Optimal Allocation of EU Emission Allowances under Imperfect Competition

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Abstract: In an effort to regulate green house gas emissions the EU makes use of a hybrid approach, which differentiates between the industry sector, regulated by emissions trading, and the traffic and households sector, regulated by different policies. A common assumption in literature is that an optimal allocation of emissions in the European system must satisfy equal marginal emission abatement costs for emitters in all sectors.

This paper shows that in case of at least one oligopolistic industry cost efficiency does not inevitably require equal marginal emission abatement costs across sectors. To achieve an optimal solution, marginal abatement costs of all emitters in the industry sector must be above or below marginal costs of other emitters, depending on the degree of competition on the commodity market and emitters' asymmetry. This has a direct impact on the optimal National Allocation Plan; e.g. an allocation favoring emitters from the industry sector can be necessary to minimize overall costs.

Keywords: European Emissions Trading Scheme, Permits Allocation, Imperfect Competition, Cost Efficiency

JEL classification: L13, Q28, Q52

1 Introduction

Due to the ratification of the Kyoto protocol the EU is committed to reduce its greenhouse gas emissions during 2008-2012 to 8% less than 1990 emissions. In order to achieve this reduction goal at minimal costs the EU makes use of a hybrid regulatory approach. It distinguishes between firms using carbon dioxide intensive technologies, and all other kinds of emitters like traffic, households, or trade. Carbon dioxide intensive firms are assigned to a sector regulated by an emissions trading scheme (referred to as *Tr*-sector).[1] All other emitters are subjected to a mix of different policies instead of emissions trading (referred to as *NTr*-sector). For example emissions of the German traffic sector are controlled through petrol taxes and tolls for freight vehicles; emissions of the German households sector are reduced e.g. through subsidies for environmental standards.[2] Due to EU regulation the amount of emissions for the emissions trading sector and the allocation of emissions to firms in the trading scheme must be determined in the so called National Allocation Plan (NAP), which differs between a macro- and a micro-plan. The macro-plan assigns a portion of the green house gas emissions allowed by the Kyoto protocol to the emissions trading sector. The remaining emissions not assigned to the *Tr*-sector are available for the *NTr*-sector. The micro-plan specifies the initial allocation of emissions to the individual firms in the *Tr*-sector. The mechanism mainly used for this allocation is grandfathering, i.e. firms get their initial emission permits for free.[3]

Recent literature on the actually decided allocation of emissions states that the trading sector receives too many emission rights. Böhringer et. al (2005) assess that with the current allocation marginal emission abatement costs for firms in the trading sector are much lower than marginal abatement costs for the non-trading sector. They, as well as Rogge et al. (2006), reason that such an allocation results in unnecessarily high costs for the restriction of carbon dioxide, which can be reduced by a shift of emissions from the *TR*- to the *NTr*-sector.

In contrast to this we show that an allocation with lower marginal abatement costs for emitters in the *Tr*-sector need not inevitably lead to higher costs for reducing emissions. Furthermore, cost efficiency can even require lower marginal abatement costs for the members of the *Tr*-sector. These results are derived from investigating the impact of imperfect competitive commodity markets on the costs minimizing allocation of emissions among sectors. Therefore, results from the literature on externalities are applied to the framework of the European emissions trading scheme. It is well known that only in the case of perfect competition cost efficiency requires that all emitters' marginal abatement costs are equal to the marginal environmental damage of emissions. But e.g. in case the externality is produced by only one single polluter being a monopolist on his commodity market this result cannot be maintained and marginal abatement costs must be less than the marginal environmental damage (Buchanan [1969], Barnett [1980],

[1]See EC (2003).

[2]See NAP (2004).

[3]Additional to a free allocation, the directive also allows to auction up to 5% (10%) of the *Tr*-sector emissions for the periode 2005-2007 (2008-2012); in the first periode most member states made no use of this option (see e.g. Betz et al., 2004 and Buchner et al., 2006).

Misiolek [1980], Baumol and Oates [1988]). Innes et al. (1991) show that even if the externality is emitted by many firms, and one of these firms is monopolistic, all firms' marginal abatement costs must be lower than marginal damage, in order to achieve a cost efficient solution in a pollution tax system.

Simpson (1995), Shaffer (1995) and Carlsson (2000) investigate the impact of oligopoly competition on the cost efficient abatement of emissions. They show that in a pollution tax system marginal abatement costs must either be lower or higher than marginal environmental damage, depending on how heterogeneous firms are. All these results are based on the fact that an environmental policy instrument is also used to correct firms' market distorting behavior. This can lead to a higher total welfare compared to an emission tax used solely to minimize emission abatement costs. In a more general way Schott (2006) states that with imperfect competition the first best solution can never be achieved with only one policy instrument. For n heterogeneous firms emitting one externality, $n+1$ different policy instruments are required to internalize the externality and to correct market distorting behavior.

