
\(\kappa\) elasticity of substitution in the CES function (final commodity aggregate)
\(\mu\) elasticity of substitution in the CES function (Armington composite)
\(\nu\) fraction of foreign debt to domestic capital
\(\Pi_t^{V,n}\) value added profit (activity \(n\))
\(\theta^m\) import tariff rate on commodity \(m\)
\(\vartheta^m\) scale parameter in the CES function (Armington composite \(m\))
\(\rho\) consumer’s rate of time preference.
\(\sigma^\epsilon\) substitution parameter in the CET function
\(\sigma^x\) substitution parameter in the CES function (final commodity aggregate)
\(\sigma^u\) substitution parameter in the CES function (Armington composite)
\(\tau\) general income tax rate
\(\tau^l\) labor income tax rate
\(\tau^k\) capital income tax rate
\(\tau^{la}\) land income tax rate
\(\vartheta^n\) indirect tax on domestic production (activity \(n\))
\(\omega^m\) import tax on imported commodity \(m\)
Ω, population at time \( t \)

\( \psi^{C,m} \) commodity \( m \)'s share parameter in the CES function (final consumption aggregate)

\( \psi^{I,m} \) commodity \( m \)'s share parameter in the CES function (final investment aggregate)

\( \psi^{G,m} \) commodity \( m \)'s share parameter in the CES function (final government consumption aggregate)

\( \zeta \) scale parameter in the CES function (final commodity aggregate)

**Appendix B. List of equations**

The model consists of equations (1) - (24) and of the equations listed in this appendix.

**B.1 Parameter relations**

\[
\varepsilon \equiv \frac{1}{\sigma^\varepsilon - 1} \quad \sigma^\varepsilon > 1 \quad (A.1)
\]

\[
\mu \equiv -\frac{1}{1 - \sigma^\mu} \quad \sigma^\mu < 1 \quad (A.2)
\]

\[
\kappa \equiv -\frac{1}{1 - \sigma^\kappa} \quad \sigma^\kappa < 1 \quad (A.3)
\]

**B.2 Prices’ relations**

\[
P^{0,n}_t = \frac{P^{I,m}_t - a^{m,n}}{1 - \nu^s} + \frac{P^{V,n}_t}{1 - \nu^s} \quad (A.4)
\]

\[
P^{V,m}_t = P^{0,n}_t \quad (A.5)
\]

\[
P^{E,m}_t = P^{E,W,m}_t \quad (A.6)
\]

\[
P^{M,C,m}_t = \left(1 + \omega^m\right) \left(1 + \theta^m\right) P^{M,W,m}_t \quad (A.7)
\]

\[
P^{A,m}_t = \frac{1}{\gamma^m} \left[ \left( P^{D,m}_t \right)^{\frac{1}{1+\mu}} + \left( P^{M,m}_t \right)^{\frac{1}{1+\mu}} \right]^\frac{1}{1+\mu} \quad (A.8)
\]
\[ P_t^{V,m} = \frac{1}{\chi^m} \left[ \left( \frac{P^{D,m}_t}{\phi^m} \right)^{1+\varepsilon} + \left( \frac{P^{E,m}_t}{(1-\phi^m)^\varepsilon} \right)^{1+\varepsilon} \right]^{1/(1+\varepsilon)} \]  

(A.9)

**B.3 Production**

**B.3.1 Armington composite**

\[ P^{A,m}_t \cdot x^{m,n}_t = P^{D,m}_t \cdot D_{t}^{E,m,n} + P^{M,m}_t \cdot M^{F,m,n}_t \]  

(A.10)

\[ x^{m,n}_t = \theta^m \left[ \phi^m \left( D_{t}^{E,m,n} \right)^{\sigma^m} \left( 1 - \phi^m \right) \left( M^{F,m,n}_t \right)^{\sigma^m} \right]^{1/\sigma^m} \]  

(A.11)

\[ \frac{D^{F,m,n}_t}{M^{F,m,n}_t} = \left( \frac{1 - \phi^m}{\phi^m} \right) \left( \frac{P^{D,m}_t}{P^{M,m}_t} \right) \]  

(A.12)

**B.3.2 Commodity supply**

\[ Q^m_t = Y^m_t \]  

(A.13)

**B.3.3 CET composite**

\[ P^{Y,m}_t \cdot Y^m_t = P^{D,m}_t \cdot D^m_t + P^{E,m}_t \cdot E^m_t \]  

(A.14)

\[ Y^m_t = \chi^m \left[ \phi^m \left( D^m_t \right)^{\sigma^m} \left( 1 - \phi^m \right) \left( E^m_t \right)^{\sigma^m} \right]^{1/\sigma^m} \]  

(A.15)

\[ \frac{D^m_t}{E^m_t} = \left( \frac{1 - \phi^m}{\phi^m} \right) \left( \frac{P^{D,m}_t}{P^{E,m}_t} \right)^{\varepsilon} \]  

(A.16)

**B.4 Consumption and leisure**

**B.4.1 Armington (consumption) composite**

\[ P^{A,m}_t \cdot c^m_t = P^{D,m}_t \cdot D^{C,m}_t + P^{M,m}_t \cdot M^{C,m}_t \]  

