Draft: May 2007

Hedonic Valuation of Health Risks Due to Residential Radon

Jakob Winstrand, Ph.D.-student
Institute for Housing and Urban Research, Uppsala University.
Box 785, 801 29 Gävle, Sweden.
E-mail: Jakob.Winstrand@ibf.uu.se
Tel: +46 26 4206514

Abstract

Radon is a radioactive gas which may be present in buildings and originates mainly from radioactive bedrock and radioactive building materials. Living in a radon contaminated house may increase the risk of suffering from lung cancer, especially among cigarette smokers. The potential risk arises due to long-term exposure to radon and the risk related to radon is thus limited to individuals living in contaminated buildings. Therefore, buyers of radon contaminated houses are likely to be willing to pay less than for non-contaminated houses, and house prices can be used to estimate the willingness-to-pay for reducing the risk due to radon. The purpose of this paper is to estimate the willingness-to-pay for reducing the risk due to radon contamination using a spatial hedonic house price model and data from the municipality of Stockholm for the period 1994 – 1996. The results imply that the percentage effect on house prices due to radon is -6.1 % for the preferred model and the corresponding willingness-to-pay for avoiding the health risks due to radon is SEK 69 196.

1 Introduction

Radon is a radioactive gas which arises when radium decays. The presence of radon in buildings originates mainly from radioactive bedrock and radioactive building materials. When radon is inhaled radioactive radon daughters (radon decay products) are caught in the lungs and increases the risk of lung cancer. The risk of developing lung cancer increases with time exposed to radon and for smokers the risks associated with radon increases considerably. (Darby *et al*, 2004)

The Swedish Radiation Protection Authority (SSI) estimates that approximately 500 cases of lung cancer per year are caused by radon in Sweden. Out of these 500 cases 90 % are smokers. (Strålskyddsnytt, 2001) In a collaborative analysis of 13 studies in 9 European countries (Darby *et al*, 2004) one of the findings is that "*In the absence of other causes of death, the absolute risks of lung cancer by age 75 years at usual radon concentrations of 0, 100, and 400 Bq/m*^{3} *would be about 0.4 %, 0.5 %, and 0.7%, respectively, for lifelong non-smokers, and about 25 times greater (10 %, 12 %, and 16 %) for cigarette smokers*." The extra risk induced by smoking is multiplicative.

Although radon is ubiquitous, outdoor concentrations are low and constitute no health risks. Indoor concentrations on the other hand can build up to potentially unhealthy levels. Since the risk arises due to long-term exposure, smoker or not, the potential health problems are restricted to individuals living in contaminated houses. Hence, house prices are likely to be affected by the presence of radon. However, the risks associated with radon mostly affect smokers and the assessment of houses by non-smokers may not be influenced by radon.

If the level of radon exceeds $200 Bq/m^3$ [1] Swedish house owners are entitled to a 50 % subsidy of the cost associated with reducing the level of radon (maximum SEK 15 000). Owners of radon contaminated houses ($\geq 200 Bq/m^3$) can also apply for a reduction of the tax assessment value. This, along with governmental information activities and measuring costs, constitutes a cost to the government. Thus, an estimate of the willingness-to-pay (WTP) for a reduction of radon in residential houses is of interest to environmental policy decision

[1] The threshold value of $400 Bq/m^3$ was reduced to $200 Bq/m^3$ in 2004

makers. The estimate of WTP for radon is interesting not only because radon constitutes a cost, it is also of interest for the allocation policies of funds between life preserving actions, and studies of how the risk is perceived in comparison with other health risks.

A frequently used method to estimate WTP for environmental goods, such as air quality, is the hedonic approach. The underlying assumption of the hedonic house price model is that a house can be seen as a bundle of attributes characterizing the house and these characteristics are assumed to determine the price of the house. The applied functional form of the hedonic price schedule differs but in many studies the price schedule is linear in parameters and estimated with OLS.

The WTP for a reduction of residential radon in Sweden has been estimated by Söderqvist (1995) to SEK 21 300. In his study, Söderqvist notes that in the estimation of the hedonic price schedule, variables correlated with the included ones may have been omitted. If omitted characteristics are correlated with the location then errors from OLS will be spatially autocorrelated, which violates the *iid*-assumption and leads to unbiased but inefficient parameter estimates and biased standard errors. Further, if the omitted characteristic is correlated with one or more of the included characteristics OLS will produce biased parameter estimates[2]. This implies that the parameter estimates of all included regressors in Söderqvists study are likely to be biased. Biased standard errors and parameter estimates is a serious problem when the estimates are used as guidance for policy decisions.

There is a vast literature where the problem with spatially dependent errors in hedonic house price models is addressed. Several studies show that OLS estimates of hedonic price schedules can be improved upon by applying models that explicitly account for the spatial structure inherent in house price data (e.g. Case *et al*, 2004, Dubin, 1992, Pace and Gilley, 1997, Wilhelmsson, 2002).

The purpose of this paper is to estimate the willingness-to-pay for reducing the risk due to radon contamination using a spatial hedonic house price model and data from the municipality of Stockholm for the period 1994 – 1996.

[2] The same problem may arise if a variable correlated with the location contains measurement errors.

After this introduction, section 2 presents the framework of the hedonic house price model. Section 3 contains a description of data and the estimation procedure. The results are presented in section 4, and section 5 concludes.

2 The hedonic house price model

The underlying assumption of the hedonic house price model is that a house can be seen as a bundle of attributes characterizing the house. The characteristics of a house can be divided into three subgroups; (i) structural characteristics that describe the building (age, living area, no. baths, etc), (ii) neighborhood characteristics that describe the neighborhood (proximity to work, schools, public transportation and other socioeconomic characteristics), and (iii) environmental characteristics that describe the surrounding environment (air quality, water quality and undesirable land uses, etc.).

The transaction price of a house is then a function of the implicit prices for all the attributes that characterizes the house. This implies that the price of the house for which some of the attributes are non-marketed goods, such as clean air, implicitly includes the prices of these non-marketed goods. Within the hedonic framework it is possible to estimate the implicit prices of the non-marketed goods using the observed house prices and the characteristics of the houses. The estimated implicit prices can then be used to estimate demand functions for the non-marketed goods.

2.1 Hedonic theory

We begin with a formal, yet brief description of the basic theory of hedonic markets based on Rosen (1974). Assume that consumers derive their utility from consumption of housing $\mathbf{z} = (z_1, \ldots, z_k)$, where z_i measures the amount of the *i*th characteristic, and consumption of a composite good x. Consumers have fix income y and their preferences are described by the utility function $U(\mathbf{z}, x)$. The hedonic price function $p(\mathbf{z})$ describes how the market price of a house depends on the characteristics of the house. Consumers are price takers and take $p(\mathbf{z})$ as given and face the budget constraint $y = p(\mathbf{z}) + x$, where the price of the composite good is normalized to unity. The consumer chooses a house with characteristics $\mathbf{z}$ and consumption of the composite good *x* such that utility is maximized subject to the budget constraint. First-order conditions of the utility maximization yields

$$\frac{\partial p(\mathbf{z})}{\partial z_i} = \frac{\partial U / \partial z_i}{\partial U / \partial x} \qquad \forall i, \tag{1}$$

where the derivative on the left-hand side is referred to as the hedonic price, or the marginal implicit price, of characteristic *i* and usually denoted p_i. The expression on the right-hand side is equal to the marginal rate of substitution of characteristic *i* and the composite good, which can be viewed as the marginal willingness to pay for the *i*th characteristic.
From the utility function we can derive a bid function, $\theta(\mathbf{z}, u, y)$, defined implicitly by $u = U(\mathbf{z}, y - \theta)$. At a given utility and income level, the bid function express the amount a consumer is willing to pay for various sets of characteristics. For a given level of utility, the partial derivative of the bid function w.r.t. characteristic *i*, i.e. $\theta_{zi} = \partial\theta / \partial z_i$, describes how a consumer is willing to change the expenditure on a house if the amount of characteristic *i* changes.