In this paper the impact of imperfect competition on the welfare maximizing allocation of permits to the *Tr*- and *NTr*-sector is examined. The analysis mainly based on the models of Simpson (1995) and Innes et al. (1991), which are extended for conjectural variations and are applied to the regulatory framework of the European emissions trading scheme. The effects of an oligopoly commodity market on other competitive industries are also investigated. The objective of this work is to derive economically reasonable suggestions for the NAPs and to show that equal marginal abatement costs for all emitters of the different sectors not necessarily lead to overall cost efficiency.

The paper is organized as follows: Section 2 defines general assumptions for the whole paper. In Section 3 firms behavior in case of perfect competitive commodity markets is investigated. In Section 4 the impact of imperfect competitive product markets on firms' abatement and production decisions are analyzed. The results of the Sections 3 and 4 are used in Section 5 to define the cost function of the permit trading sector. In Section 6 the cost minimizing allocation of permits between the different emitters is determined. Section 7 concludes.

2 General Assumptions

Due to the Kyoto protocol, we consider a country which committed itself to restrict its emission level to an amount $\bar{E} \in \mathbb{R}_+$. This country has two emission causing sectors: the industry sector, and the transport and households sector. Firms of the industry sector are committed to participate in an emission permit trading scheme to reduce emissions (*Tr*-sector). The transport and households sector is regulated by a policy mix (*NTr*-sector). The regulator allocates a certain amount of emissions to each sector; in terms of the European emission trading directive this allocation is called macro-plan. The objective of the macro-plan is to portion the total amount of emissions in a way that minimizes society's costs for achieving the reduction goal. As stated in the following assumption, the sum of both sectors emissions must not exceed the exogenously given maximal emission level.

Assumption 1 (Emission cap): *A country's total emission level is given by* $E_{Tr} + E_{NTr}$ *and must not exceed a maximum amount* $\bar{E}$.

NTr-Sector

Depending on the emissions level of the *NTr*-sector costs emerge either from avoiding emissions by less consumption, or abating emissions by applying new technologies. The costs of the *NTr*-sector are only considered in an aggregated way; we assume an optimal allocation of emissions in this sector and therefore that all individual emitters have marginal costs equal to marginal costs of the whole sector.[4] For the relevant range of parameters less emissions induce higher costs and further, cheapest abatement measures are always used first.

Assumption 2 (Non trading sector's costs): *Non trading sector's abatement costs function* $F_{NTr}(E_{NTr}) : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}$ *is twice differentiable, decreasing, and convex.*

Tr-Sector

Let $N = \{1, ..., n\}$ be the set of firms participating in the emissions trading scheme. Each firm i ($i \in N$) produces an amount of a certain commodity, $q_i \in \mathbb{R}_{\geq 0}$ ($q := (q_1, ..., q_n)$), and causes an amount of emissions $e_i \in \mathbb{R}_{\geq 0}$ ($e := (e_1, ..., e_n)$). Total emissions of the *Tr*-sector are $E_{Tr} = \sum_{i=1}^{n} e_i$.

Assumption 3 (Firms' costs functions):[5] *Firm i's cost function is given by* $C^i(q_i, e_i) : \mathbb{R}^2_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$ *and satisfies the following conditions:*[6]

- $C^i_{q_i}(\cdot) > 0, C^i_{q_i q_i}(\cdot) > 0, C^i_{e_i e_i}(\cdot) > 0, C^i_{q_i e_i}(\cdot) < 0$
- $C^i_{q_i q_i}(\cdot) C^i_{e_i e_i}(\cdot) - C^i_{q_i e_i}(\cdot)^2 > 0$
- *For all* q_i *there exists an emissions level* $\bar{e}_i(q_i)$ *such that* $C^i_{e_i}(q_i, \bar{e}_i(q_i)) = 0$, *and* $C^i_{e_i}(q_i, e_i) < 0$ *if* $e_i < \bar{e}_i(q_i)$, *and* $C^i_{e_i}(q_i, e_i) \geq 0$ *if* $e_i > \bar{e}_i(q_i)$

As quite common in economics, firms marginal production costs $C^i_{q_i}(\cdot)$ are positive and increase in output. The possibility of abating emissions is already included in the cost function, since it depends also on the firm's emission level. Similar to the *NTr*-sector we assume that cheapest abatement measures are always realized first and that the cost function is convex in emissions. Further, marginal production costs decrease

[4] See e.g. Montgomery (1972).

[5] The assumption on firms' costs functions was coined by Requate (2005).

[6] We define $F_X(X, Y, \ldots)$, $F_Y(X, Y, \ldots)$ etc. as the (partial) derivatives of the function $F(X, Y, \ldots)$. Analogously, second derivatives are abbreviated.

if firms are allowed to emit more (i.e. $C^i_{q_i e_i}(\cdot) < 0$). Assumption 3 also ensures that independently of the kind of competition the second order condition for an interior solution of firm's profit maximization is satisfied, since the Hessian of the cost function is positive definite. Moreover, it is assumed that a cost minimizing emission level in the absence of any regulation $\bar{e}_i(q_i)$ exists, i.e. marginal emission abatement costs are zero $(-C^i_{e_i}(q_i, \bar{e}_i(q_i)) = 0)$.

