Bargaining for an efficient and fair allocation of emission permits to developing countries.

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Abstract: The paper focusses on the negotiations between the developed countries currently implementing emission permit markets versus the developing countries who want to join this market. We model the negotiations according to the 'Alternating Offers Bargaining' model. The objective is to obtain an efficient and fair allocation of tradeable emission permits between these two players.

At each period, one player proposes a feasible allocation of the goods for both players. Then the other player either ends the negotiations by accepting the proposal, or prolongs them by rejecting it. The proposal is accepted if this player considers it fair. If rejected, there is a certain probability that the next round is played and the other player making a proposal. The equilibrium concept in this model is that of a subgame perfect equilibrium.

Keywords: Bargaining, alternating offers, computable general equilibrium, developing countries, emission reduction

1 Introduction

We model negotiations for the reduction of greenhouse gasses as negotiations for the division of tradable emission permits. We consider a stylized infinite sequence of one-period economies with production, here modelled by the GTAP-E model, that are stationary over time, where the emission permits enter as an endownment of the regional households. In contrast to standard general equilibrium or CGE models, the total level of permits is endogenously determined in the model. Since any agreement on the allocation of emission permits and the global emission levels affects the allocation of goods in the world economy, this means that the negotiations are over efficient allocations of goods that represent stationary contracts in our setup. Our focus is on the negotiations between the developed countries currently implementing emission permit markets versus the rapidly developing countries such as China and India.

We model these negotiations between two monolitic agents according to the alternating offers bargaining model of Rubinstein (1982) over streams of consumption and production decisions, which is a game in extensive form with perfect information. One agent, the developed world, is the aggregation of the developed regions that participate in an existing emission permit market. The other agent, the developing world, is an aggregation of the developing regions, mainly China and India, who want to join the emission permit market. This game is played between the two players over an infinite and indexed set of time periods. The objective is to obtain an efficient and fair allocation of tradeable emission permits in this two players economy. At each odd numbered period of time, the developed world proposes a feasible allocation of the goods in the economy for both players. Then the developing world either ends the negotiations by accepting the proposal, or prolongs the negotiations by rejecting it. The proposal is accepted if it is considered a fair allocation − with fair as defined in Mariotti (1999) − by the developing world. If rejected, economic life continues and the agents take inefficient decisions due to the existence of externalities in the current round before we enter the negotiations at the next (even) round, which is played with a certain probability, hence incorporating the possibility of a break-down of negotiations. At each even numbered period of time, the developing world proposes a feasible allocation of the goods in the economy for both players, which the developed world then either accepts − if considered fair − thereby ending the negotiations, or rejects, and thereby accepting an inefficient allocation for at least one more round. In this way, the negotiations are prolonged into the next (uneven) round with a certain probability. The equilibrium concept in this model is that of a subgame perfect equilibrium. As stated in Rubinstein (1982), there exists a unique pair of stationary subgame perfect equilibrium proposals in this bargaining model that requires a solution to a fixed point problem, which is computationally a hard problem.

Thus far, this fixed point problem was circumvented by either resorting to the Nash bargaining solution or to assume a finite time horizon, often limited to avoid the heavy computational burden. Recently, Houba (2005) proves that the pair of stationary subgame perfect equilibrium proposals in the alternating offers bargaining model also corresponds to the maximum of the asymmetric Nash product in a single convex program. Convergence as time between rounds vanishes is immediate by the Maximum Theorem and the axiomatization of equilibrium proposals for all discount factors becomes trivial.

More interestingly, the single program also specifies financial transfers between players and allows for an implementation of production and consumption decisions through decentralized market prices, an aspect that was thus far neglected in bargaining theory. If the sufficient conditions for uniqueness in Houba (2005) are satisfied, the negotiation process ends in a unique subgame perfect equilibrium allocation of emission permit endowments that is both efficient and fair. The model can be easily extended to allow for lobbying that affects the equilibrium proposals.

The main question with applying tradable emission permits to reduce global greenhouse gas emissions is how much permits to allocate to each member. In practice, most allocations occur to some grandfathering rule. Such an allocation follows a certain allocation rule. For example, one can provide a country a number of permits according to its historical output or based on past emissions. Providing a country with emission permits reduces the cost of adjustment but will not do enough to reduce emissions. The latter still causes too much costs on the economy due to climate related damages. Under grandfathering, permits are given away for free which results in a loss of surplus to the one distributing the permits, often the government.

As an alternative, one often proposes to auction a certain amount of permits to the polluters. In this way, these polluters are expected to express their real values for the permits and this value is transferred to the government. This positive budget effect could lead to reductions in taxes or to a possible financing of cleaning activities. Hence, auctioning could take away some of the dead weight loss associated with grandfathering rules. This however is no rule.

Under auctioning, the permit allocation is the outcome of a bidding process, but in order to avoid dead weight losses it requires that the total amount of permits auctioned is the Pareto efficient total amount. If the auctioned amount of permits falls short, the price of permits is socially too high, if permits are provided in abundance, then the permits will be priced too low to obtain the Pareto efficient emission level. So, the real issue in the auctioning of permits is to establish the Pareto efficient total amount of permits. This amount should be equal to the Pareto efficient level of emissions, which is endogenous and depends upon the countries' welfare weights. Contrary to auctioning the permits, any bargaining approach implicitly endogenizes the countries' welfare weights as the outcome of some kind of strategic bargaining process.

When choosing for the bargaining option as an alternative to auctioning or grandfathering the permits, the welfare weights of the players in a social welfare function are determined endogenously as the outcome of the bargaining process. These welfare weights provide an endogenous allocation of permisable emissions which can be subsequently translated into allocated permits. Hence, ideally, no permit trade will take place under the clearing permit price because the efficient allocation of permits has been determined already as the outcome of the bargaining process. Furthermore, there will be no deadweight losses under this rule.

In a bargaining model, the fall-back position or disagreement point plays an important role. Since this disagreement point reflects the currentÊ(meaning historical) Pareto inefficient use of energy, this point implicitly reflects an inefficient grandfathering scheme. However, the negotiated equilibrium agreement − like trade − improves upon the disagreement point and the division of the associated net gains will in general not reflect a grandfathering solution.

2 The Alternating Offers Bargaining Model

Two players bargain over the allocation of emission permits. The players' preference relations are defined on the set of ordered pairs of the type (x, t) where $x = (x_1, x_2)$ with x_1 refering to the amount of permits allocated to the developed world (player 1), and x_2 refering to the amount of permits allocated to the developing world (player 2). We assume that the preferences over (x, t) satisfy the following assumptions:

- A-1 emission permits are *desirable*,
- A-2 'time' is valuable,
- A-3 continuity,
- A-4 stationarity (the preference of (x, t) over $(y, t + 1)$ is independent of t),
- A-5 the larger the portion the more 'compensation' a player needs for a delay of one period to be immaterial to him.

Assumption A-2 implies the existence of a discount factor for each player. Let δ_i denote player i's one period discount factor, which is assumed to be fixed over time.

Nash (1953) defines a bargaining problem as consisting of a set S of alternatives described by the utilities that the players can obtain when choosing these alternatives, and a point d of disagreement. The latter point describes the utility of both players when the bargaining process does not end in agreement. A bargaining solution can then roughly be defined as the solution to one of the many possible bargaining problems $\phi \in \Pi$ defined on a compact and convex set S. The Nash Bargaining Solution $\nu(S)$ specifies this particular solution as the most efficient one on the subset of S consisting of nonnegative elements, hence the *Nash Bargaining Solution* (NBS), $\nu : \Gamma \to \mathbb{R}^2$, is defined as follows:

$$\nu(S) = \arg \max_{s \in S \cap \mathbb{R}^2_+} s_1 s_2.$$

Nash (1953) proved that the NBS ν is the only solution on Γ that satisfies the following well-known properties:

Weak Pareto Optimality: $s > \phi(S) \Rightarrow s \notin S$.

Covariance with Positive Scale Transformations: Let $\tau : \mathbb{R}^2 \to \mathbb{R}^2$ be a positive, linear, component by component transformation given by $\tau(x) = (\lambda_1 x_1, \lambda_2 x_2)$, with $\lambda_1, \lambda_2 > 0$, for all $x \in \mathbb{R}^2$, and for any $X \subset \mathbb{R}^2$ let $\tau(X) = \{y \in \mathbb{R}^2 \mid y = \tau(x) \text{ for some } x \in X\}$. Then, $\phi(\tau(S)) = \tau(\phi(S))$.

