Abstract

In this work we present ARCH and GARCH models that can be a complementary methodology to screen cartels complaints in retail gasoline markets. To show that, we performed an exercise using weekly gasoline price data for São Paulo and Florianópolis to examine the mean and the variance of these times series. The results confirm the hypothesis of bigger prices during the supposed conspiracy period just in São Paulo. Therefore, the hypothesis of a smaller variance wasn’t confirmed in neither of the cities. Finally, we believe that this work improved the discussion about how to detect tacit or secret collusion because we recommended an econometric technique which requires just data on average prices and a minimum amount of data.

Key words: Price Parallelism; Gasoline market; Antitrust

JEL: L41; L51; L95

1. Introduction

This paper discusses an empirical tool that might be used to analyze collusion on pricing behavior on retail gasoline markets. Previous research was made based in the same market as case study\(^3\) and we started from the principle that market models about non-cooperative games repeated infinitely have been used as an indication of how to improve the methodology to detect cartels. In the specific case of price parallelism, the legal rule is that this conduct is not enough to prove collusive agreement, since in the games terminology it doesn’t provide the differentiation between profit joint maximization equilibrium and Nash equilibrium.

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\(^3\) See Vasconcelos and Vasconcelos (2005).
Following this rule, the central objective of our prior paper was to offer a methodological contribution from dynamic models in the task of searching for tacit or secret collusion proofs, admitting that parallelism is just an evidence of concurrence infraction. Then we indicated a complementary method of temporal series that identifies the causal and a long run relation of the strategic variables in cartels.

However, in the current paper we take a step further by examining a more accurate time series methodology: considering the characteristics of gasoline prices, in this paper we will model volatility. With this approach, our analysis still differs from the Brazilian Antitrust Authority methodology that is used to identify which cases the cartel investigation must go forward. And this new approach also refute the appreciation of Ragazzo and Silva (2006) that describes the official methodology as more efficient and simpler than our first time series approach.

As said in our prior paper, we choose to study the gasoline markets because they have often been investigated by Antitrust Authorities given the frequent complaints of cartel agreements on prices in Brazil⁴. And, since the theoretical foundations of this subject were extensively exposed, we won’t discuss this basis again. It is just necessary remember that the theoretical studies about price parallelism predict that firms which behave strategically will maintain pricing strategies that are volatile yet similar⁵.

Another introductory comment: the methodology presented here can be classified as new because the Brazilian literature about price parallelism is still incipient and only a few papers are available as yet⁶. Consequently, we will discuss the complementary methodology mentioned before (which hasn’t been applied to analysis of the competition degree of retail

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⁴ As well as around the world. See, for example, Competition Bureau (1994); Slade (1987); Kovac, Putzová and Zemplinerová (2005); Ragazzo and Silva (2006).


gasoline market) in the light of previous research developed by Bolotova, Connor and Miller (2005).

The next section provides a further discussion of the elements of the Brazilian Antitrust Authority methodology. Section 3 discusses the new method and has an application of this approach to the Brazilian gasoline price. The results are in section 4. Section 5 concludes.

2. The Brazilian Antitrust Authority criteria to evaluate complaints on gasoline retail market

In Brazil, the Secretariat for Economic Monitoring developed a methodology to analyze the complaints on gasoline retail marketing taking into consideration pricing behaviors and profit margins. Such method, a first attempt to reach a filter which is still under discussion, has three elements: a) the profit margin tendency: if the profit margin should decrease, the market is considered to be under a competitive behavior and the complaint is dismissed; b) the linear correlation: if there isn’t a link between the margin increase to the reduction of price dispersion (or the coefficient of variation on retail prices) the case is dismissed; c) if there is such a margin increase, then it is verified whether the margin and price dispersion behaviors follow the same pattern within a State geographical area (we consider that the monitoring costs of a cartel in a State would be much too high). If they do, the case is dismissed. Therefore, the cases are prosecuted only if there is a margin increase linked to the reduction of price dispersion not following the State pattern. In these cases, the investigation continues, trying to gather more evidence through the investigative methods allowed by Brazilian law, such as inspections, dawn raids and wiretapping.