The revenue function of a firm depends on the kind of competition the firm faces. In case of a global market we assume that the commodity price is exogenously given and that firms' behavior does not influence it. Therefore, firms' marginal revenue is constant and equals the commodity price. In case of a national market a commodity price depending on industry's output can be assumed. Global, as well as national commodity markets are of relevance for the European emissions trading scheme. Hence, both cases are examined in this work. In our model the impact of imperfect competition on a national commodity market is investigated by firm 1 and 2 competing in a duopolistic market with homogeneous goods. Firms 3 to n compete with their (heterogeneous) products on global markets and therefore are modeled as price takers.

Assumption 4 (Firms' revenue functions): *The revenue functions of firms 1 and 2 are* $R_j(q_1, q_2) := P_c(Q) \cdot q_j := P_c(q_1 + q_2) \cdot q_j$ *(*$j \in \{1, 2\}$*)with*

- $P_c(Q)$ *twice differentiable,*
- $P'_c(Q) < 0$.

The revenue functions of firms 3 to n *are* $R_k(q_k) := P_k \cdot q_k$ *with* $P_k \in \mathbb{R}_+$ *being constant for all* $k \in \{3, ..., n\}$.

Comparable to the European emissions trading scheme grandfathering is used as mechanism for the initial allocation in the Tr-sector. We assume that the reduction goal is stricter than business as usual emissions and that firms use all their permits. Hence, the sum of initially allocated permits must equal the Tr-sector's emission level $(E_{Tr} = \sum_{i=1}^{n} e_i^0)$. Emitters can sell superfluous permits or buy more if they run short. Permits are traded on an allowances market on which firms are price takers, and hence take the permit price $\sigma \in \mathbb{R}_{\geq 0}$ as exogenously given.

Assumption 5 (Permits market): *Every firm* i *gets an initial amount of emission permits* $e_i^0 \in \mathbb{R}_{\geq 0}$ *(*$i \in N$*). Firms behave on the permits market as price takers, whereas the permit price is denoted as* σ.

The costs for buying or the negative costs in case of selling permits are $\sigma \cdot (e_i - e_i^0)$. Hence, a firm i's total profit is given by $\Pi_i = R_i - C^i(q_i, e_i) - \sigma \cdot (e_i - e_i^0)$ for all $i \in N$.

3 Competitive Commodity Markets

For choosing the optimal allocation among sectors, the regulator must anticipate all firms' behavior and include it in the costs function of the *Tr*-sector. Therefore, in this section the market outcome for perfect competitive firms is investigated.

On the firms' level every firm $k \in \{3, ..., n\}$ chooses its output and its emission level in a way that maximizes its total profit $\Pi_k(q_k, e_k) = R_k(q_k) - C^k(q_k, e_k) - \sigma \cdot (e_k - e_k^0)$. First order conditions for a profit maximization are

$$P_k - C^k_{q_k}(\cdot) = 0, \quad \text{and} \tag{1}$$

$$-C^k_{e_k}(\cdot) - \sigma = 0 \tag{2}$$

for all $k \in \{3, ..., n\}$. From condition (2) one can easily see that all firms have equal marginal abatement costs $-C^k_{e_k}(\cdot)$ and that their behavior is independent of the initial permits allocation within the *Tr*-sector $(e_1^0, ..., e_n^0)$, also called micro-plan.

Since the Hessian of $C^k(\cdot)$ is positive definite the second order condition for a maximum is always satisfied. Solving the system of equations yields the optimal output level $q_k^*(\sigma)$ and the optimal emission level $e_k^*(\sigma)$ depending on the permit price σ.

Lemma 1 (Competitive firms' behavior): *Firms under perfect competition decrease output and emissions due to an increasing permit price.*

Proof. Differentiating the first oder conditions with respect to the permit price σ, we have

$$e_k^{*'}(\sigma) = -\frac{1}{C^k_{e_k e_k} - {C^k_{q_k e_k}}^2 / C^k_{q_k q_k}} < 0, \quad \text{and}$$

$$q_k^{*'}(\sigma) = \frac{C^k_{q_k e_k} / C^k_{q_k q_k}}{C^k_{e_k e_k} - {C^k_{q_k e_k}}^2 / C^k_{q_k q_k}} < 0.$$

By Assumption 3 it is easy to show that the optimal output and emission level decrease if the permit price increases. □

4 Imperfect Commodity Markets

In contrast to Section 3, we now investigate an industry, where firms can influence the commodity price through their behavior. For sake of simplicity we restrict our analyzes to duopoly competition. The firms $j \in \{1, 2\}$ try to maximize their own profits $\Pi_j(q_1, q_2, e_j) = R_j(q_1, q_2) - C^j(q_j, e_j) - \sigma \cdot (e_j - e_j^0)$ for a given permit price.