At a fixed income and utility level the consumer is willing to pay $\theta(\mathbf{z}, u, y)$ for **z**, but the consumer is a price taker and faces the market price $p(\mathbf{z})$. This implies that the consumer maximizes utility when $\theta(\mathbf{z}^*, u^*, y) = p(\mathbf{z}^*)$ and $\theta_{zi}(\mathbf{z}^*, u^*, y) = p_i(\mathbf{z}^*)$, $\forall i$, at optimum quantities $\mathbf{z}^*$ and utility level u^*.

The interaction of consumers and producers of houses determines the equilibrium hedonic price schedule. Consumers maximize their utility and prefer the lowest bid given the characteristics whereas producers prefer their highest offer to be accepted. However, most markets are dominated by the stock of existing houses and we can assume that the supply of houses is fixed in the short-run. In the short-run, characteristics of a house are generally costly to change and can therefore be regarded as predetermined, which implies that the equilibrium price schedule is completely determined by consumers demand. Therefore, only the hedonic price equation and the demand structure of the consumers are relevant. Then, under the assumption of optimizing behaviour, knowledge about the hedonic price schedule and the amount chosen of the characteristics reveals information about the consumer's willingness to pay for the characteristics in the neighborhood of the chosen quantities.

The empirical approach is usually a two-step procedure. First the equilibrium price schedule is estimated using observed house prices and characteristics. Using the estimated price function, implicit marginal prices of each characteristic are calculated for each observation. In the second step these estimated marginal prices, together with data on income and possibly other socioeconomic variables, are used to estimate the demand function (or inverse demand function). From the demand function it is possible to calculate benefits or WTP for a change in a characteristic. In this study, however, the characteristic of interest is dichotomous, either the house is contaminated or not. Therefore, the implicit marginal price is not defined and a different approach is used.

2.2 The hedonic price schedule

Hedonic theory does not prescribe a functional form for the price schedule, $\mathbf{p}(\mathbf{Z})$, other than that, as Rosen (1974, p.50) says, "the best fitting functional form" should be used. The lack of theoretical guidance to the functional form of the hedonic house price schedule has led some researchers to use the Box-Cox method suggested by Halvorsen and Pollakowski (1981). However, even though the Box-Cox approach allows the model to reflect the data more accurately it has a number of drawbacks (see Ramussen and Zuehlke, 1990) and the implementation in the presence of spatial dependence is beyond the current scope. Empirically, the most frequently used form for the house price schedule is semi-log where the natural logs of prices are regressed against the unlogged characteristics (Sirmans et al, 2005). In this paper all estimates are based on the semi-log specification[3]. The hedonic price schedule in semi-log form can be expressed as

$$\mathbf{p} = \mathbf{Z}\beta + \varepsilon \, , \tag{2}$$

where $\mathbf{p}$ is an $n \times 1$ vector with logarithms of the n observed house prices, $\mathbf{Z}$ is an $n \times k$ matrix with k characteristics for each house, β is a $k \times 1$ vector with the estimated parameters and ε is an $n \times 1$ vector with error terms. The estimated parameters β can be interpreted as

[3] This decision is supported by the Box-Cox test for the linear model. Other specifications was tested but performed poorly.

the percentage effect on the price following a small change in the characteristic[4]. If the error terms are *iid* then OLS yields unbiased, efficient and consistent estimates of the price schedule in eq. (2).

That house prices are influenced by the location is common knowledge. The location factors, or neighborhood characteristics, that influence a house price will also affect prices of nearby houses. If any of these characteristics are measured with error or if characteristics correlated with the neighborhood are omitted in the regression, the error terms will be spatially correlated and thus violating the *iid*-criteria. The structure of houses in a neighborhood is often homogeneous, i.e. living area, age, building materials, etc. are often quite similar. Social services such as schools, stores, public transportation, and recreational facilities are all common to the neighborhood. Distance to the central business district and environmental characteristics are approximately the same for all houses in a neighborhood. Socioeconomic characteristics such as income, education, religion, and crime rates tend to cluster. (Militino *et al*, 2004) Many of these characteristics are often used as explanatory variables, but far from all. In empirical studies it is usually not possible or it is too costly to acquire data on all characteristics and sometimes proxies are used for unobservable characteristics. If proxies are used for neighborhood characteristics, these proxies are also likely to contain measurement errors (Dubin, 1992). Therefore, measurement errors and possibly omitted characteristics are very likely to cause spatially dependent errors and biased and inconsistent OLS-estimates of the hedonic price schedule.

Spatial dependency is a potentially serious problem in studies of WTP for housing characteristics since both the magnitude of the estimates and their significance may be affected if the spatial structure is neglected (Kim *et al*, 2003). Numerous studies have shown that when errors are spatially dependent, spatial models can be applied to improve estimation of the hedonic price schedule (Case *et al*, 2004, Dubin, 1992, Pace and Gilley, 1997, and Wilhelmsson 2002). In real estate literature it is common to model spatially dependent errors by assuming that the spatial process is generated through a spatial weight matrix. This type of spatial models is referred to as *lattice models* and is recommended by Pace *et al* (1998) for real estate modelling. Within the group of lattice models the two most widely used are the conditional autoregressive model, CAR, (also known as a spatial-lag model) and the

[4] This is not exactly true for dummy variables. If b is the estimated parameter for a dummy variable then the relative effect is $\exp(b) - 1$ (Halvorsen and Palmquist, 1980).

simultaneous autoregressive model, SAR, (also known as a spatial error model). Both methods are thoroughly described by Cressie (1993).

2.2.1. Spatial house price models

The CAR model is a spatial analogue to an AR(1) in time series econometrics. The first order spatial autoregressive process can be described as

$$\mathbf{p} = \rho \mathbf{W}_1 \mathbf{p} + \mathbf{Z}\beta + \varepsilon , \tag{3}$$

where ρ is a scalar spatial parameter and $\mathbf{W}_1$ is an $n \times n$ spatial weight matrix. The covariance matrix is $\Sigma = \sigma^2 (\mathbf{I} - \rho \mathbf{W}_1)^{-1}$ which implies that $\mathbf{W}_1$ must be symmetric to guarantee that Σ is symmetric.

