What Drives Wage Inequality?

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Abstract

The paper quantitatively evaluates the interaction between capital-skill complementarity and endogenous low-, medium- and high-skilled labour supply in generating wage inequality, using a dynamic computable general equilibrium (CGE) model of the Auerbach-Kotlikoff (1987) type.

The counterfactual analysis conducted shows that the increase in the skill premium depends crucially on the substitution elasticities between medium- and high-skilled labour and capital and the one between low-skilled labour and the capital-skill input, as well as on the employed labour supply elasticity. More specifically, the skill premium rises either if the elasticity of substitution between low-skilled labour and the capital-skill input is increased or if the substitution elasticity between medium- and high-skilled labour and capital is decreased. With regard to the labour supply elasticities we find that the rise in the skill premium will be more pronounced as labour supply becomes inelastic. Without any increase in the relative supply of medium- and high-skilled labour the rise in the German skill premium, over the decade from 1994-2004, would have been by 1.71 percentage points larger. On the contrary, if capital-skill complementarity is disregarded, the actual rise in the relative supply of medium- and high-skilled labour would have forced the German skill premium to decline by about 0.03 percent.

Keywords: Capital-skill complementarity, skill premium, wage inequality, endogenous labour supply, dynamic general equilibrium analysis.

JEL-Classification: J31, J23, C68

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1 What drives Wage Inequality?

The hypothesis of capital-skill complementarity which was first formalised by Griliches (1969) states that the elasticity of substitution between capital and unskilled labour is higher than the one between capital and high-skilled labour. A stronger version of this hypothesis even predicts that capital and unskilled labour are substitutes in production while capital and high-skilled labour are complements. As a consequence of this hypothesis, the skill premium tends to increase with the accumulation of capital.\(^1\)

The last decades were, however, also marked by a steady increase in the relative supply of medium- and high-skilled vis-à-vis low-skilled labour (Krusell et al., 2000).\(^2\) In principle, any rise in the relative supply of medium- and high-skilled labour should have an alleviating impact on the skill premium and thus on wage inequality. The rationale refers to the cost advantage of medium- and high-skilled vis-à-vis low-skilled labour, if the higher skill types are more elastically supplied than low-skilled labour.

Even though both capital-skill complementarity and the relative supply increase of medium- and high-skilled labour are empirically validated, hardly any paper analyses the joint influence of these two effects on wage inequality.\(^3\) This paper tries to fill the gap for the German case. We analyse whether capital-skill complementarity or the relative supply of medium- and high-skilled labour has a quantitatively important impact on wage inequality in Germany. In doing so, we develop a dynamic computable general equilibrium (CGE) model in the spirit of Auerbach-Kotlikoff (1987) which features capital-skill complementarity and an endogenous supply of three different labour skilltypes. Using this model, we track the evolution in the skill premium, if (1) merely capital-skill complementarity prevails but the three labour skill-types equally elastic supplied, if (2) the relative supply of medium- and high-skilled labour is more elastically relative to low-skilled labour in the absence of capital-skill complementarity, and finally if (3) both

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\(^1\)The capital-skill complementarity hypothesis has been subject of numerous empirical studies such as Griliches (1969), Fallon and Layard (1975), Bound and Johnson (1992), Goldin and Katz (1998) or Lindquist (1998) or Duffy et al. (2003) or Papageorgiou and Chmelarova (2005).

\(^2\)According to Krusell et al. (2000) in the U.S. the ratio of skilled labor hours worked (defined by at least 16 years of school attendance) to unskilled labor hours worked increased by more than 100% over the 1963-92 period.

\(^3\)In most papers, the increase in the relative supply of high skilled labour is not considered in detail at all and even the widely cited paper of Krusell et al. (2000, p. 1033) abstracts from the household sector and therewith from (endogenous) labour supply.
capital-skill complementarity and a relatively larger supply of medium- and high-skilled labour prevails. Accordingly, the first simulation quantifies the “demand side effect” for medium- and high-skilled labour resulting from capital skill complementarity. The second analysis focuses on the “supply side effect” accompanying the increase in the relative supply of medium- and high-skilled labour vis-à-vis low-skilled labour. Finally, the third simulation evaluates the combined effects of capital-skill complementarity and the relative increase in the supply of medium- and high-skilled labour over time on the evolution of wage inequality.

To summarize our results, the conducted simulations suggest that without any increase in the relative supply of medium- and high-skilled labour, the rise in the German skill premium would have been by 1.71 percentage points larger over the decade from 1994-2004. Moreover, in the absence of capital-skill complementarity, the actual rise in the relative supply of medium- and high-skilled labour would have forced the skill premium in Germany to decline by about 0.03 to 0.1 percent, depending on the assumed supply elasticity for low-skilled labour.

The remainder of the paper is as follows. The next section introduces the theoretical setup of the dynamic CGE model and outlines the calibration and empirical implementation of the model. The conducted simulations and sensitivity analyses are presented in Section three while Section four concludes.

