Dynamic Game of Transboundary Pollution Regulation and Strategic Abatement[◊]

Victoria I. Umanskaya [a,*], Charles F. Mason[b], Edward B. Barbier[c]

[a] *Department of Economics, University of California Riverside, 900 University Avenue, Riverside, CA 92521*
[b, c] *Department of Economics and Finance, University of Wyoming, 1000 E. University Avenue, Laramie, WY 82071*

Abstract

This paper examines the role of trade policy in creating incentives for cooperation in order to address transboundary stock pollution problem. We use an asymmetric dynamic model to compare outcomes of two second-best scenarios: 1) The country adversely affected by pollution, Downstream, uses a tariff policy to control for the externality, while the polluting country, Upstream, is myopic; 2) Upstream engages in strategic abatement activity to influence the level of tariff. We show that the presence of the asymmetric externality encourages strategic behavior by both players: Downstream will find it optimal to unilaterally impose a sequence of tariffs determined by the current state of pollution. And if such tariffs are imposed, Upstream will unambiguously benefit from engaging in strategic abatement activity that reduces the pollution accumulation rate and gives rise to a lower level of the tariff in the long-run. We find that feedback strategies may suggest a mechanism that supports a self-enforcing trade and environmental agreement.

JEL classification: F13; F18; Q56

Keywords: transboundary pollution, trade policy, differential games

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[◊] This version of the paper was prepared for the EcoMod Conference on Energy and Environmental Modeling, Moscow, Russia, September 13-14, and represents the current work in progress. Ensuing versions are available upon request.

[*] Corresponding author. Tel.: +1(951)827-4975; fax: +1(951) 827-5685. E-mail address: victoriu@ucr.edu.

1. Introduction

"Pollution doesn't carry a passport."[1] Indeed, often environmental problems do not stay confined within geographical borders of one nation and affect more than one country. This kind of exported externalities includes such pressing issues as acid rain, global warming, water pollution with heavy metals, and biodiversity loss. Transboundary pollution problems pose a special challenge since they have special features that distinguish them from domestic environmental issues. In cases of more than one country being involved, there often exists no international political authority with the ability to intervene and enforce cooperation. In the absence of such a regulator, collective action is difficult to obtain because of free-riding incentives. Achieving cooperative outcome becomes even more challenging in the presence of asymmetries between countries, in particular, when the damages from the externality (and thus the costs and benefits of the regulation) are asymmetric. In these cases each country recognizes that it has some influence over payoffs of the other country and their interests conflict.

If cooperation between the countries is not forthcoming, the government of the victim country has few options to control the externality. While the optimal environmental policy to address domestic externalities is a Pigouvian tax, such a tax is not feasible to combat transboundary pollution as no government can impose environmental regulations on producers located outside its political jurisdiction. It has been argued that trade policies are one of the few available instruments for creating or increasing incentives to internalize cross-border externalities (Baumol and Oates, 1988; Markusen, 1975a, 1975b).

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[1] Thomas McMillan, Canadian Environment Minister, quoted in David Francis, "Canadian Calls for Global Action on Environment," The Christian Science Monitor, 79, no. 158 (10 July 1987): p.12.

Since trade policy does not impact the source of pollution directly, it is rarely, if ever, the “first best” policy for controlling transboundary pollution, and can only serve as a “second-best” instrument, playing a role that is somewhat similar role to the role a Pigouvian tax performs within a single political jurisdiction. However, it has been acknowledged that trade measures may serve as “a useful mechanism for encouraging participation in and enforcement of multilateral environmental agreements in some instances, and for attempting to modify the behavior of foreign governments in others” (Nordstrom and Vaughan, 1999, p. 3). Therefore, there is a need to understand how this “mechanism” works. Recent literature has focused on possible linkages between trade and environmental policy promoting international cooperation in addressing transboundary pollution problems and giving rise to self-enforcing trade and environmental agreements[2]. For example, Cabo, Escudero, and Martin-Herran (2001) develop a North-South model where countries are linked by trade flow and transboundary stock pollution and show that cooperative outcome (self-enforcing agreement) is attained if non-cooperative feedback strategies are used as threat strategies to assure cooperation.

We use a dynamic model to investigate the role that international trade instruments play in the transboundary stock pollution control. Two countries, Upstream and Downstream, are characterized by two interactions: unidirectional trade and unidirectional pollution stock externality. The Upstream country produces and exports to Downstream a consumption good that generates pollution during the manufacturing process. Emissions produced in the Upstream country contribute to a pollution stock; this accumulated pollution causes damages in the Downstream country only, which uses a

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[2] Carraro (1999) defines a self-enforcing agreement as one that is profitable and stable. That is such an agreement is attained if both players are made better off by cooperating compared to the state when no players cooperate, and neither player is better of by unilaterally leaving the agreement.

tariff against imports to address this externality. Since we are interested in studying the role of trade measures in encouraging cooperation when countries have asymmetric incentives and therefore bilateral environmental agreements are difficult to forge, we construct a model characterized by strong asymmetries between the regions. In such a framework, the polluting country will be prepared to act only if its efforts serve its own interest. This simple model is used to explore and compare optimal solutions for controlling cross-border stock externality under two alternative second-best scenarios, where two countries behave non-cooperatively. In one, the Downstream decision-maker solves a dynamic problem to determine the optimal tariff rule over time, while the Upstream government acts myopically and Upstream firms maximize static profits. By contrast, in the second scenario the Upstream government realizes that the tariff her country is facing is contingent on the current stock of pollution, and thus becomes a strategic player. In this paper, we consider the case where the Upstream decision-maker engages in strategic abatement activity with the intention to reduce the pollution stock and thus indirectly influence Downstream's choice of tariff. Therefore, we use a differential game model to characterize the interactive setting with two governments.

