# Administrative Costs, Public Production and Optimal Taxation

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## Abstract

This paper analyses the structure of optimal taxation in an economy with non-separable externalities and several factors of production. The paper shows that the Diamond-Mirrlees efficiency theorem also applies to this type of economy when all commodities can be taxed, and how restrictions on the set of feasible taxes, notably on taxes to internalise the externalities, may justify diversions from production efficiency. The theoretical framework is finally used to investigate the optimal taxation of private and collective transport and the effect on the optimal tax system of the introduction of road pricing.

Keywords:
Optimal taxation, externalities, administrative costs, production efficiency, CGE model, transport, road pricing

JEL classification codes:
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# 1. Introduction

Diamond and Mirrlees (1971) have proved that when costless lump-sum transfers are not feasible production efficiency is still desirable, although the conditions for Pareto-efficiency are not attainable[1]. The assumptions that need to hold for the Diamond-Mirrlees efficiency theorem to be valid, that tax rates on all market transactions and on pure profit can be fixed at their optimal levels at no cost, are however very restrictive, as has been pointed out from the outset by Stiglitz and Dasgupta (1971). The widespread use of uniform tax rates for large groups of commodities strongly suggests that the administrative costs of such tax structures are lower than when tax rates are differentiated and thus undermines the justification of the assumption that all commodities can be taxed at their optimal level at no costs.

The implications of restrictions on the set of feasible tax instruments for optimal commodity taxation have been investigated by Munk (1980)[2] in an economy with one representative household taking as given that certain tax instruments could not be used. The present paper extends the analysis to an economy with many households and where consumption is associated with external effects, and deepens the analysis by providing an explanation in terms of administrative costs for the restrictions on the government's choice of tax rates, effectively making these restrictions endogenous.

The paper demonstrates that the Diamond-Mirrlees efficiency theorem also applies in an economy with externalities when all commodities can be taxed at their optimal level at no costs. The paper furthermore investigates how restrictions on the government's choice of tax rates influence the optimal tax system in particular in the sector producing the commodity generating the external effects. The theoretical results are used to investigate the optimal taxation of transport (private road transport and collective transport) and how the introduction of road pricing modifies the optimal tax system and changes the arguments for public ownership of the collective transport sector.

The paper is organised as follows. In section 2 the model is formulated. In section 3 the general conditions for an optimal tax structure are derived and analysed under different assumptions about which tax instruments are feasible. In section 4 the implications for the optimal taxation of private road transport and the collective transport is considered, and in this context the implication of the introduction of road pricing is discussed. A final section summarises the paper.

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[1] Strictly speaking, the theorem only claims that production efficiency outside the household sector, i.e. in production sectors transforming marketed commodities, is desirable. Since the optimal tax system requires the rates of transformation within the household sector to be different from those within the production sectors transforming marketed commodities, when comparing household production with production in the sectors transforming marketed commodities, production efficiency is clearly not desirable.

[2] Munk (1980) restated using the expenditure function approach (see Dixit and Munk 1977) and to some extent corrected the analysis in Stiglitz and Dasgupta 1971)

## 2. The model

The model considered involves the government choosing commodity and primary factor taxes and a uniform lump-sum transfer[3] to maximise a Pareto social welfare function consistent with egalitarian value judgements, subject to constraints representing the structure of the economy, including the administrative costs of taxation.

In the economy, there are production sectors labelled $k \in (1,..,N) = A$, households labelled $h \in (1,..,H) = H$ and a government labelled G. Primary factors are labelled $j \in (1,..,M) = F$ and produced commodities $i \in (M+1,..,M+N) = C$ .

The households' utility functions are $u^h(\mathbf{x^h}, \mathbf{z^h}, e)$, $h \in H$, where $\mathbf{z^h} \equiv (z_1^h,..., z_M^h)$ is the h^th^ supply of primary factors, $\mathbf{x^h} \equiv (x_{M+1}^h,..., x_{M+N}^h)$ its demand for final commodities, and $e$ the amount of an environmental good. The households' consumption of commodity M+1 (the dirty good) is negatively related to the environmental good according to $e = e\left(\sum_{h \in H} x_{M+1}^h(\mathbf{q}, \mathbf{w}, e, u^h)\right)$.
The households face prices $\mathbf{w} \equiv (w_1,.., w_M)$ for primary factors, and $\mathbf{q} \equiv (q_{M+1},..., q_{M+N})$ for produced commodities. The corresponding expenditure functions are $E^h(\mathbf{q}, \mathbf{w}, e, u^h)$, $h \in H$.

Firms maximise profit under conditions of perfect competition and constant returns to scale. $Y^k$ is the output of sector k, and $\mathbf{v}^k \equiv (v_1^k,..., v_M^k)$ the use of primary factors in that sector. Output prices are $p_{k+M} = c^k(\mathbf{p}^k)$, $k \in A$, where $c^k(\mathbf{p}^k)$ is the unit cost function in sector k, and $\mathbf{p}^k \equiv (p_1^k,..., p_M^k)$ the input prices. The output contingent primary factor input demands are $v_j^k = a_j^k(\mathbf{p}^k) Y^k$, $j \in F$, $k \in A$, where $a_j^k = a_j^k(\mathbf{p}^k)$, $j \in F$, $k \in A$ are the corresponding technical coefficients.

The government's resource requirements are $\{\mathbf{z}^G, \mathbf{x}^G\} \equiv (z_1^G, .. z_M^G, x_{M+1}^G .. x_{M+N}^G)$.

Tax rates are defined relative to market prices, $\mathbf{p} \equiv (p_1,..., p_{M+N})$. As a matter of normalisation we assume that $p_1 = 1$.

The government can levy a number of different taxes: household taxes, $(t_{M+1},..., t_{M+N})$, on the consumption of produced commodities, and, $(s_1,.., s_M)$, on the supply of primary factors; sector specific producer taxes on the use of primary factors, $(t_1^k,..., t_M^k)$, $k \in A$; and a uniform lump-sum tax, $L$.[4] The level of taxation may for the consumption of produced commodities be expressed by ratios between household prices and market prices,

$$T_i \equiv q_i / p_i = (t_i + p_i) / p_i \qquad i \in C$$

for supplies of primary factors by ratios between market prices and household prices,

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[3] This allows income taxation within the framework to be represented by a linear income tax.

