Describing the Phelix Forward Electric-Power Market using

Bayesian Estimators for Stochastic Volatility Models

Abstract

This paper applies the EMM methodology to build a stochastic volatility (SV) model of the mean and latent volatility for the Phelix future electric power market. The main objective is to find appropriate descriptions emphasising schemes for derivative pricing purposes. A Bayesian estimator is used to estimate a general scientific SV model and is computed adapting MCMC simulation proposed by Chernozhukov and Hong (2003). The approach helps circumvent the computational curse of dimensionality and is substantially superior to conventional derivative based hill climbing optimizers. The paper finds that MCMC estimation of stochastic volatility models are successful describing the energy market's two financial contracts. The success suggests that the dynamics of the market contains features known to general SV-models; that is - a preference for simulation based derivative pricing schemes mainly caused by volatility clustering. High market volatility caused by the German market's rather low transparency and credibility, the shut-down options of producers, and coal plant threshold price production decisions, induces derivative contracts both important and expensive risk management instruments. Higher market transparency and credibility may therefore suggest a potential for lower hedging costs and increased market liquidity.

Classification: C14

Keywords: Stochastic Volatility models, Efficient Method of Moments, Bayesian Estimator, Markov Chain Monte Carlo Simulation

1 Introduction and Motivation

The paper describes the base and peak partially observable one-year future electric-power financial contract series for the Phelix financial market. The main objective is to see whether versions of a stochastic volatility model (SVs) can appropriately describe the characteristics of the relatively young Phelix (German) electric-power market. The paper applies the Efficient Method of Moments (EMM) methodology to contribute to a better understanding of the commodity market's general behaviour. Appropriate SV modelling may greatly enhance derivative pricing for the energy market. The Base load contract comprises a constant delivery rate on all delivery days from Monday until Sunday and during all 24 delivery hours of any delivery day during the delivery period. The Peak load contract comprises a constant delivery rate on all delivery days from Monday until Friday and throughout 12 delivery hours from 08:00 am until 08:00 pm of any delivery day during the delivery period. Successful SV-model implementations for the forward market will indicate non-predictive market features and weak-form market efficiency introducing these commodity markets to conventional funds and enhanced risk management activities. The foundation for a non-predictive and an efficient electric-power market is also important for political acceptance of market allocations and product pricings to both private and industrial end consumers. For the market participants in general, a SV-model implementation induces market price weak-form efficiency reflecting that all historical information is accounted for, characterizing a non-predictive market.

The computational methods implemented by the EMM (and GSM) methodology, which apply discrepancy functions $s_n(\rho)$ producing asymptotically normal estimates, use the Cramer-von Mises discrepancy between the empirical distribution function of (y_t, x_{t-1}) computed from the data and the empirical distribution function of (y_t, x_{t-1}) computed from a simulation as the criterion for judging the adequacy of a fit. Therefore, for EMM implementation of the estimator, the procedure requires the computation of the estimator itself, $\hat{\rho}_n = \underset{\rho}{\arg\min}\; s_n(\rho)$, an estimate of the Hessian $\Im = \frac{\partial}{\partial\rho\partial\rho'} s^0(\rho^0)$, where $s^0(\rho) = \lim_{n\to\infty} s_n(\rho)$, and an estimate of the Fisher's information $I = Var\left[\frac{\partial}{\partial\rho'}\sqrt{n}\ s_n(\rho^0)\right] = \varepsilon\left[\frac{\partial}{\partial\rho'}\sqrt{n}\ s_n(\rho^0)\right]\left[\frac{\partial}{\partial\rho'}\sqrt{n}\ s_n(\rho^0)\right]'$. The variance of $\sqrt{n}\left(\hat{\rho}_n - \rho^0\right)$ is then of the sandwich form $V_n = Var\left[\sqrt{n}\left(\hat{\rho}_n - \rho^0\right)\right] = \mathfrak{I}^{-1}\ I\ \mathfrak{I}^{-1}$. Put $\ell(\rho) = e^{-n\, s_n(\rho)}$. Apply MCMC methods with $\ell(\rho)$ as the likelihood. From the resulting MCMC chain $\{\rho_i\}_{i=1}^{R}$ put $\hat{\rho}_n = \bar{\rho}_R = \frac{1}{R}\sum_{t=1}^{R}\rho_i$ *and* $\hat{\Im}^{-1} = \frac{n}{R}\sum_{t=1}^{R}\left(\rho_i - \bar{\rho}_R\right)\left(\rho_i - \bar{\rho}_R\right)'$.

The starting point is the usual assumption of prices following random-walk type behaviour. When linear models are used, asset prices are assumed to conform to a martingale: $E\left[S_d\left(t+1\right) | \Omega_t\right] = S_d\left(t\right) + \mu\cdot\Delta t + \sigma\cdot\varepsilon\cdot\sqrt{\Delta t}$ where *E[.]* denotes the mathematical expectation operator, $S_d(t)$ be a price of a commodity/security at the end of day *t* and for a period of time of length *T*, *S* increases by an amount $\mu\cdot T$. The $\sigma\cdot\varepsilon\cdot\sqrt{\Delta t}$ term is noise or variability of the path followed by *S*. The amount of this noise is σ times a Wiener process ($dW = \varepsilon\cdot\sqrt{dt}$). Define now the logarithm of the stock price as $ln(S_d(t))$ and let $y_d(t) = ln(S_d(t))$. A geometric Brownian-Motion model for the logarithm of a stock price *y* using Ito's lemma, the process becomes $dy = \left(\mu - \frac{\sigma^2}{2}\right)dt + \sigma\cdot dW$, where *dt* is an infinite small interval of time and *dW* is the basic Wiener process, and ε is a random drawing from a standardized normal distribution *N(0,1)*. Because the parameters μ and σ are constants, *y* follows a generalized Wiener process, with constant drift $\left(\mu - \frac{\sigma^2}{2}\right)$ and constant variance rate σ^2. The change in *y* between time *t* and some future date *T*, is, therefore normally distributed with mean $\left(\mu - \frac{\sigma^2}{2}\right)\cdot T$ and variance $\sigma^2\cdot T$. The Black's forward option pricing formula (1976) will always price such derivative correctly. However, high-frequency equity time series have shown erratic-behaviour, large negative price changes occur more often than large positive ones and the large changes tend to occur in clusters and periods of high volatility are often preceded by large negative price changes. Commodity markets show similar characteristics. The behaviour of the price changes does not agree with the frequently assumed normal distributions and the constant mean and variance assumption in generalized Brownian motion processes, which by construct reject market observed sequences of aberrant observations within which returns and volatility display changing dynamic behaviour. Based on these facts and therefore in search for more reliable forecasts, nonlinear models are plausible alternatives. A vast number of possible nonlinear time series models are available. The most persistent descriptive and forecasting devices are regime-switching models, artificial neural networks and models for stochastic volatility. This paper investigates whether a stochastic volatility specification with serial correlation in both the mean and the volatility can contribute to a higher understanding of the logarithmic price change process in energy and commodity markets. In contrast to other estimation methods, the EMM methodology can confront the empirical plausibility of specifications for stochastic volatility. The stochastic volatility model (Shephard, 2004) in the classical form with a correlation between return innovations and volatility innovations to produce asymmetric volatility effects, is fitted using a Bayesian estimator. By using EMM, systematic behaviour not contained in the stochastic model representation, will report misspecifications available from Bayesian model's moment scores.