This paper adds to the existing literature in two ways. First, it explores the role that trade policy may play in creating incentives for bilateral cooperation needed to address *dynamic* transboundary externalities. To our knowledge, this question remains unexplored, while for many important transboundary pollution problems, the damages depend primarily on the accumulated stock, rather than a flow, of pollution. Second, and more importantly, instead of considering a hypothetical cooperative scenario, assuming the existence of some kind of supranational government, we focus on two non-cooperative (second-best) scenarios where the countries are be prepared to act only if their efforts ultimately serve their own interest. Within this framework we show that feedback strategies may actually lead to a mutually beneficial outcome and thus give rise to a self-enforcing agreement.

The rest of the paper is organized as follows. Section 2 introduces a dynamic model of transboundary stock pollutant control. Section 3 presents a differential game of transboundary pollution and strategic abatement. Section 4 offers concluding remarks.

2. A Simple Model of Transboundary Pollution Control

Consider two countries, Upstream and Downstream. A single consumption good is produced only in Upstream with a given fixed endowment of factors of production and a given technology. At the trading price, there is an excess supply of this good in the Upstream country, which is being exported to the Downstream country. Consumers are homogeneous within each country, but may be heterogeneous across countries. At every instant, production of $Q(t)$ in Upstream results in a flow of emissions, $E(t)$, given by the output-emissions tradeoff function,

$$E(t) = E(Q(t)), \tag{1}$$

which indicates the amount of pollutants produced when the current output of the Upstream country is $Q(t)$. We assume that this output-emissions tradeoff function is time-invariant and increasing in the level of output produced.

The amount of pollutants emitted by the Upstream country contributes to the stock of pollution, $Z(t)$, which evolves according to the following equation of motion:

$$\dot{Z}(t) = E(Q(t)) - kZ, \quad \text{with} \quad Z(0) = Z_0, \tag{2}$$

where k represents the rate of pollution decay. Although pollution is generated by emissions in the Upstream country, we assume that the environmental damage from the stock of pollution is realized in Downstream only and that there are no damages from the flow of emissions[3]. Such externalities emerge when the pollutant is transmitted via air, rivers, lakes, or precipitation, and include important cases like the deterioration of soil and water quality attributed to acid deposition and water pollution from accumulated emissions of heavy metals or from agricultural runoff. The key feature of such exported externalities is that there exists no supra-national authority with the ability to intervene and enforce cooperation. Thus countries will act only if their efforts ultimately serve their own interest. Since the Upstream country does not suffer any damages from the pollution stock, we assume that it does not impose any environmental regulations on its firms. The Downstream government, in turn, does not have political authority to impose an environmental measure, such as Pigouvian tax, on foreign producers in order to address the source of transboundary pollution directly. However, she can indirectly tackle the externality by imposing a tariff, τ, on imports from the Upstream country. The tariff lowers the price in Upstream and forces firms to cut down the level of production and with it the flow of transboundary emissions, contributing to the pollution stock[4]. The effects of the tariff can be summarized as follows[5]:

$$\partial Q / \partial \tau < 0,\ \ \partial Y / \partial \tau < 0,\ 0 < \partial p / \partial \tau < 1,$$

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[3] Previous studies of the link between trade and environmental policies are predominantly static. However, in many important transboundary problems damages are caused by the stock rather than the flow of pollution. Thus following the recent environmental literature (e.g., Dockner and Long (1993), Mason (1997), List and Mason (2001)), we approach the problem in a dynamic framework.

[4] We assume that Downstream has market power to influence the terms of trade through its tariff.

[5] From now on, unless otherwise stated, we will suppress the time argument t.

where Y is the volume of trade and p is the Downstream market price; the price in Upstream is $p-\tau$.

For any given level of tariff, equilibrium conditions for both countries' markets imply that the Downstream policymaker's choice of the tariff determines the level of production by Upstream producers, the level of exports to Downstream, and the equilibrium after-tariff prices in both countries, so we can write $Q(\tau)$, $Y(\tau)$, $p(\tau)$. This allows us to write the net benefit functions for Upstream and Downstream, respectively, in the following form:

$$U_u(Q(\tau)-Y(\tau))-C_u(Q(\tau))+(p(\tau)-\tau)Y(\tau)\equiv W_u(\tau) \tag{3}$$

$$U_d(Y(\tau))-(p(\tau)-\tau)Y(\tau)-D(Z)\equiv W_d(\tau)-D(Z), \tag{4}$$

where U_i, $i=u,d$, represents preferences of Upstream and Downstream consumers, C_u is the cost of production in Upstream, and $D(Z)$ stands for the damage resultant from the pollution stock *Z*, $D(Z)$ is increasing and strictly convex. The tariff revenue is assumed to be redistributed among Downstream consumers.