7 This section summarizes the methodology described in OECD (2006), but can also be founded in Ragazzo and Silva (2006).
However, one central argument indicates that this method is not entirely satisfactory as a tool for analyzing intertemporal strategic pricing behavior: there is an ambiguity of reasons behind the behavior that sustains cartels. For example, if the profit margin is decreasing, how to be ensured that it isn’t a period of punishment after some firm had cheated the cartel agreement? Then, the decrease of the profit margin doesn’t must be seen only as an indication of competition, since the firms can be punishing the deviator. In summary, on one hand, a cartel can result in a profit margin increasing and, on the other hand, it can have a punishment phase with lower profits, but still it will be an anticompetitive behavior. And we can say that this aspect implies a bigger limit on the antitrust authority approach if the data is restricted to a short period of time.

Another aspect is as follows. For series exhibiting volatility, the unconditional variance may be constant even though the variance during some periods is unusually large. Casual inspection does have its perils and formal testing is necessary to substantiate any first impressions, yet the strong visual pattern is that of heteroscedasticity (inconstant variance) (Enders, 1995).

Therefore, in the next section we perform a discussion about the new methodology that can be used to complement the official procedure described here. But first, we present a summary of a study that investigates the impact of collusive conduct on the market price behavior using an econometric technique.

3. A new proposal to filter cartel complaints


The paper of Bolotova, Connor and Miller (2005) use extensions of traditional an autoregressive conditional heteroscedasticity (ARCH) model and generalized ARCH
(GARCH) models to examine the difference in the behavior of the two moments of price
distribution during collusion and the absence of it using prices from two recently discovered
conspiracies citric acid and lysine.

According to the authors, there are some advantages of using these models: *first, this*
*procedure may be used in the screening process conduct by antitrust and competition*
*authorities; second, it may be also used as an alternative to the econometric models*
*commonly employed in court proceedings to quantify the effect of conspiracy in market price;*
*third, the ARCH and GARCH models require minimum amount of data,* at least price time
series for a cartelized product before, during and after hypothesized or known conspiracy.

Following the theoretical background about cartels, the authors made two hypotheses.
The first one was related to the mean price behavior: they expected the mean price during
collusion was higher than the mean price when there was no collusion. The second one relates
to the variance behavior: they expected the variance of prices during collusion was lower than
the variance of prices when there was no collusion under the assumption of a successful
collusion (i.e. when most of the members in most of the time follow established price
discipline ad cartel may effectively address opportunistic behavior of its members)\(^8\).

The authors found support to both the mean and the variance hypotheses for lysine
prices, but for citric acid the results support the mean price hypothesis and fail to support the
variance hypothesis. They listed two explanations of this unexpected variance behavior: first,
the length of the citric acid conspiracy was longer than the length of lysine conspiracy. The
consequence of a longer period of time is the difficulty for the cartel to supervise and enforce
cartel discipline. Second, data availability problem may have had an impact on the analysis
outcome (there was less data available in citric acid case).
3.2. The econometric technique to screen cartel complaints

3.2.1. Model specification

In this section we discuss ARCH($q$) and GARCH ($p$, $q$) models that we used in our analysis as a proposal for Antitrust Authority to screen cartel complaints.

Engle apud Enders (1995, p.141) shows that it is possible to simultaneously model the mean and the variance of a series. Considering the mean equation for an observable variable $Y$ in period $t$ follows, for example, the autoregressive process of order one, denoted by AR(1),

$$Y_t = a_0 + a_1 Y_{t-1} + u_t$$  \hspace{1cm} (1)

where $u_t$ is a white noise,

$E(u_t) = 0, E(u_t, u_j) = \begin{cases} \sigma^2 & \text{for } j = 0 \\ 0 & \text{for } j \neq 0 \end{cases}$

and $|a_i| < 1$.

For this reason, the variance of $u_t$ is constant and equal to the unconditional variance, $\sigma^2$. Now suppose that the conditional variance is not constant. One simple strategy to model the conditional variance would be to estimate an AR($q$) process using the squares of the estimated residual of equation (1),

$$\hat{u}_t^2 = \alpha_0 + \alpha_1 \hat{u}_{t-1}^2 + \ldots + \alpha_q \hat{u}_{t-q}^2 + \nu_t$$  \hspace{1cm} (2)

where $\nu_t$ is a white noise process.

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8 Their hypotheses were formulated under the assumption that there was no significant change in market environment of cartel operation that could introduce an additional shock to prices.