In the SAR model spatial dependency is modelled as an autoregressive spatial error process as

$$\begin{aligned} \mathbf{p} &= \mathbf{Z}\beta + \varepsilon \\ \varepsilon &= \lambda \mathbf{W}_2 \varepsilon + \mathbf{u} \end{aligned} , \tag{4}$$

or equivalently

$$\mathbf{p} = \mathbf{Z}\beta + (\mathbf{I} - \lambda \mathbf{W}_2)^{-1} \mathbf{u} \tag{4'}$$

where λ is a scalar spatial parameter, $\mathbf{W}_2$ is an $n \times n$ spatial weight matrix, $\mathbf{u} \sim N(\mathbf{0}, \sigma^2 \mathbf{I})$ and the covariance matrix is $\Sigma = \sigma^2 \left[(\mathbf{I} - \lambda \mathbf{W}_2)^T (\mathbf{I} - \lambda \mathbf{W}_2) \right]^{-1}$. In the SAR model, given λ, the maximum likelihood estimates can be seen as an OLS on the spatially filtered variables $(\mathbf{I} - \lambda \mathbf{W}_2)\mathbf{p}$ and $(\mathbf{I} - \lambda \mathbf{W}_2)\mathbf{Z}$.

The log-likelihood for both eq. (3) and (4) is

$$\ell = -\frac{\mathrm{n}}{2} \ln(2\pi) - \frac{1}{2} \ln(|\Sigma|) - \frac{1}{2} (\mathbf{p} - \mathbf{Z}\beta)^T \Sigma^{-1} (\mathbf{p} - \mathbf{Z}\beta), \tag{5}$$

where only the covariance matrix, Σ, differs.

The spatial weight matrices determines the degree of potential spatial correlation and cause the spatial model to depart from the standard linear model and limit the application of OLS. The weights are determined exogenously and weight/element w_{ij} governs the spatial relationship between house i and house j. Often the weight matrices are row-standardized such that all rows sum to unity, the weights can then be interpreted as a spatial filter. There are several ways to specify the matrices but for any specification the matrices must be non-negative and all elements on the diagonal must be zero, since a house price cannot be used to predict itself. Usually the construction of a weight matrix is based on some measure of the distance between observations such as the nearest neighbours (see Dubin (1998) for more details and examples).

Though these two models are closely related mathematically the economic interpretation differs. In the CAR model it is implicitly assumed that, in addition to the standard explanatory variables and neighborhood characteristics, house prices are influenced by a spatially weighted average of neighbouring house prices. Thus the spatial parameter in the CAR model can be interpreted as an *indirect effect* of the neighborhood and the other explanatory variables as *direct effects* on house prices.

In the SAR model it is assumed that omitted variables correlated with the neighborhood in the house price equation cause spatially autocorrelated errors. The specification of the SAR models aims at reducing the problem with spatial autocorrelation by using a spatially weighted average of the error terms as an explanatory variable. It is reasonable to assume that a neighborhood characteristic influencing house prices has approximately the same effect on the prices of adjacent houses. Thus, omission of a neighborhood characteristic creates a spatial pattern in the residuals and spatially adjacent errors are used as “proxies” for the omitted variables. This implies that the effect on house prices of a spatially weighted average of the error terms represents a *direct effect* induced by the omitted characteristic(s).

Appendix A.1 provides a thorough description of the tests for spatial dependency that are used.

2.3 The willingness to pay

As mentioned earlier, the WTP is usually estimated by a two-step procedure, where the first step is to estimate the hedonic price schedule and the second step is to estimate the demand function for the characteristic of interest. The estimation of the demand function in step two requires calculation of the marginal price of the characteristic of interest. Unfortunately, the available data does not include the level of radon, only whether it exceeds the threshold value of $400 Bq/m^3$ or not. Therefore radon is enters the price schedule as a dummy variable and the marginal price of radon (the partial derivative of the estimated price schedule with respect to radon) is not defined.

The approach taken here is simply to calculate the WTP for reducing the risk due to radon contamination as

$$WTP_{radon} = \left(\exp\left(\hat{\beta}_{radon}\right) - 1\right)\overline{P},$$

where $\hat{\beta}_{radon}$ is the estimated parameter, using the semi-log form, for radon in the hedonic price schedule and $\overline{P}$ is the average house price.

3 Data and the estimation procedure

3.1 Data

Radon is not considered as a concealed defect, it is the buyer's responsibility to examine if the house is contaminated, not the seller. In Sweden each municipality has a responsibility to keep track of contaminated houses and if a house is classified as a contaminated house it is public information. If it is unknown whether the house is contaminated or not, the buyer can insist on having the house examined before buying it. Since it is the buyer's responsibility, the buyer also bears the cost associated with the inspection. The cost is, however, relatively small, 200 - 500 SEK (approximately € 22 – 54)[5]. Hence, following Söderqvist (1995, pp. 144), it is assumed that buyers are well informed and know if a house is contaminated or not since the sellers have "little chance of concealing the contamination".

[5] The reported cost is the current cost, but it is reasonable to assume that the cost was approximately the same during 1994 – 1996.

If it is assumed that buyers know whether houses are contaminated or not, it must also be assumed that they know when houses are classified as contaminated and tax assessment values are reduced. This implies that buyers of contaminated houses know that they receive a reduction on future taxes and are therefore willing to pay more than they would if there were no tax reduction. The tax influence on market prices must be accounted for when estimating the WTP. If not accounted for, the effect of radon will also contain the tax effect. Without the tax reduction prices for contaminated houses should be lower. All estimates and descriptive statistics are based on adjusted prices where the assessment period is ten years, real interest rate is six per cent, and the tax rate is one per cent of the tax assessment value.

The data set was collected by Statistics Sweden and used for assessment by the Swedish tax authorities and contains information on all single-family houses sold in the municipality of Stockholm during 1994 - 1996. In Sweden it is possible for owners of radon contaminated houses to apply for a reduction of the tax assessment value if the amount of Bq/m^3 exceeds the Swedish standard, which during 1994 – 1996 was $400 Bq/m^3$. The data set contains information whether a house is classified as contaminated or not. Unfortunately, there is no information on the precise level of Bq/m^3, only that it exceeds $400 Bq/m^3$.

In hedonic studies there are usually several structural characteristics included in the price equation, e.g. living area, lot size, number of bathrooms, number of bedrooms, fireplace, air condition, etc.. The purpose of these variables is to account for the structural quality of the house. In the data available for this study there is a single quality variable that captures the quality of the buildings entire structure. This variable was used for tax assessment purposes and is an index that measures the structural quality of the house based on a mandatory survey where each household answered 25 questions about the structural quality of their house. Living area, lot size, age, neighborhood characteristics and environmental characteristics are, however, not included in the quality variable. But in addition to the radon dummy and the quality variable, the data set includes selling price, lot size, living area, age, coordinates and some environmental characteristics.

Descriptive statistics are presented in Table 1. After excluding houses not suited for all-year services 5062 observations remain. Out of these observations, 139 (approximately 2.75 %) are classified as radon contaminated. Correlation coefficients for the dependent variable and the explanatory variables are presented in Table 2. For all included variables the correlation with price is significant and all are positive except radon. The highest correlation between the explanatory variables is 0.434, for age and lot size.