[4] Thus, unearned income is for all households equal to the uniform lump-sum tax, i.e. $I^h = -L$. $L$ is negative if it is interpreted as the fixed element in a progressive linear income tax schedule.

$$S_j \equiv p_j / w_j = p_j /(p_j - s_j) \qquad j \in F$$

and for the use of primary factors in the different production sectors by ratios between producer prices and market prices,

$$T_j^k \equiv p_j^k / p_j = (t_j^k + p_j)/ p_j \qquad k \in A \;\; j \in F$$

We classify tax systems $\xi \equiv \left(\mathbf{S},\mathbf{T}, (\mathbf{T}^k, k \in A), L\right)$, where $\mathbf{T} \equiv \left(T_{M+1},...,T_{M+N}\right)$, $\mathbf{S} \equiv \left(S_1,..,S_M\right)$, and $\mathbf{T}^k \equiv (T_1^k,...,T_M^k)$, $k \in A$, according to which tax structure they belong. A tax structure $\left(\xi_j^i \in \boldsymbol{\Xi}^i\right)$ is a set of tax systems where the same restrictions are imposed on the set of tax instruments. Tax systems belonging to the same tax structure are associated with the same administrative costs. The government budget including the administrative costs associated with a tax system belonging to the tax structure $i$ is $B(\boldsymbol{\Xi}^i)$. $B(\boldsymbol{\Xi}^i)$ is assumed to be homogeneous of degree 1 in $\mathbf{p}$. Other government expenditures are considered as exogenous. The government's expenditures may therefore be written as $B^i \equiv \mathbf{p}\left\{\mathbf{z}^G\left(\boldsymbol{\Xi}^i\right), \mathbf{x}^G\left(\boldsymbol{\Xi}^i\right)\right\}$. The number of tax structures, $\left(\boldsymbol{\Xi}^i, i \in \mathbb{F}\right)$, is assumed to be finite.

Using the expenditure function approach (see Dixit and Munk 1977 and Munk 1980) we specify the government's maximisation problem for a given tax structure as the maximisation of the social welfare function, $W\left(u^1,u^2,..,u^H\right)$, with respect to $u^1,u^2,..,u^H$; $L$; $\mathbf{w} \equiv \left(w_1,..,w_M\right)$; $\mathbf{q} \equiv \left(q_{M+1},...,q_{M+N}\right)$; and $\mathbf{p}^k \equiv \left(p_1^k,.., p_M^k\right)$, $k \in A$, subject to the following constraints:

1) $E^h(\mathbf{q},\mathbf{w},e,u^h) = -L$, $h \in H$, which requires lump-sum taxation to be uniform and the levels of individual utilities to be consistent with the level of unearned income, $I^h = -L$, for each household;

2) $\sum_{h \in H} z_j^h\left(\mathbf{q},\mathbf{w},e,u^h\right) - \sum_{i \in C} a_j^{i-M}\left(\mathbf{p}^{k-M}\right)\left(\sum_{h \in H} x_i^h\left(\mathbf{q},\mathbf{w},e,u^h\right) + x_i^G\right) - z_j^G = 0$, $j \in F$, which represents the general equilibrium conditions of the economy;[5]

3) The environmental good is related to the consumption of the dirty good by

$$e = e\left(\sum_{h \in H} x_{M+1}^h\left(\mathbf{q},\mathbf{w},e,u^h\right)\right)$$

and depending of the tax structure a subset of the following tax constraints:

[5] For a price-tax vector to be consistent with an equilibrium situation it has to satisfy the conditions for profit maximisation, utility maximisation and the material balances. The conditions for profit maximisation and utility maximisation are substituted into the material balance conditions, and finally the material balance conditions for the produced commodities are substituted into the material balance conditions for the primary factors. Notice that since we consider more than one primary factor we cannot without a priori imposing productive efficiency incorporate the material balance conditions into the government's budget constraint.

4) $\{ p_j^k / w_j = T_j^k S_j ,\ j \in F,\ k \in A \}$, which for primary factors constrain the differences between sector specific producer prices, $p_j^k$, and household prices; $w_j$ ;

5) $\{ q_i / c^{i-M}(\mathbf{p}^{i-M}) = T_i ,\ i \in C \}$, which for produced commodities constrain the differences between household prices, $q_i$ , and producer prices (represented by unit costs, $c^i(\mathbf{p}^i)$ ).

The government solves its maximisation problem in a two-step procedure. First, it calculates the optimal tax system, $\xi^{i*}$, $i \in \boldsymbol{\Xi}^i$, for each tax structure, $i \in \mathbb{F}$. Then, in the second step, it chooses as the overall optimal tax system, $\xi^*$, that of the solutions to the maximisation problems for the different tax structures, which is associated with the highest level of social welfare.

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## 3. The conditions for an optimal tax system

In this section we derive the optimality conditions for a tax system belonging to a given tax structure, i.e. when certain tax rates are constrained due to the administrative costs involved in differentiating the tax rates.

The Lagrangian expression corresponding to the government's maximisation problem for a given tax structure, $\boldsymbol{\Xi}^s$, may be formulated as

$$
\begin{aligned}
\pounds = {} & W\left(u^1, u^2, .., u^H\right) \\
& + \sum_{h \in H} \psi^h \left(-E^h(\mathbf{q}, \mathbf{w}, e, u^h) - L\right) \\
& + \sum_{j \in F} \lambda_j \left( \sum_{h \in H} z_j^h\left(\mathbf{q}, \mathbf{w}, e, u^h\right) - \sum_{i \in C} a_j^{i\text{-}M}\left(\mathbf{p}^{i\text{-}M}\right)\left( \sum_{h \in H} x_i^h\left(\mathbf{q}, \mathbf{w}, e, u^h\right) + x_i^G\left(\boldsymbol{\Xi}^s\right)\right) - z_j^G\left(\boldsymbol{\Xi}^s\right)\right) \\
& + \sum_{j \in F} \sum_{k \in A} \gamma_j^k \left( p_j^k / w_j - T_j^k S_j \right) + \sum_{i \in C} \gamma_i \left( q_i / c^{i-M}(\mathbf{p}^{i-M}) - T_i \right) \\
& + \rho \left( e \left( \sum_{h \in H} x_{M+1}^h\left(\mathbf{q}, \mathbf{w}, e, u^h\right)\right) - e \right) \qquad (1)
\end{aligned}
$$

where

$\psi^h$ is the net social value of transferring income lump-sum wise from the government to household h

$\lambda_j$ is the opportunity cost price of primary factor j[6]

$\gamma_j^k$ is the social welfare cost price of increasing the tax on the use of primary factor j in sector k

[6] Since by normalisation $p_1 = 1$, $\lambda_1$ may also be interpreted as the cost of government funds in terms of social welfare.