9 In this section we are following Enders (1995).
The expression (2) implies that the linear projection of the squared error of a forecast of $Y_t$ on the previous $q$ squared forecast errors is given by

\[ \hat{E}(u_t^2 | u_{t-1}^2, u_{t-2}^2, ...) = \alpha_0 + \alpha_1 u_{t-1}^2 + \alpha_2 u_{t-2}^2 + ... + \alpha_q u_{t-q}^2 \] (3)

Therefore, an equation like (2) is called an autoregressive conditional heteroscedastic process of order $q$, denoted $u_t \sim ARCH(q)$.

A more parsimonious representation of higher order ARCH($q$) model can be obtained using Bollerslev’s GARCH($p$, $q$) model. Bollerslev apud Enders (1995) extended Engel’s original ARCH process by developing a technique that allows the conditional variance to be an ARMA process.

Suppose an ARCH($q$) process of $u_t$ characterized by

\[ u_t = \nu_t \sqrt{h_t} \]

where $\sigma_\nu^2 = 1$ and

\[ E[u_t^2] = h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i u_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j} \] (4)

Since $\nu_t$ is a white-noise process that is independent of past realization of $u_{t-i}$, the conditional and unconditional means of $u_t$ are equal to zero. The point is that the conditional variance of $u_t$ is given by $E_{t-}u_t^2 = h_t$. Thus, the conditional variance of $u_t$ is given by $h$ in equation (4). This generalized ARCH($p$, $q$) model, called GARC($p$, $q$), allows for both autoregressive and moving average components in the heteroscedastic variance.
For a well defined GARCH\((p, q)\) process we must have \(\alpha_0 > 0, \alpha_i \geq 0, \beta_j \geq 0\) and
\[
\sum_{j=1}^{q} \alpha_j + \sum_{j=1}^{p} \beta_j \leq 1.
\]

3.2.2. Source, nature of data and descriptive statistic analyses

As we said before, our methodology followed the work of Bolotova, Connor and Miller (2005). Phrased differently, we estimated an extension of traditional ARCH \((q)\) and GARCH \((p, q)\) models (introducing a conspiracy dummy variable) to analyze the mean and the variance behavior of gasoline price.

We used weekly common gasoline average prices reported by ANP (Petroleum National Agency) (2006), for São Paulo and Florianópolis cities, starting from January until December 2006. For the purpose of this study we assumed that March 2006 was the beginning of the conspiracy and April 2006 was the ending date of the conspiracy in São Paulo (dummy takes the value one over this period). And that January 2006 was the beginning of the first period of conspiracy and March 2006 was the ending of it; we still supposed a second period of the conspiracy in Florianópolis, restarting in September 2006 going until December 2006 (dummy takes the value one over this two separated periods).

Figure 1 illustrates the behavior of the São Paulo gasoline prices and the first impression is a visual pattern of significant fluctuation of the data. Figure 2 illustrates the behavior of the Florianópolis gasoline prices with two moments showing a decreasing of the prices that can be seen as a price war. The visual patterns of these two time series were the indicatives for our hypothetical periods of cartel and the choices of these periods were made just with the intention to clarify the methodology proposed here.
Figure 1 – Gasoline prices behavior, São Paulo City, 2006

Source: ANP (2006)

Figure 2 – Gasoline prices behavior, Florianópolis City, 2006

Source: ANP (2006)
Descriptive analysis can reveal some evidence of the presence of collusive behavior on both gasoline markets. In São Paulo, the mean “cartel price” was R$ 2,466, higher than the “pre-cartel mean price” (R$ 2,376) and the “pos-cartel mean price” (R$ 2,408). Also, the variance during the hypothetical “cartel period” was smaller than the variances of the “pre and pos cartel price” (0.00003, 0.000261 and 0.000246, respectively). In Florianópolis, the mean “cartel price” in the first and second periods were R$ 2.59 and R$ 2.60 respectively, higher than the “war price” period (R$ 2.45). Also, the variances during the hypothetical cartel periods were smaller than the variance of the “war price” period (0.001707; 0.001617; and 0.057208, respectively).

Nevertheless, since the antitrust authority is interested in the price parallelism and tacit or secret collusion, the long run value of the variance and the mean can be important to confirm these kinds of signals of anticompetitive behavior, as we will see in the next section.

4. Modeling price volatility in Brazilian gasoline markets

The first step to verify the existence of conditional variances in São Paulo and Florianópolis gasoline markets was the estimation of an AR(q) process for the price and variance. In São Paulo’s case, we reached an AR(1)\textsuperscript{10} for the mean equation of price and the “conspiracy dummy” variable showed be statistically significant. Thus, the mean price of gasoline in São Paulo during collusion is 0.02 cents per liter higher than the mean price during the period without collusion (Table 1). Phrased differently, we accepted the hypothesis of bigger prices in São Paulo during the supposed conspiracy period. However, we didn’t reach a statistical significant coefficient for the “conspiracy dummy” in mean equation of Florianópolis price (Table 2).