Symmetry: Suppose that the problem is symmetric (that is, $s \in S \Rightarrow (s_2, s_1) \in S$). Then $\phi_1(S) = \phi_2(S)$.

Independence of Irrelevant Alternatives: $S \subseteq T$ and $\phi(T) \in S \Rightarrow \phi(T) = \phi(S)$.

There exists a significant body of research concerning bargaining within the field of economics and game theory. In this paper, we use one of the models proposed in this literature, the Alternating Offers model. The original alternating offers model has been defined in Rubinstein (1982). It concerns a stylized example of a procedure to divide a dollar among two players by letting the players alternatingly propose such a division. Rubinstein (1982) formulated this problem as a game in extensive form assuming that both players have perfect information. Instead of representing the game by a matrix (normal form), an extensive form represents the game in a tree form where each node denotes a state in the game. Play begins at a unique initial node, and flows through the tree along a path determined by the players until a terminal node is reached, where play ends and payoffs are assigned to all players. Each non-terminal node belongs to a player; that player chooses among the possible moves at that node, each possible move is an edge leading from that node to another node. Under perfect information, at any stage of the game, every player knows exactly what has taken place earlier in the game.

Negotiations for the reduction of greenhouse gases are modelled as negotiations between two monolithic agents for the division of tradable emission permits. One agent, *the developed world*, is the aggregation of the developed regions that participate in an existing emission permit market. The other agent, *the developing world*, is the aggregation of the developing regions, mainly China and India, who want to join the emission permit market. This game is played between these two players over an infinite and indexed set of time periods. We assume it is a game in *extensive form* with *perfect information*. The assumption of perfect information implies that each player has complete information about the preference of the other. The bargaining costs of each player is therefore assumed known to the other. The objective of the game is to obtain an *efficient* and *fair* allocation of tradable emission permits.

This paper applies the alternating offers model of Rubinstein (1982) to possible negotiations for letting developing countries China and India enter an emission permit market with the Annex B countries. We model the assignment of carbon emission targets as a negotiation game for the division of tradable emission permits. The economic model consists of an infinite repetition of the static GTAP-E model. In contrast to the standard computable general equilibrium, the total level of permits and its distribution over the regional households are endogenously determined by the negotiation game. Since any agreement on the allocation of emission permits and the global emission levels affect the efficient allocation of goods in the world economy, this means that the negotiations are over efficient allocations of goods that represent stationary contracts in our setup. Our focus is on the negotiations between a player that represents the developed countries currently implementing emission permit markets versus another player representing the rapidly developing countries such as China and India.

The negotiations between Annex B, and China and India follow the alternating offers bargaining model of Rubinstein (1982) extended to allow for an infinite stream of consumption and production decisions in an economy that is modelled using the GTAP-E model introduced in Burniaux and Truong (2002). One agent, Annex B, is the aggregation of the developed regions that participate in an existing emission permit market. The other agent, the developing world, is an aggregation of the developing regions, mainly China and India, who contemplate joining the emission permit market. This game is played between these two self-interested agents (or players) over an infinite and indexed set of time periods. The objective is to obtain an efficient allocation of tradable emission permits over these agents in the economy.

At each odd numbered period t of time, the developed world proposes a *feasible allocation* (x^*, t) of emission permits in the economy for both players. Then the developing world either ends the negotiations by accepting the proposal x^*, or prolongs the negotiations by rejecting it. The proposal is accepted if x^* is considered a *fair* allocation $-$ with fair as defined in Mariotti (1999) $-$ by the developing world. If rejected, economic life continues and the agents take inefficient decisions due to the existence of externalities in the current round before we enter the negotiations at the next (even) round. In this equilibrium, there is an excess supply of emission permits, requiring a permit price equal to zero. The regions do not take account of their emissions causing the associated externalities to persist. The next round is played with a certain probability π_{t+1}, hence incorporating the possibility of a break-down of negotiations with probability $1 - \pi_{t+1}$.

At each even numbered period t of time, the developing world proposes a feasible allocation (x^*, t) of the emission permits in the economy for both players, which the developed world then either accepts x^* $-$ if considered fair $-$ thereby ending the negotiations, or rejects, and thereby accepting an inefficient allocation for at least one more round. In this way, the negotiations are prolonged into the next (uneven) round $t+1$ with a certain probability π_{t+1}.

The equilibrium concept in this game is that of a subgame perfect equilibrium. Subgame perfectness is a refinement of the Nash equilibrium. A set of strategies is a Nash equilibrium, if no player can do better by unilaterally changing his or her strategy. A strategy profile is a subgame perfect equilibrium, if it represents a Nash equilibrium of every subgame of the original game. More informally, this means that if (1) the players played any smaller game that consisted of only one part of the larger game and (2) their behaviour represents a Nash equilibrium of that smaller game, then their behaviour is a subgame perfect equilibrium of the larger game. The concept of subgame perfectness is attributed to the work of the German Nobel prize winner Reinhard Selten (see Selten (1965)).

Rubinstein (1982) proves that the alternating offers procedure has a unique subgame perfect equilibrium and that this equilibrium is stationary. A stationary strategy is history- and time-independent. Stationary equilibria represent the simplest forms of behaviour that is consistent with rationality. Equilibrium proposals are Pareto efficient and there will be agreement on the first proposals. Under stationarity, a player will make the same proposal which is then accepted by the other player. The proposals of the players can differ. The computation of such a stationary subgame perfect equilibrium can be formulated as a fixed point problem.

Houba (2007) proves that the fixed point problem representing a subgame perfect equilibrium in the alternating offers model is equivalent to a certain convex optimization program with the Nash product as its objective function. This equivalence extends to general equilibrium models with convex production technologies. Houba (2007) also proves that the Pareto-efficient proposals resulting from such a bargaining process can be supported by Walrasian prices.

Each player accepts a proposal that he thinks is *fair*. Here we interprete fairness of a bargaining solution in the sense of Mariotti (1999). A *bargaining problem* can be described as a set $S \subseteq \mathbb{R}^2$. A *bargaining solution* to the collection Π of bargaining problems, is a function $\phi : \Pi \rightarrow \mathbb{R}^2$ such that $\phi(S) \in S$ for all $S \in \Pi$, with S restricted to the set Γ of compact and convex sets such that there exists an $s \in \bar{S}$ such that $s > 0$, for each $\bar{S} \in \Gamma$.

Mariotti (1999) states that a possible interpretation of these axioms is as properties that should be satisfied by the choices of a fair arbitrator. The included axiom on the independence of irrelevant alternatives may be viewed as a relevant criterion on rationality or consistency in choice, but it hardly has no ethical interpretation, ant therefore it can hardly be seen as a requirement of fair arbitration. Mariotti (1999) therefore replaces the independence of irrelevant alternatives axiom by a criterion of impartiality in distributive justice known as *Suppes-Sen Proofness*, see Suppes (1966) and Sen (1970).

Suppes-Sen Proofness: $(s_2, s_1) > \phi(S)$ or $s > \phi(S) \Rightarrow s \notin S$.

The equilibrium concept in this model is that of a *subgame perfect equilibrium*. A perfect equilibrium is one where not only the strategies chosen at the beginning of the game form an equilibrium, but also the strategies planned in every subgame. As stated in Rubinstein (1982), there exists a unique pair of stationary subgame perfect equilibrium proposals in this bargaining model that requires a solution to a fixed point problem.

3 The Global Trade and Analysis Project: Energy (GTAP-E) model.

The GTAP-E model is a multi-sector, multi-regional, computable general equilibrium model. For information on computable general equilibrium models I refer to Shoven and Whalley (1992) or, more recently, Ginsburgh and Keyzer (1997), which are a sort of standard works on the area. The GTAP-E economic model is based on the GTAP5 database, for which we refer to Dimaranan and McDougall (2002). The GTAP5 database is a product of the Global Trade Analysis Project at Purdue University (see GTAP (2007)).