$\gamma_i$ is the social welfare cost price of increasing the tax on the consumption of commodity i

$\rho$ is the marginal social value of the environmental good

To simplify the notation and facilitate interpretation we define for h$\in H$

$$E_u^h \equiv \frac{\partial E^h}{\partial u}; \quad E_e^h = \frac{\partial E^h}{\partial e}; \quad E_{j,u}^h \equiv \frac{\partial^2 E^h}{\partial q_j \partial u}, \ j\in C; \quad E_{j,u}^h \equiv \frac{\partial^2 E^h}{\partial w_j, \partial u}, \ j\in F;$$

$$E_{ij}^h \equiv \frac{\partial^2 E^h}{\partial q_i \partial q_j}, \ i, j\in C; \ E_{is}^h \equiv \frac{\partial^2 E^h}{\partial q_i \partial w_s}, \ i\in C, \ s\in F; \quad E_{rs}^h \equiv \frac{\partial^2 E^h}{\partial w_r \partial w_s}, \ r, s\in F;$$

$$E_{ij} \equiv \sum_{h\in H} E_{ij}^h \ \ i, j \in C \cup F; \quad E_{ie} \equiv \sum_{h\in H} E_{i,e}^h; \ Z_j \equiv z_j^h; \quad X_i \equiv x_i^h$$

The first order conditions with respect to $u^1, u^2, .., u^H$; $L$; $\mathbf{w} \equiv (w_1, .., w_M)$; $\mathbf{q} \equiv (q_{M+1}, ..., q_{M+N})$; and $\mathbf{p}^k \equiv (p_1^k, .., p_M^k)$, k$\in A$, may therefore be expressed as:

$$\frac{\partial £}{\partial u^h} = \frac{\partial W}{\partial u^h} - \psi^h E_u^h + \sum_{j\in F} \lambda_j \left( -E_{ju}^h - \sum_{k=C} a_j^{k-M} E_{ku}^h \right) + \rho \frac{de}{dX_{M+1}} \sum_{h\in H} E_{M+1,u}^h = 0 \qquad h\in H \quad (2)$$

$$\frac{\partial £}{\partial L} = -\sum_{h\in H} \psi^h = 0 \qquad (3)$$

$$\frac{\partial £}{\partial w_s} = \sum_{h=H} \psi^h z_s^h + \sum_{j\in F} \lambda_j \left( -E_{js} - \sum_{k=C} a_j^{k-M} E_{ks} \right) + \sum_{k\in A} \frac{\gamma_s^k}{w_s} T_s^k S_s + \rho \frac{de}{dX_{M+1}} E_{M+1,s} = 0 \qquad s \in F \quad (4)$$

$$\frac{\partial £}{\partial q_i} = -\sum_{h=H} \psi^h x_i^h + \sum_{j\in F} \lambda_j \left( -E_{ji} - \sum_{k=C} a_j^{k-M} E_{ki} \right) + \frac{\gamma_i}{p_i} + \rho \frac{de}{dX_{M+1}} E_{M+1,i} = 0 \qquad i \in C \quad (5)$$

$$\frac{\partial £}{\partial e} = -\sum_{h\in H} \psi^h E_e^h + \sum_{j\in F} \lambda_j \left( -E_{je} - \sum_{k\in C} a_j^{k-M} E_{ke} \right) + \rho \frac{de}{dX_{M+1}} E_{M+1,e} - \rho = 0 \qquad (6)$$

$$\frac{\partial £}{\partial p_s^k} = -\sum_{j\in F} \lambda_j a_{js}^k Y^k + \frac{\gamma_s^k}{w_s} + \frac{\gamma_{k+M}}{p_{k+M}} T_{k+M} a_s^k = 0 \qquad k\in A, \ s\in F \quad (7)$$

where $a_{js}^k = \frac{\partial^2 c^k(\mathbf{p}^k)}{\partial p_j^k \partial p_s^k}$ and where some $\gamma_i$ and $\gamma_j^k$ are zero according to the tax structure considered.

From (2) we obtain that the net marginal social value of a lump-sum transfer from the government to household h may be expressed as

$$\psi^h = \mu^h - \lambda_1 \qquad h\in H \quad (8)$$

where (in analogy with the definition by Diamond 1975 for an economy without tax restrictions) the *net marginal social welfare of income for household h* we defined as

$$\mu^h \equiv \beta^h \text{-} \lambda_1 \sum_{j \in F} \left( w_j - \tilde{p}_j \right) \frac{\partial z_j^h}{\partial I^h} \quad + \lambda_1 \sum_{i \in C} \left( q_i - \tilde{p}_i \right) \frac{\partial x_i^h}{\partial I^h} + \rho \frac{de}{dX_1} \sum_{h \in H} \frac{\partial x_i^h}{\partial I^h} \qquad \mathrm{h} \in H \quad (9)$$

and where $\beta^h \equiv \frac{\partial W}{\partial u^h} \frac{\partial v^h}{\partial I^h}$ is the marginal social welfare of income for household h and $\tilde{p}_j \equiv \lambda_j / \lambda_1$ , $\mathrm{j} \in F$ , $\tilde{p}_i \equiv \left( \mathrm{c}^1 \left( \tilde{p}_j, j \in F \right), ..., \mathrm{c}^{\mathrm{N}} \left( \tilde{p}_j, j \in F \right) \right)$ , $\mathrm{i} \in \mathrm{C}$ the opportunity cost prices of primary factors and produced commodities, respectively, relative to the opportunity cost price of primary factor 1.

By normalisation $p_1 = 1$, hence $p_1 \equiv \tilde{p}_1$.

From (3) we have (when a uniform lump-sum tax is feasible) that

$$\tilde{R}_{\mathrm{L}} = \lambda_1 , \qquad (10)$$

where $\tilde{R}_{\mathrm{L}} = \frac{\sum_{\mathrm{h} \in H} \mu^{\mathrm{h}}}{\mathrm{H}}$ is the distributional characteristic if the uniform lumpsum tax.

Using these definitions we obtain from (4) and (5), respectively, the *indices of discouragement* in analogy with Mirrlees 1976)[7]

$$d_s = \frac{\sum_{i=C} \left( q_i - \tilde{p}_i \right) E_{is} + \sum_{j=F} \left( w_j - \tilde{p}_j \right) E_{js}}{-Z_s} = - \frac{\lambda_1 - \tilde{R}_{\mathrm{s}}}{\lambda_1} - \frac{\rho \frac{de}{dX_{M+1}} \mathrm{E}_{\mathrm{M+1,s}}}{\lambda_1 (-Z_s)} + \sum_{k \in A} \frac{\gamma_{\mathrm{s}}^k}{w_{\mathrm{s}} \lambda_1 (-Z_s)} T_{\mathrm{s}}^k S_s \qquad \mathrm{s} \in \mathrm{F} \ (12)$$

$$d_k = \frac{\sum_{i=C} \left( q_i - \tilde{p}_i \right) E_{ik} + \sum_{j=F} \left( w_j - \tilde{p}_j \right) E_{jk}}{X_k} = - \frac{\lambda_1 - \tilde{R}_{\mathrm{k}}}{\lambda_1} - \frac{\rho \frac{de}{dX_{M+1}} \mathrm{E}_{\mathrm{M+1,s}}}{\lambda_1 X_k} - \frac{\gamma_k}{p_k \lambda_1 X_k} \qquad \mathrm{k} \in \mathrm{C} \ (13)$$

where $\tilde{R}_{\mathrm{j}} = \frac{\sum_{\mathrm{h} \in H} \mu^{\mathrm{h}} E_j^h}{E_j}$ ($\mathrm{j} \in \mathrm{C} \cup \mathrm{F}$) and $\tilde{R}_{\mathrm{e}} = \frac{\sum_{\mathrm{h} \in H} \mu^{\mathrm{h}} E_e^h}{E_e}$ are respectively the *net distributional characteristic* of commodity j and the public god externality.