In order to consider different kinds of firms' behaviour we model the competition on the commodity market of firms 1 and 2 by means of "conjectural variations".[7] Every

[7]For a survey on conjectural variations see Dixit (1986).

firm j ($j \in \{1,2\}$) has believes about the other firm's behavior and its rival's *expected* output change due to an increasing firm's output, represented by a constant v^j.[8]

$$\frac{dq_{-j}}{dq_j} := v^j$$

The "conjectural variations" framework offers the possibility to investigate different expected kinds of competition on the product market in one model. In case of $v^j = -1$ for all $j \in \{1,2\}$, firms behave like price takers in the product market, for e.g. symmetric firms $v^j = 1$ for all $j \in \{1,2\}$ corresponds to a profit maximizing cartel. For $v^j = 0$ for all $j \in \{1,2\}$, firms behave à la Cournot. We restrict our analysis to all possible kinds of expected competition between price takers and cartel, therefore, $v^j \geq -1$ for all $j \in \{1,2\}$ must hold.

Considering the expected rival's response to an output change implies (as a system of first order conditions for a profit maximization) for all $j \in \{1,2\}$

$$P_c(\cdot) + (1+v^j)q_j P_c'(\cdot) - C^j_{q_j}(q_j, e_j) = 0, \tag{3}$$

$$-C^j_{e_j}(q_j, e_j) - \sigma = 0. \tag{4}$$

Solving equation (4) yields the cost minimizing emissions level $\hat{e}_j(q_j, \sigma)$, which depends on firm j's output and the permit price. Equation (4) also shows that in equilibrium all firms have equal marginal abatement costs $-C^j_{e_j}(\cdot)$, and that the cost minimizing emissions level $\hat{e}_j(\cdot)$ is still independent of the micro-plan $(e^0_1, ..., e^0_2)$, analog to Section 3. Applying the theorem of implicit functions to (4), we have

$$\frac{\partial \hat{e}_j}{\partial q_j} = -\frac{C^j_{q_j e_j}}{C^j_{e_j e_j}} > 0 \quad \text{and} \quad \frac{\partial \hat{e}_j}{\partial \sigma} = -\frac{1}{C^j_{e_j e_j}} < 0$$

for $j \in \{1,2\}$. These results show that the cost minimizing emission level increases if production is expanded and decreases if permits getting more expensive, which is quite intuitive. Substituting $\hat{e}_j(q_j, \sigma)$ in lhs of (3) yields firm's marginal profit solely depending on firms' output levels and the permit price:

$$\mu^j(q_1, q_2, \sigma) := P_c(\cdot) + (1+v^j)q_j P_c'(\cdot) - C^j_{q_j}(q_j, \hat{e}_j(q_j, \sigma))$$

Firm j's "implicit reaction function", anticipating cost minimizing emission behavior, is then

$$\mu^j(q_1, q_2, \sigma) = 0. \tag{5}$$

From this conditions we can derive the equilibrium output level $q^*_j(\sigma)$ for all $j \in \{1,2\}$. Including this result in $\hat{e}_j(q_j, \sigma)$ we have the equilibrium emissions level $e^*_j(\sigma)$.

[8] In the following we denote the competitor of firm j by $-j$.

Lemma 2 (Imperfect competitive firm's production behavior): *The effect of an increasing permit price on the output level of a firm under imperfect competition is ambiguous.*

Proof. Using the notation of Dixit (1986), we define

$$\begin{aligned} a_i := \mu^i_{q_j} &= (2+v^j)P_c'(\cdot) + (1+v^j)q_j P_c''(\cdot) - C^j_{q_j q_j} - C^j_{q_j e_j}\frac{\partial \hat{e}_j}{\partial q_j} \\ &= (2+v^j)P_c'(\cdot) + (1+v^j)q_j P_c''(\cdot) - C^j_{q_j q_j} + \frac{C^{j2}_{q_j e_j}}{C^j_{e_j e_j}} \end{aligned}$$

and

$$b_i := \mu^i_{q_{-j}} = P_c'(\cdot) + (1+v^j)q_j P_c''(\cdot).$$

Then, second corder conditions for an interior solution are $a_j + v^j b_j < 0$ for $j \in \{1, 2\}$.[9] The slope of firm j's reaction function is

$$r_j := -\frac{b_j}{a_j}.$$

We assume stability of the equilibrium and therefore, following Dixit (1986), that $a_j < 0$ and $\Delta := a_1 a_2 - b_1 b_2 > 0$. Due to Assumption 3, the Hessian of $C_j(q_j, e_j)$ is positive definite and since $v_j \geq -1$, we can show that $r_j > -1$ holds.[10]

Following Dixit total differentiation of the implicit reaction functions (5) with respect to the permit price σ yields firm j's change in output

$$\frac{dq_j^*}{d\sigma} = \frac{a_{-j}\frac{\partial \hat{e}_j}{\partial q_j} + r_j a_j \frac{\partial \hat{e}_{-j}}{\partial q_{-j}}}{\Delta}. \tag{6}$$