2 Theoretical Setup & Calibration of the CGE Model

In the spirit of Auerbach and Kotlikoff (1987) we develop a dynamic, two country CGE model based on neoclassical growth theory. The representative firm’s optimal behaviour is derived from an intertemporal investment model with convex adjustment costs. The production function features capital-skill-complementarity and uses low-, medium- and high-skilled labour, next to capital, as inputs. On the households side, a representative, infinitively lived individual maximizes his life time utility via an optimally chosen intertemporal consumption path and optimal low-, medium and high-skilled labour supply. The dynamic CGE model thus mimics the most important behavioural margins at

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4 A comprehensive model documentation can be found by Stimmelmayr (2007) or Radulescu (2007).
the firm and household level which drive the evolution of the skill premium. Moreover, since capital accumulation and labour supply are dynamic phenomena, the applied model is especially suitable for quantifying the impact of economic growth on the skill premium under the presence of both capital-skill complementarity and a varying relative supply of the different labour skill-types.\(^5\)

One important limitation of the model refers to the fact that there is no scope for “involuntary” unemployment in this type of general equilibrium analysis, since (1) labour markets are assumed to be competitive and since (2) we impose no constraints on the behaviour of firms or workers.

Moreover, as frequently discussed, any increase in the skill-premium provides an incentive to invest in human capital. Even though the model does not feature the household’s investment in human capital explicitly, we solve this shortcoming by applying appropriate supply elasticities for the different labour skill-types. Therefore, we are able to map the pattern in the labour supply of the different skill types as shown in the German data. Additionally, we perform several sensitivity analyses with regard to the applied supply elasticities of the different labour skill categories.

### 2.1 Production Technology

The representative firm relies on a neoclassical, well-behaved, linearly homogenous production technology, \( Y = F(K, L_M, L_H) \) with capital, \( K \), low-skilled, \( L_L \), medium skilled, \( L_M \), and high-skilled labour, \( L_H \), as inputs:

\[
Y = F(K, L_L, L_M, L_H) = F_K K + F_{LL} L_L + F_{LM} L_M + F_{LH} L_H,
\]

with \( F(0) = 0; \quad F' > 0; \quad F'' < 0 \).  

The respective wage rates for each skill-type of labour are denoted by \( w_i \), with \( i \in \{L, M, H\} \). The price of the output good is normalized to unity. Investments, \( I \), incur adjustment costs of size \( J(I, K) \), which imply decreasing returns from capital accumulation.\(^6\) Accounting for total wage costs, \( \sum w_i L_i \), and the depreciation of capital, \( \delta K \), the

\(^5\)Compared to a mere steady state analysis, the applied model solves the economy’s dynamic transition path under perfect foresight from the initial to the final steady state.

\(^6\)The adjustment cost function, \( J(I, K) \) is assumed to be linearly homogeneous in investments, \( I \), and capital, \( K \), and convex in investments, \( J_I \) and \( J_{II} > 0 \). Moreover, the steady state adjustment costs are
net of tax profits, \( \pi \), of the representative firm read:

\[
\pi = Y_t - J_t - \sum w_{i,t}L_{i,t} - \delta K_t - T^P_t,
\]

with \( T^P_t = \tau^P [Y_t - J_t - \sum w_{i,t}L_{i,t} - \delta K_t]. \) (2)

Furthermore, firm profits are subject to a profit tax, \( \tau^P \), from which various costs accompanying factor inputs can be deducted.\(^7\)

**Capital-Skill Complementarity**

The feature of capital-skill complementarity is embedded into the production function by the means of different elasticities of substitution between capital and the three different skill types of labour. As proposed by Griliches (1969), we apply a larger value for the elasticity of substitution between capital and low-skilled labour than for the one between capital and medium- or high-skilled labour.

To be more specific, there are two possible ways of implementing capital skill complementarity using two CES aggregates, namely \( \Upsilon^1 \) and \( \Upsilon^2 \) (see also Krussel et al. 2000). The first specification, \( \tilde{Y}_1 = \Upsilon^1 [L_M, L_H, \Upsilon^2(K, L_L)] \) suggests that the elasticity of substitution between medium- or high-skilled labour and capital is the same as the one between medium- or high-skilled labour and low-skilled labour since substitution elasticities are the same within each CES aggregates.

The second formulation \( \tilde{Y}_2 = \Upsilon^1 [L_L, \Upsilon^2(K, L_M, L_H)] \) implies similar substitution elasticities between low-skilled labour and capital as well as between low-skilled labour and medium- or high-skilled labour. Since the first specification is, however, not consistent with the empirical findings (Hamermesh 1993)\(^8\) we incorporate the second version of capital-skill complementarity in our CGE model. The functional form of the production

\(^7\)Any further taxation of firm profits on the household level (via the dividend or capital gains tax) is neglected since we are primarily interested in the effects of capital accumulation and endogenous labour supply on the evolution of the skill premium.

