Energy Saving Technical Progress:How about The Solow Model?

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Abstract

This paper attempts to make an analysis of the different effects of the direction of technical change on share of energy in the national income. We extend the standard Solow Model by adding the energy factor and allowing for technical progress induced by biased R&D activities. For determining the direction of technical change we use Kennedy's modified innovation possibilities frontier. The rate of capital augmenting technical progress is ex ante determined. Based on this, firms decide on the share of expenditures in labour augmenting technical progress and the rate of innovation in energy augmenting technologies is determined accordingly. The energy demand in this model depends on the depreciation rate of capital. We show that Harrod neutral technical progress is a necessary condition for the steady-state. We point out also that the impact of the marginal propensity to save and the depreciation rate of capital is negative on capital and labour prices whereas it is positive on energy price.

Keywords: Technical change; Efficiency; Factor bias
JEL classification: O33; Q40

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1 Introduction

> In the February 1956 issue of this Journal (Quarterly Journal of Economic Growth), Robert M. Solow presented an ingeniously simple yet extremely useful model for the examination of various aspects of the problem of economic growth. It does not seem to be generally realized, however, that this model, after a minor change, may be used to make sense out of some other assertions. (Buttrick (1958))

As cited above, the Solow model (1956) made an important contribution to the economic growth literature. This model of long-run growth accepts all the Harrod-Domar assumptions except the assumption that there is no possibility of factor substitution. Under the standard neoclassical conditions Solow (1956) discusses growth paths where technical change is neutral and analyses long-run behaviour of interest and wage rates. The theoretical literature concerning the factor bias gives evidence of different effects from the direction of technical change on the factor prices and factor shares in the national income. Research on the direction of technical change started with the pioneering work of Hicks (1932) which concluded that the new inventions is directed to economizing the use of a factor which has become relatively expensive. As pointed out by Habakkuk (1962), when the elasticity of substitution is low then the scarcity of a factor (labour in his case) will increase the price of this factor (wage) and as a result the technical progress will be biased on this scarce factor (*labour augmenting technical progress*). Kennedy (1964) emphasizes the trade-off between different types of innovation and introduces the concept of "innovation possibilities frontier". In a more recent study, using an endogenous technical change model where the technical progress is given by the increase in the variety of machines, Acemoglu (2001) shows that the elasticity of substitution between different factors determines the pattern of technical change.

Dasgupta and Heal (1974), Stiglitz (1974) and Solow (1974), among others, analysed the technological conditions under which economy could have a positive long-run growth in the presence of a non-renewable natural resource. Krautkraemer (1998) indicates that new technologies increase the efficiency of use of non-renewable resources which means that the technical progress is *energy augmenting*. More recently, in an empirical work, Jones (2002) finds for the US that the relatif energy prices and the share of energy in the GNP decreased over the passed 50 years. This is an distributive consequence of an energy saving technical progress.[1] However, these recent works do not consider the classical growth literature which is able to give some interesting results on the subject. This paper attempts to provide such an analysis extending the standard Solow Model (1956) by adding energy factor and allowing the technical progress, induced by R&D activities, to be biased.

The composition of the paper is as follows. Section 2 introduces the set-up of the model. Section 3 contains some preliminary results on the transitional dynamics of the model. Section 4 presents functional income distribution analysis. Section 5 gives some concluding remarks.

2 The Model

We consider the following production function with constant returns to scale and diminishing return to each input.

$$Y_t = F(A_t K_t, B_t L_t, C_t E_t) \tag{1}$$

[1] We use the term *energy augmenting technical progress* to say that new technologies increase the efficiency of use of energy. Furthermore, this innovation process can be called *energy saving technical progress*, as increasing energy efficiency can decrease energy use.