From (6) we obtain an expression for the opportunity value of an increase in the environmental good.

$$\rho = \left[ -\tilde{R}_e E_e + \sum_{i \in C} \left( q_i - \tilde{p}_i \right) E_{ie} + \sum_{j \in F} \left( w_j - \tilde{p}_j \right) E_{je} \right] \frac{1}{1 - \frac{de}{dX_{M+1}} \mathrm{E}_{\mathrm{M+1,e}}} \qquad (14)$$

[7] The *indices of discouragement* provide an approximation to the reduction in compensated demand from the first best situation where household prices are equal to producer prices to the second best situation. Notice that the indices of discouragement here are defined for differences between household prices and opportunity cost prices rather than the difference between household prices and producer prices.

Rewriting (12 ) and (13) the conditions for an optimal tax system may in matrix notation be written as

$$\mathbf{Et} = \mathbf{b} \tag{15}$$

where

$$\mathbf{E} \equiv \left[ \left\{ E_{ij}, i, j \in F \cup \mathrm{C} \right\} \right], \ \tilde{\mathbf{t}} = \left\{ \tilde{t}_i, i \in F \cup \mathrm{C} \right\} \text{ and}$$

$$\mathbf{b} = \left\{ b_i, i \in F \cup \mathrm{C} \right\}$$

and where

$$b_s = -\frac{\lambda_1 - \tilde{R}_\mathrm{s}}{\lambda_1}(-Z_s) - \frac{\rho \dfrac{de}{dX_1}\mathrm{E}_{\mathrm{M+1,s}}}{\lambda_1} - \sum_{k \in A} \frac{\gamma_\mathrm{s}^k}{w_\mathrm{s}\lambda_1} T_\mathrm{s}^k S_s \qquad \mathrm{s} \in \mathrm{F}$$

$$b_k = -\frac{\lambda_1 - \tilde{R}_\mathrm{k}}{\lambda_1} X_\mathrm{k} - \frac{\rho \dfrac{de}{dX_1}\mathrm{E}_{\mathrm{M+1,k}}}{\lambda_1} - \frac{\gamma_k}{p_k \lambda_1} \qquad \mathrm{k} \in \mathrm{C}$$

By Cramer's Rule

$$\tilde{t}_k = \frac{\left|\mathbf{E}_k\right|}{\left|\mathbf{E}\right|} \qquad \mathrm{k} \in F \cup \mathrm{C} \tag{16}$$

where $\left|\mathbf{E}\right|$ is the determinant of $\mathbf{E}$ and where $\mathbf{E}_k$ is obtained from $\mathbf{E}$ by replacing the k$^{\text{th}}$ column by $\mathbf{b}$.

Let $C_{ij}$ be the i,j cofactor of $\mathbf{E}$. We then have by general properties of matrices that

$$\left|\mathbf{E}_k\right| = -\sum_{i \in C \cup F} C_{ik} \frac{\lambda_1 - \tilde{R}_\mathrm{i}}{\lambda_1} E_i - \frac{\rho}{\lambda_1}\frac{de}{dX_1} \sum_{i \in C \cup F} C_{ik} E_{M+1,i} + \sum_{j \in F} C_{jk} \sum_{r \in A} \frac{\gamma_\mathrm{j}^r}{w_\mathrm{j}\lambda_1} T_\mathrm{i}^k S_i - \sum_{i \in C} C_{ik} \frac{\gamma_i}{p_i \lambda_1} \quad \mathrm{k} \in F \cup \mathrm{C} \tag{17}$$

$$\left|\mathbf{E}\right| = \sum_{i \in C \cup F} C_{i,M+1} E_{M+1,i} \tag{18}$$

$$0 = \sum_{i \in C \cup F} C_{ik} E_{M+1,i} \qquad k \neq M+1 \tag{19}$$

We may therefore rewrite (16)

$$\tilde{t}_s = \frac{-\sum\limits_{i \in C \cup F} C_{is} \dfrac{\lambda_1 - \tilde{R}_\mathrm{i}}{\lambda_1} \mathrm{E}_\mathrm{i}}{\left|\mathbf{E}\right|} + \frac{\sum\limits_{j \in F} C_{is} \sum\limits_{k \in A} \dfrac{\gamma_\mathrm{j}^k}{w_\mathrm{j}\lambda_1} - \sum\limits_{i \in C} C_{is} \dfrac{\gamma_i}{p_i \lambda_1}}{\left|\mathbf{E}\right|} \qquad \mathrm{s} \in F \tag{20}$$

$$\tilde{t}_{M+1} = \frac{-\sum_{i\in C\cup F} C_{i,M+1} \frac{\lambda_1 - \tilde{R}_i}{\lambda_1} \mathrm{E}_i}{|\mathbf{E}|} - \frac{\rho}{\lambda_1} \frac{de}{dX_{M+1}} + \frac{\sum_{j\in F} C_{j,M+1} \sum_{r\in A} \frac{\gamma_j^r}{w_j \lambda_1} - \sum_{i\in C} C_{i,M+1} \frac{\gamma_i}{p_i \lambda_1}}{|\mathbf{E}|} \tag{21}$$

$$\tilde{t}_k = \frac{-\sum_{i\in C\cup F} C_{ik} \frac{\lambda_1 - \tilde{R}_i}{\lambda_1} \mathrm{E}_i}{|\mathbf{E}|} + \frac{\sum_{j\in F} C_{jk} \sum_{r\in A} \frac{\gamma_j^r}{w_j \lambda_1} - \sum_{i\in C} C_{ik} \frac{\gamma_i}{p_i \lambda_1}}{|\mathbf{E}|} \qquad \mathrm{k} \neq M+1, \mathrm{k} \in C \tag{22}$$

Rewriting (7) we get the following expression to be satisfied by the taxes on the use of primary factors in the various production sectors in the case of only two primary factors, i.e. F=(1,2),[8]

$$\frac{p_2^k}{p_1^k} - \tilde{p}_2 = -\frac{\gamma_s^k}{w_s} / \lambda_1 a_{2s}^k Y^k + \frac{\gamma_{M+k}}{p_{M+k}} T_{M+k}\, a_s^k / \lambda_1 a_{2s}^k Y^k \qquad \mathrm{s}\in F,\ \mathrm{k}\in A \tag{23}$$

Equation (23) shows that the Diamond and Mirrlees theorem that productive efficiency is desirable when all commodities can be taxed therefore also applies to an economy with externalities and that production efficiency in general is not desirable when it is not possible to tax all market transactions at there optimal level (cf. Stiglitz and Dasgupta 1971 and Munk 1980).