By $a_j < 0$, $\frac{\partial \hat{e}_j}{\partial q_j} > 0$ and the possibility that $r_j < 0$ the sign of (6) is ambiguous. □

In contrast to the results in Section 3, the sign of (6) can also be positive even if firms behave like price takers.[11] Hence, although it is not very intuitive an increasing output can be the consequence of a higher permit price. Due to stability reasons the denominator Δ is always positive, whereas a_j must be negative. As aforementioned, the cost minimized emission level always increases with the output level, $\frac{\partial \hat{e}_j}{\partial q_j} > 0$. Therefore, a necessary condition for (6) being positive is that the slope of the reaction function is negative, $r_j < 0$. In terms of Bulow et al. (1985) this means that a necessary condition for an increasing output is that firms regard their output as a *strategic substitute.* In case of output being a *strategic complement* ($r_j > 0$) the effect of an increasing permits price is unambiguous and the change in output is always negative.

[9]This condition corresponds to the Hessian of the profit functions being negative definite.

[10]Seade (1980) shows for somewhat different stability conditions that the assumption $|r_j| < 1$ assures stability of the equilibrium. Tirole (1988) points out that this assumption guaranties uniqueness of the equilibrium.

[11]After applying the somewhat stricter assumptions of Simpson (1995), equation (6) coincides with his results.

Since $r_j > -1$, another necessary condition for an increasing output level is that

$$\left| a_{-j} \frac{\partial \hat{e}_j}{\partial q_j} \right| < \left| a_j \frac{\partial \hat{e}_{-j}}{\partial q_{-j}} \right| .$$

Therefore, the conditions for $\frac{dq_j^*}{d\sigma} > 0$ are satisfied if $r_j < 0$ and e.g. $\frac{\partial \hat{e}_j}{\partial q_j}$ is sufficiently smaller than $\frac{\partial \hat{e}_{-j}}{\partial q_{-j}}$; especially as a lower $\frac{\partial \hat{e}_j}{\partial q_j}$ also coincides with a greater $|a_j|$. Hence, if we have an increasing output, it is not unreasonable in the following to assume that $\frac{\partial \hat{e}_j}{\partial q_j} < \frac{\partial \hat{e}_{-j}}{\partial q_{-j}}$.[12]

This can also be interpreted from an economic perspective. If a firm's output increases due to a higher permits price, its competitor is likely to be more dependent on emissions causing raw materials for an additional output unit. In other words the competitor makes use of a technology which is to a greater extent dependent on the usage of fossil fuels.

Lemma 3 (Imperfect industry's production behavior): *If one firm's output increases due to a higher permit price, the competitor's output must decrease to an even greater extent.*

Proof. Due to $r_i > -1$, we have

$$\frac{dq_1^*}{d\sigma} + \frac{dq_2^*}{d\sigma} = \frac{(1+r_1)a_1 \frac{\partial \hat{e}_2}{\partial q_2} + (1+r_2)a_2 \frac{\partial \hat{e}_1}{\partial q_1}}{\Delta} < 0. \tag{7}$$

□

Therefore, if firms are sufficiently different, only one firm can increase its output due to a higher permits price. In case of identical, or nearly identical firms, both decrease their outputs in consequence of an increasing permits price, analog to the results of Section 3.

Lemma 4 (Imperfect firm's emissions): *The effect of an increasing permit price on the emission level of a firm under imperfect competition is ambiguous.*

Proof. Considering (6), the slope of the equilibrium emission level with regard to the permit price yields

$$\frac{de_j^*}{d\sigma} = \frac{\partial \hat{e}_j}{\partial q_j} \frac{dq_j^*}{d\sigma} + \frac{\partial \hat{e}_j}{\partial \sigma}. \tag{8}$$

[12]If inverse demand is linear or both firms have equal conjectures and output levels, $\frac{\partial \hat{e}_j}{\partial q_j} < \frac{\partial \hat{e}_{-j}}{\partial q_{-j}}$ is even a necessary condition for $\frac{dq_j^*}{d\sigma} > 0$, since in this case $a_{-j} + r_j a_j = a_{-j} - b_j < 0$ always holds. In case both firms regard their output as strategic substitutes and firm j is not a price taker the derivation $\frac{dq_j^*}{d\sigma}$ can be positive with $\frac{\partial \hat{e}_j}{\partial q_j} \geq \frac{\partial \hat{e}_{-j}}{\partial q_{-j}}$ if the inverse demand function is *concave* and firm j produces at *sufficiently lower marginal production costs* ($C_{q_{-j}}^{-j}(q_{-j}^*, e_{-j}^*) > C_{q_j}^{j}(q_j^*, e_j^*)$). This means that firm j has lower marginal production costs if emissions are constant, although it prefers to cause more emissions with the next output unit compared to its competitor. This special case, $\frac{dq_j^*}{d\sigma} > 0$ and $\frac{\partial \hat{e}_j}{\partial q_j} \geq \frac{\partial \hat{e}_{-j}}{\partial q_{-j}}$, is not considered in the following.