where Y is the flow of output, A, B and C denote the technical progress augmenting the factors of production capital K, labor L and energy E respectively. We suppress time arguments to simplify the notation. We use a circumflex to note efficient input quantity. The production function becomes

$$Y = F(\hat{K}, \hat{L}, \hat{E}) \tag{2}$$

We assume that all savings are invested $I = sY$, and the increase in the stock of physical capital is:

$$\dot{K} = (1 - \varphi)I - \delta K \tag{3}$$

where the depreciation rate of capital δ and the saving rate s are constant with $0 < s < 1$ and $\delta > 0$. A dot over a variable denotes the differentiation with respect to time. The investment demand is equal to the saving and we have the equilibrium condition for the good market;

$$I = sY \tag{4}$$

The share of total investment I in physical capital is denoted by $1 - \varphi$. So the variation in total investment in R&D is

$$\dot{D} = \varphi I \tag{5}$$

with $0 < \varphi < 1$ The energy demand in this model depends on the depreciation rate of capital, thus

$$\frac{\dot{E}}{E} = j(\delta) \tag{6}$$

with $j^{'}(\delta) < 0$. The more the capital is depreciated the lower is the energy demand. This implies that firms producing with new machines demand less energy. This is of course an interpretation of vintage capital model. We will introduce the other expressions before we pass to the equilibrium analysis. Population grows at a constant exogenous rate, n, so the growth rate of labor force is given by

$$\frac{\dot{L}}{L} = n \tag{7}$$

For determining the direction of technical change we use Kennedy's modified inno-

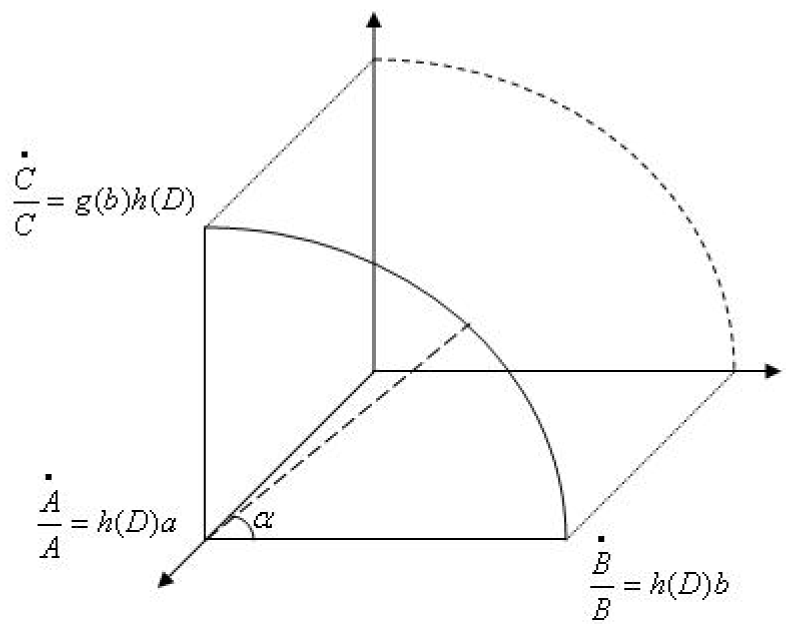

Figure 1: Innovation possibilities frontier

vation possibilities frontier. The rate of capital augmenting technical progress is ex ante determined. Based on thus firms decide on the share of expenditures in labor augmenting technical progress, b, and the share of expenditures in energy augmenting technologies, $g(b)$ is determined accordingly. In Figure 1 we illustrate the direction of technical change given by the value of α. The technical change is neutral if $\alpha = 45$, on the other hand we have an energy biased pattern of technical change if $\alpha > 45$. The rates of technical progress for each factor are given below.

$$\frac{\dot{A}}{A} = ah(\dot{D}) \qquad \frac{\dot{B}}{B} = bh(\dot{D}) \qquad \frac{\dot{C}}{C} = g(b)h(\dot{D}) \tag{8}$$

As in Solow (1956), we introduce the ratio of capital to labor, $k = \frac{K}{L}$. In addition, we have here two other ratios; the ratio of energy to labor, $e = \frac{E}{L}$ and the ratio of capital to energy, $z = \frac{k}{e} = \frac{K}{E}$.