Figure 1 in appendix A.2 show the spatial spread of contaminated houses and regular houses in the municipality of Stockholm. In Figure 2, also in appendix A.2, average price of houses sold each month are plotted. The monthly average ranges from slightly above 1 million SEK to approximately 1.25 million SEK. The top and bottom are within a few months at the end of 1995. The top is caused by sales of unusually expensive houses.

Table 1. Descriptive statistics

	Mean	Median	Max	Min	Std.
Price*	1129.374	950.000	5600.000	350.000	512.239
Lot size**	570.716	561.000	2956.000	54.000	305.404
Living area**	118.313	117.000	366.000	45.000	38.596
Sea front***	0.003	0.000	1.000	0.000	0.051
Sea view***	0.008	0.000	1.000	0.000	0.087
Radon***	0.027	0.000	1.000	0.000	0.163
Quality	27.124	27.000	57.000	13.000	5.051
Age	44.257	45.000	95.000	0.000	20.561
No. observations		5062	**No. contaminated houses**		139

Note:*Price is measured in thousands of SEK. **Lot size and Living area are measured in m^3. ***Dummy variable.

Table 2. Correlation coefficients.

	Price	Lot size	Living area	Sea front	Sea view	Radon	Quality	Age
Price	1							
Lot size	0.459	1						
Living area	0.671	0.270	1					
Sea front	0.155	0.039	0.085	1				
Sea view	0.160	-0.000*	0.095	-0.004*	1			
Radon	-0.050	-0.050	0.037	-0.009*	0.040	1		
Quality	0.385	0.198	0.366	0.027*	0.038	0.006*	1	
Age	0.293	0.434	-0.109	-0.016*	-0.051	-0.102	-0.036	1

* Not significant at 5 %.

3.2 The estimation procedure

All models estimated in this paper use, in addition to a constant, the following explanatory variables (**Z**):

Lot size – Measured in square meters

Living area – Measured in square meters

Sea front	– 1 if the house is adjacent to water, zero otherwise
Sea view	– 1 if the house has a sea view, zero otherwise
Radon	– 1 if the house is classified as radon contaminated, zero otherwise
Quality	– A measure of the structural quality
Age	– The age of the house measured in full years
Age^2	– The age variable squared
YD95	– Year dummy for 1995
YD96	– Year dummy for 1996

Sometimes very old houses are sold at a higher price than houses built more recently. Reasons for that may be that better building materials were used and/or attractive locations was easier to find. To capture such effects the house age squared (Age^2) is included as an explanatory variable.

Since there are several exogenous variables in **Z**, equal weight matrices could have been used for the spatial-lag structure and the spatial error structure. However, two different specifications are used. The weight matrix $\mathbf{W}_1$ is row-standardized and constructed by assigning equal weights to the 20 nearest neighbours (Euclidean distance). $\mathbf{W}_2$ is similar with the difference that only neighbouring houses sold at a previous date are assigned weights. Other specifications of the weight matrices were tested, as well as using two equal matrices, the results were relatively stable but the matrices used was preferred on the basis of the standard error of the regressions.

Due to the nonlinearity induced by the spatial structure the spatial-lag and spatial error regression models are estimated using maximum likelihood (ML) assuming normality for the error terms. The ML estimation is conducted using concentrated likelihood functions such that for the CAR and SAR models a univariate optimization procedure over the spatial parameter can be employed. Note that when these models are estimated the spatial parameters ρ and λ are restricted to the interval $\left(1/\gamma_{\min}, 1/\gamma_{\max}\right)$ where $\gamma_{\min}$ and $\gamma_{\max}$ are the smallest and the largest eigenvalues, respectively, of the weight matrix (Anselin & Florax, 1994). Both algorithms below are described in by Anselin & Bera (1998).

The CAR model is estimated using following procedure:

1. Estimate $\mathbf{p} = \mathbf{Z}\beta_0 + \varepsilon_0$ with OLS.
2. Estimate $\mathbf{W}_1\mathbf{p} = \mathbf{Z}\beta_L + \varepsilon_L$ with OLS.
3. Calculate $\varepsilon_0 = \mathbf{p} - \mathbf{Z}\hat{\beta}_0$ and $\varepsilon_L = \mathbf{W}_1\mathbf{p} - \mathbf{Z}\hat{\beta}_L$.
4. Find the MLE $\hat{\rho}$ of ρ that maximizes the concentrated log-likelihood
   $L_C = -(n/2)\ln(\pi) + \ln(|\mathbf{I} - \rho\mathbf{W}_1|) - (n/2)\ln[(1/n)(\varepsilon_0 - \rho\varepsilon_L)^T(\varepsilon_0 - \rho\varepsilon_L)]$
   given ε_0 and ε_L.
5. Use $\hat{\rho}$ to calculate $\hat{\beta} = \hat{\beta}_0 - \hat{\rho}\hat{\beta}_L$ and $\hat{\sigma}_\varepsilon^2 = (1/n)(\varepsilon_0 - \hat{\rho}\varepsilon_L)^T(\varepsilon_0 - \hat{\rho}\varepsilon_L)$.

The algorithm used to estimate the SAR model is:

1. Choose a start value λ_0, preferably such that $1/\gamma_{\min} < \lambda_0 < 1/\gamma_{\max}$.
2. Calculate $\varepsilon_0 = (\mathbf{I} - \lambda_0\mathbf{W}_2)\mathbf{p} - (\mathbf{I} - \lambda_0\mathbf{W}_2)\mathbf{Z}\beta_0$ where β_0 is the OLS estimate of β in $(\mathbf{I} - \lambda_0\mathbf{W}_2)\mathbf{p} = (\mathbf{I} - \lambda_0\mathbf{W}_2)\mathbf{Z}\beta + \varepsilon$.
3. Find the value $\hat{\lambda}$ of λ that maximizes the concentrated log-likelihood
   $L_C = -(n/2)\ln(\pi) + \ln(|\mathbf{I} - \lambda\mathbf{W}_2|) - (n/2)\ln(\varepsilon_0^T\varepsilon_0/n)$.
4. Repeat step 2 and 3 until $\hat{\lambda}$ converge.
5. Use $\hat{\lambda}$ to calculate
   $$\hat{\beta} = \left[\mathbf{Z}^T(\mathbf{I} - \hat{\lambda}\mathbf{W}_2)^T(\mathbf{I} - \hat{\lambda}\mathbf{W}_2)\mathbf{Z}\right]^{-1}\mathbf{Z}^T(\mathbf{I} - \hat{\lambda}\mathbf{W}_2)^T(\mathbf{I} - \hat{\lambda}\mathbf{W}_2)\mathbf{p}$$
   and $\hat{\sigma}_\varepsilon^2 = \hat{\varepsilon}^T\hat{\varepsilon}/n$ where $\hat{\varepsilon} = (\mathbf{I} - \hat{\lambda}\mathbf{W}_2)\mathbf{p} - (\mathbf{I} - \hat{\lambda}\mathbf{W}_2)\mathbf{Z}\hat{\beta}$.

All estimations were performed in MATLAB and the CAR and SAR was estimated using, slightly modified, code provided by J.P. LeSage at www.spatial-econometrics.com. Asymptotic t-statistics are calculated using a numerical hessian.