(A.17)

\[ c^m_t = \theta^m \left[ \phi^m \left( D^{C,m}_t \right)^{\sigma^m} \left( 1 - \phi^m \right) \left( M^{C,m}_t \right)^{\sigma^m} \right]^{1/\sigma^m} \]  

(A.18)

\[ \frac{D^{C,m}_t}{M^{C,m}_t} = \left( \frac{1 - \phi^m}{\phi^m} \right) \left( \frac{P^{D,m}_t}{P^{M,m}_t} \right)^{\mu} \]  

(A.19)

**B.4.2 Final aggregate consumption good**
\[ P^C_i \cdot c_i = \sum_{m=1}^{M} P^A_{t,m} \cdot c^m_i \]  
(A.20)

\[ c_i = \zeta \cdot \left[ \sum_{m=1}^{M} \psi^{C,m} \cdot (c^m_i)^{\alpha^m} \right]^{\frac{1}{\sigma^m}} \]  
(A.21)

\[ c^m_i = \frac{1}{\zeta^{1+\kappa}} \left( \frac{P^A_{t,m}}{\psi^{C,m} \cdot P^C_t} \right)^{\kappa} \]  
(A.22)

\[ P^C_t = \frac{1}{\zeta} \cdot \left[ \sum_{m=1}^{M} \left( P^A_{t,m} \right)^{1+\kappa} \right]^{\frac{1}{1+\kappa}} \]  
(A.23)

B.4.3 Armington (investment) composite

\[ P^A_{t,m} \cdot I^m_i = P^D_{t,m} \cdot D^I_{t,m} + P^M_{t,m} \cdot M^I_{t,m} \]  
(A.24)

\[ I^m_i = \vartheta^{m} \left[ \phi^{m} \left( D^I_{t,m} \right)^{\sigma^m} + \left(1 - \varphi^{m}\right) \left( M^I_{t,m} \right)^{\sigma^m} \right]^{\frac{1}{\sigma^m}} \]  
(A.25)

\[ \frac{D^I_{t,m}}{M^I_{t,m}} = \left( \frac{1 - \varphi^{m}}{\varphi^{m}} \right) \frac{P^D_{t,m}}{P^M_{t,m}} \]  
(A.26)

B.4.4 Final aggregate investment good

\[ P^I_i \cdot I = \sum_{m=1}^{M} P^I_{t,m} I^m_i \]  
(A.27)

\[ I_i = \zeta \cdot \left[ \sum_{m=1}^{M} \psi^{I,m} \cdot (I^m_i)^{\sigma^m} \right]^{\frac{1}{\sigma^m}} \]  
(A.28)

\[ I^m_i = \frac{1}{\zeta^{1+\kappa}} \left( \frac{P^I_{t,m}}{\psi^{I,m} \cdot P^I_t} \right)^{\kappa} I_i \]  
(A.29)

\[ P^I_i = \frac{1}{\zeta} \cdot \left[ \sum_{m=1}^{M} \left( P^I_{t,m} \right)^{1+\kappa} \right]^{\frac{1}{1+\kappa}} \]  
(A.30)

B.5 Government behavior
B.5.1 Armington composite

\[ P_{i}^{A,m} \cdot G_{i}^{m} = P_{i}^{D,m} \cdot D_{i}^{G,m} + P_{i}^{M,m} \cdot M_{i}^{G,m} \]  
(A.31)

\[ G_{i}^{m} = G^{m} \left[ \phi^{m} \left( D_{i}^{G,m} \right)^{\sigma_{m}} + \left( 1 - \phi^{m} \right) \left( M_{i}^{G,m} \right)^{\sigma_{m}} \right]^{\frac{1}{\sigma_{m}}} \]  
(A.32)

\[ \frac{D_{i}^{G,m}}{M_{i}^{G,m}} = \left( \frac{1 - \phi^{m} P_{i}^{D,m}}{\phi^{m} P_{i}^{M,m}} \right)^{\sigma_{m}} \]  
(A.33)

B.5.2 Final aggregate government consumption good

\[ P_{i}^{G} \cdot G_{i} = \sum_{m=1}^{M} P_{i}^{A,m} G_{i}^{m} \]  
(A.34)

\[ G_{i} = \zeta \cdot \left[ \sum_{m=1}^{M} \psi^{G,m} \cdot \left( G_{i}^{m} \right)^{\sigma_{m}} \right]^{\frac{1}{\sigma_{m}}} \]  
(A.35)

\[ G_{i}^{m} = \frac{1}{\zeta^{1+\kappa}} \left( \frac{P_{i}^{A,m}}{\psi^{G,m} \cdot P_{i}^{G}} \right)^{\kappa} G_{i} \]  
(A.36)

\[ P_{i}^{G} = \frac{1}{\zeta} \left[ \sum_{m=1}^{M} \left( \frac{P_{i}^{A,m}}{\psi^{G,m}} \right)^{1+\kappa} \right]^{\frac{1}{1+\kappa}} \]  
(A.37)