For the governments maximisation problem to be interesting a number of tax constraints must be non-binding and hence a number of the corresponding opportunity cost prices, $\gamma_j^k$ and $\gamma_i$ , must be zero. The opportunity cost price of a tax constraint is not only zero when the corresponding transactions can be taxed freely, but also when, according to tax equivalence theorems, a tax on the transaction is equivalent to taxes on other transactions which are feasible

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8 $$-\lambda_1 a_{1s}^k Y^k - \lambda_2 a_{2s}^k Y^k + \frac{\gamma_s^k}{w_s} + \frac{\gamma_{M+k}}{p_{M+k}} T_{M+k}\, a_s^k = 0 \qquad \mathrm{s}\in F,\ \mathrm{k}\in A$$

$$-\lambda_1 \left[ \frac{a_{1s}^k}{a_{2s}^k} + \frac{\lambda_2}{\lambda_1} \right] a_{2s}^k Y^k + \frac{\gamma_s^k}{w_s} + \frac{\gamma_{M+k}}{p_{M+k}} T_{M+k}\, a_s^k = 0 \qquad \mathrm{s}\in F,\ \mathrm{k}\in A$$

By the assumption of constant returns to scale production functions and cost minimisation

$$1 = f^k\left(a_1^k\left(p^k\right), a_2^k\left(p^k\right)\right) \qquad \mathrm{k}\in A$$

By differentiation with respect to $p_s^k$ and by implications of cost minimization we have

$$\frac{a_{1s}^k}{a_{2s}^k} = -\frac{f_2^k}{f_1^k} = -\frac{p_2^k}{p_1^k} \qquad \mathrm{s}\in F,\ \mathrm{k}\in A$$

where $a_{js}^k \equiv \dfrac{\partial^2 c^k\left(p^k\right)}{\partial p_j^k \partial p_s^k}$

Substituting for $\dfrac{a_{1s}^k}{a_{2s}^k}$ and $\dfrac{\lambda_2}{\lambda_1}$ we therefore get

$$\left[\frac{p_2^k}{p_1^k} - \tilde{p}_2\right] = -\left(\frac{\gamma_s^k}{w_s} + \frac{\gamma_{M+k}}{p_{M+k}} T_{M+k} a_s^k\right) / \lambda_1 a_{2s}^k Y^k \qquad \mathrm{s}\in F,\ \mathrm{k}\in A$$

(see Munk 1980). For example, that the consumption of only one produced good or the supply of only one primary factor cannot be taxed does not in itself impose a binding constraint. Similarly that one of the inputs in an industry cannot be taxed does not impose a constraint if all other inputs and the consumption of the commodity produced in the sector can. However, when adding tax constraints, a tax constraints which taken in isolation could be considered as normalisation rule, also may become a biding constraint.

We consider three different tax structures

1) when there are no tax restrictions,
2) when the supply of different primary factors need to be taxed according to the same rate ( as under a general income tax) and when in one production sector the use of the primary factors cannot be taxed, and
3) when in addition, the consumption of the commodity associated the externality cannot be taxed.

For ease of exposition we assume that there are only two primary factors.

### *The optimal tax system without tax restrictions*

When the government can tax all market transactions, (23) becomes

$$\frac{p_2^k}{p_1^k} - \tilde{p}_2 = 0 \qquad \mathrm{k} \in A \quad (24)$$

and, we may, as a matter of normalisation, assume that the use of primary factors in all production sectors is untaxed, i.e. $p_s^k = p_s$ , $\mathrm{s} \in F, \mathrm{k} \in A$ .

All market prices are therefore equal to the corresponding opportunity cost prices, i.e.

$$p_i = \tilde{p}_i \equiv \sum_{j \in F} \mathrm{a}_{\mathrm{j}}^{\mathrm{i\text{-}M}} \left(\mathbf{p}\right) p_j \qquad \mathrm{i} \in C \quad (25)$$

and the optimal tax system becomes

$$t_{\mathrm{s}}^{\mathrm{k}} = 0 \qquad \mathrm{s} \in \mathrm{F}, \mathrm{k} \in \mathrm{A} \quad (26)$$

$$t_s = \frac{-\sum_{i \in C \cup F} C_{is} \frac{\lambda_1 - \tilde{R}_{\mathrm{i}}}{\lambda_1} \mathrm{E}_{\mathrm{i}}}{\left|\mathbf{E}\right|} \qquad \mathrm{s} \in F \quad (27)$$

$$t_{M+1} = \frac{-\sum_{i \in C \cup F} C_{i,M+1} \frac{\lambda_1 - \tilde{R}_i}{\lambda_1} \mathrm{E_i}}{|\mathbf{E}|} - \frac{\rho}{\lambda_1} \frac{de}{dX_{M+1}} \tag{28}$$

$$t_k = \frac{-\sum_{i \in C \cup F} C_{ik} \frac{\lambda_1 - \tilde{R}_i}{\lambda_1} \mathrm{E_i}}{|\mathbf{E}|} \qquad \mathrm{k} \neq M+1, \mathrm{k} \in C \tag{29}$$

### ***The optimal tax system with restrictions on taxation of the supply of primary factors***

In this case we assume that the supply of the two primary factors, labour and capital, must be taxed at the same rate, i.e. $S \equiv p_j / w_j$, $\mathrm{s} \in F$, and that the use of the primary factors in sector N cannot be taxed, i.e. $t_s^N = 0$, $\mathrm{s} \in F$.

Since by normalisation $p_1 \equiv \tilde{p}_1$ and by assumption $p_2^N = p_2$,

$$p_2 - \tilde{p}_2 = -\frac{\gamma_1^N}{w_1} / \lambda_1 \mathrm{a}_{21}^{\mathrm{N}} Y^{\mathrm{N}} \qquad \mathrm{k} \neq N\, \mathrm{k} \in A \tag{30}$$

Since $\mathrm{a}_{21}^{\mathrm{N}} = \lambda_1 \mathrm{a}_1^{\mathrm{N}} \mathrm{a}_2^{\mathrm{N}} \sigma^{\mathrm{N}} Y^{\mathrm{N}} / c^{\mathrm{N}}(\mathbf{p}^{\mathrm{N}})$ and $p_N = c^{\mathrm{N}}(\mathbf{p}^{\mathrm{N}})$ the optimal taxes on the use of labour and capital in the other production sectors must therefore satisfy

$$\frac{p_2^k}{p_1^k} - p_2 = \frac{\gamma_1^N p_N}{w_1} / \lambda_1 \mathrm{a}_1^{\mathrm{N}} \mathrm{a}_2^{\mathrm{N}} \sigma^{\mathrm{N}} Y^{\mathrm{N}} \qquad \mathrm{k} \neq N\, \mathrm{k} \in A \tag{31}$$