\(^8\)According to Hamermesh’s (1993) findings, the substitution elasticity between skilled labour and unskilled labour is higher than the one between skilled labour and capital. This confirms the second specification of capital-skill complementarity.
function thus states:

\[
Y = \left\{ \alpha_L L_L^{-\rho_1} + (1 - \alpha_L) \left[ A(\alpha_M L_M^{-\rho_2} + \alpha_H L_H^{-\rho_2} + \alpha_K K^{-\rho_2})^{-\frac{1}{\rho_2}} \right]^{-\frac{1}{\rho_1}} \right\},
\]

with \( \alpha_K = (1 - \alpha_M - \alpha_H) \),

where \( \alpha_i, i \in \{L, M, H\} \), governs the distribution parameter and \( A \) denotes neutral technological change. The variable \( \rho_j, j \in \{1, 2\} \), denotes the substitution parameter. Hence the elasticity of substitution between low-skilled labour and the capital-skill input is \( \sigma_1 \), with \( \sigma_1 = 1/(1 + \rho_1) \) and the one between medium- or high-skilled labour and capital is \( \sigma_2 \), where \( \sigma_2 = 1/(1 + \rho_2) \).

As discussed, the phenomena of capital-skill complementarity requires \( \sigma_1 > \sigma_2 \). Moreover, if capital and low-skilled labour are assumed to be substitutes while capital and medium- or high-skilled labour are complements, the substitution elasticities and substitution parameters have the following properties: \( \sigma_1 > 1 \) with \( 1 < \rho_1 < \infty \) and \( \sigma_2 < 1 \) with \( -1 < \rho_2 < 0 \).

**No Capital-Skill Complementarity**

In case we abstract form capital-skill complementarity, the different skill-types of labour are all complementary to capital and each input features an identical substitution elasticity of \( \sigma = 1/(1 + \rho) \). To keep the model setup as simple as possible in the case of no capital-skill complementarity, we introduce a labour composite, \( \overline{L} \), which consists of the three different labour skill-types. The latter enters besides capital the production function as follows: \(^9\)

\[
Y = A \left[ (\alpha \overline{L}^{-\rho} + (1 - \alpha)K^{-\rho}) \right]^{-\frac{1}{\rho}},
\]

with \( \overline{L} = \left[ \sum_{i} \frac{\alpha_i w_i L_i^{-\rho}}{\rho_L} \right]^{-\frac{1}{\rho_L}} \), with \( i \in \{L, M, H\} \).

the parameter \( \rho_L \) denotes the substitution elasticity within the labour composite.

\(^9\)The specification of no capital-skill complementarity is applied when we quantify the alleviating impact of an increase in the relative supply of medium- and high-skilled labour in the counterfactual analysis.
Given the above formulation, the optimal unit labour demands, \( l_i, i \in \{L, M, H\} \), for each skill-type follow from the cost minimization problem:

\[
\min_{l_i, i \in \{L, M, H\}} \sum_1^3 \left[ w_i l_i \right] \quad s.t. \quad L = \left[ \sum_1^3 \frac{\alpha_i (1 + r \tau)/r \tau L_i^0}{1 + r \tau} \right]^{r \tau} = 1.
\]

**Firm Value & Optimal Firm Behaviour**

The firm value, \( V_t \), is determined by the present value of all future payouts, \( \chi \), which are residually determined by net of tax profits less investment expenditures:

\[
V_t = \sum_{S=1}^\infty \left[ \frac{\chi S}{1 + r_S} \prod_{z=t}^{S+1} \frac{G}{1 + r_Z} \right],
\]

with \( \chi_t = (1 - \tau^P) \left[ Y_t - J_t - \sum w_{i,t} L_{i,t} - \delta K_t \right] - I_t \).

The variable \( G \) represents the steady state growth factor, \( 1 + g \), and the discount rate \( r^V \) denotes the return on firm equity. Optimal firm behaviour follows from the firm’s maximization problem:

\[
V_t(K_t) = \max_{L_{i,t}, I_t, i \in \{L, M, H\}} \left[ \chi_t + \frac{G}{1 + r_{t+1}} V(K_{t+1}) \right],
\]

\( s.t. \quad GK_{t+1} = I_t + (1 - \delta) K_t, \)

and yields the following first order conditions:

\[
(a) \quad L_i : \quad F_{L_i,t} = w_i, \\
(b) \quad I : \quad q_{t+1}^c = (1 + r_{t+1}) (1 - \tau^P) [1 + J_t],
\]

Following equation (7a), optimal labour demand for each skill type is determined by the point of equality between the marginal product of the respective labour skill type, \( F_{L_i} \), and the corresponding wage rate.