The results show that the quasi-Bayesian estimator using statistical, non-likelihood base criterion functions, provide a useful alternative to the usual extremum estimators. In particular, the two energy market series are well represented applying these quasi-Bayesian criterion functions. The overall score functions from the MCMC chains are far from rejection of the stochastic volatility representation. The optimal scored models report both serial correlation in mean and volatility, as well as a small negative asymmetric volatility specification for an optimal score. Importantly, error-transformations or long memory et cetera are not necessary for an EMM-based appropriate model fit. The rest of this article is therefore organised as follows. Section 2 studies the auxiliary model specification. Section 3 conducts EMM stochastic volatility specifications and evaluates validity. Section 4 implements the MCMC estimated SV-model, Section 5 interprets the models result and Section 6 contain summarises and conclusions.

2 Quasi-Bayesian estimations

The class of estimators referred to as quasi-Bayesian estimators (QBEs), which are defined similarly to Bayesian estimators but use general statistical functions in place of the parametric likelihood function, are statistical motivated. QBEs are typically means or quartiles of a quasi-posterior distribution, hence can be estimated (computed) at the parametric rate $1/\sqrt{B}$, where B is the number of draws from the distribution (functional evaluation). Moreover, QBE estimation includes point estimates, confidence intervals, prior information, and simple imposition of constraints.

Classical average-like criterion functions $L_n(\theta)$ are highly non-convex, almost everywhere flat, and have numerous discontinuities and local optima, inducing that the QBE approach will yield a computable and theoretically attractive alternative to the extremum-based estimation and inference. QBE estimators, generally not a log-likelihood functions, the transformations $p_n(\theta) = \dfrac{e^{L_n(\theta)}\pi(\theta)}{\int_\Theta e^{L_n(\theta)}\pi(\theta)d\theta}$ are proper distribution densities over the parameters of interest (quasi-posterior). The mean is defined as $\hat{\theta} = \int_\Theta \theta p_n(\theta)d\theta = \int_\Theta \theta \left(\dfrac{e^{L_n(\theta)}\pi(\theta)}{\int_\Theta e^{L_n(\theta)}\pi(\theta)d\theta} \right) d\theta$, where Θ is the parameter space. Formally, let $p_n(u)$ be a penalty or loss function associated with making an incorrect decision. Three examples of $p_n(u)$ are (1) $p_n(u) = |\sqrt{n}u|^2$, the squared loss function, (2) $p_n(u) = \sqrt{n}\sum_{j=1}^{d}|u_j|$, the absolute deviation loss function, and (3)

$p_n(u) = \sqrt{n}\sum_{j=1}^{d}(\tau_j - 1(u_j \le 0))u_j$, $\quad for\ \tau_j \in (0,1)$ for each *j*, the check loss function of Koenker and Bassett (1978). The parameter is assumed to belong to a subset Θ of the Euclidean space. Using the above quasi-posterior $p_n(\theta) = \dfrac{e^{L_n(\theta)}\pi(\theta)}{\int_\Theta e^{L_n(\theta)}\pi(\theta)d\theta}$ density from above, define the quasi-posterior risk function as: $Q_n(\xi) = \int_\Theta p_n(\theta-\xi)p_n(\theta)d\theta = \int_\Theta p_n(\theta-\xi)\left(\dfrac{e^{L_n(\theta)}\pi(\theta)}{\int_\Theta e^{L_n(\theta)}\pi(\theta)d\theta}\right)d\theta$.

The class of QBEs that minimizes the functions $Q_n(\xi)$ for various choices of p_n are defined as: $\hat{\theta} = \underset{\xi \in \Theta}{\arg\inf}\left[Q_n(\xi)\right]$. The estimator can be interpreted as a decision rule for the least unfavourable given the statistical information provided by the probability measure p_n, using the loss function ρ_n. In particular, the loss function ρ_n may asymmetrically penalize deviations from the truth, and π may give differential weights to different values of θ. The solution to the above *arg inf* function for loss functions (1)-(3) includes, respectively[1] quasi posterior means, medians, and marginal τ_j th quartiles

3. Data set and Score Generator

The EMM methodology is used to estimate stochastic volatility models for two financial series from the German energy market (base and peak load contracts). Both models are estimated from two daily percentage change data sets consisting of 1156 observations from 2002 to the start of 2007. The time series consists of all available daily observations from this financial market. Characteristics of the two data sets are reported in Table 1. For both commodity market series the *KPSS* statistic cannot reject stationary series. One of the features which stand out most

[1] The formulation implies that conditional on data, the decision $\hat{\theta}$ satisfies Savage's axioms of choice under uncertainty with subjective probabilities given by p_n.

Table 1. Characteristics for the raw OBX Index series and the Forward Commodity Market

Panel A: Base Load One-Year Forward Contract

	Mean / Median	Std.dev.	Maximum / Minimum	Moment Kurt/Skew	Quantile* Kurt/Skew	Quantile* Normal	K-S Z-test	Serial dep. Q(12)
Phelix Base	0.06624	1.00463	9.8555	18.5360	0.3385	15.8646	3.77630	45.8547
	0.07065		-7.0490	0.39324	0.0285	{0.0004}	{0.0000}	{0.0000}
	BDS-statistic (ε = 1)			KPSS (Stationary)		ARCH		Serial dep.
	m=2	m=3	m=4	Level (12)	Trend (12)	(12)		$Q^2(12)$
Phelix Base	5.5580	6.4706	6.8379	0.12280	0.10910	32.6356		200.13
	{0.0000}	{0.0000}	{0.0000}	{0.4854}	{0.1305}	{0.0000}		{0.0000}

Panel B: Peak Load One-Year Forward Contract

	Mean / Median	Std.dev.	Maximum / Minimum	Moment Kurt/Skew	Quantile Kurt/Skew	Quantile Normal	K-S Z-test	Serial dep. Q(12)
Phelix Peak	0.06720	0.95790	9.0275	14.0756	0.2627	10.5622	2.91950	25.4296
	0.08083		-6.4281	0.29721	-0.0485	{0.0051}	{0.0000}	{0.0129}
	BDS-statistic (ε=1)			KPSS (Stationary)		ARCH		Serial dep.
	m=2	m=3	m=4	Level (12)	Trend (12)	(12)		$Q^2(12)$
Phelix Peak	5.2893	5.7941	5.2632	0.13850	0.14160	38.4785		93.00
	{0.0000}	{0.0000}	{0.0000}	{0.4281}	{0.0602}	{0.0000}		{0.0000}