We may as a matter of normalisation choose the level of tax/subsidy on the primary factors in the production of private road transport and collective transport such that the unit cost at opportunity cost prices and at producer prices are the same, i.e. such that $\tilde{p}_i = \sum_{j \in F} \mathrm{a_j^{i\text{-}M}} \tilde{p}_j = \sum_{j \in F} \mathrm{a_j^{i\text{-}M}} p_j^{\mathrm{i\text{-}M}} = p_i$. The optimal tax system then becomes

$$t_s = (1 - S) / w_s \qquad \mathrm{s} \in F \tag{32}$$

$$t_3 = \frac{-\sum_{i \in C \cup F} C_{i,3} \frac{\lambda_1 - \tilde{R}_i}{\lambda_1} \mathrm{E_i}}{|\mathbf{E}|} - \frac{\rho}{\lambda_1} \frac{de}{dX_{M+1}} - \frac{\sum_{i \in F} C_{i,3} \frac{\gamma_{\mathrm{i}}^N}{w_{\mathrm{i}} \lambda_1}}{|\mathbf{E}|} \tag{33}$$

$$t_k = \frac{-\sum_{i \in C \cup F} C_{ik} \frac{\lambda_1 - \tilde{R}_i}{\lambda_1} \mathrm{E}_i}{|\mathbf{E}|} - \frac{\sum_{i \in F} C_{ik} \frac{\gamma_i^N}{w_i \lambda_1}}{|\mathbf{E}|} \qquad k \neq 3, k \in C \quad (34)$$

### *The optimal tax system with additional restrictions on the taxation of the commodity associated with external effects*

In this case, in addition to the tax restrictions in the previous case, we assume that the consumption of the “dirty good” cannot be taxed, i.e. $t_3 = 0$, and that in the production of the “dirty good” the use of the primary factor 1 cannot be taxed either, i.e. $t_1^1 = 0$. The optimal tax system then becomes

$$t_2^1 = \frac{\gamma_1^N}{w_1} / \lambda_1 \mathrm{a}_{21}^{\mathrm{N}} Y^{\mathrm{N}} - \left( \frac{\gamma_1^1}{w_1} + \frac{\gamma_3}{p_3} \mathrm{a}_1^1 / \lambda_1 \mathrm{a}_{21}^1 Y^1 \right) \qquad \mathrm{s} \in F$$

$$\frac{p_2^k}{p_1^k} - p_2 = \frac{\gamma_1^N p_N}{w_1} / \lambda_1 \mathrm{a}_1^{\mathrm{N}} \mathrm{a}_2^{\mathrm{N}} \sigma^{\mathrm{N}} Y^{\mathrm{N}} \qquad \mathrm{k} \neq 1 \text{ and N}, \mathrm{k} \in A \ (36)$$

$$t_s = (1 - S) / w_s \qquad \mathrm{s} \in F \ (37)$$

$$\tilde{t}_k = \frac{-\sum_{i \in C \cup F} C_{ik} \frac{\lambda_1 - \tilde{R}_i}{\lambda_1} \mathrm{E}_i}{|\mathbf{E}|} + \frac{\sum_{i \in F} C_{ik} \sum_{r \in A} \frac{\gamma_i^r}{w_i \lambda_1} - C_{3,k} \frac{\gamma_3}{p_3 \lambda_1}}{|\mathbf{E}|} \qquad k \neq 3, k \in C \quad (38)$$

### *Interpretation*

When all commodities can be taxed, production efficiency is desirable. Commodities which are relatively complementary with the untaxed use of the primary factors (leisure in the case of labour) will tend to be taxed at a relatively high rate.

Commodities, which are predominantly consumed by high-income households, and primary factors, which are predominantly supplied by high-income households, will also tend to be taxed at relatively high rates.

When all commodities can be taxed only the tax on the *dirty good* is adjusted to take into account the externality, however, when the dirty good cannot be taxed, then the optimal tax rates for the dirty good is modified indirectly to compensate for this (see Sandmo 1975).

When it is not possible to differentiate the rates of tax on the supply of the primary factors, sector specific taxes on the use of primary factors in those sectors where it is possible are used to approach the marginal rate of transformation between labour and capital to that which would have applied if it had been possible to differentiate the rates of tax on the supply of the primary factors. The extent to which the sector specific factor taxes will be differentiated in the production sectors will depend on the elasticity of substitution between the primary factors in the sector where it is not possible to differentiate, i.e. sector N.

When the dirty good cannot be taxed and the use of labour cannot be taxed in the production of the dirty good, the tax on capital in that sector also depends also on the tax constraints $\gamma_1^1$ and $\gamma_3$. The tax formulas for other goods than the dirty good are thus modified to take into account the effect of congestion externality.

## 4. Taxation of transport

In this section we use the insight obtained in the previous section to analyse the optimal taxation of transport. We analyse, based on the three different set of assumptions considered above with respect to which commodities can be taxed, the optimal tax structure in an economy with three products, *private road transport, collective transport and other goods*, and two primary factors *capital* and *labour*. On this background we identify the likely consequences of the introduction of road pricing with respect to the optimal tax structure.

In order to focus the analysis we base the analysis on a number of stylised facts:

*Private road transport*

- is the result of home production and more difficult to observe for tax purposes than market transactions,
- is produced using the same technology by all households,
- is consumed disproportionately by the high income households,
- is relatively complementary with leisure,
- is associated with congestion externalities, and
- is produced with a relatively high input of capital.

*Collective transport*

- is consumed disproportionately by the low income households, and
- is less complementary with leisure than private road transport (being used predominately for commuting).

*Capital*

- is supplied disproportionately by high income households.

*Administrative costs*

- the administrative costs associated with a tax structure without tax restrictions are so large that such a tax structure is not desirable
- the administrative cost of taxing private road transport are large but have fallen significantly with the introduction of modern monitoring technology

We assume that the government is *inequality-averse*, and that the net distributional characteristics for capital, $\tilde{R}_2$, is lower than for labour, $\tilde{R}_1$, and that the net distributional characteristic for private road transport, $\tilde{R}_3$, is lower than for collective transport, $\tilde{R}_4$.

We consider the optimal tax system under two alternative tax structures, $\Xi^R$ and $\Xi^T$.