4 Estimation results

First eq.(2) was estimated with OLS, results are presented in Table 4 below[6]. Using the residuals from the regression Moran's I-test, Z_I, and the joint LM-test, $LM_{\lambda\rho}$, for $H_0 : \rho = 0,\ \lambda = 0$, was carried out and the test results are presented in Table 3. (All tests for spatial dependency are described in appendix A.1.) Both tests are significant at 5 %[7] indicating existence of spatial dependency. Thus, a spatial model is likely to improve the estimates.

In order to detect which type of spatial dependence the restricted LM-test statistics for $H_0 : \lambda = 0$ and $H_0 : \rho = 0$, LM_λ and LM_ρ respectively, was calculated. Both test results are presented in Table 3 and are significant, thus indicating existence of both spatial-lag and spatial error dependence. However, since both tests have power against the each others null, these results are not reliable. To achieve more reliable results the LM-tests based on the OLS residuals without restrictions on λ and ρ, LM_λ^* and LM_ρ^* respectively, was carried out. The results are presented in Table 3. Both tests are in line with previous tests and show significant result for both types of spatial dependency.

To further examine the type of dependency the two LM-tests LM_λ^E and LM_ρ^E which are based on ML estimates of the CAR and the SAR model, respectively, was carried out. The result from the ML estimation of respective model is presented in Table 4. The spatial parameter is significant in both models. However, the test results presented in Table 3 for these two LM-tests contradicts the previous results, only the test for spatial error dependence, LM_λ^E, is significant. Hence, the residuals from the SAR model show no spatial pattern whereas the residuals from the CAR model indicate a spatial pattern. The result supports the SAR specification. Thus, based on all the results presented the preferred model for the hedonic house price schedule is the SAR specification.

[6] As a simple experiment a model where longitude and latitude and the cross product are included is estimated with OLS. The polynomial, or surface, is included to capture neighborhood effects (see e.g. Case *et al* 2004). The estimation suffered from multicollinearity and, thus, the results are not reliable and will not be commented. Nevertheless, the result from the estimation of the model is presented (in Table 4). Using a higher order polynomial in longitude and latitude rendered the model impossible to estimate due to singularity of the $\mathbf{Z}^T\mathbf{Z}$ matrix.

[7] Henceforth results will only be referred to as significant or not significant and the level used is 5 %.

The results of tests for spatial dependence presented in Table 3 are unambiguous, spatial dependency is a problem. Thus, a spatial model is likely to improve the estimates. A closer look at the results in Table 4 reveals that the SAR model has a better fit, based on the standard error of the regression, than both the OLS and the CAR model. In all estimated models, the parameters have correct sign, besides the parameter for *Age* which is positive in the OLS and the CAR model.

The parameter for radon i significant in all models and ranges from -0.092 for the CAR model to -0.063 for the SAR model. These estimates imply that the percentage effect on house prices due to radon is -8.8 % in the CAR model and -6.1 % in the SAR model[8]. Calculation of WTP based on the estimate of the SAR model yields a WTP of SEK 69 196. Assuming an assessment period of ten years and a real interest rate of six per cent, households annual WTP for avoiding the health risks due to radon is SEK 8 869[9].

Table 3. Tests for spatial dependence.

Null	**Restriction**	**Test statistic**	**Estimated statistic**	**p-value**
$\lambda = 0$	$\rho = 0$	Z_I	78.423†	0.000
$\lambda = 0, \rho = 0$	$\mathbf{W}_1 = \mathbf{W}_2$	$LM_{\lambda\rho}$	5984.997†	0.000
$\lambda = 0$	$\rho = 0$	LM_{λ}	5881.896†	0.000
$\rho = 0$	$\lambda = 0$	LM_{ρ}	39.631††	0.000
$\lambda = 0$	ρ is unrestricted and not estimated	LM_{λ}^{*}	5860.385	0.000
$\rho = 0$	λ is unrestricted and not estimated	LM_{ρ}^{*}	18.120	0.000
$\lambda = 0$	ρ is unrestricted but estimated	LM_{λ}^{E}	5901.487	0.000
$\rho = 0$	λ is unrestricted but estimated	LM_{ρ}^{E}	0.005	0.946

Note: †Results for $\mathbf{W}_2$ is presented, the result using $\mathbf{W}_1$ was similar. †† Results for $\mathbf{W}_1$ is presented, the result using $\mathbf{W}_2$ was similar. The statistics are distributed as $Z_I \sim N(0, 1)$, $LM_{\lambda\rho} \sim \chi_2^2$ with a critical value of 5.99 at 5 % and all the other statistics are $\sim \chi_1^2$ with a critical value of 3.84 at 5 %.

[8] As noted in footnote 4, if *b* is the estimated parameter for a dummy variable then the relative effect is exp(*b*) – 1 (Halvorsen and Palmquist, 1980).
[9] For the estimates from the CAR model the calculation of WTP yields SEK 99 207 for the entire assessment period and SEK 12 716 annually.

Table 4. Estimation results.

	OLS 1	OLS 2	CAR	SAR
Constant	5.563 [222.802]	35563.295 [20.451]	5.493 [267.547]	6.188 [136.958]
Lot size	2.2e-4 [18.058]	2.2e-4 [18.380]	2.3e-4 [17.614]	2.8e-4 [25.207]
Living area	0.006 [60.117]	0.005 [59.591]	0.006 [61.922]	0.004 [45.649]
Sea front	0.462 [7.511]	0.421 [7.145]	0.460 [47.513]	0.262 [5.472]
Sea view	0.406 [11.380]	0.379 [11.073]	0.397 [11.192]	0.191 [6.929]
Radon	-0.090 [-4.702]	-0.098 [-5.326]	-0.092 [-4.833]	-0.063 [-4.316]
Quality	0.014 [20.387]	0.014 [21.249]	0.014 [20.546]	0.012 [23.636]
Age	0.004 [5.683]	7.6e-4* [1.018]	0.004 [25.887]	-0.008 [-70.930]
Age squared	1.2e-5* [1.551]	3.1e-5 [3.957]	1.3e-5 [10.480]	7.2e-5 [45.349]
YD95	4.6e-4* [0.059]	0.006* [0.834]	-0.002* [-0.297]	0.012 [2.014]
YD96	0.014* [1.816]	0.021 [2.944]	0.009* [1.191]	0.031 [5.295]
ρ			0.014 [5.777]	
λ				0.986 [53.521]
Latitude		-0.005 [-20.450]		
Longitude		-0.022 [-20.437]		
Latitude*Longitude		4.2e-10 [20.439]		
$\mathbf{R}^2_{adj}$	0.662	0.691	0.663	0.804
Log-likelihood			2249.770	3479.244
Std. error of regression	0.220	0.211	0.219	0.168

Note: * Not significant at 5 %. Asymptotic t-values in square brackets.

5 Conclusion

The purpose of this paper was to estimate the WTP for reducing the risk due to radon contamination in the municipality of Stockholm, during the period 1994 – 1996, using a spatial hedonic house price model. Tests for spatial dependency in the estimation of the hedonic price schedule strongly indicated that a spatial specification is preferred. The results imply that the percentage effect on house prices due to radon is -6.1 % for the preferred model and the corresponding willingness-to-pay for avoiding the health risks due to radon is SEK 69 196 for the entire assessment period. Assuming an assessment period of ten years the annual amount is SEK 8 869.