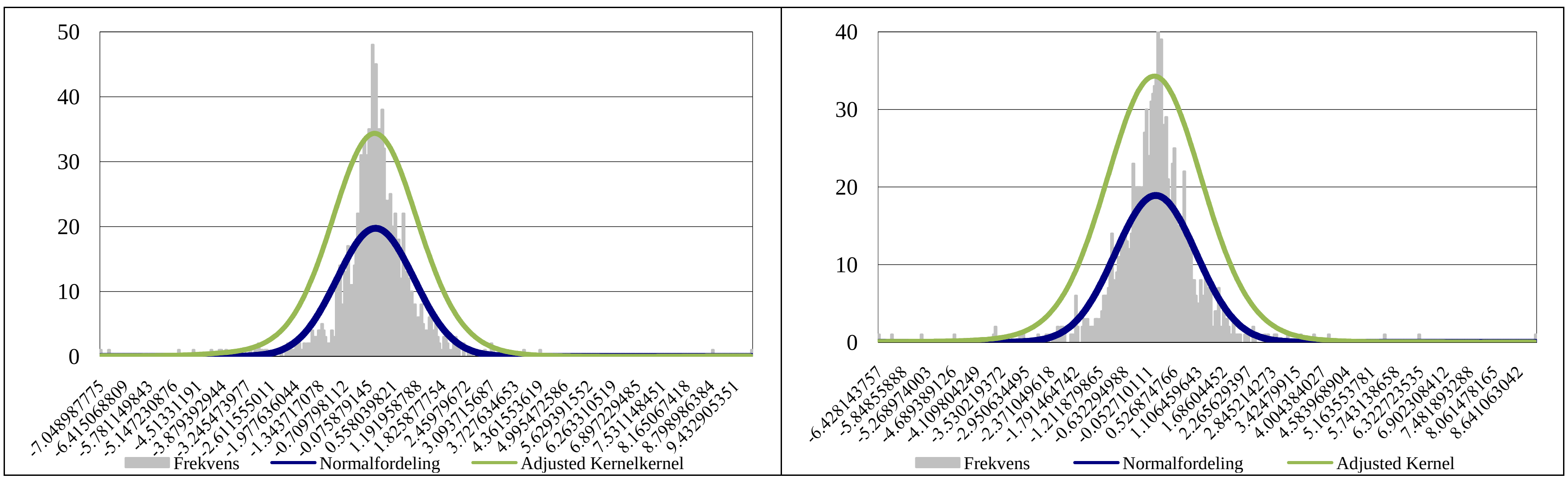


Figure 1. Frequency distribution for the Base Load (left) and Peak Load (right) Future Commodity contracts

prominently is the kurtosis of the two series which are larger than normally distributed values. The features of the series reflect the fact that the tails of the distribution are thicker than the tails of the normal distribution. Differently stated, large observations occur more often than one might expect for a normally distributed variable. The first important observation is the two data sets distributional similarities. The mean is almost identical with the standard deviation greater for the base load contract most likely due to prices during low demand periods (from 8.00 evening to 08:00 morning). The maximum and minimum will therefore naturally show a higher distance for the base contract. The two series skewness measures are positive suggesting some form of capacity constraint features in the market (higher volatility from high positive price changes). The quartile measure of skewness is generally, relative to the moment measure, smaller suggesting influence from outliers in the classical moment measure. The Q, Q^2 and *ARCH* statistics (12 lags)[2] induce serial correlation in both mean and volatility (clustering) for both commodity series. Figure 1 plots the frequency distributions of the two raw data series together with a normal distribution and a kernel density. The kernel density estimator, $\hat{f}(y) = \frac{1}{nh}\sum_{t-1}^{n} K(\frac{y_t - y}{h})$, where $K(z)$ is a function that satisfies $\int K(z)\, dz = 1$ and h is the so-called bandwidth. $K(z)$ is the Gaussian kernel. By inspection of the plots, one see that the distributions are more peaked and have fatter tails than the corresponding normal distribution. Hence, both very small and very large observations occur more often compared to a normally distributed variable with the same first and second moments. Importantly, these features found in the two commodity series suggest nonlinear models, simply because linear models would not be able to generate these data.

The first step to implement the EMM estimator is the SNP score generator model (moments). The SNP model, described in Gallant and Tauchen (1989), appropriately used provide a

[2] See McLLeod and Li (1983) for Q and Q^2 and Engle (1982) for the *ARCH* statistics

Table 2. Score generator Model selection values

Panel A: Base Load One-Year Future Contract

Case	L_μ	L_g	L_r	L_p	K_z	I_z	K_x	I_x	p_q	s_n	BIC	HQ	AIC	
1	1	0	0	1	0	0	0	0	3	1.4033	1.4094	1.4067	1.4050	
2	2	0	0	1	0	0	0	0	4	1.3984	1.4076	1.4035	1.4010	
3	3	0	0	1	0	0	0	0	5	1.3958	1.4080	1.4026	1.3993	
4	2	0	1	1	0	0	0	0	5	1.2494	1.2616	1.2562	1.2529	
5	2	1	1	1	0	0	0	0	8	1.1138	1.1351	1.1256	1.1198	
6	2	1	1	1	4	0	0	0	12	1.0588	1.0923	1.0774	1.0683	
7	2	1	1	1	6	0	0	0	14	1.0517	1.0914	1.0737	1.0630	**
8	2	1	1	1	8	0	0	0	16	1.0477	1.0934	1.0723	1.0601	
9	2	1	1	1	6	0	1	0	19	1.0472	1.1021	1.0776	1.0628	

Panel B: Peak Load One-Year Future Contract

Case	L_μ	L_g	L_r	L_p	K_z	I_z	K_x	I_x	p_q	s_n	BIC	HQ	AIC	
1	1	0	0	1	0	0	0	0	3	1.4026	1.4087	1.4060	1.4043	
2	2	0	0	1	0	0	0	0	4	1.3987	1.4078	1.4037	1.4012	
3	3	0	0	1	0	0	0	0	5	1.3975	1.4097	1.4042	1.4009	
4	2	0	1	1	0	0	0	0	5	1.3031	1.3153	1.3098	1.3065	
5	2	1	1	1	0	0	0	0	7	1.2216	1.2429	1.2334	1.2276	
6	2	1	1	1	4	0	0	0	11	1.1487	1.1823	1.1673	1.1583	
7	2	1	1	1	6	0	0	0	13	1.1324	1.1721	1.1544	1.1437	
8	2	1	1	1	8	0	0	0	15	1.1252	1.1679	1.1489	1.1373	**
9	2	1	1	1	10	0	0	0	17	1.1252	1.1710	1.1506	1.1382	
10	2	1	1	1	8	0	1	0	20	1.1236	1.1846	1.1574	1.1409	