Under $\Xi^R$ factor incomes; i.e. the incomes form labour and capital are taxed according to a general linear income tax schedule, i.e. $S_j \equiv \frac{p_j}{w_j} = S$ , j∈*F*, and the use of labour and capital cannot be differentiated in the other good sector, i.e. $\mathbf{T}^3 = (1,1)$. The tax structure may therefore be expressed as $\Xi^R \equiv \{ (S_1 = S_2 = S), \mathbf{T}^3 = (1,1) \}$,

Under $\Xi^T$, in addition to the tax restrictions imposed under $\Xi^R$, private road transport and the use of labour in the production of private road transport cannot be taxed. This tax structure may therefore be expressed as $\Xi^T \equiv \{ (S_1 = S_2 = S), (T_3 = 1), (T_1^1 = 1), \mathbf{T}^3 = (1,1) \}$.

We interpret a switch from $\Xi^T$ to $\Xi^R$ as the introduction of road pricing.

We want specifically to address two questions:

1) What if any are the optimal diversions from production efficiency under the two tax structures?

2) How will the optimal tax system change as a consequence of the reduction of the administrative cost of taxing private road transport?

### *The optimal taxation of transport without tax restrictions*

For reference we first consider the optimal taxation of transport when there are no tax restrictions.

There are three reasons why road transport under this tax structure would be taxed at a relatively high level: 1) because it is associated with a congestion externality, 2) because it is complementary to leisure and 3) because the share of consumption of high-income households is relatively large.

Collective transport will be taxed at a lower level than road transport: 1) because it is not associated with a congestion externality, 2) because it is less complementary to leisure and 3) because the share of consumption of high-income households is relatively low.

Productive efficiency is desirable under this tax structure.

Notice that neither the tax on collective transport nor the producer primary factor taxes are used neither to stimulate the demand for labour nor to reduce the external damage associated with the consumption of road transport.

### *The optimal taxation of transport under a general income tax* $\Xi^R$

Under the tax structures, $\Xi^R$, it is not possible to tax the supply of capital and labour at different rates nor the use of capital and labour in the production of other goods. By the stylised facts it would be desirable to tax the supply of labour at a lower rate if it had been possible differentiate the rate of tax on the supply of capital and labour, i.e. by decreasing $S_1$. By properties of Lagrange multipliers we thus have

$$\frac{dW}{dS_1} = -\gamma_1^3 < 0 \qquad s \in \mathrm{F} \quad (39)$$

i.e. that $\gamma_1^3 > 0$.

The optimal tax on the use of capital and labour in the production of private road transport and collective transport must therefore satisfy

$$\frac{p_2^k}{p_1^k} - p_2 = \gamma_1^3 \frac{p_5}{w_1} / \lambda_1 \mathrm{a}_1^3 \mathrm{a}_2^3 \sigma^3 Y^3 > 0 \qquad k \in (1,2) \quad (40)$$

In differentiating the tax on capital and labour the government face a trade off between on the one hand the objective of increasing the demand for labour relative to the demand for capital, and on the other hand the objective of limiting the distortionary costs due to the loss in production efficiency. The larger the elasticity of substitution between labour and capital in the production of other goods, the larger are the distortionary costs, and the smaller is the differentiation of the rates of tax on capital and labour in the private road transport and the collective transport sectors. Notice that the taxation of the use of primary factors in the production of private road transport and collective transport is not used as an instrument to reduce the externality in the private road pricing sector, but only to stimulate the demand for labour.

We may as a matter of normalisation (since we can set the tax rates on consumption without restrictions) choose the level of tax/subsidy on the primary factors in the production of private road transport and collective transport such that the unit cost at opportunity cost prices and at producer prices are the same, i.e. such that $\tilde{p}_i = \sum_{j \in F} \mathrm{a}_j^{i-2} p_j = \sum_{j \in F} \mathrm{a}_j^{i-2} p_j^{i-2} = p_i$ , i=3,4. The optimal tax system then becomes

$$t_s = (1 - S) / w_s \qquad s \in F \quad (41)$$

$$t_3 = \frac{-\sum_{i\in C\cup F} C_{i,3}\frac{\lambda_1 - \tilde{R}_i}{\lambda_1}\mathrm{E}_i}{|\mathbf{E}|} - \frac{\rho}{\lambda_1}\frac{de}{dX_3} + \frac{\sum_{i\in F} C_{i,3}\frac{\gamma_i^3}{w_i\lambda_1}}{|\mathbf{E}|} \tag{42}$$

$$t_4 = \frac{-\sum_{i\in C\cup F} C_{i4}\frac{\lambda_1 - \tilde{R}_i}{\lambda_1}\mathrm{E}_i}{|\mathbf{E}|} + \frac{\sum_{i\in F} C_{i4}\frac{\gamma_i^3}{w_i\lambda_1}}{|\mathbf{E}|} \tag{43}$$

$$t_5 = \frac{-\sum_{i\in C\cup F} C_{i5}\frac{\lambda_1 - \tilde{R}_i}{\lambda_1}\mathrm{E}_i}{|\mathbf{E}|} + \frac{\sum_{i\in F} C_{i5}\frac{\gamma_i^3}{w_i\lambda_1}}{|\mathbf{E}|} \tag{44}$$

### *The optimal taxation of transport under a general income tax when road transport cannot be taxed, $\Xi^T$*

Under $\Xi^T$ it is, as under $\Xi^R$, not possible to tax at different rates the supply of capital and labour nor the use in production of other goods. Furthermore, it is assumed to be possible neither to tax road transport, i.e. $T_1 = 1$, nor the use of labour in the production of road transport, i.e. $T_1^1 = 1$.[9]

By the assumption made it would be desirable if possible to tax capital at a higher rate than labour and to tax road transport at a higher rate than other goods. By properties of Lagrange multipliers we thus have

$$\frac{dW}{dS_1} = -(\gamma_1^1 + \gamma_1^3) < 0 \tag{45}$$

$$\frac{dW}{dT_1^1} = -\gamma_1^1 S_1 \; ? \tag{46}$$

$$\frac{dW}{dT_3} = -\gamma_3 > 0 \tag{47}$$

We may assume that $\gamma_1^3 > 0$ as under $\Xi^R$. However, the sign of $\gamma_1^1$ cannot be determined on the basis of the assumptions made; on the one hand it would be desirable to decrease the tax on the use of labour in the production of private road transport in order to increase the demand for labour, on the other hand it would be desirable to increase the tax in order to increase the price and thus the discouragement of private road transport.

[9] Notice that not being able to tax the dirty good would not be a binding constraint if it were possible to tax the use of both capital and labour in the production of the dirty good.

The optimal tax on the use of capital in the production of private road transport is

$$t_2^1 = p_2^1 - p_2 = \frac{\gamma_1^3 p_5}{w_1} / \lambda_1 \mathrm{a}_1^3 \mathrm{a}_2^3 \sigma^3 Y^3 - \left( \frac{\gamma_1^1 p_5}{w_1} - \frac{\gamma_3 p_5}{p_3} \mathrm{a}_1^1 \right) / \lambda_1 \mathrm{a}_1^1 \sigma^1 Y^1 \tag{48}$$

It is desirable to tax the use of capital in the production of private road transport for two reasons: to increase the demand for labour and to discourage the consumption of private road transport. The distortionary cost of achieving this objective for the first reason depends on the elasticity of substitution in the other good sector, and for the second reason on the elasticity of substitution in the private road transport sector.