\textsuperscript{10} To choose the order of AR process we use Akaike (AIC) and Swartz (SC) information criteria.
Table 1 – Modeling conditional heteroscedasticity of Sao Paulo’s retail gasoline market

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>Const</td>
<td>0.75652</td>
<td>0.18557</td>
<td>4.072 [0.000]</td>
<td>Const</td>
<td>0.00094</td>
<td>0.00049</td>
<td>1.8948[0.064]</td>
</tr>
<tr>
<td>Price(-1)</td>
<td>0.68500</td>
<td>0.07723</td>
<td>8.869 [0.000]</td>
<td>Res(-1)</td>
<td>0.34036</td>
<td>0.14508</td>
<td>2.3461[0.023]</td>
</tr>
<tr>
<td>Dummy</td>
<td>0.02311</td>
<td>0.0062</td>
<td>3.712 [0.001]</td>
<td>Dummy</td>
<td>-0.0006</td>
<td>0.00119</td>
<td>-0.563[0.576]</td>
</tr>
</tbody>
</table>

Diagnostic Test

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>Serial Correlation</td>
<td>CHSQ(1)= 3.8218[0.051]</td>
<td>Serial Correlation</td>
<td>CHSQ(1)= 0.5164[0.472]</td>
</tr>
<tr>
<td>Functional Form</td>
<td>CHSQ(1)= 6.7619[0.009]</td>
<td>Functional Form</td>
<td>CHSQ(1)= 0.7329[0.392]</td>
</tr>
<tr>
<td>Normality</td>
<td>CHSQ(2)= 52.139[0.000]</td>
<td>Normality</td>
<td>CHSQ(2)= 663.43[0.000]</td>
</tr>
<tr>
<td>Heteroscedasticity</td>
<td>CHSQ(1)= 1.3952[0.238]</td>
<td>Heteroscedasticity</td>
<td>CHSQ(1)= 0.2335[0.629]</td>
</tr>
</tbody>
</table>

Source: Elaborated by authors

Table 2 - Modeling conditional heteroscedasticity of Florianópolis’ retail gasoline market

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<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>Const</td>
<td>1.4484</td>
<td>0.32839</td>
<td>4.4106 [0.000]</td>
<td>Const</td>
<td>0.04632</td>
<td>0.02427</td>
<td>1.9085[0.062]</td>
</tr>
<tr>
<td>Price(-1)</td>
<td>0.4118</td>
<td>0.13216</td>
<td>3.1162 [0.003]</td>
<td>Rest(-1)</td>
<td>0.02869</td>
<td>0.14517</td>
<td>0.1976[0.884]</td>
</tr>
<tr>
<td>Dummy</td>
<td>0.0599</td>
<td>0.04953</td>
<td>1.2105 [0.232]</td>
<td>Dummy</td>
<td>-0.0355</td>
<td>0.03105</td>
<td>-1.143[0.258]</td>
</tr>
</tbody>
</table>

Diagnostic Test

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Serial Correlation</td>
<td>CHSQ(1)= 2.3664[0.124]</td>
<td>Serial Correlation</td>
<td>CHSQ(1)= 0.0053[0.942]</td>
</tr>
<tr>
<td>Functional Form</td>
<td>CHSQ(1)= 3.5617[0.059]</td>
<td>Functional Form</td>
<td>CHSQ(1)= 0.0006[0.994]</td>
</tr>
<tr>
<td>Normality</td>
<td>CHSQ(2)= 479.32[0.000]</td>
<td>Normality</td>
<td>CHSQ(2)= 2769.8[0.000]</td>
</tr>
<tr>
<td>Heteroscedasticity</td>
<td>CHSQ(1)= 2.0131[0.156]</td>
<td>Heteroscedasticity</td>
<td>CHSQ(1)= 1.0093[0.315]</td>
</tr>
</tbody>
</table>

Source: Elaborated by authors

The results of the variance equation estimation were as follows: the signal of the dummy variable showed a negative effect of the conspiracy on variance. This result is according with we hypothesized but this coefficient was not statistically significant for both cities (Tables 1 and 2).