Marginal profuctivities of the production factors gives the factor prices. Thus we have;

$$r = Af' \tag{9}$$

$$w = Bf' \tag{10}$$

$$v = Cf - Azf' - B\frac{1}{e}f' \tag{11}$$

where r, w and v denote the cost of capital, wage and energy price respectively.

We can have the same notation for the efficient capital to efficient energy ratio, that is $\hat{z} = \frac{\hat{k}}{\hat{e}} = \frac{\hat{K}}{\hat{E}}$

$$v = C\left(f - \hat{z}f' - \frac{1}{\hat{e}}f'\right) \tag{12}$$

We can rewrite the equation (2) as follows:

$$\frac{Y}{\hat{L}} = \hat{y} = f\left(\hat{z}, 1, \hat{e}\right) \tag{13}$$

We assume that this function satisfies the well known Inada conditions that imply $\lim_{\hat{z}\to\infty} f'\left(\hat{z}, 1, \hat{e}\right) = 0$ and $\lim_{\hat{z}\to 0} f'\left(\hat{z}, 1, \hat{e}\right) = \infty$. For this equation system we have 15 unknown variables with 15 equation, thus there is a solution.

3 Transitional dynamics

To analyse the equilibrium conditions we start by giving the growth rate of efficient capital-energy ratio $\hat{z} = \frac{AK}{CE}$. Using the previous system of equations, the growth rate of the efficient capital-energy ratio can be written as follows;

$$\frac{\dot{\hat{z}}}{\hat{z}} = \frac{(1-\varphi)sCf}{z} - (\delta + j(\delta)) + (a - g(b))h(\dot{D}) \tag{14}$$

or in another fashion,

$$\frac{\dot{\hat{z}}}{\hat{z}} = \frac{(1-\varphi)sAf(\hat{z}, \frac{1}{\hat{e}})}{\hat{z}} - (\delta + j(\delta)) + (a - g(b))h(\dot{D}) \tag{15}$$

By definition, on a balanced growth path (BGP) efficient capital labour ratio and efficient energy labor ratio are constant. Thus the efficient capital energy ratio $\hat{z} = \frac{\hat{k}}{\hat{e}}$ is *ipso facto* constant. It can be seen from (14) that the stationarity condition requires $\frac{\dot{\hat{z}}}{\hat{z}} = 0$. In order to satisfy this stationarity condition we should have

- $\dot{A} = 0$. Moreover, from (8) it follows that for $\dot{A} = 0$, $a = 0$.
- $(a - g(b))h(\dot{D}) = 0$. Consequently, we have $a = g(b) = 0$.

At steady-state, technical progress does not affect neither energy nor capital. In fact, as Uzawa (1961) demonstrated for the capital-labor ratio (k), in our case, Harrod-neutral (labor augmenting) technical progress is the only type of technical progress consistent with a stable steady-state ratio $\hat{z}^*$.

As it is shown in Figure 2, for all $\hat{z}^- < \hat{z}^*$ or $\hat{z}^+ > \hat{z}^*$, $\hat{z}$ converges to the steady-state effective capital-energy ratio $\hat{z}^*$.

Remark 1

$$\hat{z}^- \Rightarrow \frac{(1-\varphi)sAf(\hat{z}, \frac{1}{\hat{e}})}{\hat{z}} > (\delta + j(\delta)), \frac{\dot{\hat{k}}}{\hat{k}} > \frac{\dot{\hat{e}}}{\hat{e}} \Rightarrow \frac{\dot{\hat{z}}}{\hat{z}} > 0 \Rightarrow \hat{z} \textit{ increases.}$$