Table 3. Characteristics of Semiparametric Score Residuals

Panel A: Base Load One-Year Future Contract

Panel A.		Standard	Max.	Moment	Quantile	Quantile	Serial dependence	
	Mean	deviation	Min.	Kurt/Skew	Kurt/Skew	Normal	Q(12)	$Q^2(12)$
Residual	0.0811234	1.2067539	7.444263	11.306346	0.1510	1.1104	11.312	6.1988
			-11.85469	-0.755695	-0.0120	{0.5740}	{0.5024}	{0.9057}
Panel B.	BDS-statistic (ε=1)				ARCH	K-S	RESET	Joint
	m=2	m=3	m=4	m=5	(12)	Z-test	(12;6)	Bias
Residual	0.57764	1.052113	0.60607	0.34263	0.9525	1.6471754	16.819893	2.545375
	{0.3376}	{0.2294}	{0.3320}	{0.3762}	{0.3291}	{0.0088}	{0.1565}	{0.4671}

Panel B: Peak Load One-Year Future Contract

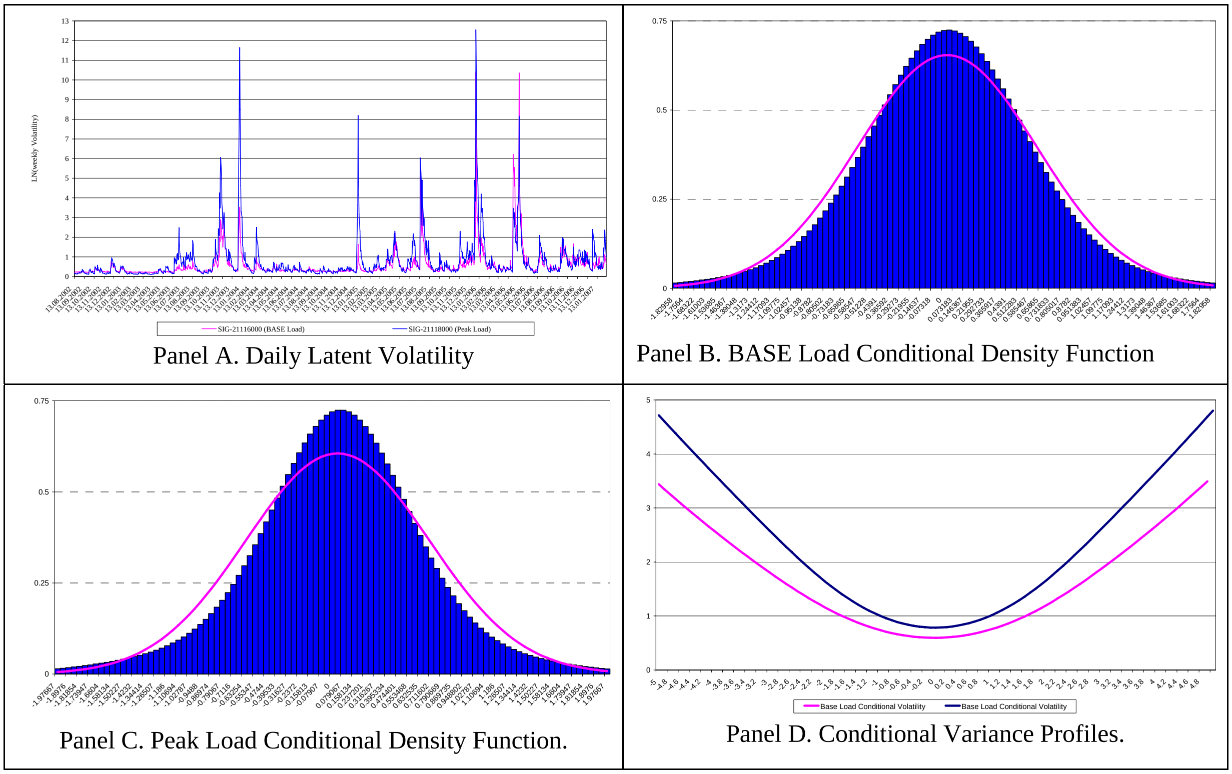
Panel A.		Standard	Max.	Moment	Quantile	Quantile	Serial dependence	
	Mean	deviation	Min.	Kurt/Skew	Kurt/Skew	Normal	Q(12)	$Q^2(12)$
Residual	0.0486345	1.1619544	10.885409	14.782357	0.1384	1.2994	14.1147	2.9319
			-10.2333	0.3013201	-0.0453	{0.5222}	{0.2934}	{0.9960}
Panel B.	BDS-statistic (ε=1)				ARCH	K-S	RESET	Joint
	m=2	m=3	m=4	m=5	(12)	Z-test	(12;6)	Bias
Residual	0.505008	-0.104009	-0.380573	-0.048984	1.0436	1.6471754	14.818666	0.45413
	{0.3512}	{0.3968}	{0.3711}	{0.3985}	{0.3070}	{0.0088}	{0.2515}	{0.9288}

reasonably good statistical description of the data set. Starting from a VAR model, the SNP methodology if necessary, elaborates the description of the data set from VAR, to Normal (G)ARCH, to Semiparametric GARCH, and to Nonlinear Nonparametric. Applying the BIC (Schwarz, 1978) values for model selection, the preferred model for the Base load data set is: L_μ=2, L_g=1, L_r=1, L_p=1, K_z=6, K_x=0. Hence, the model is an AR(2) model for $\{y_t\}$ with a GARCH(1,1) conditional scale function and time homogenous nonparametric innovation density with thick tails accommodated via K_z=6. The Peak load data set is: L_μ=2, L_g=1, L_r=1, L_p=1, K_z=8, K_x=0. Note that the dependence on the past is through the linear location function and GARCH scale function. The SNP models define the GSM criterion function. The model elaborations are reported in Table 2. The identification process for both series therefore suggests a linear mean using ARMA(p,q) modelling and a (G)ARCH(m,n) regime switching specification for the latent volatility. Moreover, the SNP methodology describes the GARCH process using a BEKK (Engle & Kroner, 1995) formulation, which includes parameters for asymmetry and level effects in the conditional volatility. Hence, for the base both asymmetric volatility and level effects are present, while the peak load show no level effects but asymmetric volatility is significant. The SNP specifications are adjusted for maximum likelihood as well as parameter correlations. For a formal evaluation of the final models, the residuals are exposed to enhanced specification tests. Specification tests are shown in Table 3 for the two optimal semi-parametric GARCH models. The test statistics suggest no data dependence for any test statistics. Hence, misspecifications are minimised and both models may be used for descriptive and forecasting purposes in the commodity market.

4 Characterizing the Phelix forward Electric-Power Market

4.1 Characteristic details

Some characteristics of the two time series from the SNP configuration are reported in Figure 2. The daily conditional volatility is plotted in panel A. The plot suggest an constantly increasing volatility since the start in 2002. The one-step-ahead density $f_K(\tilde{y}_t | x_{t-1}.\hat{\theta})$, conditional on the



Panel A. Daily Latent Volatility

Panel B. BASE Load Conditional Density Function

Panel C. Peak Load Conditional Density Function.

Panel D. Conditional Variance Profiles.