Regional aggregation: The GTAP5 database aggregates the world into 66 regions. We take a further regional aggregation into 3 regions as depicted in Table 1. Let $\mathcal{R}$ denote the set of all regions in the second column of Table 1, indexed with r. The developed regions are given by the countries that signed the Annex B to the Kyoto Protocol. We denote this region with `AnnexB` and number it with 1 in the set $\mathcal{R}$. `CHIND` refers to China and India, as the fast developing regions that will be the focus of this paper, and number it with 2 in the set $\mathcal{R}$. The remaining regions constitute the underdeveloped world, and we aggregate them into the 'Rest of the World' (`RoW`) region.

Nr.	r	Region Description	Comprising GTAP5 Regions:
1	`AnnexB`	Annex B regions	United States, Austria, Belgium, Denmark, Finland, France, Germany, United Kingdom, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal, Spain, Sweden, Australia, New Zealand, Japan, Canada, Switzerland, Rest of EFTA, Hungary, Poland, Rest of Central European Association, Former Soviet Union
2	`CHIND`	China and India	China, India
3	`RoW`	Rest of the World	Hong Kong, Korea, Taiwan, Indonesia, Malaysia, Philippines, Singapore, Thailand, Vietnam, Bangladesh, Sri Lanka, Rest of South Asia, Mexico, Central America and Caribbean, Colombia, Peru, Venezuela, Rest of Andean Pact, Argentina, Brazil, Chile, Uruguay, Rest of South America, Turkey, Rest of Middle East, Morocco, Rest of North Africa, Botswana, Rest of SACU, Malawi, Mozambique, Tanzania, Zambia, Zimbabwe, Rest of Southern Africa, Uganda, Rest of Sub-Saharan Africa, Rest of World

Table 1: The regional aggregation.

Sectoral aggregation: The GTAP5 database distinguishes 57 tradable goods in each region. GTAP-E takes a further aggregation into the production sectors, summarized in Table 2. Among these tradable goods, we distinguish the fossil fuels coal, oil, gas, and petroleum. The economic model further contains a global bank and a transport sector, the latter one containing air, land, and sea as transport modes. These three transport modes are aggregated into one production sector. Let $\mathcal{S}$ denote the set of all goods indexed with s, whose elements are depicted in the second column of Table 2. This set contains a subset $\mathcal{S}_{trad}$ of tradeable goods, which excluded the nontradeable capital good (`cgds`). Then there is a subset of non-coal fossil fuels, $\mathcal{S}_{ncoal} = \{\texttt{gas}, \texttt{p\_c}, \texttt{oil}\}$, a subset $\mathcal{S}_f = \mathcal{S}_{ncoal} \cup \{\texttt{coal}\}$ of fossil fuels, a subset

$\mathcal{S}_e = \mathcal{S}_f \cup \{\texttt{ely}\}$ of energy goods, and a subset $\mathcal{S}_{ne} = \mathcal{S}_{trad} \setminus \mathcal{S}_e$ of non-energy goods. The set $\mathcal{S}_{margin} = \{\texttt{ois}\}$ denotes the set of margin goods, or transport sectors. These transport sectors only add a margin to the export price of each good.

Nr.	s	Sector Description	Comprising GTAP5 Sectors:
1.	`rice`	Rice	paddy rice
2.	`crops`	Primary Agriculture, and Fishing	wheat; cereal grains n.e.c.; vegetables, fruit, nuts; oil seeds; sugar cane, sugar beet; plant-based fibers; crops n.e.c.; fishing
3.	`livestock`	Livestock products	bovine cattle, sheep and goats; animal products n.e.c.; raw milk; wool, silk-worm cocoons;
4.	`forestry`	Forestry	forestry
5.	`coal`	Coal Mining	Coal
6.	`oil`	Crude oil	Oil
7.	`gas`	Natural gas extraction	Gas; gas manufacture, distribution
8.	`pc`	Refined oil products	Petroleum, coal products
9.	`ely`	Electricity	Electricity
10.	`ois`	Other industries and services	minerals n.e.c.; bovine cattle, sheep and goat; meat products; vegetable oils and fats; dairy products; processed rice; sugar; food products n.e.c.; beverages and tobacco products; textiles; wearing apparel; leather products; wood products; paper products, Chemical, rubber, and plastic prod.; publishing; mineral products n.e.c.; ferrous metals; metals n.e.c.; metal products; motor vehicles and parts; transport equipment n.e.c.; electronic equipment; machinery and equipment n.e.c.; manufactures n.e.c.; water; construction; trade; transport n.e.c.; water transport; air transport; communication; financial services n.e.c.; insurance; business services n.e.c.; recreational and other services; public administration and defence, education; ownership of dwellings
			n.e.c.= 'not elsewhere classified'

Table 2: The sectoral aggregation in GTAP-E, Traded Commodities.

Production factors: GTAP considers the goods in Table 3 as the primary means of production. They are allocated as initial endowments to the consumer household representing a region as its source of income.

Capital is considered internationally mobile, land and natural resources are tied to the production sector in which they serve as an input. Land is currently only used as an input to the agricultural production sectors. Labour is only regional mobile

Nr.	Production factor	Sector Description
1	`capital`	Capital
2	`land`	Land
3	`natres`	Natural resources
4	`SkLab`	Skilled labour
5	`UnSkLab`	Unskilled labour

Table 3: The production factors in GTAP-E.

Functional forms: Computable general equilibrium modelling extensively makes use of constant elasticities of substitution (CES) functional forms and its special cases such as Cobb-Douglas (CD) and Leontief functions. In order to differentiate among groups of consumption and input goods of different substitution elasticities, often nested CES functions are applied. Such nested CES functions could be represented in a tree structure where the expenditure or cost to obtain a certain good is disaggregated along different levels of aggregation towards the set of goods and endowments in Tables 2 and 3. The use of such nested CES structures dates back to Armington (1969). The model in this section is written down in its dual form, i.e. using cost functions to describe the producers behaviour and using expenditure functions to describe the consumer's behaviour.

The simplest functional form is a so-called Leontief functional form, denoted with $\mathrm{leontief}(p_1, \ldots, p_n)$. We define the associated cost or expenditure function by

$$\textsc{leontief}(p_1, \ldots, p_n) = \sum_{i=1}^{n} \alpha_i p_i$$

for certain productivity parameters $\alpha_1, \ldots, \alpha_n$. The function $\textsc{leontief}$ denotes the cost c.q. expenditure to obtain one unit of the output good given prices $p_1, \ldots, p_n$ of the n input goods, assuming a production function or utility function of leontief type. The elasticity of substitution between each pair of input goods is uniformly equal to infinity.

The Cobb-Douglas functional form is denoted with $\textsc{cd}(p_1, \ldots, p_n)$. We define the associated cost or expenditure function by

$$\textsc{cd}(p_1, \ldots, p_n) = \prod_{i=1}^{n} p_i^{\alpha_i}$$

for certain productivity parameters $\alpha_1, \ldots, \alpha_n$ such that $\sum_i \alpha_i = 1$. $\textsc{cd}$ denotes the cost c.q. expenditure to obtain one unit of the output good given prices $p_1, \ldots, p_n$ of the n input goods, assuming a production function or utility function of Cobb-Douglas type. The elasticity of substitution between each pair of input goods is uniformly equal to one.

The Constant Elasticity of Substitution functional form is denoted with $\textsc{ces}(p_1, \ldots, p_n)$. We define the associated cost or expenditure function by

$$\text{CES}(p_1, \ldots, p_n) = \left(\sum_{i=1}^{n} \alpha_i p_i^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$$

for certain productivity parameters $\alpha_1, \ldots, \alpha_n$ and elasticity of substitution σ. CES denotes the cost c.q. expenditure to obtain one unit of the output good given prices $p_1, \ldots, p_n$ of the n input goods, assuming a production function or utility function of CES type. The elasticity of substitution between each pair of input goods is uniformly equal to σ.