The level of emissions generated by production of $Q(t)$ can also be written as a function of the tariff imposed by the Downstream government,

$$E(\tau)=E(Q(\tau)) \tag{5}$$

Since $E'(Q)>0$, the flow of emissions is decreasing with tariff, that is $E'(\tau)<0$.

The objective of the Downstream government is to choose the sequence of tariffs that maximizes the discounted stream of net benefits (5) taking into account the evolution of the pollution stock:

$$\max_{\tau}\int_0^{\infty} e^{-rt}\left[W_d(\tau)-D(Z)\right]dt$$

subject to $$\dot{Z}(t) = E(\tau) - kZ, \qquad Z(0) = Z_0 \tag{6}$$

where r is the positive discount rate that is assumed constant and identical for both countries. The current-value Hamiltonian for this optimization problem can be written as

$$H = W_d(\tau) - D(Z) + \theta\left[E(\tau) - kZ\right],$$

where θ is the co-state variable representing the shadow price of pollution for the Downstream government and thus θ is presumably negative.

The necessary conditions for the maximum principle require that the optimal tariff sets the marginal welfare cost associated with the tariff equal to the marginal benefit represented by the shadow value of the marginal pollution reduction,

$$W_d'(\tau) = -\theta E'(\tau), \tag{7}$$

and that the shadow price of pollution evolves at the rate equal to the marginal damage from pollution less the opportunity cost of cutting down emissions by one unit,

$$\dot{\theta} = (r+k)\theta + D'(Z). \tag{8}$$

Total differentiation of the first-order condition (7) and substitution from the adjoint equation (8) allows us to characterize the equilibrium state and trajectories leading to that state. Thus the evolution of the tariff can be written as

$$\dot{\tau} = \frac{W_d'(\tau)(r+k) - E'(\tau)D'(Z)}{W_d''(\tau) - \dfrac{W_d'(\tau)}{E'(\tau)}E''(\tau)} \tag{9}$$

Combining equations (9) and (6), and using the fact that $\dot{\tau} = \tau'(Z)\dot{Z}$, we obtain the feedback decision rule for the optimal tariff:

$$\tau'(Z) = \frac{W_d'(\tau)(r+k) - E'(\tau)D'(Z)}{\left(W_d''(\tau) - \dfrac{W_d'(\tau)}{E'(\tau)}E''(\tau)\right)(E(\tau) - kZ)} \tag{10}$$

Note that the equilibrium tariff function is characterized by a non-linear first-order ordinary differential equation. Existence of a solution over some compact set of stocks follows from standard theorems (see, e.g., Boyce and DiPrima 2005, pp. 68-70). Expression (10) shows that the optimal tariff at every instant is determined by the current amount of accumulated pollution. Therefore, although the tariff targets the flow of goods and not pollution directly, its magnitude varies with the level of the pollution stock.

It can be demonstrated that the solution to this optimization problem leads to a unique saddle point steady state characterized by the following conditions[6]:

$$W_d'(\tau^e)(r+k) = E'(\tau^e)D'(Z^e) \tag{11}$$

$$Z^e = E(\tau^e)/k \tag{12}$$

$$\theta^e = -\frac{D'(Z^e)}{r+k} \tag{13}$$

Equations (11)-(13) characterize the equilibrium that is determined by unilateral and non-cooperative actions of the Downstream policymaker. While she is only able to levy a tax on the flow of imports and not on the flow of emissions directly, the magnitude of the optimal tariff is determined by the level of accumulated pollution. It follows from (3) that the tariff choice determines the level of welfare in the Upstream country. Hence any adjustment of the tariff level to an increase in the pollution stock also leads to a reduction in Upstream country's net surplus, and therefore creates incentives for Upstream country to seek the influence over Downstream authority's policy decisions in its favor. The next section considers this possibility.

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[6] The superscript *e* stands for the steady state equilibrium.