Figure 2. Characteristics for the Semi-parametric GARCH Score model

values for $x_{t-1} = (\tilde{y}'_{t-L}, \cdots, \tilde{y}'_{t-2}, \tilde{y}'_{t-1})'$, is plotted in panel B and C, respectively. All lags are set at the unconditional mean of the data. These results suggest small non-normal features in the one-step-ahead density. The conditional variance function is plotted in panel D where we show the average over all $x_{t-1} = (y_{t-L}, \cdots, y_{t-2}, y_{t-1})$ in the data of the conditional variance $VAR(y_t | y_{t-L}, \cdots, y_{t-1} + \delta)$ plotted against δ, the percentage growth. The plots suggest small asymmetric volatility from the shocks (δ). The information in these plots for both the mean and the volatility is useful for the implementation of the stochastic volatility model below.

4.2 EMM and MCMC Estimated Stochastic Volatility models

Let y_t denote the first difference (logarithmic) over a short time interval of the price of a financial asset traded on an active speculative market. The stochastic volatility model in the form used by

Gallant, Hsieh, and Tauchen (1997) with a slight modification to produce a leverage effect (correlation between return innovations and volatility innovations) is

$$\begin{aligned} y_t &= a_0 + a_1\left(y_{t-1} - a_0\right) + \exp(\nu_t)\cdot u_{1t} \\ \nu_t &= b_0 + b_1\left(\nu_{t-1} - b_0\right) + u_{2t} \\ u_{1t} &= z_{1t} \\ u_{2t} &= s\left(r\cdot z_{1t} + \sqrt{1-r^2}\cdot z_{2t}\right) \end{aligned}$$

where $z_{1t}; z_{2t}$ are iid Gaussian random variables. The parameter vector is $\rho = (a_0, a_1, b_0, b_1, s, r)$.

Early references are Clark (1973) and Tauchen and Pitts (1983). More recent references are Gallant and McCulloch (2006), Andersen (1994), and Durham (2003). See Shephard (2004) for more background and references.

The score generator's (SNP) expectation under of the structural model is used as the vector of moment conditions. The SNP score is the derivative of the log density with respect to the SNP parameters. The SNP parameters are replaced by the quasi-maximum likelihood estimates, which are computed by maximizing the SNP pseudo-likelihood. Assuming a data generation process and computation of the expectation of a nonlinear function given values of the structural parameters, the structural model provides a means to generate simulated realization for given values of the structural parameters: $\rho \mapsto \left\{\hat{y}_\tau(\rho), \hat{x}_{\tau-1}(\rho)\right\}_{\tau=1}^{N}$, where ρ is the vector structural parameters to be estimated, $\hat{y}_\tau$ are endogenous variables, and $\hat{x}_\tau$ are lagged endogenous variables (dependent on ρ). Let $\left\{\tilde{y}_t, \tilde{x}_{t-1}\right\}_{t=1}^{n}$ denote the observed data set, where $\tilde{x}_{t-1} = \left(\tilde{y}_{t-1}, ..., \tilde{y}_{t-L}\right)$, $L \geq 1$. The first step is quasi-maximum likelihood estimation of the SNP

Table 4. Score diagnostics for rho ρ -parameters

Panel A: Base Load Future Contract

Score diagnostics:

Index	normalized mean score	standard error	t-statistic	Descriptor	
1	-1.71225	1.91954	-0.892010	$a_0[1]$	1
2	1.42405	1.79202	0.794660	$a_0[2]$	2
3	1.83808	1.70793	1.076200	$a_0[3]$	3
4	0.78931	1.62422	0.485960	$a_0[4]$	4
5	-0.2932	1.46695	-0.199870	$a_0[5]$	5
6	0.8736	1.70494	0.512390	$a_0[6]$	6
7	0	0	0.000000	A(1,1)	0 0
8	0.08875	0.93529	0.094890	B(1,1)	
9	1.42646	1.02402	1.393010	B(2,1)	
10	-0.1652	2.4469	-0.067510	$R_0[1]$	
11	-0.18906	1.37795	-0.137200	P(1,1)	s
12	-5.0884	9.30471	-0.546860	Q(1,1)	s
13	-1.02402	1.0263	-0.997780	V(1,1)	s
14	-0.08701	0.29575	-0.294220	W(1,1)	s

Panel B: Peak Load Future Contract

Score diagnostics:

Index	normalized mean score	standard error	t-statistic	Descriptor	
1	-1.21781	1.86141	-0.654240	a0[1]	1
2	-0.41935	1.67197	-0.250810	a0[2]	2
3	1.61802	1.54826	1.045060	a0[3]	3
4	-0.1638	1.35437	-0.120940	a0[4]	4
5	-0.7301	1.18829	-0.614410	a0[5]	5
6	-1.23612	1.35063	-0.915220	a0[6]	6
7	-0.47136	1.53112	-0.307850	a0[7]	7
8	-1.45956	1.57213	-0.928400	a0[8]	8
9	0	0	0.000000	A(1,1)	0 0
10	0.11652	0.99924	0.116610	B(1,1)	
11	1.95082	1.0074	1.936490	B(1,2)	
12	-0.80866	1.79847	-0.449640	R0[1]	
13	-1.78948	1.32152	-1.354110	P(1,1)	s
14	-9.63493	7.92441	-1.215850	Q(1,1)	s
15	-1.01182	0.6348	-1.59393	V(1,1)	s

Table 5. MCMC estimation EMM primary results

Panel A: Base Load Future Contract

			Standard errors:			Hessian
Rho(ρ)	Mean	Mode	Sandwich	Hessian	Information	t-ratios
a0	0.071699	0.061523	0.002603	0.007371	0.000843	8.346688
a1	0.113480	0.118164	0.004599	0.016402	0.002504	7.204467
b0	-0.669818	-0.663086	0.002946	0.020113	0.005617	-32.968847
b1	0.874645	0.907227	0.002653	0.007969	0.000995	113.844468
σ	0.247081	0.211914	0.001046	0.006344	0.002544	33.403856
ρ	-0.053480	-0.051758	0.002021	0.021357	0.002705	-2.423459

The log posterior value at the mode:	8.181400	ltheta (SNP)	12
		lrho	6
		Degrees of freedom of model	5
		χ^2	0.146518
		The degrees of freedom for seinfo are:	2377

Panel B: Peak Load Future Contract

			Standard errors:			Hessian
Rho(ρ)	Mean	Mode	Sandwich	Hessian	Information	t-ratios
a0	0.072387	0.067871	0.019510	0.009785	0.026543	6.936026
a1	0.195553	0.181152	0.015927	0.013965	0.025452	12.971883
b0	-0.727810	-0.702637	0.099236	0.027993	0.035249	-25.100444
b1	0.709858	0.832031	0.045146	0.007768	0.002015	107.107341
σ	0.234794	0.238281	0.029900	0.006271	0.002540	38.000359
ρ	-0.137902	-0.113281	0.154540	0.024251	0.006826	-4.671199