These results are higher than those reported by Söderqvist (1995), where the percentage effect on house prices due to radon is -4.0 % and the estimated WTP is SEK 21 300 during the period 1981 – 1987. The difference in the estimates is likely to be influenced by the application of different methods, but it may also be a result of an increase in households perception of the risk associated with radon.

It should be noted that the estimate of SEK 69 196 for the WTP is based on a rather crude calculation and not, as would be preferred, on a household demand function. The reason for this is that radon is a dichotomous variable, which cause the standard methods to derive the demand function to be inappropriate. At the moment the author is searching the literature to find a solution to this problem.

References

Anselin, L., 2001, "Rao's Score Test in Spatial Econometrics" *Journal of Statistical Planning and Inference*, Vol. 97, pp. 113-139.

Anselin, L., Bera, A.K., Florax, R. & Yoon, M.J., 1996, "Simple Diagnostic Tests for Spatial Dependence" *Regional Science and Urban Economics*, Vol. 26, pp. 77-104.

Anselin, L. & Bera, A.K., 1998, "Spatial Dependence in Linear Regression Models with an Introduction to Spatial Econometrics". In: A. Ullah & D.E.A. Giles (Eds), *Handbook of Applied Economic Statistics*. New York: Marcel Dekker.

Anselin, L. & Florax, R.J.G., 1994, "Small Sample Properties of Tests for Spatial Dependence in Regression Models: Some Further Results", Research Paper 9414, *Regional Research Institute*, West Virginia University, Morgantown, West Virginia.

Burridge, P., 1980, "On the Cliff-Ord Test for Spatial Autocorrelation" *Journal of the Royal Statistical Society*, Series B, Vol. 42, No. 1, pp. 107-108.

Case, B., Clapp, J., Dubin, R. & Rodriguez, M., 2004, "Modeling Spatial and Temporal House Price Patterns: A Comparison of Four Models" *Journal of Real Estate Finance and Economics*, Vol. 29, No. 2, pp. 167-191.

Cliff, A.D. & Ord, J.K., 1981, *Spatial Processes. Models and Applications.* London: Pion Limited.

Cressie, N. A.C., 1993, *Statistics for Spatial Data. Revised Edition.* New York: Willey.

Darby, S., *et al,* 2004, "Radon in Homes and Risk of Lung Cancer: Collaborative Analysis of Individual Data From 13 European Case-control Studies", BMJ, doi:10.1136/bmj.38308.477650.63, http://bmj.bmjjournals.com/cgi/content/short/bmj.38308.477650.63v1

Dubin, R.A., 1992, "Spatial Autocorrelation and Neighborhood Quality" *Regional Science and Urban Economics*, Vol. 22, pp. 433-452.

Dubin, R.A., 1998, "Spatial Autocorrelation: A Primer" *Journal of Housing Economics*, Vol. 7, pp. 304-327.

Halvorsen, R. & Palmquist, R., 1980, "The Interpretation of Dummy Variables in Semilogarithmic Equations" *The American Economic Review*, Vol. 70, No. 3, pp. 474-475.

Halvorsen, R. & Pollakowski, H.O., 1981, "Choice of Functional Form for Hedonic Price Equations" *Journal of Urban Economics*, Vol. 10, pp. 37-49.

Kim, C.W., Phipps, T.T. & Anselin, L., 2003, "Measuring the Benefits of Air Quality Improvement: A Spatial Hedonic Approach" *Journal of Environmental Economics and Management*, Vol. 45, pp24-39.

Militino, A.F., Ugarte, M.D. & García-Reinaldos, L., 2004, “Alternative Models for Describing Spatial Dependence among Dwelling Selling Prices” *Journal of Real Estate Finance and Economics*, Vol. 29, No. 2, pp. 193-209.

Pace, R.K. & Gilley, O.W., 1997, “Using the Spatial Configuration of the Data to Improve Estimation” *Journal of Real Estate Finance and Economics*, Vol. 14, No. 3, pp. 333-340.

Pace, R.K., Barry, R. & Sirmans, C.F., 1998, “Spatial Statistics and Real Estate” *Journal of Real Estate Finance and Economics*, Vol. 17, No. 1, pp. 5-13.

Palmquist, R.B., 1992, “Valuing Localized Externalities” *Journal of Urban Economics*, Vol. 31, pp. 59-68.

Ramussen, D.W. & Zuehlke, T.W., 1990, “On the Choice of Functional Form for Hedonic Price Functions” *Applied Economics*, Vol. 22, pp. 431-438.

Rosen, S., 1974, ”Hedonic Prices and Implicit Markets: Product Differentiation in Pure Competition” *Journal of Political Economy*, Vol. 82, No. 1, pp. 34-35.

Sirmans, G.S., Macphearson, D.A. & Zietz, E.N., 2005, “The Composition of Hedonic Pricing Models” *Journal of Real Estate Literature*, Vol. 13, No. 1, pp. 3-43.

Strålskyddsnytt, 2001, Vol.19, No. 1, “Cancerrisker Från Radon i Inomhusluft”, www.ssi.se.

Söderqvist, T., 1995, “Property Values and Health Risks: The Willingness to Pay for Reducing Residential Radon Radiation” *Scandinavian Housing & Planning Research*, Vol. 12, pp. 141-153.

Wilhelmsson, M., 2002, “Spatial Models in Real Estate Economics” *Housing, Theory and Society*, Vol. 19, pp. 92-101.

Appendix

A.1 Tests for spatial dependence

Our interest is whether we have a problem with spatial autocorrelation in the residuals from OLS or not. If so, which model should we use to capture this spatial dependency, the CAR model with a spatially lagged dependent variable or the SAR with spatial error dependence? The majority of the tests described in this appendix are LM-tests derived under different assumptions.

A.1.1 Tests for spatial error dependence $(H_0 : \lambda = 0)$

A widely used test for spatial dependence in the disturbances of a regression model is Moran's I-statistic $I = \mathbf{e}^T \mathbf{W}\mathbf{e} / \mathbf{e}^T \mathbf{e}$, where **e** are regression residuals and **W** is a row-standardized weight matrix. Cliff and Ord (1973) modify the statistic based on OLS residuals, by subtracting its mean and dividing by its standard deviation, and show that the asymptotic distribution of the modified I-statistic is standard normal. When **W** is standardized the modified test statistic is

$$Z_I = \frac{I - E(I)}{[\mathrm{var}(I)]^{1/2}} \tag{A.1}$$

where

$$I = \mathbf{e}^T \mathbf{W}\mathbf{e} / \mathbf{e}^T \mathbf{e}$$

$$E(I) = tr(\mathbf{MW})/(n-k)$$

$$\mathrm{var}(I) = \frac{tr(\mathbf{MWMW}^T) + tr\left[(\mathbf{MW})^2\right] + [tr(\mathbf{MW})]^2}{(n-k)(n-k+2)} - [E(I)]^2$$

$$\mathbf{M} = \left(\mathbf{I} - \mathbf{Z}(\mathbf{Z}^T\mathbf{Z})^{-1}\mathbf{Z}^T\right),$$

e are OLS residuals and *tr* is the trace operator. An asymptotically equivalent test is the LM-test derived by Burridge (1980) which takes the form

$$LM_\lambda = \frac{\left(\mathbf{e}^T \mathbf{W}\mathbf{e} / \hat{\sigma}^2\right)^2}{tr\left(\mathbf{WW} + \mathbf{W}^T\mathbf{W}\right)} \sim \chi_1^2, \tag{A.2}$$

where $\hat{\sigma}^2 = \mathbf{e}^T \mathbf{e} / n$. Though asymptotically equivalent, simulation experiments have shown that Moran's I is slightly superior in small samples (Anselin, 2001). In both test it is assumed that $\rho = 0$ in eq. (3) and the null is $H_0 : \lambda = 0$ [10]. Since this test also has power against $\rho = 0$ it is desirable to consider a test where possible spatial-lag dependence is accounted for (Anselin & Bera, 1998).