3. Trade, Transboundary Pollution Control, and Strategic Abatement

In the preceding section we have characterized the solution that can be achieved if the Downstream government unilaterally chooses a sequence of tariffs while myopic firms in the Upstream country choose the output levels (or equivalently, emission levels) to maximize static profits. We now assume that the Upstream government no longer acts myopically and now realizes that the level of tariff her country faces depends on the current stock of pollution. This perceived feedback effect creates an incentive for the Upstream government to choose a course of action to influence the evolution of the pollution stock and, ultimately, the optimal tariff rule selected by the Downstream government over time. Although the Upstream country does not suffer any direct damages from the accumulated emissions (or, for some reason, does not care about these damages), it now conjectures how the level of pollution indirectly affects the welfare level of Upstream's constituents. The stock of pollution becomes a payoff-relevant variable for Upstream, and the Upstream government may now decide to choose a pollution control strategy of her own. Specifically, suppose that the Upstream authority can establish an abatement facility by paying a sunk cost *SC*. The sunk cost can be thought of as expenditures, necessary to build the abatement plant, and may be interpreted as the determinant of the abatement plant size: higher sunk cost implies larger abatement facility. In a general case, the size of the plant may be treated as a choice variable for the Upstream government, however, in this model we make a simplifying assumption of a fixed sunk cost, that is the abatement facility of only one given size can be established by the Upstream country. Operating this abatement plant allows the Upstream country to reduce the flow of emissions, contributing to the pollution stock. To introduce the possibility of abatement very simply, suppose that the Upstream authority selects a sequence of abatement standards and at every instant, abatement reduces the rate of accumulation of the pollution stock, that is

$$\dot{Z}(t) = E(\tau) - a - kZ \ , \tag{14}$$

where a is the rate of abatement. Thus at every point in time, the Upstream country offers a strategic substitute for the Downstream country's tariff in a sense that both the tariff and abatement are designed to decrease the rate of pollution accumulation. However, abatement is costly, and the Upstream country faces the cost $\pi(a)$ of abating at the rate of a, that reduces the flow of Upstream's net benefits. This cost of abatement is assumed to be strictly convex, increasing in a, and if the rate of abatement a goes down to zero, so does the marginal cost of abatement, $\pi'(a)$.

Since both governments' decisions now determine the evolution of the pollution stock, the Upstream government is able to influence Downstream's payoffs and thus both governments are acting strategically. The appropriate framework for the analysis of these strategic interactions is a differential game. We assume that both governments use Markov perfect (feedback) strategies. Both governments design their optimal policies as decision rules dependent on the current level of the stock of pollution. Since the current state of pollution summarizes the history of play, using feedback decision rules allows the players to introduce memory into their strategies. A Markov perfect equilibrium is probably the most realistic solution concept for dynamic games because at every time *t* and for all possible values of *Z(t)*, the strategy defines an equilibrium set of decisions independent of previous actions. Thus the choice of Markov strategies implies that the corresponding outcomes are subgame-perfect.

The goal of each authority is to maximize her citizens' welfare. Assuming that the Downstream policymaker plays the Markov strategy $\tau(Z(t))$, the Upstream government determines the optimal level of abatement by solving the following optimization problem:

$$\max_a \int_0^\infty e^{-rt}[W_u(\tau(Z)) - \pi(a)]dt - SC$$

subject to $\dot{Z} = E\left(\tau(Z)\right) - a - kZ\,,\ \ Z(0) = Z_0$

The Upstream country will set up the abatement plant if the present discounted value of the flow of its citizens' net benefits in the presence of abatement exceeds the sunk cost *SC*. In the following analysis we will assume that building the abatement facility is profitable for Upstream and focus on the dynamic game between the two countries given that the abatement plant has been established. The current-value Hamiltonian function for this problem is the sum of the instantaneous current value of payoffs plus the value of the instantaneous growth of pollution stock:

$$H_u = W_u\left(\tau(Z)\right) - \pi(a) + \theta_u[E\left(\tau(Z)\right) - a - kZ]\,,$$

where θ_u represents the shadow price of pollution for the Upstream country. One could refer to the H_u as a dynamic value function because it also takes into account the effect of current stock and control on the size and valuation of future stock. The maximum principle conditions are

$$\pi'(a) = -\theta_u \tag{15}$$

$$-\dot{\theta}_u = -(r+k)\theta_u + \tau'(Z)[W_u'(\tau) + \theta_u E'(\tau)] \tag{16}$$

Equation (15) states that at every instant, the optimal level of abatement sets the marginal cost of abatement equal to the highest hypothetical price which the Upstream decision-maker would be willing to pay to have one less infinitesimally small unit of pollution. The derivative $\tau'(Z)$ captures the reaction of the Downstream government to a change in the pollution stock, and is expected to be positive in equilibrium: when the level of accumulated pollution is high, the Downstream decision-maker will have some incentive to reduce the rate of common emissions and therefore to increase the tariff. Equation of motion for the co-state variable illustrates how the Upstream government's conjecture about the Downstream country's strategy affects the value that Upstream places on pollution. The adjoint equation (16) shows that the shadow price of pollution depreciates at the rate equal to the opportunity cost of decreasing total emissions by one unit, $-(r+k)\theta_u$, and the marginal effect of that unit on the current-value function of Upstream's problem, $\tau'(Z)[W_u'(\tau)+\theta_u E'(\tau)]$. The last term illustrates the perceived indirect effect of the pollution stock on the Upstream country's payoffs.