The log posterior value at the mode:	-10.751000	ltheta (SNP)	15
		lrho	6
		Degrees of freedom of model	8
		χ^2	0.216211
		The degrees of freedom for seinfo are:	2254

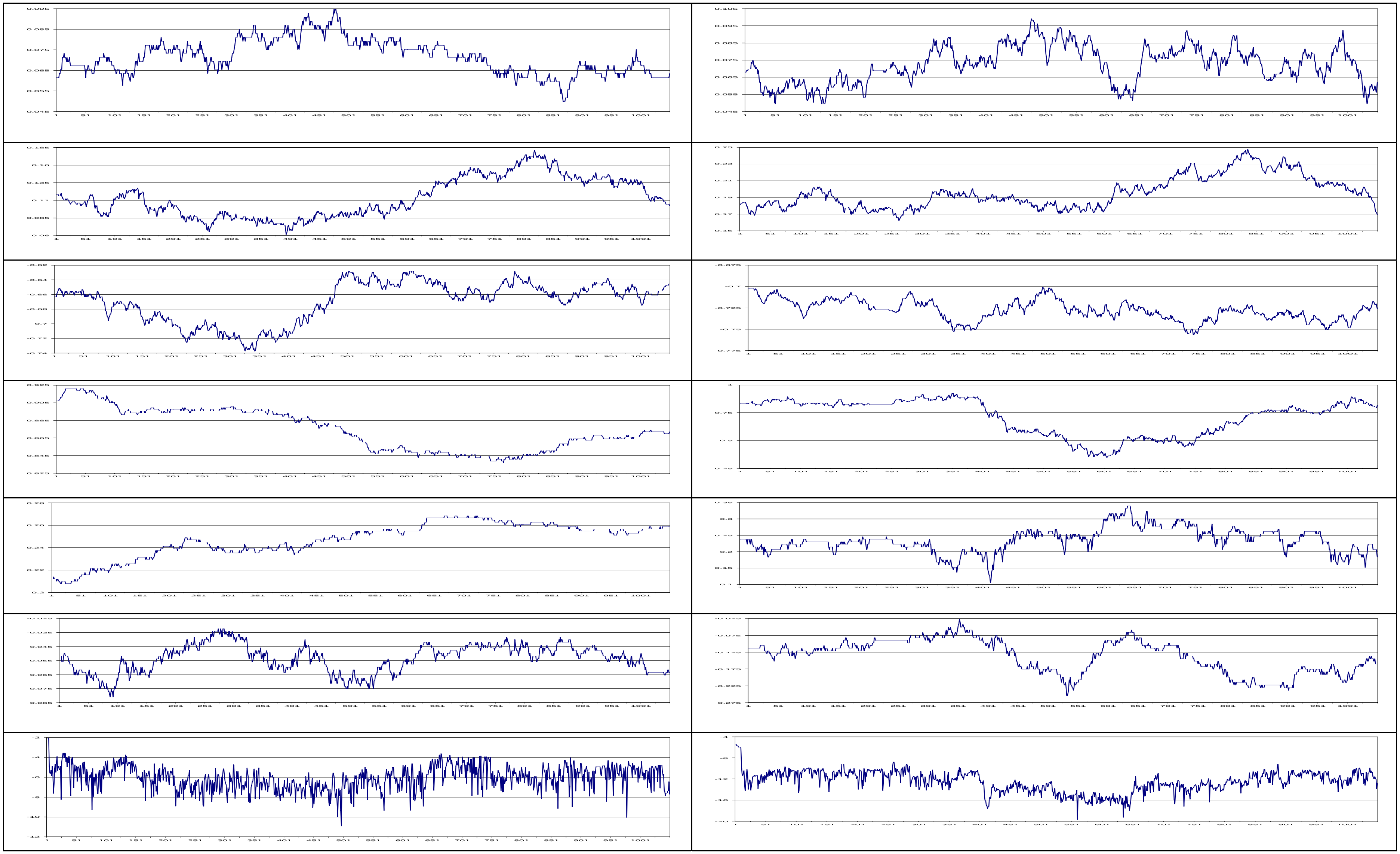


Figure 3. MCMC Chain from the Optimal Parameter Files. The panels are from top to bottom a_0, a_1, b_0, b_1, σ, ρ, and π. Base Load to the left and Peak Load to the right.

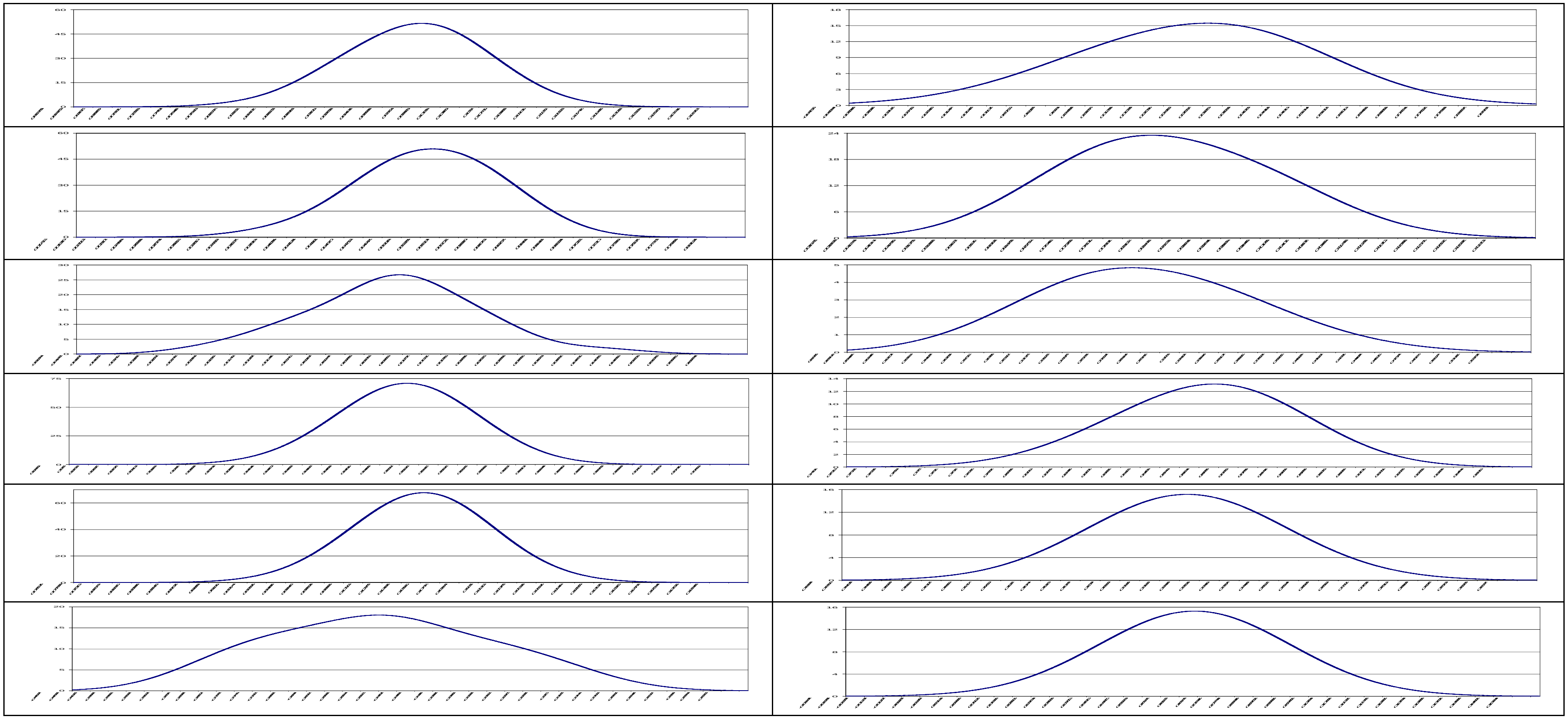


Figure 4. Kernel Density Estimates of Chain from the Optimal Parameter File (Base and Peak One-Year Forward)