The optimal tax on the use of primary factors in the use of private road transport and collective transport must as under $\Xi^R$ satisfy

$$t_2^2 = p_2^2 - p_2 = \gamma_1^3 \frac{p_5}{w_1} / \lambda_1 \mathrm{a}_1^3 \mathrm{a}_2^3 \sigma^3 Y^3 > 0 \tag{49}$$

We may as a matter of normalisation choose the level of tax/subsidy on the primary factors in the production of collective transport such that the unit cost at market prices and at producer prices are the same, i.e. such that $\tilde{p}_4 = \sum_{j \in F} \mathrm{a}_\mathrm{j}^{\mathrm{i}\text{-}2} p_j = \sum_{j \in F} \mathrm{a}_\mathrm{j}^{\mathrm{i}\text{-}2} p_j^{\mathrm{i}\text{-}2} = p_4$. The optimal tax system then becomes

$$t_s = (1 - S) / w_s \qquad \mathrm{s} \in F \quad (50)$$

$$t_3 = 0 \tag{51}$$

$$t_4 = \frac{- \sum_{i \in C \cup F} C_{i4} \frac{\lambda_1 \text{-} \tilde{R}_\mathrm{i}}{\lambda_1} \mathrm{E}_\mathrm{i}}{|\mathbf{E}|} - \frac{\sum_{i \in F} C_{i4} \frac{\gamma_\mathrm{i}^3}{w_\mathrm{i} \lambda_1} - C_{3,4} \frac{\gamma_3}{p_3 \lambda_1}}{|\mathbf{E}|} \tag{52}$$

$$t_5 = \frac{- \sum_{i \in C \cup F} C_{i5} \frac{\lambda_1 \text{-} \tilde{R}_\mathrm{i}}{\lambda_1} \mathrm{E}_\mathrm{i}}{|\mathbf{E}|} - \frac{\sum_{i \in F} C_{i5} \frac{\gamma_\mathrm{i}^3}{w_\mathrm{i} \lambda_1} - C_{3,5} \frac{\gamma_3}{p_3 \lambda_1}}{|\mathbf{E}|} \tag{53}$$

The tax on collective transport may thus be lower than under $\Xi^R$. A reduction of the tax on collective transport is now an instrument in reducing the consumption of private road transport. In contrast to under $\Xi^R$, the size of the congestion externality now in a significant way affect the tax on collective transport ( through the value of $\gamma_3$).

### *The optimal taxation of transport and the implication of the reduction of the administrative costs associated with road pricing*

We first consider the situation prior to the invention of the technologies which have very significantly reduced the administrative costs of monitoring private road transport for tax purposes. It is reasonable to assume that the overall optimal tax system before the introduction of these technologies belonged to the $\Xi^T$ tax structure rather than to the $\Xi^R$ tax structure. Under this tax structure it is desirable to differentiate the taxation of primary factors both in the road pricing and the collective transport sector in favour of labour when the administrative costs involved are disregarded. It is therefore a relevant question whether it is realistic to assume that the administrative costs of differentiating the tax on the use of labour and capital are such that it is justified in the private road transport and collective transport sectors, but not in other sectors. In the case of the private road transport sector the benefit from taxing capital is particularly large because it is not possible to tax the consumption of private road transport and because it is desirable to tax private road transport for three reasons 1) to discourage the consumption of leisure, 2) to reduce the congestion externalities and 3) for distributional reasons. In the case of the collective transport sector the reason why it may be less difficult to differentiate the tax on labour and capital than in the other good sector may be that the sector is under public ownership[10]. Public ownership may also make it easier to subsidise the consumption of collective transport. Put in another way, the desirability of differentiating the taxation of labour and capital in the collective transport sector and to discourage the consumption of private road transport at low administrative costs may provide an extra argument for public ownership.

We now consider the situation after the invention of low cost technologies to monitor private road transport. A significant reduction of the costs of taxing private road transport make the switch from the tax structure $\Xi^T$ to $\Xi^R$ a desirable proposition which is likely not only to result in a reduction in the congestion externality, but also a *Double dividend* by improving the efficiency of the tax system as it will discourage the consumption of leisure and have a desirable impact on the income distribution. Introduction of road pricing will change the optimal level of taxation not only within the road transport sector, but also in the collective transport sector. Within the private road transport sector the justification for taxing capital will have been reduced to the same level as in other sectors. Within the collective transport sector the rationale for subsidising collective transport in order to reduce the consumption of private road transport and thus the congestion externalities will have gone.

The taxing of the consumption of private road transport by road pricing thus reduces to some extent the justification for public ownership of collective transport.

[10] Increasing returns to scale in collective transport is not assumed in the present paper, but can easily be introduced in the model (see Munk 1980) and provide a well known additional supplementary reason for public ownership.

## 5. Summary and concluding remarks

We have shown that production efficiency is desirable when consumption of all commodities and the supply of all factors can be taxed at their optimal level at no cost in a model with congestion externalities. The assumptions required for production efficiency to be desirable are however not often satisfied in reality. We have derived the optimal tax structure assuming that the supply of primary factors cannot be differentiated (as the cannot under a general income tax) showing that in these cases productive efficiency is not desirable, if the administrative costs of differentiating the taxation of the use of primary factors are lower than the benefit. We have shown that the justification of diverting from productive efficiency is even stronger if the consumption of the good associated with congestion externalities cannot be taxed at its optimal level.

We have used this framework to analyse the taxation of transport. The framework provides a justification for the system of taxation found in countries such as Denmark, where car ownership is taxed at a rather high level, and where the consumption of collective transport is subsidised and under public ownership. We have shown that public ownership of collective transport may be justified not only by increasing returns to scale technologies, but also by reduction of the administrative costs of subsidising collective transport and stimulating the demand for (low skilled) labour.

The framework also has provided an interesting insight into the likely impact of the introduction of road pricing on the optimal tax system. Road pricing removes most of the justification for a tax on car ownership, but not entirely as the fact that the supply of capital and the supply of labour cannot be taxed at separate rates still will provide a justification for taxing the use of capital as long as the benefits exceed the extra administrative costs). The introduction of road pricing also reduces the justification for subsidies to the consumption of collective transport and hence reduces the justification for public ownership.

The introduction of road pricing is not only likely to result in a reduction of the congestion externalities associated with road transport, but also to provide a Double dividend as the change in the tax system following the introduction of road pricing will move in the same direction as would be desirable even disregarding the environmental benefit viz. discouragement of the consumption of leisure and redistribution of real income form high to low income households.

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