Assuming that the Upstream decision-maker plays the Markov strategy $a(Z(t))$, the Downstream authority determines the optimal tariff rule by solving the maximization problem

$$\max_{\tau} \int_0^{\infty} e^{-rt}[W_d(\tau)-D(Z)]dt$$

subject to $$\dot{Z} = E(\tau) - a(Z) - kZ\,,\quad Z(0) = Z_0$$

The current-value Hamiltonian for this problem is

$$H_d = W_d(\tau) - D(Z) + \theta_d[E(\tau) - a(Z) - kZ]$$

and the maximum principle conditions can be formulated as

$$W_d'(\tau) = -\theta_d E'(\tau) \tag{17}$$

$$\dot{\theta}_d = (r + k + a'(Z))\theta_d + D'(Z) \tag{18}$$

The objective functional of the Downstream government's problem is no different than it was in the simple model in Section 2 as the policymaker continues to maximize the total welfare of her country's citizens. She does it by choosing the sequence of tariffs that set the marginal loss of Downstream's net surplus induced by the tariff equal to the value of reduced emissions; thus the first order condition (17) is parallel to the optimal tariff rule (7) in the simple version of the model. However, the possibility of abatement affects the value that the Downstream government assigns to the pollution stock. The derivative $a'(Z)$ expresses the reaction of the Upstream government to a change in the pollution stock. Intuitively, we would expect that $a'(Z) > 0$ in equilibrium, i.e. since the Upstream government now places a negative value on the pollution stock, it will have some incentive to increase the abatement rate if the pollution stock is large. Equation (18) illustrates how this abatement strategy is being taken into account by the Downstream policymaker. While the shadow price of pollution still evolves at the rate given by the difference between the marginal damage generated by a unit of emissions, $D'(Z)$, and the opportunity cost of reducing emissions by that unit, $-(r + k + a'(Z))\theta_d$, the latter opportunity cost now also incorporates the expected strategic response by the Upstream government to an increase in the pollution stock. If the Downstream government conjectures that the Upstream country will increase the rate of abatement as pollution stock rises, she has some incentives to keep the pollution stock high in order to encourage higher level of abatement by the Upstream country. Thus the Downstream government faces a tradeoff: by decreasing total emissions through the tariff, she is also giving up an additional, infinitesimally small unit of abatement.

A steady state equilibrium is defined as the solution to the equation system $\dot{Z} = \dot{\theta}_u = \dot{\theta}_d = 0$ and is characterized by[7]

$$\theta_u^* = W_u'(\tau^*)\tau'(Z^*)/[r + k - E'(\tau^*)\tau'(Z^*)] \tag{19}$$

$$\theta_d^* = -D'(Z^*)/[r + k + a'(Z^*)] \tag{20}$$

$$Z^* = [E(\tau^*) - a^*]/k \tag{21}$$

LEMMA $W_d'(\tau) \big/ E'(\tau)$ *is a positive and increasing function of the tariff* τ.

Proof. See Appendix A.

PROPOSITION 1 *If the Upstream government's feedback strategy* $a(Z)$ *has* $a'(Z) > 0$ *and the Downstream government's feedback strategy* $\tau(Z)$ *has* $\tau'(Z) > 0$ *in the neighborhood of the steady state, then these strategies constitute a positive Markov perfect equilibrium. The steady state tariff corresponding to the Markov prefect equilibrium is lower than the tariff chosen by the Downstream government in the absence of abatement.*

Proof. See Appendix B.

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[7] The superscript * denotes the Markov perfect equilibrium.

Proposition 1 provides us with two important results. Firstly, it establishes the existence of the steady state equilibrium under plausible conditions. While we cannot guarantee the uniqueness of the equilibrium feedback strategies, our result verifies that the strategies that are intuitively meaningful give rise to a positive long-run equilibrium. Secondly, it demonstrates how strategic interactions between the two countries alter the long-run policy outcome. The use of tariffs to address the pollution externality imposes a cost on the Downstream country in the form of reduced net benefits. Abatement offers the Downstream authority a substitute for the tariff pollution control, and thus she is willing and able to commit to a lower level of tariff. By engaging in abatement, the Upstream country increases the opportunity cost of raising the tariff for its trading partner. The Downstream government perceives that higher pollution stock will lead to a higher level of abatement undertaken by Upstream; therefore, in equilibrium, the Downstream policymaker reduces the tariff.

An important implication of Proposition 1 is that the Upstream government can improve the social welfare of her country by using the feedback abatement strategy. A positive steady state abatement rate implies that the benefits from the abatement activity in terms of the tariff reduction exceed the cost of undertaking abatement activity. If that was not the case, the Upstream government would find it optimal to chose a zero abatement rate.

While we were able to compare the long-run tariff rates in the presence and in the absence of strategic abatement, it is not obvious how the steady state pollution levels measure up to each other. A lower steady state tariff in the case of the strategic Upstream government implies that the long-run level of total emissions will be higher in the presence of abatement. At the same time, as it follows from expression (21), the equilibrium level of pollution under abatement scenario is being reduced by the steady state rate of abatement. The ultimate effect of strategic abatement on the pollution stock is thus determined by the relative strength of these two offsetting factors, and the evaluation of the extent of a divergence in steady state pollution stocks may require numerical analysis. Because of this uncertainty about the change in the steady state level of pollution, we leave (for now) the effect of the strategic abatement on the social welfare in the Downstream country as ambiguous.