score generator: $\tilde{\theta}_n = \underset{\theta \in \Theta}{\arg\max} \frac{1}{n}\sum_{t=1}^{n} \ln f_t\left(\tilde{y}_t \mid \tilde{x}_{t-1}, \theta\right)$. For the second step, the moment criterion is

$m_n(\rho, \tilde{\theta}_n) = \frac{1}{N}\sum_{\tau=1}^{N} \frac{\partial}{\partial\theta} \ln f\left[\hat{y}_\tau(\rho) \mid \hat{x}_{\tau-1}, \tilde{\theta}_n\right]$, and the GMM estimator of the structural parameter

vector is $\hat{\rho}_n) = \underset{\rho \in \Re}{\arg\min}\, m_n'(\rho, \tilde{\theta}_n)\left(\tilde{I}_n\right)^{-1} m_n(\rho, \tilde{\theta}_n)$, where $\left(\tilde{I}_n\right)^{-1}$ is the weighting matrix. If the

SNP score generator is a good statistical approximation to the data generating process, then the

estimator is $\tilde{I}_n = \frac{1}{n}\sum_{t=1}^{n}\left[\frac{\partial}{\partial\theta}\ln f_t(\tilde{y}_t \mid \tilde{x}_{t-1}, \tilde{\theta}_n\right]\left[\frac{\partial}{\partial\theta}\ln f_t(\tilde{y}_t \mid \tilde{x}_{t-1}, \tilde{\theta}_n\right]'$. The standard errors computed

by adaptation of the Chernozhukov and Hong (op.cit.) are asymptotically correct. However, in finite samples certain tuning parameters need to be chosen carefully to get reliable results. The normalized mean SNP score vector under the model, which is

$\sqrt{n} m_n'(\tilde{\rho}_n, \tilde{\theta}_n) = \frac{1}{N}\sum_{\tau=1}^{N}\sqrt{n}\frac{\partial}{\partial\theta}\ln f\left[\hat{y}_\tau(\tilde{\rho}_n) \mid \hat{x}_{\tau-1}(\tilde{\rho}_n), \tilde{\theta}_n\right]$, along with the unadjusted standard errors

of the normalized scores, and the corresponding quasi-*t*-statistics, are reported in Table 4. The quasi-t-ratios are not actually asymptotically $N(0,1)$ because they take only into account the randomness in $\tilde{\theta}_n$, while treating $\hat{\rho}_n$ as if it were a fixed value ρ_0. As shown by Newey (1985) and Tauchen (1985), the unadjusted standard errors are biased upwards so the quasi-*t*-ratios are downward biased relative to 2.0. The *t*-ratios are used to assess model fits along all dimensions. Table 4 reports that all the scores have values less than 2, suggesting a well model fits. For informative diagnostics Figure 3 plots the MCMC chain for all ρ's and π's. The chains look like they are stable. The rejection rate is about 15-20% inducing an appropriate scaling. Other indicators of reliability are also available. Kernel density plots of the marginal of the MCMC chain for ρ are shown in Figure 4. The figures show no obvious departure from normality, inducing reliability. The kernel plots of stats, which are the statistics $\sigma(\rho)$ computed from the simulations $\rho \mapsto \left\{\hat{y}_\tau(\rho)\right\}_{\tau=1}^{N} \mapsto s\left(\rho\right)$ for each ρ in the MCMC chain, are available for the mean and the volatility. The density plots show no obvious deviation from normality[3]. For both series

[3] All tables, plots etc. are available from author upon request.

the chain reports low and close to zero serial correlation for all ρs and the scatterplots report correlation matrices indicating no linear tightly packed relationship. In summary, the chains look well tuned for both series.

5 Empirical Findings

From the Score generator and the BIC-optimal Semi-Parametric GARCH specification, the models show no obvious mean regime shifts. The Scores show serial correlation in mean and volatility (clustering), asymmetry and level influence on the volatility. Several lags of hermite polynoms suggest deviation from the normal distributional return features. Specially constructed conditional volatility plots do not suggest significant asymmetric volatility for neither the base nor the peak load series. Due to the BEKK formulation of conditional volatility, volatility persistence can be measured using the SNP relationship $\varphi = \eta^2_{ARCH} + \eta^2_{GARCH}$. The base and peak contracts report a persistence of 0.975 and 0.893, respectively[4]. Half the shocks are therefore dissipated after approximately 20 and 6 trading days, respectively. The base contract seems to have higher volatility persistence than the peak load contract. This difference in volatility features between these two contracts may be attributed to the international energy markets physical interconnectors. The feature may therefore stem from the fact that the peak load is much more difficult to solve at day-time using the import option due to transfer constraints between European energy markets as well as supply/demand restrictions at day-time for potential exporters' in there respective home markets.

The reported model residuals in Table 3 report no systematic features. The moment scores should therefore be relevant and be well described for the EMM methodology's SV model building process. The primary results from running EMM is the ρ-parameter mode, which is the suggested estimate for EMM, the mean, and three sets of standard errors: sandwich $V = \mathfrak{I}^{-1}\, I\, \mathfrak{I}^{-1}$, information matrix I^{-1}, and Hessian $\mathfrak{I}^{-1}$. The sandwich estimator involves numerical

[4] The SNP-parameters are adjusted by the GSM modelling procedures to resolve SNP parameter correlations.

differentiation and several nonlinear optimizations inducing accuracy concerns. When the SNP fit is a good approximation to the true data generating process, the standard error from the Hessian is preferred for *t*-ratios. The correct chi-square statistic is the negative of the normalized value of the optimized objective function, which is $nm_n'(\tilde{\rho}_n,\tilde{\theta}_n)\left(\tilde{I}_n\right)^{-1}m_n(\tilde{\rho}_n,\tilde{\theta}_n)$, and

$\tilde{I}_n=\frac{1}{n}\sum_{t=1}^{n}\left[\frac{\partial}{\partial\theta}\ln f_t(\tilde{y}_t\,|\,\tilde{x}_{t-1},\tilde{\theta}_n\right]\left[\frac{\partial}{\partial\theta}\ln f_t(\tilde{y}_t\,|\,\tilde{x}_{t-1},\tilde{\theta}_n\right]'$. Under correct specification of the structural model, the normalized value of the optimized EMM objective function is asymptotically χ^2 with degrees of freedom equal to the length of θ minus the length of ρ minus one to account for the SNP normalization rule ($A(1,1)=1$). The objective function value for the base and peak load contracts are -8,18 and -10.75 respectively, indicating good model fits to actual data series. The results are summarized in Table 5 for both contracts.