Regional (consumption) household: Final demand in each region is modelled by introducing a regional consumption household. Each region is endowed with a certain amount of each production factor. GTAP-E distinguishes among capital, labour, land, and natural resources in Table 3 as the economy's production factors. He obtains income from the payments on these endowments. The consumer's income consists of the value of his total time endowment offered for labour, $p_r(\texttt{UnSkLab})\mathit{UnSkLab}_r + p_r(\texttt{SkLab})\mathit{SkLab}_r$, the value of his (sector-specific) land endowments $\sum_s p^F_{s,r}(\texttt{land})L_{s,r}$, the value of the region's available natural resources $\sum_s p^F_{s,r}(\texttt{natres})R_{s,r}$, and the value of his capital endowment $p_r(\texttt{capital})K_r$. Furthermore, we assume that the income from taxes and tariffs accrue as income to the regional household.

The regional consumption household obtains utility from spending his income on private consumption goods, on government consumption goods, and on savings. Government consumption refers to the region's consumption of publicly provided goods.

Let p^U_r denote the consumer price per unit of util to regional household r. This consumer price is determined as the minimal expenditure to obtain one unit of util at given prices. GTAP-E specifies this relation by

$$p^U_r = \text{CD}\left(p^G_r(\texttt{cons}), p^P_r(\texttt{cons}), p^I\right) \quad \perp \quad u_r \tag{1}$$

where $p^h_r(\texttt{cons})$ denotes the consumer price for government consumption ($h = G$), or private consumption ($h = P$). p^I denotes the price of the investment good. GTAP-E assumes that the decomposition uses a Cobb-Douglas utility function. The complementary variable to this equation is the amount of utils, u_r, demanded by region r. This implies a constant share of government expenditures, and a constant share of private expenditures, in the regional household r's total expenditures. The remaining (fixed) share of income is spent on savings. In GTAP, these savings are modelled by assuming that the regional household buys the output good of a global bank. Utility maximizing amounts of government consumption, $u^G_r(\texttt{cons})$, private consumption, $u^P_r(\texttt{cons})$, and savings, u^I_r, per unit of util, follow by Shepherd's Lemma as the first-order derivative of the expenditure function in equation (1) to their respective prices.

The set $\mathcal{C} := \{\texttt{cons}, \texttt{ec}, \texttt{nec}\}$ denotes the set of aggregates of all consumption goods in each consumer household. In $\mathcal{C}$, $\texttt{cons}$ refers to an aggregate consumption good, which is disaggregated into an energy composite $\texttt{ec}$ and a nonenergy composite $\texttt{nec}$. Let $p_r^h : \mathcal{C} \rightarrow \mathbb{R}$ map each composite $\ell \in \mathcal{C}$ onto its consumer price $p_r^h(\ell)$ respectively. It follows with expenditure minimization, that each composite's price is determined by the minimum expenditure to obtain one unit of this good. Hence,

$$\begin{aligned} p_r^h(\texttt{cons}) &= \text{CES}\left(p_r^h(\texttt{ec}), p_r^h(\texttt{nec})\right) & \perp \quad & c_r^h(\texttt{cons}) \\ p_r^h(\texttt{ec}) &= \text{CD}\left(p_r^h(s) \mid s \in \mathcal{S}_e\right) & \perp \quad & c_r^h(\texttt{ec}) \\ p_r^h(\texttt{nec}) &= \text{CD}\left(p_r^h(s) \mid s \in \mathcal{S}_{ne}\right) & \perp \quad & c_r^h(\texttt{nec}) \end{aligned} \tag{2}$$

for household's consumption $h \in \{G, P\}$. Utility maximizing amount of household h's consumption $c_r^h(s)$ of each good $s \in \mathcal{S}$ and consumption $c_r^h(\ell)$ of each of its composites $\ell \in \mathcal{C}$, per unit of util, follow by Shepherd's Lemma as the first-order derivative of the appropriate expenditure function in equation (2) to the corresponding consumer price.

Production sector households: Each commodity in Table 2 is assumed to be the unique output good of a particular production sector in each region. A production sector is endowed with a constant returns to scale production technology that produces its output good using the goods in Table 2 as intermediate inputs and the economy's endowments as primary goods. Under these conditions, the usual assumption of profit maximization is equivalent to cost minimization. The production structure is derived from the GTAP-E model in Burniaux and Truong (2002).

Let $p_{s,r}^O$ denote the producer price per unit of output of production sector s in region r. Then $p_{s,r}^O = (1 + t^O(s,r))p_{s,r}$ including output taxes $t^O(s,r)$ put onto the market price $p_{s,r}$ of region r's good s. This producer price is determined as the minimal cost to produce one unit of the output good at given prices. GTAP-E specifies this relation by

$$p_{s,r}(1 + t^O(s,r)) = \text{LEONTIEF}\left(p_{s,r}^F(\texttt{eva}), p_{s,r}^F(\mathcal{S}_{ne})\right) \quad \perp \quad y_{s,r} \tag{3}$$

where $p_{s,r}^F(\texttt{eva})$ denotes the producer specific price per unit of the composite 'Energy-Value-Added' good and $p_{s,r}^F(\mathcal{S}_{ne})$ the producer specific price of each non-energy input good.

Value-added is obtained from the costs of including the natural resource good, land, labour, capital, and energy into the production process. Notice that GTAP-E modifies the original GTAP modelling of the energy input into the production technologies. The standard GTAP model as described in Hertel and Tsigas (1997) treats energy inputs in the same manner as non-energy intermediate inputs. Burniaux and Truong (2002) proposes to shift energy from being an intermediate input to being a value-added component in the production technology. To this end, Burniaux and Truong (2002) introduces a capital-energy component in the value-added nest of production. This component is a composite of capital and an energy composite to allow for the substitution between capital and energy in the production process on the long term. On the short term, capital and energy are complementary goods. GTAP-E assumes a positive elasticity of substitution between the energy composite and capital, making these goods substitutes inside this nest.

The set $\mathcal{P} := \{\texttt{eva}, \texttt{labour}, \texttt{ke}, \texttt{e}, \texttt{ff}, \texttt{nc}\}$ denotes the set of aggregates of all input goods in the production nested function. The coal good and a `ncoal` composite of all noncoal fossil fuels constitute an aggregate fossil fuel good `ff`. The fossil fuel aggregate, together with the electricity input makes up an energy composite good `e`. Energy `e` and capital combine into a capital-energy composite `ke`, which forms a part of the Energy-Value-added composite `eva` together with the primary factor composite `Labour` and goods `natres`. Let $p^F_{s,r} : \mathcal{P} \to \mathbb{R}_+$ map each producer's composite input in $\mathcal{P}$ to its associated producer specific price, such that

$$
\begin{aligned}
&p^F_{s,r}(\texttt{eva}) = \textsc{ces}\left(p^F_{s,r}(\texttt{ke}), p^F_{s,r}(\texttt{labour}), (1+t^F_{s,r}(\texttt{land}))p_{s,r}(\texttt{land}), (1+t^F_{s,r}(\texttt{land}))p^F_{s,r}(\texttt{natres})\right) && \perp && a_{s,r}(\texttt{eva})\\
&p^F_{s,r}(\texttt{labour}) = \textsc{leontief}\left((1+t^F_{s,r}(\texttt{SkLab}))p_{s,r}(\texttt{SkLab}), (1+t^F_{s,r}(\texttt{UnSkLab}))p_{s,r}(\texttt{UnSkLab})\right) && \perp && a_{s,r}(\texttt{labour})\\
&p^F_{s,r}(\texttt{ke}) = \textsc{ces}\left((1+t^F_{s,r}(\texttt{capital}))p_{s,r}(\texttt{capital}), p^F_{s,r}(\texttt{e})\right) && \perp && a_{s,r}(\texttt{ke})\\
&p^F_{s,r}(\texttt{e}) = \textsc{leontief}\left(p^F_{s,r}(\texttt{ff}), p^F_{s,r}(\texttt{ely})\right) && \perp && a_{s,r}(\texttt{e})\\
&p^F_{s,r}(\texttt{ff}) = \textsc{ces}\left(p^F_{s,r}(\texttt{coal}), p^F_{s,r}(\texttt{nc})\right) && \perp && a_{s,r}(\texttt{ff})\\
&p^F_{s,r}(\texttt{nc}) = \textsc{ces}\left(p^F_{s,r}(\mathcal{S}_{ncoal})\right) && \perp && a_{s,r}(\texttt{nc})
\end{aligned}
\tag{4}
$$

Then, we can obtain the cost minimizing amount $a_{s,r}(\bar{s})$ of input good $\bar{s} \in \mathcal{S}$ or amount $a_{s,r}(\ell)$ of input composite $\ell \in \mathcal{P}$ from the first-order derivative of the appropriate cost function in either equation (3) or equations (4) to the corresponding producer specific price.