Optimal investment behaviour, as specified in (7b), requires that the shadow price of capital, \( q_{t+1}^c \), equals the incurred cost of an additional investment. These costs include the tax adjusted cost of the investment as well as adjustment costs.
2.2 Household Side & Labour Supply

The household sector is modelled in the spirit of an infinitely lived, representative agent who’s maximization problem involves the optimal inter-temporal choice of consumption, \( C \), and optimal individual labour supply \( l_i, i \in \{L, M, H\} \). The rate of time preference, \( \rho \), determines the weight the household assigns to future consumption. Additionally, the supply of labour involves some disutility of work in the size of \( \varphi(l_i) \):

\[
U(A_t) = \max_{C_t, l_i, i \in \{L, M, H\}} \left[ u(C_t - \sum_i^3 \varphi(l_{i,t})) + \rho U^*(A_{t+1}) \right],
\]

s.t. \( GA_{t+1} = (1 + \tau_t)A_t + \sum_i^3 [(1 - \tau^{L_i})w_{i,t}L_{i,t}] + T_t - C_t. \)

Total wealth of the representative household consists of his capital endowment, \( A \), and capital income, \( \tau A \), earned on the various types of assets held, \(^10\) net of tax labour income, \( \sum_i^3 [(1 - \tau^{L_i})w_{i,t}L_{i,t}] \), earned by each of the different skill types and lump sum transfers, \( T \). The variable, \( \tau^{L_i} \), denotes the respective tax on labour income.

Optimal labour supply of each skill type requires that the marginal disutility from labor supply equals the net of tax compensation payment received in the form of net of tax wages:

\[
\varphi'(l_i) = \left(1 - \tau^{L_i}\right) w_i. \tag{9}
\]

Applying a specific functional form for the disutility of work, which is contingent on the respective labour supply elasticity \( \varepsilon_i \) and a scaling parameter, \( \gamma \), the individual labour supply of each skill category is given by:\(^11\)

\[
\varphi(l_i) = \gamma^{-1/\varepsilon_i} \frac{l_i^{1+1/\varepsilon_i}}{1+1/\varepsilon_i} \Rightarrow l_{i,t} = \gamma \left[ \frac{1 - \tau^{L_i}}{1 + \tau C} w_i \right]^{\varepsilon_i}. \tag{10}
\]

\(^10\)The household’s portfolio consists of four different types of assets, including domestic firm equity, domestic business debt and domestic as well as foreign government bonds. The variable \( \tau \) denotes the average portfolio return since all assets are assumed to be imperfect substitutes. A detailed discussion on the household’s portfolio problem is provided in Stimmelmayr (2007).

\(^11\)Aggregate labour supply of each skill category, \( L^S_i \), is achieved via aggregation: \( L^S_i = l_{i,t} \cdot N_t \), where \( N \) denotes the size of the labour force in the economy.
Finally, the household’s optimal consumption path is determined by the *Euler Equation*:

\[
\frac{u'(Q_t)}{u'(Q_{t+1})} = \frac{p(1 + \tau_{t+1})}{G}, \quad \text{with} \quad Q_t = C_t - \sum_i \varphi(l_{i,t}).
\]  

(11)

The latter specifies the marginal rate of substitution between present and future consumption.

### 2.3 Empirical Implementation of the CGE Model

The computational procedure of any numeric CGE model requires the specification of functional forms and the choice of appropriate behavioural parameters and elasticities from the empirical literature. In this context, the calibration implies that the initial steady-state of the model replicates the stationary long-run macroeconomic equilibrium of the considered economy.\(^{12}\) All behavioural parameters applied in our model are standard results in line with the empirical literature. The most important ones are summarized in Table 1.

The real annual growth rate of the German economy is assumed to be 1.1 per cent, which is, according to Bandholz et al. (2005), a fair estimate for Germany after reunification. Economic depreciation reaches 6 per cent of the capital stock and is just one percentage point below the value applied by Fehr (1999). The adjustment speed towards the new steady-state is determined by the half-life of investment. As in the study of Cummins et al. (1996), we take a value of 8.0, implying that during the following 8 years after the policy shock half of the long-run increase in the capital stock is accumulated.\(^{13}\)

One of the key parameters describing the production side of the economy is the elasticity of substitution between the different labour skill types and capital. The empirical literature provides extensive evidence of different estimates for this parameter.\(^{14}\) We fol-

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\(^{12}\) Even though hardly any model is able to reproduce the macroeconomic equilibrium as detailed as provided by the national accounts data, each model should at least reflect the stylised facts of the considered economy.

\(^{13}\) To achieve this pattern of the half-life of investment, the adjustment cost parameter is set equal to 2. Such a value is also applied by Valkonen (1999) and represents a lower end value of available estimates (see Whithed, 1994).