Anselin *et al* (1996) derives two different LM-tests for the same null, $H_0 : \lambda = 0$, but with ρ unrestricted; one requires ρ to be estimated whereas the other does not. The test statistic when ρ is estimated is

$$LM_\lambda^E = \frac{\left(\mathbf{e}^T \mathbf{W}_2 \mathbf{e} / \hat{\sigma}^2\right)^2}{T_{22} - \left[T_{21}^A\right]^2 \operatorname{var}(\hat{\rho})} \sim \chi_1^2, \tag{A.3}$$

where $T_{ij} = tr\left(\mathbf{W}_i \mathbf{W}_j + \mathbf{W}_i^T \mathbf{W}_j\right)$, $T_{21}^A = tr\left(\mathbf{W}_2 \mathbf{W}_1 (\mathbf{I} - \hat{\rho}\mathbf{W}_1)^{-1} + \mathbf{W}_2^T \mathbf{W}_1 (\mathbf{I} - \hat{\rho}\mathbf{W}_1)^{-1}\right)$, and $\mathbf{e}$ is the vector with ML residuals from the null model in eq. (3). The test statistic that does not require estimation of ρ is calculated as

$$LM_\lambda^* = \frac{\left(\mathbf{e}^T \mathbf{W}_2 \mathbf{e} / \hat{\sigma}^2 - T_{21} \Psi \mathbf{e}^T \mathbf{W}_1 \mathbf{p} / \hat{\sigma}^2\right)^2}{T_{22} - (T_{21})^2 \Psi} \sim \chi_1^2, \tag{A.4}$$

where $\Psi = \hat{\sigma}^2 \left[T_{11}\hat{\sigma}^2 + \left(\mathbf{W}_1 \mathbf{Z}\hat{\beta}\right)^T \mathbf{M} \left(\mathbf{W}_1 \mathbf{Z}\hat{\beta}\right)\right]^{-1}$, $\mathbf{e}$ is the vector with OLS residuals from eq. (2), $\hat{\sigma}^2 = \mathbf{e}^T \mathbf{e} / n$ and $\hat{\beta}$ is the OLS estimate of β, i.e. $\hat{\beta} = \left(\mathbf{Z}^T \mathbf{Z}\right)^{-1} \mathbf{Z}^T \mathbf{p}$.

A.1.2 Tests for spatial autoregressive dependence $(H_0 : \rho = 0)$

Assuming that $\lambda = 0$ the LM-test statistic for $H_0 : \rho = 0$ can be calculated as

$$LM_\rho = \Psi\left(\mathbf{e}^T \mathbf{W} \mathbf{p} / \hat{\sigma}^2\right)^2 \sim \chi_1^2, \tag{A.5}$$

[10] Note that the LM test for an MA error process, i.e. $\mathbf{u} = \lambda \mathbf{W}_2 \varepsilon + \varepsilon$ in eq. (6), is identical to the test for the AR process in eq. (A.2) (Anselin *et al*, 1996).

where **e** is the vector with OLS residuals from eq. (A.2) and Ψ is defined as above. Analogous to the test in eq. (A.2) for spatial error dependence, this test has power against $\lambda = 0$ and it is desirable to consider a test where possible spatial error dependence is accounted for. As in the previous case, we can relax the restrictions on the spatial error process, and tests corresponding to LM_{λ}^{E} and LM_{λ}^{*} above are derived by Anselin *et al* (1996). The test statistic for $H_0 : \rho = 0$ in the presence of λ can be calculated as

$$LM_{\rho}^{E} = \frac{\left(\mathbf{e}^T \mathbf{B}^T \mathbf{B} \mathbf{W}_1 \mathbf{p}\right)^2}{H_{\rho} - H_{\theta\rho} \operatorname{var}\left(\hat{\theta}\right) H_{\theta\rho}^{T}} \sim \chi_1^2 \qquad \text{(A.6)}$$

where **e** is a vector with residuals from the ML estimation of the null model in eq. (4) with parameters $\theta^T = \left(\beta^T \;\; \lambda \;\; \sigma^2\right)$ and $\mathbf{B} = \mathbf{I} - \hat{\lambda}\mathbf{W}_2$. The terms in the denominator are

$$H_{\rho} = tr\left(\mathbf{W}_1 \mathbf{W}_1\right) + tr\left[\left(\mathbf{B}\mathbf{W}_1\mathbf{B}^{-1}\right)^T \left(\mathbf{B}\mathbf{W}_1\mathbf{B}^{-1}\right)\right] + \left(\mathbf{B}\mathbf{W}_1\mathbf{Z}\hat{\beta}\right)^T \left(\mathbf{B}\mathbf{W}_1\mathbf{Z}\hat{\beta}\right) / \hat{\sigma}^2,$$

$$H_{\theta\rho}^{T} = \begin{bmatrix} \left(1/\hat{\sigma}^2\right)\left(\mathbf{B}\mathbf{Z}\right)^T \mathbf{B}\mathbf{W}_1\mathbf{Z}\hat{\beta} \\ tr\left[\left(\mathbf{W}_2\mathbf{B}^{-1}\right)^T \mathbf{B}\mathbf{W}_1\mathbf{B}^{-1}\right] + tr\left(\mathbf{W}_2\mathbf{W}_1\mathbf{B}^{-1}\right) \\ 0 \end{bmatrix},$$

and $\operatorname{var}\left(\hat{\theta}\right)$ is the estimated variance matrix for $\theta^T = \left(\beta^T \;\; \lambda \;\; \sigma^2\right)$[11]. The test statistic in eq. (A.6) requires eq. (4) to be estimated but Anselin *et al* (1996) also derives a test statistic for $H_0 : \rho = 0$ which is based on OLS estimation of eq. (2). The test statistic is calculated as

$$LM_{\rho}^{*} = \frac{\left[\left(1/\hat{\sigma}^2\right)\left(\mathbf{e}^T \mathbf{W}_1 \mathbf{p} - T_{12}\left(T_{22}\right)^{-1} \mathbf{e}^T \mathbf{W}_2 \mathbf{e}\right)\right]^2}{\Psi^{-1} - \left(T_{21}\right)^2 \left(T_{22}\right)^{-1}} \sim \chi_1^2, \qquad \text{(A.7)}$$

where **e** is a vector with residuals from OLS estimation of eq. (2).