Savings: The global bank is an abstract production sector which produces the investment good in the economy, using net investments of each region as inputs. The investment of a production sector is modelled as the part of the output level of this sector which is produced specifically for investment purposes. Regional net investments are given by the total investments by each production sector, net of capital goods supplied by the regional household. The cost of each unit of the investment good is decomposed into the costs of obtaining the output share of each production sector meant for investments.

$$p^I = \text{LEONTIEF}\left(p_{\texttt{cgds},r} \mid r \in \mathcal{R}\right) \quad \perp \quad I \tag{5}$$

The first-order derivatives of the cost function in (5) to the price of capital services in region r provides the global bank's cost minimizing demand $a^I(r)$ for region r's capital services good. The variable complementary to this equation is the output level of the global bank, I, which represents global investments.

Transport: The GTAP5 database distinguishes three transport modes, air, land, and sea transport. Each transport mode is an abstract production sector that produces a composite transport good which is an aggregate of the supply of this transport mode by the production sector in each region. Each regional margin production sector produces an amount meant for transport with this mode of transport.

$$p^T_m = \text{CD}\left(p^X_{m,r} \mid m \in \mathcal{S}_{margin}\right) \quad \perp \quad y^T_m \tag{6}$$

The first-order derivatives of the cost function in (6) to the price of transport services of type m in region r provides transport's cost minimizing demand $a^T_{m,r}$ for transport of type m by region r. The variable complementary to this equation is the output level of the global transport sector m, y^T_m.

Foreign trade: The tradable goods produced by a production sector are traded internationally. This implies that, for each tradable good, there exists a variant produced in each region. Following Armington (1969), we assume that these goods are substitutable but not perfectly. Hence, each tradable good has a domestically produced equivalent and imported equivalents. The literature often refers to such goods as *Armington goods.*

$$\begin{array}{rcl} p^F_{s,r}(trad) = \text{CES}((1+\text{td}^F_{s,r}(trad))p^D_{trad,r}, p^m_{trad,r}) & \perp & a_{s,r}(trad) \\ p^h_r(trad) = \text{CES}((1+\text{td}^h_{s,r}(trad))p^D_{trad,r}, p^m_{trad,r}) & \perp & c^h_r(trad) \end{array} \tag{7}$$

for households $h \in \{G, P\}$. This indicates that the price of each 'Armington good' is dependent on the household that consumes it. Then we can obtain the expenditure c.q. cost minimizing amount $c^h_{trad,r}(\texttt{d})$ for households $h = G, P$ and $a_{trad,s,r}(\texttt{d})$ for producer s, of each domestically purchased or $c^h_{trad,r}(\texttt{i})$ for households $h \in \{G, P\}$ and $a_{trad,s,r}(\texttt{i})$ for producer s, of each imported version of each tradeable good in $\mathcal{S}_{trad}$ from the first-order derivatives of the appropriate expenditure or cost function to the corresponding price, by Shepherd's Lemma.

The price of the import aggregate in each region is constructed from the regionally different export prices of this good, using a CES functional form.

$$p^m_{trad,r} = \text{CES}\left((1+\text{tms}_{trad,r}(\bar{r}))p^{cif}_{trad,r}(\bar{r}) \mid \bar{r} \in \mathcal{R} \setminus \{r\}\right) \quad \perp \quad y^m_{trad,r} \tag{8}$$

We can now obtain the cost minimizing amounts $b_{trad,r}(\bar{r})$ of region $\bar{r}$'s good *trad* per unit of region r's *trad* import composite. This specification assumes that the import composite is equal among consumer and producer households in region r.

The traded goods are often supposed to have different prices depending on whether they are produced for domestic use or for export. The revenue per unit of a traded good is decomposed into the revenue of selling this good on the domestic market at a domestic price $p^D_{trad,r}$, and the revenue of selling the composite export good abroad at an export price $p^X_{trad,r}$. The exported goods are sold on the world market.

$$p_{trad,r} = \text{CET}\left(p^D_{trad,r}, p^X_{trad,r}\right) \quad \perp \quad y_{s,r} \tag{9}$$

We can now obtain the revenue maximizing amounts $a^X_{trad,r}$ of region r's good *trad* exported to the world market or $a^D_{trad,r}$ produced for domestic sales from the first-order derivative of the unit revenue function in equation (9) to the corresponding price.

The export of region r's good *trad* incurs a transport margin, depending on the region $\bar{r}$ where it is exported to. The value of the transport margin to region $\bar{r}$ is assumed to be a fixed fraction of the total export value of region r's good *trad*. We therefore use a CD function to determine the price region r's good *trad* when arriving in region $\bar{r}$:

$$p^{cif}_{trad,r}(\bar{r}) = \text{LEONTIEF}\left((1+\text{txs}_{trad,r}(\bar{r}))p^X_{trad,r} + p^T_m \mid \bar{r} \in \mathcal{R} \setminus \{r\}\right) \quad \perp \quad a^X_{trad,r}(\bar{r}) \tag{10}$$

Market clearing equations. While the prices of each tradeable good are determined by the marginal cost to produce these goods, the prices of the endowments are such that they clear the market for the underlying endowment good. Land is taken as specific to the production sector in which it is used as an input good. GTAP currently only considers agricultural land. Similar to the land market, GTAP assumes a market for natural resources which also is specific to the production sector in which it is used. The labour market is assumed to be region specific. GTAP assumes that capital is not sector-specific, and only regional mobile.

$$\begin{aligned}
L_{s,r} &= a_{s,r}(\texttt{land})y_{s,r} && \perp \quad p^F_{s,r}(\texttt{land}),\\
R_{s,r} &= a_{s,r}(\texttt{natres})y_{s,r} && \perp \quad p^F_{s,r}(\texttt{natres}),\\
\mathit{SkLab}_r &= \textstyle\sum_s a_{s,r}(\texttt{SkLab})y_{s,r} && \perp \quad p_r(\texttt{SkLab}),\\
\mathit{UnSkLab}_r &= \textstyle\sum_s a_{s,r}(\texttt{UnSkLab})y_{s,r} && \perp \quad p_r(\texttt{UnSkLab}),\\
K_r &= \textstyle\sum_s a_{s,r}(\texttt{capital})y_{s,r} && \perp \quad p_r(\texttt{capital}).
\end{aligned} \tag{11}$$

Domestic production of good *trad* in region r suffices to cover total domestic demand by the private and government households as well as the total demand for this good as an intermediate in the other production sectors in region r.

$$a^D_{trad,r} y_{trad,r} = \left[c^P_{trad,r}(\mathtt{d}) + c^G_{trad,r}(\mathtt{d})\right] u_r + \textstyle\sum_s a^F_{trad,s,r}(\mathtt{d}) y_{s,r} \quad \perp \quad p^D_{trad,r}. \tag{12}$$

Total imports of good *trad* into region r equals the demand for this import good by private and government households as well as the total demand for this imported good as an intermediate in the production sectors in region r.

$$y^m_{trad,r} = \left[c^P_{trad,r}(\mathtt{i}) + c^G_{trad,r}(\mathtt{i})\right] u_r + \textstyle\sum_{trad} a^F_{trad,s,r}(\mathtt{i}) y_{s,r} \quad \perp \quad p^m_{trad,r}. \tag{13}$$

Total exports of good *trad* of region r equal the demand for this good as import in other regions $\bar{r}$ and, in case the good is a margin good, as a part of transport demand.

$$a^X_{trad,r}(\bar{r}) y_{trad,r} = b_{trad,r}(\bar{r}) y^m_{trad,\bar{r}} + a^T_{trad,r} y^T_{trad,r} \quad \perp \quad p^{cif}_{trad,r}(\bar{r}) \tag{14}$$

The output of the transport sector m is determined by the use of this transport mode in the export of each good between regions.

$$y^T_m = \textstyle\sum_{trad} \sum_r \sum_{\bar{r}} \phi_{trad,r,\bar{r}}(m) a^X_{trad,r}(\bar{r}) y_{trad,r} \quad \perp \quad p^T_m \tag{15}$$

where $\phi_{trad,r,\bar{r}}(m)$ denotes the share of transport mode m in the export of good *trad* from region r to $\bar{r}$. This parameter is determined independently from the SAM.