\(^{14}\) Most estimates for the elasticity of factor substitution range between 0 and 1, depending on the underlying estimation technique. For instance, the study by Chirinko, Fazzari and Meyer (1999), which is based on panel data, suggests an elasticity of 0.25, whereas a higher value of 0.7 is calculated by Jorgenson and Yun (2001).
low Krussel et al. (2000) who suggest a value of 1.67 for the elasticity of substitution between low-skilled labour and capital and a value of 0.67 for the one between high-skilled labour and capital. These data are very close to Lindquist’s (2005) estimates of 1.4 and 0.52 for the Swedish economy. The empirical evidence for the German economy by FitzRoy and Funke (1995) deviates, however, quite significantly from the above numbers. They find a much lower elasticity of substitution of only 0.50 between low-skilled labour and capital and of 0.21 between high-skilled labour and capital using data on 32 West German manufacturing industries between 1975-1995.

Table 1: Behavioural Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual Growth Rate (Bandholz et al. 2005)</td>
<td>0.011</td>
</tr>
<tr>
<td>Economic Depreciation (Fehr 1999)</td>
<td>0.06</td>
</tr>
<tr>
<td>Half-life of Capital Accumulation (Cummins et al. 1996)</td>
<td>8.0</td>
</tr>
<tr>
<td>Elasticity of Substitution (Krussel et al. 2000)</td>
<td></td>
</tr>
<tr>
<td>low-skilled labour and capital</td>
<td>1.67</td>
</tr>
<tr>
<td>high-skilled labour and capital</td>
<td>0.67</td>
</tr>
<tr>
<td>Elasticity of Factor Substitution (German Central Bank 1995)</td>
<td>0.80</td>
</tr>
<tr>
<td>Compensated Labour Supply Elasticity (Fehr 1999)</td>
<td></td>
</tr>
<tr>
<td>low-skilled (short/long run)</td>
<td>0.55/0.49</td>
</tr>
<tr>
<td>medium-skilled (short/long run)</td>
<td>0.66/0.54</td>
</tr>
<tr>
<td>high-skilled (short/long run)</td>
<td>0.71/0.59</td>
</tr>
<tr>
<td>aggregate (short/long run)</td>
<td>0.67/0.56</td>
</tr>
<tr>
<td>Intertemporal Elasticity of Substitution (Flaig 1988)</td>
<td>0.40</td>
</tr>
</tbody>
</table>

In case we abstract from capital-skill complementarity, we apply a value of 0.8 for the elasticity of factor substitution which is based on the estimates by the German Central Bank (1995). This value corresponds to the various empirical studies for West German industries which present estimates ranging between 0.3 and 1.3 (see Roskamp, 1977).

Even though the empirical literature on labour supply elasticities is manifold, the empirical evidence on the labour supply elasticities of different labour skill types is rather scarce. Therefore, we follow Fehr (1999), who computes compensated labour supply elasticities\(^\text{15}\) for the lowest, the third and the top quintile of the German income distribution using a similar dynamic CGE model as the one applied here. In general, a rise in the wage rate might result in an increase or a decrease of labour supply, depending on the relative size of the substitution and the income effect. The income effect, however, becomes

\(^{15}\)The compensated supply elasticity characterises the relevant substitution effect between labour and leisure.
more important in the long run, since the share of human capital in total wealth gains more weight as the lifetime horizon is extended. Hence, the presented estimates by Fehr are throughout smaller for the long-run compared to the short-run. These estimates are also in line with the one by Feldstein (2005) who suggests an aggregated, compensated labour supply elasticity of 0.51.

Finally, the inter-temporal elasticity of substitution is set to 0.4. This value is based on Flaig’s (1988) empirical research for Germany, and is just slightly lower than the values applied for instance by Keuschnigg and Dietz (2004) or Valkonen (1999).

Macroeconomic Data

Table 2 reports the main German macroeconomic aggregates for the year 2004, the medium-run averages, and the aggregates replicated by our CGE model.16

<table>
<thead>
<tr>
<th>Table 2: Replicated Macroeconomic Structure (in Bn. Euro)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Germany</td>
</tr>
<tr>
<td>GDP</td>
</tr>
<tr>
<td>Private Consumption</td>
</tr>
<tr>
<td>Compensation of Employees</td>
</tr>
<tr>
<td>Capital Stock</td>
</tr>
<tr>
<td>Depreciation</td>
</tr>
<tr>
<td>Gross Capital Formation</td>
</tr>
</tbody>
</table>

Note: Variables marked with a * are set exogenously to replicate the initial steady-state of the German economy.

Source: Destatis (2005), own calculation.

The wage income of low-, medium-, and high-skilled labour amounts to 156.7, 601.3 and 411.8 Bn. Euro, respectively (Destatis, 2005). If we add up these group specific wage earnings we compute an aggregate total labour income of 1169.8 Bn. Euro as reported in Table 2.

The changes in the capital stock, the skill distribution of the German working force and the gross monthly incomes of the various skill types are presented in Table 3.