[11] The variance matrix is calculated as the inverse of the negative hessian. The hessian is calculated numerically .

A.1.3 Joint tests for spatial dependence $\left(H_0 : \rho = 0,\ \lambda = 0\right)$

The tests for spatial lag dependence, LM_ρ, and spatial error dependence, LM_λ, are not independent, not even asymptotically. Only using either type of test will therefore lead to doubtful inference in the presence of both and/or other kinds of correlation (Anselin & Bera 1998). Anselin & Bera (1998) gives a joint test for $H_0 : \rho = 0,\ \lambda = 0$ based on OLS estimation and assuming that $\mathbf{W}_1 = \mathbf{W}_2 = \mathbf{W}$. The test statistic is calculated as

$$LM_{\lambda\rho} = \frac{\left[\left(1/\hat{\sigma}^2\right)\left(\mathbf{e}^T\mathbf{W}\mathbf{p} - \mathbf{e}^T\mathbf{W}\mathbf{e}\right)\right]^2}{\Psi - T} + \frac{\left[\left(1/\hat{\sigma}^2\right)\mathbf{e}^T\mathbf{W}\mathbf{e}\right]^2}{T} \sim \chi_2^2, \tag{A.8}$$

where $T = tr\left(\mathbf{W}\mathbf{W} + \mathbf{W}^T\mathbf{W}\right)$. It should be noted, however, that a rejection of the null gives no guidance to the type of dependence and will therefore only be useful to detect spatial dependence.

A.2 Figures

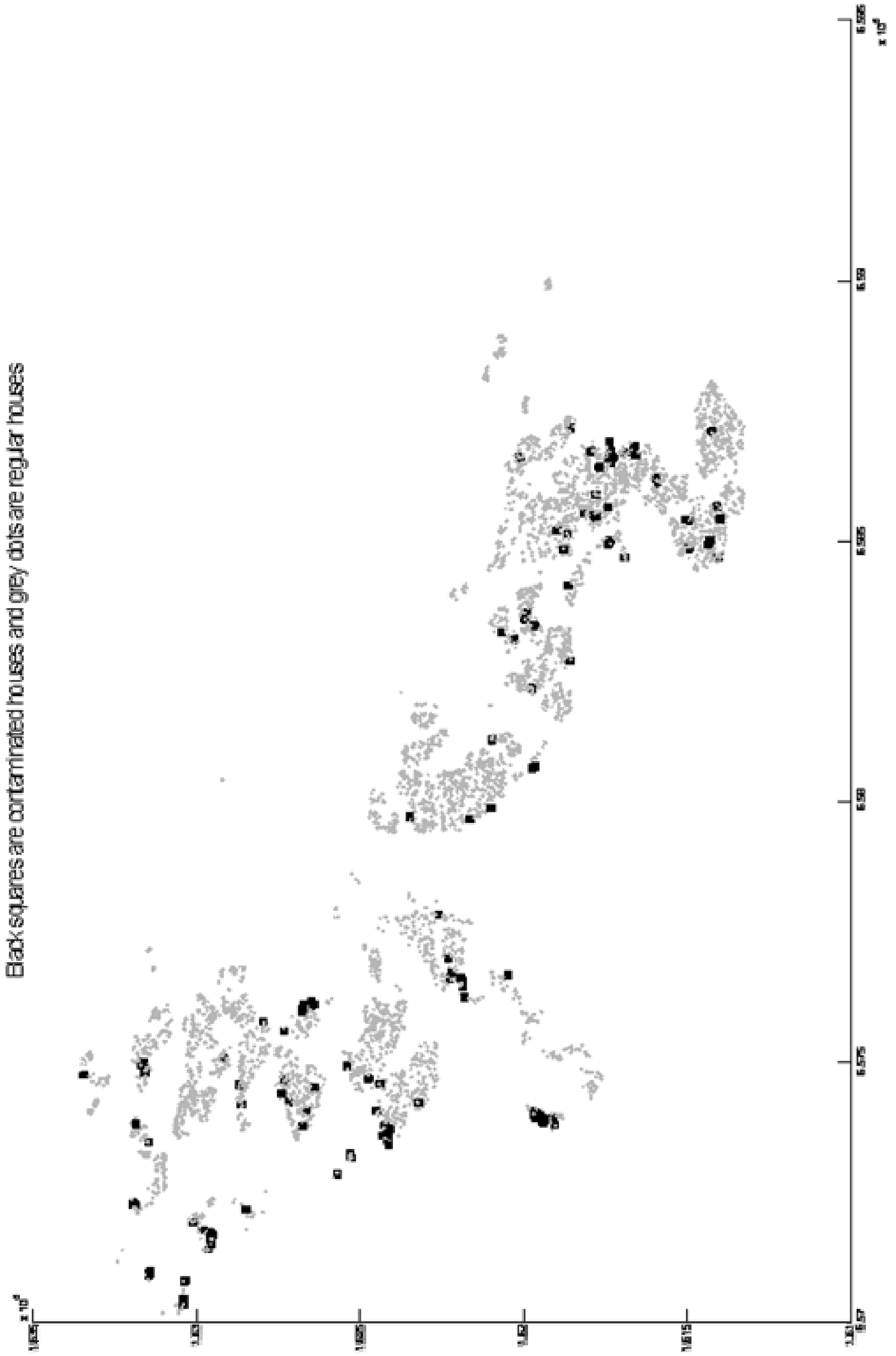
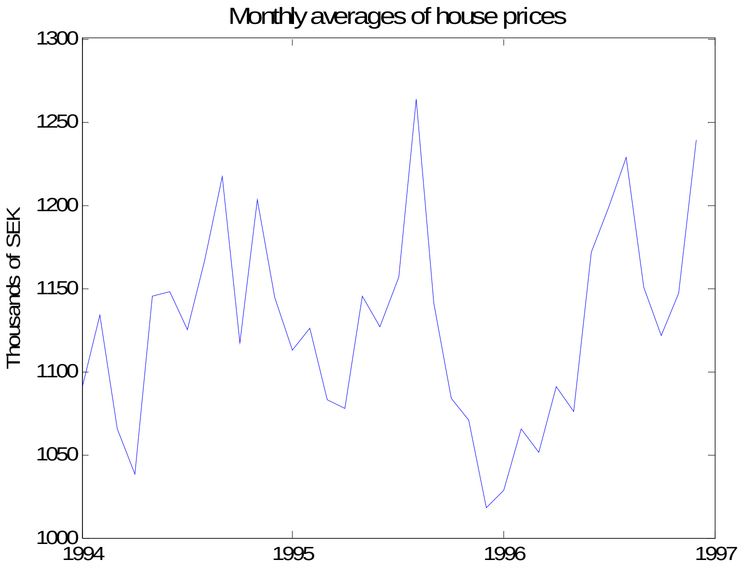


Figure 1



Monthly averages of house prices
Thousands of SEK
1300
1250
1200
1150
1100
1050
1000
1994
1995
1996
1997

Figure 2.