The production of capital goods services in region r suffices to fulfill the demand for region r's capital goods by the global bank.

$$y_{\mathtt{cgds,r}} = a^I(r) I \quad \perp \quad p_{\mathtt{cgds},r}. \tag{16}$$

The output in investment goods by the global bank covers all demand originating from the savings by each regional household. This is a form of closure of the model. We could also have chosen to let investments equal savings on the regional level instead of global level.

$$I = \textstyle\sum_r u^I_r \quad \perp \quad p^I \tag{17}$$

Total expenditure in each region, $p^U_r u_r$ should in equilibrium equal this region's total income. Let M_r denote region r's real income, using p^U_r as this region's price index. Then,

$$u_r = M_r \quad \perp \quad p_r^u \tag{18}$$

Equilibrium: We assume that there is perfect competition on the markets. In our model this means that the prices of these goods equal their marginal costs of production. The markets are then cleared by the output levels of the production sectors. We take the capital market and energy markets to be global markets, while the labour market is a regional market.

In the vector p we collect all the producer prices and in the vector q we collect all the consumer prices of the goods in the economy. We can split these vectors into a part p_G of prices referring to goods, and p_ω of prices referring to the consumption household's endowments in production factors. In the vector y we collect all the activity levels of the production sectors. Output of each production sector is either used for domestic production or for exports, according to a matrix H. Total output for domestic and export purposes is then given by $H(p)y$. From the production tree of each production sector, we construct an input-output matrix $A(p)$ where each column refers to the input-output vector that minimizes the cost of producing one unit of this production sector's output good at prices p in the economy. We split the input-output matrix $A(p)$ into a submatrix $A_G(p)$ referring to the goods input-output submatrix, and $A_\omega(p)$ referring to the production factor inputs. In the vector u we collect all the utility levels of the regional households in the economy. From the consumption tree of each regional household, we construct a consumption matrix $C(p)$ where each column refers to the consumption vector that minimizes the expenditure on goods to obtain one unit of utility at prices p in the economy. Let ω_r denote consumer r's endowment vector of production factors. Take $\omega = (\omega_1, \ldots, \omega_R)$. Total expenditure, $p_r^U u_r$, should equal region r's income, $p_{\omega,r}^\top \omega_r$, according to this region's budget constraint. We define real income $M_r(p_{\omega,r})$ equal to $p_{\omega,r}^\top \omega_r / p_r^U$. Take $M(p_\omega) = (M_1(p_{\omega,1}), \ldots, M_R(p_{\omega,R}))^\top$.

Definition 1 *The producer prices p^*, activity levels y^*, and utility levels u^* constitute an equilibrium if,*

1) (goods market clearing) *the activity levels and consumption levels are such that demand is met by total supply for each good:*

$$C(p^*)u^* - (H(p^*) - A_G(p^*))y^* \leq 0 \quad \perp \quad p_G^*. \tag{19}$$

(factor market clearing) *factor prices p_ω^* are such that demand for each production factor is met by its supply:*

$$\omega - A_\omega(p^*))y^* \leq 0 \quad \perp \quad p^*_\omega. \tag{20}$$

2) (income) *for each consumer r, total expenditure on goods equals total income obtained from selling its factor endowments:*

$$u^* - M(p^*_\omega) \leq 0 \quad \perp \quad p^{U*}. \tag{21}$$

3) (zero profits) *the producer price of each good is determined by the minimum cost to produce one unit of this good:*

$$p^{*\top}\left[H(p^*) - A(p^*)\right] \leq 0 \quad \perp \quad y^*, \tag{22}$$

and the consumer price of each good is determined by the minimum expenditure to obtain one unit of this good:

$$p_G^{*\top} C(p^*_G) \geq p^{U*} \quad \perp \quad u^*. \tag{23}$$

Numeraire: Due to the homogeneity of degree zero in the excess demand and the supply functions in the equilibrium equations, any positive multiple of an equilibrium price vector will result in an equilibrium. We therefore have to choose a numeraire good. We could choose one of the goods as the numeraire good, or fix a certain price index thereby imposing an extra equation on the equilibrium. GTAP chooses the price of the savings good as its numeraire.

Computation of an equilibrium: Ginsburgh and Keyzer (1997) propagate the use of a so-called *Negishi format* to compute an equilibrium. The Negishi theorem, Negishi (1960), shows that a competitive equilibrium can be represented through a welfare optimum with nonzero welfare weights α_r, which are such that each consumer r satisfies his budget constraint. We define the Negishi format in Definition 2.

Definition 2 *The* Negishi format *is defined as the welfare optimum:*

$$\begin{array}{rll} W(\alpha) = & \max_{x_r \geq 0, \forall_r, y_j \geq 0, \forall_j} & \sum_i \alpha_r U_r(x_r), \\ & s.t. & \sum_r x_r - \sum_j y_j \leq \sum_r \omega_r \\ & & y_j \in Y_j, \end{array}$$

with welfare weights α such that the budget constraints

$$p^\top x_r^* = p^\top \omega_r + \sum_j \theta_{rj} \Pi_j(p),$$

with x_r^* solving for the welfare optimum, hold for every consumer r.

The Negishi theorem, Negishi (1960), can now be stated as

Theorem 3.1 *Under the assumption that the consumer household's preferences can be described by a continuous, strictly concave, nonsatiated utility function U_r that satisfies $U_r(0) = 0$, whose endowments in each good are strictly positive, and the assumption that each producer has a compact and convex production set containing the possibility of inaction, there exists nonzero welfare weights α^* in Definition 2 such that the resulting allocation is a competitive equilibrium.*

The Negishi format provides a direct link to welfare analysis and the format makes it possible to use weaker assumptions on the production technology. Sometimes, e.g. with externalities or nonconvexities, it is easier to formulate a centralized welfare program such as the Negishi format than to specify its decentralized counterpart, the excess demand or CGE format. Notice that, choosing the Negishi format implies that only primal forms can be used.

Writing the equilibrium problem stated in Definition 1 in the Negishi format gives the following optimization problem to solve in a price vector $\bar{p}$:

$$\begin{array}{lll} W(\alpha) = & \max_{u,y} & \sum_r \alpha_r u_r, \\ & \text{s.t.} & u - M(\bar{p}) \leq 0 \\ & & C(\bar{p})u - (H(\bar{p}) - A_G(\bar{p}))y \leq 0 \quad \text{(goods market equilibrium)} \\ & & -\omega + A_\omega(\bar{p})y \leq 0 \qquad\qquad\quad\; \text{(factor market equilibrium)} \\ & & u,\ y \geq 0 \end{array} \tag{24}$$

Notice that the prices p are the dual variables to the market equilibrium constraints. Solving optimization problem (24) results in an optimal solution (u^*, y^*, p^*) such that $p^{*\top}[H(\bar{p} - A(\bar{p}) \leq 0$ and $p^{*\top} C_r(\bar{p}) - \bar{p}_r^U \geq \alpha_r$ for any consumer r. While solving (24), we should only consider prices $p \in \mathcal{S}_A^n$ to prevent (24) from becoming unsolvable.

The equilibrium in an economy with constant returns to scale production technologies is now computed by solving the optimization problem (24) recursively. Take an initial value for α^0, for example give each consumer the same weight $(1/R) * 100$. Choose an initial price vector $\bar{p}^0$. In standard CGE modelling, $\bar{p}^0$ could equal e. For these initial values, we can solve optimization problem (24), to obtain an equilibrium (u^{*0}, y^{*0}) and associated prices p^{*0} as the values of the dual variables to the inequalities defining the feasible set. For this value, we can compare expenditure $p^{*0}C(\bar{p})$ with income $p^{*0}_{\Omega}\omega^r$ for each consumer r. In case expenditure, exceeds consumer r's income, then he obviously was assigned a too high value for α_r and this value should be reduced, and v.v.. This adjustments provides a new value for α. Take $\bar{p}$ equal to p^{*0}. For the new values, we can compute a new solution to equation (24).