As shown above, the last term rhs of (8) is always negative. The sign of the first term rhs solely depends on the sign of $\frac{dq_j^*}{d\sigma}$ since $\frac{\partial \hat{e}_j}{\partial q_j}$ is positive. In case the equilibrium output decreases with the permits price the equilibrium emission level also decreases, $\frac{de_j^*}{d\sigma} < 0$. If the equilibrium output increases with the permit price the effect on firm's emissions is ambiguous. Only if the output increases sufficiently strong ($\frac{dq_j^*}{d\sigma} > -\frac{1}{C^j_{q_j e_j}}$) the emission level also increases, $\frac{de_j^*}{d\sigma} > 0$. □

Lemma 5 (Imperfect industry's emissions): *If one firm's emission level increases due to a higher permit price, the competitors emission level decreases to an even greater extent.*

Proof. Summing up the effects of a increasing permit price on firms' emissions yields

$$\frac{de_1^*}{d\sigma} + \frac{de_2^*}{d\sigma} = \frac{\partial \hat{e}_1}{\partial q_1}\frac{dq_1^*}{d\sigma} + \frac{\partial \hat{e}_2}{\partial q_2}\frac{dq_2^*}{d\sigma} + \frac{\partial \hat{e}_1}{\partial \sigma} + \frac{\partial \hat{e}_2}{\partial \sigma}. \tag{9}$$

The last two terms of (9) are negative, only the first two terms are ambiguous. If both firm decrease their output total emissions must decrease. The case of interest is if one firm's output increases. Let us assume w.l.o.g. that $\frac{dq_j^*}{d\sigma} > 0$, then it must hold due to (7) that $\frac{dq_{-j}^*}{d\sigma} < 0$ and $\left|\frac{dq_j^*}{d\sigma}\right| < \left|\frac{dq_{-j}^*}{d\sigma}\right|$. Futhermore, as shown above, we assume in this case $0 < \frac{\partial \hat{e}_j}{\partial q_j} < \frac{\partial \hat{e}_{-j}}{\partial q_{-j}}$. Therefore, we can state that (9) must always be negative.□[13] [14]

5 The Trading Sectors' Cost Function

In this section the results from analyzing firms' behavior are used to derive the cost function of the *Tr*-sector. Therefore, we combine the optimal emission levels of all firms $e_i^*(\sigma)$ $i \in N$ with the market clearing condition, $E_{Tr} = \sum_{i=1}^n e_i^*(\sigma)$. From this equation we get a permit price only depending on the total amount of emissions, $\sigma^*(E_{Tr})$. It can easily be shown that the permit price function has a negative slope,

$$\sigma^{*'}(E_{Tr}) = \frac{1}{\sum_{i=1}^n e_i^{*'}(\sigma)} < 0. \tag{10}$$

Before we can introduce the cost function, we have to define some additional notation. The output level $\bar{q}_i$ represents firm i's profit maximizing output in the absence of any

[13] If we would consider the case $\frac{dq_j^*}{d\sigma} > 0$ and $\frac{\partial \hat{e}_j}{\partial q_j} > \frac{\partial \hat{e}_{-j}}{\partial q_{-j}}$ it can be possible that industry's total emissions rise due to an increasing permit price. See e.g. Requate (2005).

[14] Analyzing the effect of an increasing permit price on firms' profits yields that it is also ambiguous. In contrast to Simpson (1995), both firms can benefit from a higher permit price. For a detailed analysis see Ehrhart et al. (2006).

emission regulation.

$$\bar{q}_i := \arg\max_{q_i} R_i(q) - C^i(q_i, \bar{e}_i(q_i))$$

for all $i \in N$. The output level $\tilde{q}_i$ and the emission level $\tilde{e}_i$ represent the equilibrium levels in case of a permit trading scheme with E_{Tr} emission permits.

$$\tilde{q}_i := q_i^*(\sigma^*(E_{Tr})) \quad \wedge \quad \tilde{e}_i := e_i^*(\sigma^*(E_{Tr}))$$

The costs of the Tr-sector are defined as the difference between social welfare before emission regulation and social welfare after the introduction of the permit trading scheme. For the industry consisting of firms 1 and 2 social welfare is the sum of consumers' surplus and firms' profits. Since the firms 3 to n face exogenously given commodity prices, social welfare for these industries is the sum of firms' profits.

Assumption 6 (Trading sector's costs): *The trading sector's cost function is*

$$F_{Tr} = W(\bar{q}, \bar{e}(\bar{q})) - W(\tilde{q}, \tilde{e})$$

with $W(q,e) = [\int_0^{q_1+q_2} P_c(Z)dZ - \sum_{j=1}^2 C^j(q_j, e_j)] + [\sum_{k=3}^n (P_k \cdot q_k - C^k(q_k, e_k))]$.