So far we have examined the differences that Upstream government' strategic behavior introduces in the Downstream country's tariff choice in the long run, i.e. when the equilibrium tariff path over time approaches the steady state as time goes to infinity. Since without further information we cannot tell how fast the dynamic system will approach the steady state, it is also important to characterize how the equilibrium tariff path changes in the presence of strategic abatement. Because the maximum principle conditions (15) and (17) hold at all points in time, we may differentiate both sides of these equations with respect to time to derive the following differential equations that describe the equilibrium abatement and tariff path:

$$\dot{a} = (r+k)\frac{\pi'(a)}{\pi''(a)} + \tau'(Z)\frac{W_u'(\tau) - \pi'(a)E'(\tau)}{\pi''(a)} \tag{22}$$

$$\dot{\tau} = \frac{W_d'(\tau)(r+k) - E'(\tau)D'(Z)}{SOC_d} + a'(Z)\frac{W_d'(\tau)}{SOC_d}, \tag{23}$$

where $SOC_d = W_d''(\tau) - \frac{W_d'(\tau)}{E'(\tau)}E''(\tau)$ is the second order condition for the Downstream country's maximization problem. Note that the first term in equation (23) describes the tariff evolution over time in the absence of strategic abatement (see equation (9)), and thus the second term shows how Upstream's abatement strategy affects the equilibrium tariff time path. If $a'(Z) > 0$, then the rate of increase in tariff over time is greater than in the simple version of the model.

Combining equations (22) and (23) with the equation of motion for the pollution stock (14), and using the fact that $\dot{a} = a'(Z)\dot{Z}$ and $\dot{\tau} = \tau'(Z)\dot{Z}$, we obtain the following system of the first-order differential equations, characterizing the equilibrium feedback strategies:

$$a'(Z) = (r+k)\frac{\pi'(a)}{\pi''(a)[E(\tau)-a-kZ]} + \tau'(Z)\frac{W_u'(\tau)-\pi'(a)E'(\tau)}{\pi''(a)[E(\tau)-a-kZ]} \tag{24}$$

$$\tau'(Z) = \frac{W_d'(\tau)(r+k)-E'(\tau)D'(Z)}{SOC_d[E(\tau)-a-kZ]} + a'(Z)\frac{W_d'(\tau)}{SOC_d[E(\tau)-a-kZ]} \tag{25}$$

The solution to the system (24)-(25) yields the equilibrium feedback strategies $a(Z)$ and $\tau(Z)$. Intuitively, we expect that both the tariff and abatement strategies are increasing in *Z*; that is if the stock of pollution is increasing, both countries will react by strengthening their pollution control. Appendix C derives the conditions on exogenous parameters of the model under which such plausible strategies will emerge.

PROPOSITION 2 *If the initial stock of pollution is small, then the optimal tariff in the presence of strategic abatement increases faster with the stock of pollution than that of the simple version of the model.*

Proof. See Appendix D.

The intuition behind Proposition 2 is as follows. If the Upstream government is strategic, she places a negative value on the pollution stock, reflected by her own shadow price of pollution. The magnitude of the shadow price translates into the level of abatement that the Upstream country undertakes. It follows from equation (19), that in the long run the magnitude of the shadow price of pollution derived by the Upstream government increases with $\tau'(Z)$. Higher marginal increase in tariff in response to the rising pollution stock implies larger welfare reduction for the Upstream country. As a result, the Upstream government derives a larger equilibrium shadow price of pollution. Thus by reinforcing the marginal tariff response to an increase in the pollution stock, the Downstream country achieves a higher level of abatement undertaken by the Upstream country.

Derivation of the Markov perfect equilibrium in a general framework is quite difficult, since it requires solving two simultaneous non-linear differential equations, (24) and (25). One resolution of this difficulty is to assume players use linear feedback strategies; such strategies can be defended in a linear-quadratic game such as the one we considered in Section 2. Assuming that the cost of abatement, $\pi(a)$, is quadratic, we can extend the posed linear-quadratic example to examine the evolution of the pollution stock in the presence of strategic abatement. The class of linear-quadratic differential games is characterized by the equilibrium strategies that are linear with respect to the state variable. Therefore, we will assume that our differential game gives rise to the following pair of linear Markov strategies:

$$\tau(Z) = \mu_1 + \mu_2 Z \tag{26}$$

$$a(Z) = \eta_1 + \eta_2 Z \tag{27}$$

To guarantee the existence of the Markov perfect equilibrium, we will focus on the strategies that are increasing in Z: $\mu_2 > 0$ and $\eta_2 > 0$. Upon substitution of these strategies in the linear version of the state equation, $\dot{Z} = E^0 - \lambda\tau - a - kZ$, we find that the time path for the stock of pollution follows the differential equation

$$\dot{Z} = (E^0 - \lambda\mu_1 - \eta_1) - (\lambda\mu_2 + \eta_2 + k)Z \tag{28}$$