Carbon emissions: CGE modelling practice associates carbon emissions with the demand for fossil fuels by the economy's production sectors. For each production sector s in region r, it defines a coefficient $\mathrm{co_2shr}(f, s, r)$ associated with the sector's demand $a_{s,r}(f)$ for fossil fuel f. Define, for any set $\mathcal{R}$, the matrix $\mathrm{CO}_2^F(\mathcal{R})$ such that $\mathrm{CO}_2^F(s, r) = \sum_{f\in\mathcal{S}_f}\sum_s \mathrm{co_2shr}(f, s, r)a_{s,r}(f)$ if $r \in \mathcal{R}$ and $\mathrm{CO}_2^F(s, r) = 0$ otherwise. Each of these coefficients measure the amount of carbon emissions per unit of fossil fuel use.

Let us define an emission permit by the amount of carbon emissions that it allows to the owner. Then we can refer to the total emissions of a production sector as the amount of emission permits demanded by this sector. We assume that there is a market for emission permits among the developed regions. Region AnnexB has been provided with an initial endowment of emission permits E under the Annex B of the Kyoto Protocol. These endowments refer to the emissions allowed to the developed regions under the Kyoto Protocol. On the emission permit market, there exists a price p_E that equilibrates the market, i.e. such that

$$-E_1 + e^\top \mathrm{CO}_2^F(\{1\})y = 0 \quad \perp \quad p_E \tag{25}$$

with e the $\mid \mathcal{S} \mid$-dimensional vector with all unit components.

We have assumed that the endowments of emission permits are allocated to the regional households of the developed regions. This allocation adds an income of $p_E E_1$ to the developed region. Real income for the Annex B region's consumer then becomes $M_1(p_{\omega,1}, p_E; E_1) = {}^{p_{\omega,1}\omega_1 + p_E E_1}/_{p^U_1}$. There is a lively debate on how to reallocate the permit endowments over the different housholds in each regions. There is mentioning of grandfathering, i.e. allocation according to some rule, or auctioning.

The emissions of each production sector can be included into the nested cost functions described above by letting the cost of each fossil fuel input $f \in \mathcal{S}_f$ consist of the cost on this fossil fuel and the cost on emissions related to the use of this fossil fuel using a leontief cost function with a parameter equal to ${}^1/_{\mathrm{co_2shr}(f,s,r)}$ associated with the fossil fuel use in production input, and a parameter equal to 1 associated with the fossil fuel use itself in these sectors:

$$p^F_{s,1}(f) = \text{LEONTIEF}(p^F_{s,1}(f), p_E). \tag{26}$$

Notice that the formulation in equation (26) is equivalent with putting a tax equal to ${}^{p_E}/_{\mathrm{CO_2shr}(f,s,1)}$ on the use of fossil fuel f in production input.

The economy's CO_2 emissions add to the CO_2 concentrations in the Earth's atmosphere. These increased concentrations are responsible for changes in the climate indicated by changes in the mean global temperature, regional precipitation, sun radiation, and sea level rises. If $\bar{E}$ denotes existing concentrations of CO_2 in the atmosphere, then economic activities will increase these concentrations to a level of $\bar{E} + e^\top \mathrm{CO}_2^F(\{1,2,3\})y$ through its intensive use of fossil fuels in the production processes. Climate models are applied to use these concentrations and compute its effects on the global climate. The consequences of climate change are expected to impose significant costs on the current economy in the form of reduced productivity or loss of land, the decrease in population due to changed birth and mortality rates following health risks, or changes in vegetation. We define what is commonly known as a damage function, but would be better expressed as an impact function, D, that relates the economy's total CO_2 emissions with the net damage on the economy. These damages are often measured as a percentage of real income, but we temporarily follow a more general approach. D is a composite of a reduced form representing a climate model, and a traditional damage function. We choose a convex function D such that $D' > 0$, assuming that increased emissions cause more damage than benefits to the economy.

We distinguish between a damage on endowments matrix, D_ω, and the damage to productivity matrices, D_A and D_C. All these matrices are determined as functions of global emissions, $e^\top \mathrm{CO}_2^F(\{1,2,3\})y^*$, and result in regional, c.q. sectoral damages. We can rewrite the equilibrium equations in Definition 1 to include such damages and we thus obtain the following equilibrium problem: find utility levels u^*, production levels y^*, and prices p^* such that

$$\begin{array}{rcl} (E - D_c) \odot C(p^*)u^* - (H(p^*) - (E - D_{A_G}) \odot A_G(p^*))y^* \leq 0 & \perp & p^*_G \\ (E - D_\omega) \odot \omega - (E - D_{A_\omega}) \odot A_\omega(p^*)y^* \leq 0 & \perp & p^*_\omega \\ u^* - (E - D_\omega) \odot M(p^*_\omega) \leq 0 & \perp & p^*_U \end{array} \tag{27}$$

where E denotes the matrix with all components equal to one and $\odot$ is a matrix operator refering to component-wise matrix multiplication, i.e. $A \odot B = [a_{ij} \times b_{ij}]$. We assume that the damage functions D_ω, D_A, and D_C are chosen such that the underlying assumptions in this exchange economy with constant returns to scale production remain valid.

4 Bargaining for permits

The bargaining model applied in Houba (2005) extends the alternating offers model with discounting in Rubinstein (1982) by replacing the dollar by a multi-dimensional bundle of endowments in a two-person economy, called the economic environment in Roemer (1988). The two players are the developed regions AnnexB, and the rapidly developing region CHIND on the other hand. These two players negotiate the amount of permits allocated as an endowment to each of them. The allocation of the other endowments of production factors over the regional households are assumed to remain the same. The subject of negotiations is a feasible allocation in the economy.

We index the set $\mathcal{T}$ of bargaining rounds with a time index t. We often refer to the developed regions, or Annex B regions, as player 1 and the developing world, here China and India together, as player 2. At t odd, the developed regions propose a feasible allocation of emission permits $\epsilon^t = (\epsilon^{1,t}, \epsilon^{2,t})$, with $\epsilon^{i,t}$ denoting the total amount of emission permits allocated to player $i \in \{1, 2\}$. Then the developing world either ends the negotiations by accepting the proposal or prolongs the negotiations by rejecting it. If rejected, then the probability of a next (even) round is $e^{-r_2\Delta}$, $r_2 \geq 0$, and $\Delta \geq 0$, which implies a probability of breakdown $1 - e^{-r_2\Delta}$. At t even, the developing world proposes the feasible allocation $\epsilon^t = (\epsilon^{1,t}, \epsilon^{2,t})$, which is either accepted or rejected by the developed world. The probability of the next (odd) round is $e^{-r_1\Delta}$, $r_1 \geq 0$. This alternating offers procedure represents a game in extensive form with perfect information and, therefore, the subgame perfect equilibrium (SPE) concept is appropriate.

The bargaining problem in utility presentation is denoted as (S, d), with $S \subset \mathbb{R}^2$ the nonempty, compact and convex set of feasible utility pairs (u_1, u_2), the disagreement point $d \in S$, and the existence of feasible utility pairs $u \in S$ such that $u > d$. In the case of a disagreement concerning the allocation of permits, only the developed regions in Annex B engage in emission permit trading using the permit endowments allocated under the Kyoto Protocol. The disagreement point d consists of the utility levels (u_1^d, u_2^d) obtained when only the Annex B regions trade. This represents the original equilibrium to which we refer with the superindex 'd'. Within the model, $u_1^d = M_1(p_{\omega,1}^d, p_E^d; E)$ and $u_2^d = M_2(p_{\omega,2}^d, p_E^d; 0)$, refering to the Kyoto Protocol allocation of permits.

With each allocation $\epsilon = (\epsilon_1, \epsilon_2)$ of permit endowments over the developed and developing regions, we can associate an equilibrium $(p^{\epsilon*}, y^{\epsilon*}, u^{\epsilon*})$ in Definition 1. So, we represent the alternating offers procedure in terms of utility but as a function of the proposed permit endowments. This function is given by the computed equilibrium in the GTAP-E model.