Differentiating F_{Tr} with respect to E_{Tr}, we have

$$\begin{aligned} F'_{Tr}(E_{Tr}) &= \textstyle\sum_{j=1}^2 (-P_C + C^j_{q_j}) q_j^{*'}(\sigma^*)\sigma^{*'}(E_{Tr}) + \sum_{k=3}^n (-P_k + C^k_{q_k}) q_k^{*'}(\sigma^*)\sigma^{*'}(E_{Tr}) \\ &\quad + \textstyle\sum_{i=1}^n C^i_{e_i} e_i^{*'}(\sigma^*)\sigma^{*'}(E_{Tr}). \end{aligned}$$

Including (1)-(4) and (10) yields

$$F'_{Tr}(E_{Tr}) = \left((1+v^1) q_1^* \frac{dq_1^*}{d\sigma} + (1+v^2) q_2^* \frac{dq_2^*}{d\sigma} \right) \frac{P'_C(\cdot)}{\sum_{i=1}^n e_i^{*'}(\sigma^*)} - \sigma^*(E_{Tr}).$$

6 Efficient Allocation Plan

Considering the results of Section 5 we now can solve the cost minimization problem the regulator faces:

$$\begin{aligned} \min_{E_{Tr}, E_{NTr}} \; & F_{Tr}(E_{Tr}) + F_{NTr}(E_{NTr}) \\ \text{s.t. } \; & \bar{E} \geq E_{Tr} + E_{NTr} \end{aligned} \tag{11}$$

Due to Assumption 2 a higher allocation of emissions for the NTr-Sector coincides with lower costs, from this fact we can derive that the condition $\bar{E} \geq E_{Tr} + E_{NTr}$ must always be satisfied with equality. From the first order condition for an interior solution we can derive the following equation, which must be satisfied in a cost minimum.[15]

$$\sigma^*(E_{Tr}) = -F'_{NTr}(\cdot) + \left((1+v^1) q_1^* \frac{dq_1^*}{d\sigma} + (1+v^2) q_2^* \frac{dq_2^*}{d\sigma} \right) \frac{P'_C(\cdot)}{\sum_{i=1}^n e_i^{*'}(\sigma^*)} \tag{12}$$

With (2), (4) and (12) we can derive the main result of this paper, which is summarized in Proposition 1.

[15] We only consider the case of an interior solution of (11), i.e. we assume that $F_{Tr}(E_{Tr})$ is convex with respect to E_{Tr} and that the permit price σ^* is non-negative (i.e. E_{Tr} is sufficiently small).

Table 1: Overview for $v^1 = v^2 \neq -1$

	output strategic substitute $\exists r_j < 0$	output strategic complement $\forall r_j > 0$
$q_1^{*2}\epsilon_\sigma(q_1^*) + q_2^{*2}\epsilon_\sigma(q_2^*) < 0$	$\sigma^* < -F'_{NTr}(\cdot)$	$\sigma^* < -F'_{NTr}(\cdot)$
$q_1^{*2}\epsilon_\sigma(q_1^*) + q_2^{*2}\epsilon_\sigma(q_2^*) = 0$	$\sigma^* = -F'_{NTr}(\cdot)$	-
$q_1^{*2}\epsilon_\sigma(q_1^*) + q_2^{*2}\epsilon_\sigma(q_2^*) > 0$	$\sigma^* > -F'_{NTr}(\cdot)$	-

Proposition 1 (Cost minimizing allocation): *In a cost minimizing allocation with imperfect product markets the marginal abatement costs of firms in the Tr-sector do not have to equal the marginal abatement costs of the NTr-sector.*

For further analysis on the cost efficient allocation plan we assume that both firms, 1 and 2, have identical conjectures ($v^1 = v^2$). Then it holds for price taking firms ($v^j = -1$) that all emitters in the *Tr*-sector and the *NTr*-sector must have equal marginal abatement costs. Therefore, if firms behave like price takers on their commodity markets all marginal abatement costs must be equal, irrespective if commodity prices are endogenous or exogenous.

In all other considered cases of expected competition ($v^j > -1$), emitters' marginal abatement costs can differ between sectors. The first term rhs and the fraction in the last term rhs of (12) are per assumption positive. Hence, if the permit price is above or below the marginal abatement costs of the *NTr*-sector, solely depends on the sign of the term $(1+v^1)q_1^*\frac{dq_1^*}{d\sigma} + (1+v^2)q_2^*\frac{dq_2^*}{d\sigma}$ in equation (12).