The second step was to test the ARCH effect, i.e. the existence of conditional variance. In both cases, we proceeded the ARCH (1) until ARCH (4). For the price series of São Paulo, the Lagrange Multiplier – LM – version of the test yields a statistic of 4.9611 in ARCH(1)11.

11 The same results is reached if one considers ARCH (2), but in ARCH (3) and ARCH (4) we can’t reject the hypothesis the there are no ARCH.
which is above the 95 per cent critical value of $\chi^2_{(1)}$, and hence we reject hypothesis that there are no ARCH effects in AR (1) process of series price. The same conclusion is reached if one considers the F version of the test (Table 3).

Table 3 - Autoregressive Conditional Heteroscedasticity Test of Residuals (OLS case): São Paulo’s price

<table>
<thead>
<tr>
<th>Dependent variable is Price</th>
<th>List of the variables in the regression: Constant</th>
<th>Price(-1)</th>
<th>Dummy</th>
</tr>
</thead>
<tbody>
<tr>
<td>51 observation used for estimation from 2 to 52</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lagrange Multiplier Statistic</th>
<th>ARCH(1)</th>
<th>ARCH(2)</th>
<th>ARCH(3)</th>
<th>ARCH(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi^2(1)$ = 4.9611</td>
<td>(0.026)</td>
<td>$\chi^2(2)$ = 5.6959</td>
<td>(0.058)</td>
<td>$\chi^2(3)$ = 5.5894</td>
</tr>
</tbody>
</table>

| F Statistic | F(1, 47) = 5.0647 | (0.029) | F(2, 46) = 2.8917 | (0.066) | F(3, 45) = 1.8463 | (0.152) | F(4, 44) = 1.3758 | (0.258) |

Source: Elaborated by authors

However, the rejection of the hypotheses that there are no ARCH effects does not necessarily imply that conditional variance of AR process is variable, since we should verify if the residuals of AR process are serially uncorrelated. We performed the test of serial correlation for residuals and the LM version for it yielded a rejection of the hypothesis which the residuals are serially uncorrelated\(^{12}\). This result means that the existence of ARCH effect on gasoline prices series of São Paulo isn’t conclusive. In others words, given the price variance pattern, we could not confirm a smaller price volatility.

Considering now the series prices of gasoline for Florianópolis, we couldn’t reject the hypothesis that there are no autoregressive conditional heteroscedasticity of residuals in both statistics, LM and F (Table 4).

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\(^{12}\) Test of Serial Correlation of Residuals (OLS case) for São Paulo’s price: Lagrange Multiplier Statistic $\chi^2(1) = 3.8218$ and $p – \text{value equal 0.051}$. 
Table 4 - Autoregressive Conditional Heteroscedasticity Test of Residuals (OLS case): Florianópolis’ price

<table>
<thead>
<tr>
<th>Lagrange Multiplier Statistic</th>
<th>ARCH(1)</th>
<th>ARCH(2)</th>
<th>ARCH(3)</th>
<th>ARCH(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi^2(1)$</td>
<td>0.13776</td>
<td>0.13021</td>
<td>0.1401</td>
<td>0.18872</td>
</tr>
<tr>
<td></td>
<td>[0.771]</td>
<td>[0.937]</td>
<td>[0.987]</td>
<td>[0.996]</td>
</tr>
<tr>
<td>F Statistic</td>
<td>F(1, 47) = 0.12730</td>
<td>F(2, 46) = 0.0588</td>
<td>F(1, 47) = 0.0413</td>
<td>F(4, 44) = 0.0408</td>
</tr>
<tr>
<td></td>
<td>[0.723]</td>
<td>[0.943]</td>
<td>[0.989]</td>
<td>[0.997]</td>
</tr>
</tbody>
</table>

Source: Elaborated by authors

5. Final considerations

The central objective of this work was to present a complementary methodology to detect collusion in retail gasoline markets. Following the literature, we performed an exercise to show how the ARCH and GARCH models can be used to examine the behavior of the mean and the variance of price distribution.

Comparing with our previous work in area, we believe had improved the analyses of this problem in two directions: first, this econometric technique can be used to test the presence of collusive behavior on markets where collusion is likely to take place just with data on average prices; second, these models require minimum amount of data (price time series for a cartelized product before, during and after hypothesized or known conspiracy).

Finally, a suggestion to future works is include more cities in the analyses and compare the results of the official procedure with the results of the methodology proposed here.
6. Bibliography