Let $\epsilon = (\epsilon_1, \epsilon_2)$ denote the proposed allocation of permits by the developed regions, and let $\nu = (\nu_1, \nu_2)$ denote the developing region's proposal of allocating the permits over the Annex B region and China and India. In any stationary subgame perfect equilibrium, player 1, or the Annex B region, accepts proposal ν if and only if $u_1^\nu \geq (1 - e^{-r_1\Delta})u_1^d + e^{-r_1\Delta}u_1^\epsilon$. The developing region `CHIND` will accept any proposal ϵ, if $u_2^\epsilon \geq (1 - e^{-r_2\Delta})u_2^d + e^{-r_2\Delta}u_2^\nu$. Allocations of permits that will be acceptable to both players under this definition, are called *individually rational*.

$\hat{\mathcal{M}}(r_1, r_2, \Delta)$ provides the subgame perfect proposals for both players with associated utility levels. It is determined as follows, see also Houba (2005).

$$
\begin{aligned}
\hat{\mathcal{M}}(r_1, r_2, \Delta) = \quad & \arg\max_{u, u^\epsilon, y^\epsilon, D^\epsilon, u^\nu, y^\nu, D^\nu, \epsilon, \nu} u_1^{\frac{r_2}{r_1+r_2}} u_2^{\frac{r_1}{r_1+r_2}} \\
\\
\text{s.t. } & u^\epsilon - (E - D_\omega^\epsilon) \odot M(\bar{p}_\omega^\epsilon, \bar{p}_E^\epsilon; \epsilon) \leq 0 && \perp \; p_U^\epsilon \\
& (E - D_C^\epsilon) \odot C(\bar{p}^\epsilon)u^\epsilon - (H(\bar{p}^\epsilon) - (E - D_{A_G}^\epsilon) \odot A_G(\bar{p}^\epsilon))y^\epsilon \leq 0 && \perp \; p_G^\epsilon \\
& -(E - D_\omega^\epsilon) \odot \omega + (E - D_{A_\omega}^\epsilon) \odot A_\omega(\bar{p}^\epsilon)y^\epsilon \leq 0 && \perp \; p_\omega^\epsilon \\
& -(\epsilon_1 + \epsilon_2) + e^\top \mathrm{CO}_2^F(\{1,2\})y^\epsilon \leq 0 && \perp \; p_E^\epsilon \\
\\
& u^\nu - (E - D_\omega^\nu) \odot M(\bar{p}_\omega^\nu, \bar{p}_E^\nu; \nu) \leq 0 && \perp \; p_U^\nu \\
& (E - D_C^\nu) \odot C(\bar{p}^\nu)u^\nu - (H(\bar{p}^\nu) - (E - D_{A_G}^\nu) \odot A_G(\bar{p}^\nu))y^\nu \leq 0 && \perp \; p_G^\nu \\
& -(E - D_\omega^\nu) \odot \omega + (E - D_{A_\omega}^\nu) \odot A_\omega(\bar{p}^\nu)y^\nu \leq 0 && \perp \; p_\omega^\nu \\
& -(\nu_1 + \nu_2) + e^\top \mathrm{CO}_2^F(\{1,2\})y^\nu \leq 0 && \perp \; p_E^\nu \\
\\
& \Phi_\omega(e^\top \mathrm{CO}_2^F(\{1,2,3\})y^\epsilon) - D_\omega^\epsilon \leq 0 && \perp \; \Gamma_\omega^\epsilon \\
& \Phi_C(e^\top \mathrm{CO}_2^F(\{1,2,3\})y^\epsilon) - D_C^\epsilon \leq 0 && \perp \; \Gamma_C^\epsilon \\
& \Phi_A(e^\top \mathrm{CO}_2^F(\{1,2,3\})y^\epsilon) - D_A^\epsilon \leq 0 && \perp \; \Gamma_A^\epsilon \\
\\
& \Phi_\omega(e^\top \mathrm{CO}_2^F(\{1,2,3\})y^\nu) - D_\omega^\nu \leq 0 && \perp \; \Gamma_\omega^\nu \\
& \Phi_C(e^\top \mathrm{CO}_2^F(\{1,2,3\})y^\nu) - D_C^\nu \leq 0 && \perp \; \Gamma_C^\nu \\
& \Phi_A(e^\top \mathrm{CO}_2^F(\{1,2,3\})y^\nu) - D_A^\nu \leq 0 && \perp \; \Gamma_A^\nu \\
\\
& u_1 \leq u_1^\epsilon - u_1^d && \perp \; \mu_1 \\
& u_2 \leq u_2^\nu - u_2^d && \perp \; \mu_2 \\
& (1 - e^{-r_1\Delta})u_1^d + e^{-r_1\Delta}u_1 \leq u_1^\nu - u_1^d && \perp \; \lambda_1 \\
& (1 - e^{-r_2\Delta})u_2^d + e^{-r_2\Delta}u_2 \leq u_2^\epsilon - u_2^d && \perp \; \lambda_2 \\
\\
& u, u^\epsilon, y^\epsilon, D^\epsilon, u^\nu, y^\nu, D^\nu, \epsilon, \nu \geq 0
\end{aligned}
\tag{28}
$$

where Φ_ω, Φ_C, and Φ_A are damage functions that relate total global emissions to damages expressed as a percentage of income, consumption, and input efficiency respectively. Notice that these functions contain economic variables, output levels, as their arguments. To prevent problems with possible nonconvexities, we therefore introduce the variables D_ω, D_C, and D_A for each proposal.

In Table 4, we depicted the first results of a simulation using a value of 0.99 for the probability $e^{-r\Delta} = 0.99$ with which the next round in the bargaining process takes place. These results concern the calculated allocation of emission permits over the Annex B and China and India regions under proposal A by the Annex B regions and proposal B by China and India. Table 5 provides the outcomes of this simulation with respect to the consequences on welfare in these regions.

$e^{-r\Delta} = 0.99$	Benchmark emissions	Permit Endowment A $p^{CO_2} = 0.0003$	Permit Endowment B $p^{CO_2} = 0.0003$
China and India	227004	226794	221313
EU	687296	685499	680845
Rest of Annex1	720412	716824	716824
USA	623286	622516	620066

Table 4: Optimal allocation of emission permits (in $GtCO_2$).

The endowments allocated to the regions under both proposals do not differ that much from the benchmark emissions, hence creating a low excess demand for emission permits, and subsequently, the permit price is relatively low. Permit prices are equal under both proposals confirming the symmetric aspects of the bargaining game. The rather small differences between benchmark emissions and allocated permitted emissions is due to the choice of damage functions. Choosing damage functions that have a larger climate impact on the economy undoubtedly will result in a lower allocation of permits to the regions under both proposals as more benefit is to be taken out of this.

A well-known result for the subgame perfect equilibrium in the alternating offers model is that it is advantageous to be the proposing player. This means that each player gets a better deal according to his own equilibrium proposal compared to what he gets from accepting his opponent's equilibrium proposal. In terms of dividing a single dollar, a proposing player gets a larger share if he proposes compared to what is offered to him, if he is the responding player. This extends to the current setting. The intuition is that the responding player compares immediate acceptance with one period of inefficient delay followed by his proposed (efficient) agreement.

Table 5 gives an overview of the welfare effects of implementing both proposals. Depicted are the benchmark levels of indirect utility, the utility of the regions under Proposal A, and the utilities of the regions under Proposal B. Furthermore, we calculated the minimal increases in utility u for each region that participates in the bargaining process. As indicated by the values of the variable u, the bargaining countries all win a welfare gain resulting from the bargaining process.

$e^{-r\Delta} = 0.99$	u^d	u	u^A	u^B
China and India	1133455	97876	1237213	1242674
EU	7011843	111742	7763476	7725894
Rest of Annex1	5744384	366653	6209859	6695345
USA	7218402	287770	7530327	7604397
Rest of the World	4587013	0	4886771	4748858

Table 5: Indirect utilities.

China and India gain more in welfare under there own proposal than when accepting player A's proposal. This is in concordance with the aformentioned result obtained from the divide one dollar game. As to player A, only the EU would gain, contrary to the other regions. This might be the consequence of the impact of damage functions on the respective regions. The rest of the world, or the underdeveloped world is also gaining from the bargaining. This is due to trade effects, when energy intensive products of the Annex B and China and India become relatively more expensive causing consumers and producers to take more of these regions' alternatives into their product mix.

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