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16The medium-run average is computed over the period form 1998 to 2004.
Table 3: Capital Stock, Labour Force and Gross Monthly Income

<table>
<thead>
<tr>
<th></th>
<th>1984#)</th>
<th>1994</th>
<th>2004</th>
<th>%-age Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital Stock (Bn. Euro)</td>
<td>6,481</td>
<td>9,168</td>
<td>11,162</td>
<td>41.5 72.3</td>
</tr>
<tr>
<td>Labour Force (in %)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low-Skilled</td>
<td>42.8</td>
<td>28.4</td>
<td>28.8</td>
<td>-33.6 -32.7</td>
</tr>
<tr>
<td>Medium-Skilled</td>
<td>45.7</td>
<td>53.8</td>
<td>51.3</td>
<td>17.7 12.3</td>
</tr>
<tr>
<td>High-Skilled</td>
<td>11.5</td>
<td>17.8</td>
<td>19.9</td>
<td>54.8 73.0</td>
</tr>
<tr>
<td>Monthly Income#) (in Euro)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low-Skilled</td>
<td>1131</td>
<td>1704</td>
<td>2100</td>
<td>50.7 85.7</td>
</tr>
<tr>
<td>Medium-Skilled</td>
<td>1432</td>
<td>2045</td>
<td>2550</td>
<td>42.8 78.0</td>
</tr>
<tr>
<td>High-Skilled</td>
<td>2244</td>
<td>3247</td>
<td>3672</td>
<td>44.6 63.6</td>
</tr>
</tbody>
</table>

#) Data for West-Germany.


The capital stock rose by 41.5 percent between 1984 and 1994 and by 21.8 percent in the period from 1994 to 2004. In total, the capital stock grew by more than 73 per cent over the last twenty years.

The skill distribution of the German labour force was also marked by significant changes during the reported period from 1984 to 2004. While the supply of low-skilled labour dropped by more than 30 per cent, the supply of medium- and high-skilled labour rose by 12.3 and 73.0 per cent, respectively. As opposed to this, the change in the monthly gross income of the different labour skill types showed a more uniform pattern. The largest increase in monthly gross income was registered by the low-skilled workers with around 86 percent, followed by medium- and high-skilled workers with 78 and 63.6 per cent, respectively.

The skill-premium of medium and high-skilled German workers (defined as the ratio of medium- or high-skilled wages and low-skilled wages) amounted to 1.27 and 1.98 in 1984. In the succeeding two decades the change in the skill premium for high-skilled German workers was, however, marked by a continuous downward trend. It declined by 3.9 per cent in the period from 1984 to 1994 and fell once more by 8.4 per cent in the subsequent period from 1994 to 2004. The respective changes in the skill premium of medium-skilled German workers amounted to -5.2 per cent and 1.2 per cent.
3 Simulation Results

3.1 The Demand Side Effect

The first class of simulations is devoted to the “demand side effect” which identifies the change in the demand for the different labour skill types resulting from capital accumulation and due to capital-skill complementarity. According to the specification of the production technology as stated in equation (3), medium- and high-skilled labour constitute a complement to capital while low-skilled labour is rather a substitute, such that the accumulation of capital increases the demand for medium- and high-skilled labour.

Whether the demand for low-skilled labour is affected by the accumulation of capital depends on the relative sizes of the marginal product and the factor price for low-skilled labour on the one hand and the marginal product and factor price of the capital-skill input on the other hand. Since the accumulation of capital increases the demand for medium- and high-skilled labour and therewith the price for the capital-skill input, it might also be valuable for the profit maximizing firm to substitute the capital-skill input by low-skilled labour.

Nevertheless, under the feature of capital-skill complementarity the increase in the demand for medium- and high-skilled labour will constitute the predominating effect, if the substitution elasticity between low-skilled labour and the capital-skill input is larger than the one between medium- or high-skilled labour and capital.

To quantify the demand side effect for the period from 1994 to 2004, we consider the following strategy. We disturb the initial steady state equilibrium by reducing the capital stock by 21.8 per cent, what approximates the amount by which the capital stock actually increased in Germany between 1994 and 2004 (see Table 3). The estimates for the demand side effect resulting from capital-skill complementarity are reported in Table 4. The three Scenarios displayed vary with regard to the applied labour supply elasticities, \( \varepsilon_i \), \( i \in \{L, M, H\} \) as well as the substitution elasticity between low-skilled labour and the capital-skill input, \( \sigma_1 \), and the elasticity within the capital-skill input, \( \sigma_2 \). Moreover, since we apply identical values for the labour supply elasticity of each skill type, capital accumulation affects medium- and high-skilled labour, the two components of the capital-skill input, in the same way.
Scenario 1 employs the estimates for the substations elasticities $\sigma_j$, $j \in \{1, 2\}$ from Krussel et al. (2000), while the results under Scenario 2 and Scenario 3 are derived using the elasticities identified by Lindquist (2005) and FitzRoy and Funke (1995). Furthermore, under Scenario 1 we additionally apply different values for the labour supply elasticity $\varepsilon_i$.