If imperfect competitive firms consider their outputs as strategic complements ($r_j > 0$), we have $\frac{dq_j^*}{d\sigma} < 0$ for $j \in \{1,2\}$ and therefore $\sigma^* < -F'_{NTr}(\cdot)$. Only if a firm considers its output as strategic substitute ($r_j < 0$) the permit price can be higher than the marginal abatement costs of the *NTr*-sector. Another necessary condition is that one firm's output must increase with the permit price, $\frac{dq_j^*}{d\sigma} > 0$. Since we then have $\frac{dq_{-j}^*}{d\sigma} < 0$ and $\left|\frac{dq_j^*}{d\sigma}\right| < \left|\frac{dq_{-j}^*}{d\sigma}\right|$, firm j must have a sufficiently greater market share ($q_j^* > q_{-j}^*$) for $\sigma^* > -F'_{NTr}(\cdot)$ to hold.

With the elasticity being defined as $\epsilon_x(f(x)) = \frac{df(x)}{dx} \cdot \frac{x}{f(x)}$ and $v^1 = v^2$ the condition for $\sigma^* > -F'_{NTr}(\cdot)$ can be rewritten as

$$q_1^{*2} \cdot \epsilon_\sigma(q_1^*) + q_2^{*2} \cdot \epsilon_\sigma(q_2^*) > 0.$$

The main results for the social cost minimizing emissions allocation are summarized in Table 1. The optimal allocation depends on output being a strategic substitute or not and how asymmetric firms are.

The allocation of emissions between sectors has also an impact on the perfect competitive firms 3 to n in the *Tr*-sector. If the optimal allocation induces $\sigma^* < -F'_{NTr}(\cdot)$ the firms face a lower permit price. In this case the firms 3 to n benefit from the imperfect competition between firms 1 and 2 in a way that their profits increase. We then have the strange situation that firms in the *Tr*-sector are interested in imperfect competition in other industries also participating in emissions trading.

If $\sigma^* > -F'_{NTr}(\cdot)$ leads to a cost minimizing solution, perfect competitive firms suffer from other industries in the *Tr*-sector being imperfect competitive. In this case they face a higher permit price and therefore their profits decrease.

If we assume not too asymmetric imperfect competitive industries in the European emissions trading scheme, marginal abatement costs of emitters in the *NTr*-sector exceed the optimal permit price for emitters in the *Tr*-sector. In this case one can say that the correction of the market distorting behavior of firms 1 and 2 are *financed* by the emitters in the *NTr*-sector and that all other firms in the *Tr*-sector benefit from this correction.

7 Conclusion

This paper shows that cost efficiency in the European emissions trading scheme does not inevitable require equal marginal emission abatement costs for all emitters. Depending on commodity markets competition and firms' asymmetry marginal abatement costs of emitters in the trading sector must be above or below marginal costs of the non trading sector to minimize overall costs. This result contradicts the common assumption in literature that an optimal macro-plan must satisfy equal marginal costs.

An allocation resulting in unequal marginal costs can increase welfare since market distorting behavior of imperfect competitive firms is also mitigated. Certainly, if only one industry is imperfect competitive and the number of price taking firms in the *Tr*-sector is sufficiently large the deviation of an optimal allocation from equal marginal costs will be negligible. But, in case a large part of the industries in the *Tr*-sector are imperfect competitive an unequal allocation can increase welfare significantly.

The hybrid approach of the EU provides the possibility of unequal marginal costs through its *inter-sector-flexibility*, i.e. the regulator can determine the marginal costs for emitters in each sector by choosing a certain macro-plan. The question is, if an instrument like tradable permits should be used to correct the negative effects of imperfect competition. Admittedly, it would be more preferable if the regulator achieves perfect competition without abusing environmental policies. But if this is not possible why not using the allocation of emissions to achieve at least a second best solution?

But even if overall welfare is improved one must recognize that an allocation with unequal marginal costs between sectors influences profits of all firms. In case the optimal allocation leads to a reduced permit price all competitive firms in the trading sector benefit from this allocation. This effect on their profits can give an incentive for institutional abuse of the emission trading system. For example firms can make use of the opt-in or opt-out rule of the European emissions trading law to change from the *NTr*-sector to the *Tr*-sector if marginal costs differ between them.[16]

For verifying if an allocation plan favoring emitters of one sector increases overall welfare in the European Union of course further theoretical and empirical research is

[16]See EC(2003).

needed. For example the stylized model in this paper neglects the future integration of the flexible mechanisms of the Kyoto protocol into the European system. The flexible mechanism approach enables countries to obtain additional emission allowances from e.g. transferring carbon dioxide reducing technology into third world countries or investing in reforestation. If firms in the emissions trading sector are allowed to freely obtain such additional allowances, marginal abatement costs are only determined by the world price for emission rights. Hence, the regulator looses its steering-wheel to influence the European permit price through the National Allocation Plan. Therefore, the question is if the usage of additional allowances from flexible mechanisms should be restricted in order to maintain the inter-sector-flexibility of the European regulator. Also from a legal perspective it must be clarified if an abuse of the emissions trading scheme for correcting market distortions is in line with European law. The initial allocation of emission permits is not allowed to conflict state aid law, which is maybe the case if it leads to unequal marginal abatement costs.

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