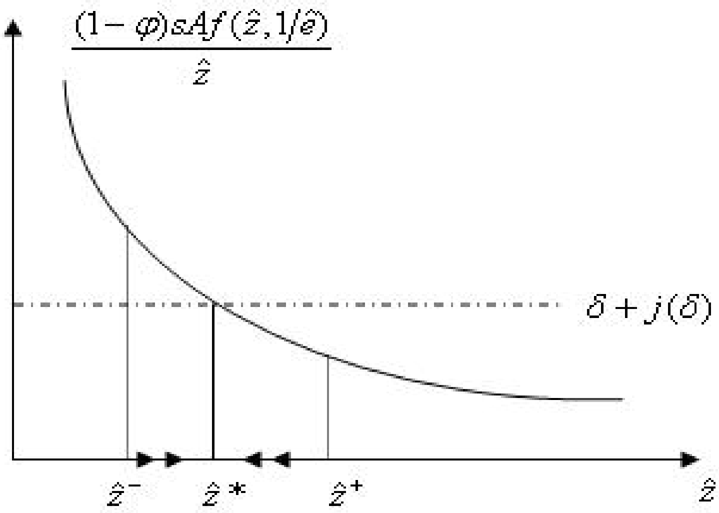

Figure 2: Phase diagram

By the same way we have[2]

$$\hat{z}^- \Rightarrow \frac{\dot{\hat{z}}}{\hat{z}} > 0 \Rightarrow \hat{z} \text{ increases.}$$

In what follows, we examine factor bias and technical progress in a functional income distribution analysis.

4 Distributional effects of factor bias

We now develop a basic framework for analysing the factor bias of technical change and the focus is on the energy saving technical progress.

[2]In order to conserve space we do not analyze in detail the effects from the variations in the parameters of the model (saving rate s, share of R&D in total investment γ and depreciation rate of capital δ). For a detailed discussion about the subject, see for example Barro and Sala-i-Martin (1995).

Table 1: Effects of the parameters of the model on the factor prices

	r	w	v
s	-	-	+
δ	-	-	+

$\frac{dr}{ds} = \frac{dr}{d\hat{z}}\frac{d\hat{z}}{ds}$ $\frac{dr}{d\hat{z}} = \dot{A}f' + Af'' < 0$. Because $f'' < 0$ and $\dot{A} = 0$. From the phase diagram we know that $\frac{d\hat{z}}{ds} > 0$. Thus we have *in fine* $\frac{dr}{ds} < 0$; if the marginal propensity to save increases then the cost of capital (or interest rate) decreases. Similarly we have $\frac{dw}{ds} < 0$ which means that larger amount of saving yields to a decrease in the wage level. On the other hand, $\frac{dv}{ds} = \frac{dv}{d\hat{z}}\frac{d\hat{z}}{ds}$ where $\frac{dv}{d\hat{z}} = \dot{C}\left(f - \hat{z}f' - \frac{1}{\hat{e}}f'\right) + C\left(f' - f'' - \frac{1}{\hat{k}}f''\right)$. As we found that $g(b) = 0$, then it follows that $\dot{C} = 0$. We obtain $\frac{dv}{ds} > 0$.

The same analysis should be conducted for the depreciation rate of capital δ. For this purpose we start with the interest rate; $\frac{dr}{d\delta} = \frac{dr}{d\hat{z}}\frac{d\hat{z}}{d\delta}$ which is negative. We have also $\frac{dw}{d\delta} < 0$. However we obtain $\frac{dv}{d\delta} > 0$ which means that if the depreciation rate of capital is high, then new technologies (that are relatively more energy saving) are used in the production and the energy efficiency increases. In our model factors are paid according to their productivities. Then increasing productivity of energy increases energy prices. Table 1 summarizes the results we obtained in this section.

5 Concluding remarks

In this paper we presented a simple growth model using Solow's (1956) framework. Our purpose has been to investigate the stability conditions of the standard Solow Model that is extended by adding energy factor and allowing for technical progress induced by R&D activities. The main conclusions that can be drawn here are: first, Harrod neutral (labor augmenting) technical progress is the only type of technical progress consistent with a stable steady-state ratio; second, increasing savings increases energy prices and decreases other factor prices; third, if the capital is depreciated rapidly, then the new energy saving machines are used in the production, this would increase energy prices.

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