The solution to this equation is $Z^a(t)$, the time path for pollution stocks under the strategic abatement scenario,

$$Z^a(t) = e^{-(\lambda\mu_2 + \eta_2 + k)t}[Z_0 - Z^*] + Z^*, \tag{29}$$

where Z_0 is the initial pollution stock and $Z^* = (E^0 - \lambda\mu_1 - \eta_1)/(\lambda\mu_2 + \eta_2 + k)$ is the steady state level of pollution. Since $\lambda\mu_2 + \eta_2 + k$ is positive, $Z^a(t)$ is globally convergent to the steady state Z^*. The optimal path (29) shows that given that the initial pollution stock is small, $Z^a(t)$ rises monotonically from the initial level to the steady state level; and linear strategies (26) and (27) imply that so do the tariff and the abatement rate. Clearly, if we start with the initial level of pollution that is above the steady state level, then stocks, tariffs, and abatement decrease monotonically to their steady state equilibrium levels.

4. Conclusion

This paper contributes to the literature on the role of trade policy in addressing international environmental problems. We develop an asymmetric dynamic model of transboundary pollution and trade and then compare the outcomes of two second-best scenarios: strategic and non-strategic. We show that if the government of the exporting and polluting Upstream country is myopic, the importing and polluted Downstream country will find it optimal to unilaterally impose a sequence of tariffs determined by the current state of pollution, while Upstream firms will respond to these tariffs by selecting emission levels to maximize static profits. In this setting, we demonstrate that both the stock of pollution and tariff rise monotonically from the initial level to the steady state level, which is unique and globally and asymptotically stable.

Under the second scenario, the Upstream government realizes the feedback tariff rule used by the Downstream government and thus the indirect effect of the pollution stock on her country's welfare. In this context, we show that the Upstream country may find it optimal to engage in strategic abatement, so the question is analyzed in a game theoretic model. Our findings indicate that in the presence of strategic abatement, the Downstream government will increase the tariffs in response to the rising pollution stock faster than in the non-strategic setting, since higher marginal tariffs encourage a higher rate of abatement by the Upstream country. We show that strategic abatement increases the opportunity cost of raising the tariff for the Downstream government, and therefore she commits to a lower level of optimal tariff in the long run. In contrast to earlier papers that use Markov strategies to characterize the non-cooperative scenario, in which each country's environmental policy is chosen to further its own interest, this model indicates that strategic considerations and the use of feedback strategies bring about a degree of multilateral cooperation and create incentives for the Upstream country to at least partially internalize the environmental externality. While a general finding of previous studies is that cooperative pollution control Pareto dominates non-cooperative (Markov perfect) pollution control, our results imply that strategic behavior of the Upstream government leads to an increase in her country's welfare and that the outcome for the Downstream country's welfare and environmental quality is ambiguous. This conclusion is crucial for the ongoing debate about the benefits of tying environmental and trade international agreements together. If under certain conditions feedback strategies give rise to an increase in the Downstream country's welfare, then our strategic model suggests a self-enforcing mechanism for attaining a mutually beneficial global trade and environmental agreement. A sufficient condition for such a mechanism to emerge would be a condition for the Markov perfect equilibrium pollution stock not to deteriorate compared to the non-strategic scenario. Such a condition would ensure that Downstream's net surplus gain from a lower long-run tariff is not offset by greater than before damages from the increased pollution stock.

Appendix A

Proof of Lemma

It is clear from the maximum conditions for the Downstream country that marginal net benefits, $W_d'(\tau)$, and marginal total emissions, $E'(\tau)$, have the same sign since the shadow price of pollution is negative. Intuitively, it means that a reduction in the net surplus induced by the tariff is the opportunity cost of a decrease in emissions flow that this tariff allows to achieve. To show that $W_d'(\tau)/E'(\tau)$ is an increasing function of the tariff, we differentiate this function with respect to τ:

$$\left(\frac{W_d'(\tau)}{E'(\tau)}\right)' = \frac{1}{E'(\tau)}\left[W_d''(\tau) - \frac{W_d'(\tau)}{E'(\tau)}E''(\tau)\right] \tag{A1}$$

In (A1), the expression in brackets is the second order condition for the Downstream country's maximization problem, and therefore is negative. Since $E'(\tau) < 0$, the whole expression (A1) is positive; thus $W_d'(\tau)/E'(\tau)$ is an increasing function of τ.

Appendix B

Proof of Proposition 1

If both the abatement and the tariff strategies in equilibrium are increasing functions of the pollution stock, it is clear from expressions (19) and (20) that the long run equilibrium is characterized by the negative shadow price of pollution realized by both governments. That is not only the Downstream government, but also the Upstream policymaker now realizes that the stock of pollution has a negative effect on her country's welfare, and uses this information when formulating her feedback pollution regulation strategy. It then follows from the first order conditions (15) and (17) that both the steady state tariff, τ^*, and the steady state abatement rate, a^*, are positive.

To verify how the long-run tariff rate obtained when both countries are strategic players, τ^*, compares to the steady state tariff for the case where the Upstream government acts myopically, τ^e, (obtained in Section 2), suppose that $\tau^* \geq \tau^e$.