The model's parameters for the base and peak load contracts show both positive drift (a_0) and serial correlation (a_1) in the mean. The base contract shows a much higher drift than the peak contract; that is the energy prices between 08:00 pm and 08:00 am has increased more than day-time prices. The peak contract shows the highest daily serial correlation in the mean. The conditional volatility parameters report both a negative constant parameter (b_0); the peak contract is highest in absolute terms. That is, low demand periods (night) induce higher volatility. The volatility shows rather low persistence (b_1), with values around 0.84 and 0.9 for base and peak contracts, respectively. The result suggests that only 6 days are needed for half the shocks to day out for the base contract. The volatility parameter (σ) show highest values for the base contract. The base and peak load contracts report an instantaneous volatility of 0.233 and 0.212 respectively. Finally, the two asymmetric volatility factors are negative (ρ), suggesting higher volatility from positive shocks than negative. However, both parameters are small and insignificant as indicated from plots in Figure 2. The volatility is rather high relative to other financial markets. Hence, the higher commodity market volatility relative to equity/currency markets is confirmed. The prices of derivatives (the risk market) will therefore be higher in commodity markets, reflecting the positive price-formula volatility effects. For both series, derivative formulas should adjust for serial correlation in the mean, volatility clustering and level effects.

Monte Carlo Simulations may therefore lead us to a deeper insight of the nature of the price processes describable for stochastic volatility models. The results are close to the moment based (nonlinear optimizers) techniques adjusting for a more robust model specification. The QBE techniques help to keep the model's parameters in the region where predicted shares are positive. Therefore, to evaluate parameter stability, we compute confidence intervals by inverting the criterion difference test based on the asymptotic chi-square distribution of the optimized objective function. The criterion difference confidence intervals reflect asymmetries in the objective function and are to be preferred. They are also safer from a numerical analysis point of view because they only require that the mode be accurately determined by the MCMC chain, which requires neither careful tuning to try to get *I* accurately determined nor excessive length to get *J* accurately computed. The easiest way to find the boundary $q_i(\rho_i) - q_i(\hat{\rho}_i) = \chi^2_{1-\alpha}$ is to compute $q_i(\rho_i)$ for values near where the standard errors for the optimal solution ought to be and interpolate. Table 6 reports these results for all six model coefficients (ρs) for both series. For the base series, using quadratic interpolation, the criterion difference confidence interval for ρ_4 is (0.8998, 0.9253) whereas the interval from the standard errors is (0.8913, 0.9232).

Table 6. Confidence Intervals for Base and Peak One-Year Forward Energy Contracts

Using quadratic interpolation between Rho(ρ) points, the criterion difference confidence interval (95%) is:

Rho(ρ)	Confidence Intervals: Criterion Difference *		Hessian Interval	
ρ_1	0.05125	0.07526	0.04678	0.07627
ρ_2	0.08744	0.13254	0.08536	0.15097
ρ_3	-0.72134	-0.61231	-0.70331	-0.62286
ρ_4	0.89976	0.92531	0.89129	0.92316
ρ_5	0.20012	0.22312	0.19923	0.22460
ρ_6	-0.07452	0.00121	-0.09447	-0.00904

* The 95% critical point of a χ^2 on 1 degrees freedom is: 3.841

Using quadratic interpolation between Rho(ρ) points, the criterion difference confidence interval (95%) is:

Rho(ρ)	Confidence Intervals: Criterion Difference *		Hessian Interval	
ρ_1	0.05895	0.83755	0.04830	0.08744
ρ_2	0.16329	0.19523	0.15322	0.20908
ρ_3	-0.76343	-0.63452	-0.75862	-0.64665
ρ_4	0.82219	0.84538	0.81649	0.84757
ρ_5	0.22599	0.24790	0.22574	0.25082
ρ_6	-0.19353	-0.03459	-0.16178	-0.06478

* The 95% critical point of a χ^2 on 1 degrees freedom is: 3.841

6 Summary and Conclusions

This work has used the QBE estimators (using common statistical, non-likelihood criterion functions) for a stochastic volatility representation for the base and peak load one-year forward Phelix (German) energy market contracts. The results show that these estimators are useful alternatives to the usual extremum estimators. For both data series, model output as scores and kernel plots for parameters and statistics, suggests reliable confidence intervals for all parameters. The MCMC approach therefore extends model findings relative to nonlinear optimisers. For both marketed series, the MCMC QBE approach suggests reliable goodness of fit measures. Due to these stochastic volatility findings, the price of a derivative of a forward/future may not be possible to determine using closed form formulas. The reason is that there may not exist a self-financing strategy involving forwards/futures and risk-less bonds applying a tracking portfolio approach perfectly replication the derivatives pay-off (the arbitrage argument). In absence of hedging possibilities (perfect tracking) the only available methodology for perfect derivative pricing is simulation based methodologies that closely replicate market characteristics and dynamic equilibrium models[5]. Consequently, the stochastic volatility results suggest that as for valuation of path dependent options, simulation may be considered as the best numerical method

[5] The results of Broadie and Glasserman (1996) give the direct path wise estimates for the hedge parameters within a single simulation run (greeks).

for derivative valuation[6]. Hence, any derivative can be priced using the preferred specifications and parameters together with simulation techniques in Mathematica® or any other programming tools[7].

The success of this version of the SV model, suggests positive serial correlation in the mean and that volatility tends to cluster. Thus, although price processes are hardly predictable, the variance of the forecast error is time dependent and can be estimated by means of observed past variations. The observed volatility clustering induce that the unconditional distribution of returns is at odds with the hypothesis of normally distributed price changes. The stochastic volatility models are therefore an area in empirical financial data modelling that is fruitful as practical descriptive and forecasting device together with applications such as Value-at-Risk, option pricing schemes and portfolio management. SV models allow us to explain empirically observed departures from Black-Scholes-Merton prices for options and understand why we should expect to see occasional dramatic moves in financial markets. This paper doesn't want to claim the practical adequacy of a (nonlinear) time series model refutes, for example, the market efficiency hypothesis. However, the empirical evidence must be taken seriously. Serial correlation per se cannot reject market efficiency. Modern econometrics should bring the application of economics closer to the empirical reality of the world, allowing us to make better decisions, inspire new theory and improve model building.

[6] Simulations of varying lengths are available from author upon request.
[7] The *t*-ratios, serial correlation plots and the Mathematica® implementation for derivative pricing are all available from author upon request.

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