### Table 4: The Demand Side Effect (in %)

<table>
<thead>
<tr>
<th></th>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Scenario 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\varepsilon_i= 0.01 / 0.37 / 0.56$</td>
<td>$\varepsilon_i= 0.56$</td>
<td>$\varepsilon_i= 0.56$</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>1.67</td>
<td>1.40 / 1.50 / 1.40</td>
<td>0.50</td>
</tr>
<tr>
<td>$\sigma_2$</td>
<td>0.67</td>
<td>0.52 / 0.52 / 0.62</td>
<td>0.21</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Labour Demand</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Low-Skilled</td>
<td>0.05 / 1.89 / 2.79</td>
<td>3.36 / 3.20 / 3.24</td>
<td>7.78</td>
</tr>
<tr>
<td>Medium-, High-Skilled</td>
<td>0.13 / 3.88 / 5.31</td>
<td>6.41 / 6.42 / 5.62</td>
<td>11.1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Wage Rates</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Low-Skilled</td>
<td>5.22 / 5.18 / 5.04</td>
<td>6.09 / 5.78 / 5.86</td>
<td>14.3</td>
</tr>
<tr>
<td>Medium-, High-Skilled</td>
<td>14.1 / 10.8 / 9.67</td>
<td>11.7 / 11.8 / 10.25</td>
<td>20.7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Skill Premium (in %)</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Medium-, High-Skilled</td>
<td>8.41 / 5.37 / 4.41</td>
<td>5.33 / 5.65 / 4.14</td>
<td>5.56</td>
</tr>
</tbody>
</table>

*Source:* Own calculations.

Overall, Table 4 shows that for a given degree of capital-skill complementarity, as reflected by the values of the substitution elasticities $\sigma_1$ and $\sigma_2$, the wage inequality will be more pronounced the less elastic the labour supply is. For instance, if we compare column one and three under Scenario 1, we find that the skill-premium is nearly twice as large if the labour supply equals 0.01 instead of 0.56. This finding can be easily explained by the features of capital-skill complementarity. Since the accumulation of capital is accompanied by an increase in the demand of medium- and high-skilled labour, the wage of medium- and high-skilled labour will increase faster than for low-skilled labour. Moreover, if we assume that the supply of low-skilled labour is rather inelastic, this input become more costly. Accordingly, it is less valuable for a profit maximizing firm to substitute the capital-skill input by low-skilled labour if the latter is rather inelastic.

Our second finding illustrates the importance of the substitution elasticities $\sigma_j$. The larger the elasticity of substitution between low-skilled labour and the capital-skill input, $\sigma_1$, the larger is the increase in the skill premium. In this case low-skilled labour and the capital-skill input can be substituted more easily. As opposed to this, if the substitution
elasticity within the capital-skill input, $\sigma_2$, is raised the skill premium declines (see column 3, Scenario 2).

An increase in $\sigma_2$ implies that medium- or high-skilled labour and capital are less complementary and hence the accumulation of additional capital results in a lower demand for medium- and high-skilled labour. Accordingly, the increase in medium- and high skilled wages is less pronounced explaining the rather modest change in the skill premium.

### 3.2 The Supply Side Effect

The second class of simulations focuses on the “supply side effect” which specifies the impact of an increasing supply of low-, medium-, and high-skilled labour on the wage rates - or more generally speaking, on the skill premium. Since we are solely interested on the impact of an increased labour supply, we now abstract from capital-skill complementarity. Each version of our CGE model now treats the three different labour skill types as perfect substitutes, as formalized in equation (4).

From a theoretical point of view, the direction of the supply side effect is straightforward. If the supply of high-skilled labour increases in the course of time while the supply of low-skilled labour stays constant or declines, high-skilled labour will becomes relatively abundant. Therefore, the wage rate of high skilled labour will record a downward trend while the wage rate for low-skilled labour will rise. Consequently, a larger increase in the supply of high-skilled labour vis-à-vis low-skilled labour will have an alleviating impact on the skill premium.

In the applied dynamic CGE model we, however, can not map the transitional dynamics of the labour force in detail, since the model features only a single representative individual. To resolve this shortcoming, we assign a lower value to the supply elasticity of low-skilled labour than for medium- and high-skilled labour.\(^{17}\) Hence, an identical increase in the wage rate for low- and high-skilled labour will have a much larger effect on the supply of high-skilled labour than on low-skilled labour, as illustrated in Figure 1.

\(^{17}\)From an overall perspective this approach is similar to one where the individual decision of investing in human capital is modelled explicitly. The challenge of this approach is, however, to select the appropriate labour supply elasticities for the different skill types as the empirical evidence on these parameters is extremely poor.
This result is in line with the stylized facts for the German economy reported in Table 3 for the analysed period from 1994 to 2004.