Then using Lemma, we can induce that

$$\frac{W_d'(\tau^*)}{E'(\tau^*)} \geq \frac{W_d'(\tau^e)}{E'(\tau^e)},$$

which implies from the first order conditions (7) and (17) that the shadow price of pollution that the Downstream government derives in the absence of abatement exceeds that of the strategic abatement scenario. Then from the equilibrium conditions (13) and (20):

$$\frac{D'(Z^*)}{r+k+a'(Z^*)} \geq \frac{D'(Z^e)}{r+k}.$$

If the Upstream government's Markov perfect strategy is such that $a'(Z) > 0$ in equilibrium, it follows that

$$\frac{D'(Z^*)}{r+k} \geq \frac{D'(Z^e)}{r+k}.$$

Since the damage function $D(Z)$ is increasing and strictly convex, it is clear from the last expression that $Z^* > Z^e$. Using the steady state pollution stock expressions (12) and (21), we may write

$$[E(\tau^*) - a^*]/k > E(\tau^e)/k$$

We have shown that the Downstream government's strategy that has $\tau'(Z) > 0$ in equilibrium ensures an interior solution for the steady state abatement rate: $a^* > 0$. Thus it follows from the last expression that $E(\tau^*) > E(\tau^e)$. Given that emissions are decreasing in tariff, we obtain $\tau^* < \tau^e$, which contradicts our initial assumption. Thus we have proven that the steady state tariff derived for the abatement scenario is smaller than the tariff chosen by the Downstream government in the absence of abatement.

Appendix C

Derivation of conditions for plausible equilibrium feedback strategies $a'(Z) > 0$ and $\tau'(Z) > 0$ to emerge

Using equations (24) and (25) from the text, we can rewrite the differential equations characterizing equilibrium Markov strategies:

$$\tau'(Z) = \frac{\pi''(a)[(r+k)W_d'(\tau) - D'(Z)E'(\tau)](E(\tau) - a - kZ) + (r+k)\pi'(a)W_d'(\tau)}{\pi''(a)SOC_d(E(\tau) - a - kZ)^2 - W_d'(\tau)[W_u'(\tau) - \pi'(a)E'(\tau)]} \tag{C1}$$

$$a'(Z) = \frac{(r+k)\pi'(a)SOC_d(E(\tau) - a - kZ) + [(r+k)W_d'(\tau) - D'(Z)E'(\tau)][W_u'(\tau) - \pi'(a)E'(\tau)]}{\pi''(a)SOC_d(E(\tau) - a - kZ)^2 - W_d'(\tau)[W_u'(\tau) - \pi'(a)E'(\tau)]} \tag{C2}$$

Since the term $W_u'(\tau) - \pi'(a)E'(\tau)$ is the marginal effect of the tariff on the Upstream country's value function, it is negative. Thus it follows from equation (C1) that if the initial stock of pollution is below the steady state level, $\tau'(Z) > 0$.

To ensure that $a'(Z) > 0$, we need the nominator of expression (C2) to be negative or

$$(r+k)\pi'(a) + \frac{(r+k)W_d'(\tau) - D'(Z)E'(\tau)}{SOC_d(E(\tau) - a - kZ)}[W_u'(\tau) - \pi'(a)E'(\tau)] > 0 \tag{C3}$$

Condition (C3) can be substituted by a stricter and thus sufficient condition for $a'(Z) > 0$:

$$\frac{(r+k)W_d'(\tau) - D'(Z)E'(\tau)}{SOC_d(E(\tau)-kZ)} < -\frac{(r+k)\pi'(a)}{W_u'(\tau) - \pi'(a)E'(\tau)} \tag{C4}$$

Note that the left hand side of expression (C4) is the marginal tariff under scenario with the myopic Upstream government. Thus condition (C4) states that the Upstream government will adopt a strategy that has $a'(Z) > 0$ provided that the non-strategic marginal tariffs are relatively small. The intuition behind this condition is as follows. As shown by Proposition 2 in the text, if the Upstream government increases uses the strategy $a'(Z) > 0$, the Downstream country will respond to the rising pollution levels by a larger increase in tariff. Since the Upstream government can foresee this increase in marginal tariff rates along the equilibrium path, she will find it optimal to use the $a'(Z) > 0$ strategy only if the non-strategic marginal tariffs are sufficiently low.

Appendix D

Proof of Proposition 2

Provided that the Upstream government's abatement strategy has $a'(Z) > 0$ and that the initial stock of pollution is below its steady state level, expression (25) implies that

$$\tau'(Z) > \frac{W_d'(\tau)(r+k) - E'(\tau)D'(Z)}{SOC_d[E(\tau)-a-kZ]} > \frac{W_d'(\tau)(r+k) - E'(\tau)D'(Z)}{SOC_d[E(\tau)-kZ]} \tag{D1}$$

Note that the right hand side of (D1) is the same as expression (10) and it represents the reaction of the Downstream's tariff strategy to a change in the pollution stock in the absence of strategic abatement. Hence, we have proved that the marginal change in the equilibrium tariff induced by an increase in pollution is greater if the Upstream government is acting strategically.

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