![Figure 1: The Supply Side Effect](image)

Similar to the first set of simulations, we quantify the supply side effect by disturbing the initial steady state in the sense that once again we reduce the initial capital stock by 21.8 per cent while the final steady state is left unchanged. The different scenarios conducted vary solely with regard to the employed values for the supply elasticities of low-, medium- and high-skilled labour. Table 5 displays the impact of the differentiated labour supply elasticities on the labour supply per se, on the wage rates and the skill premium.

In Scenario 1 and 2 we apply the short-run and long-run labour supply elasticities for the different skill types, respectively, as suggested by Fehr (1999). In a first version of Scenario 3 we just reduce the labour supply elasticity for low-skilled individuals to 0.01. In the second version, we cut the labour supply elasticity for both medium- and high-skilled labour by half and in a third version we only reduce the supply elasticity of high-skilled individuals by 50 per cent.

Under Scenario 1 and 2, the differences in labour supply across skill types results in wage compression even though the quantitative effect is of only rather small magnitude.

Albeit the increase in the demand for medium- and high-skilled labour (5.07 and 5.48
per cent, respectively) is significantly larger than for low skilled-labour (4.23 percent),
the relatively high supply elasticity of medium- and high-skilled labour alleviates the
increase in the respective wages rates. As a consequence, the skill premium of medium-
and high-skilled labour declines by nearly 0.3 and 0.2 per cent, respectively.

Table 5: The Supply Side Effect (in %)

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Scenario 2</th>
<th>Scenario 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_L = 0.55$</td>
<td>$\varepsilon_L = 0.49$</td>
<td>$\varepsilon_L = 0.01$</td>
</tr>
<tr>
<td>$\varepsilon_M = 0.66$</td>
<td>$\varepsilon_M = 0.54$</td>
<td>$\varepsilon_M = 0.54$</td>
</tr>
<tr>
<td>$\varepsilon_H = 0.71$</td>
<td>$\varepsilon_H = 0.59$</td>
<td>$\varepsilon_H = 0.59$</td>
</tr>
</tbody>
</table>

Labour Supply / Wage Rates

| Low-Skilled  | 4.23 / 7.83 | 3.95 / 8.23 | 0.08 / 8.57 | 4.42 / 9.22 | 4.14 / 8.62 |
| Medium-Skilled  | 5.07 / 7.81 | 4.35 / 8.21 | 4.50 / 8.47 | 2.42 / 9.27 | 4.56 / 8.61 |
| High-Skilled  | 5.48 / 7.80 | 4.76 / 8.21 | 4.91 / 8.46 | 2.70 / 9.26 | 2.52 / 8.66 |

Skill Premium (in %)

| High- vs. Low-Skilled | -0.026 | -0.017 | -0.10 | 0.036 | 0.043 |
| Medium- vs. Low-Skilled | -0.018 | -0.009 | -0.09 | 0.042 | -0.009 |
| High- vs. Medium-Skilled | -0.008 | -0.008 | -0.01 | -0.00 | 0.034 |

Source: Own calculations.

The simulation results in Scenario 2 follow in principal the same pattern as in Scenario 1, however, at a lower magnitude. This is the case, since the dispersion in the labour supply elasticity is lower in Scenario 2 compared to Scenario 1. Overall, we can conclude that the magnitude of the supply side effect increases with the spread between the supply elasticity for medium- or high-skilled labour and low-skilled labour.

From Scenario 3 we learn that the supply side effect enhances the skill premium, if medium- and high skilled-labour are supplied less elastically compared to low-skilled labour. Accordingly, the supply side effect might go in either direction depending on the constellation of the different labour supply elasticities.

Thus, the impact of the supply side effect is rather small, even if we assume rather low values for the respective labour supply elasticities.

### 3.3 The Overall Effect

The final set of simulation deals with the “overall effect” by simultaneously considering the demand side effect and the supply side effect. Accordingly, we incorporate both capital-
skill complementarity and additionally apply differentiated labour supply elasticities for the various skill types.

We apply under Scenarios 1 and 2 the substitution elasticities suggested by Krussel et al. (2000), while the respective elasticities applied in Scenario 3 are derived from the estimates by FitzRoy and Funke (1995). In Scenario 1 we additionally use the estimates by Fehr (1999) for the different short-run labour supply elasticities while we employ Fehr’s long-run estimates for Scenarios 2 and 3.

Table 6: The Overall Effect (in %)

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Scenario 2</th>
<th>Scenario 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_{L/M/H}$</td>
<td>$\varepsilon_{L/M/H}$</td>
<td>$\varepsilon_{L/M/H}$</td>
</tr>
<tr>
<td>0.55 / 0.66 / 0.71</td>
<td>0.49 / 0.54 / 0.59</td>
<td>0.49 / 0.54 / 0.59</td>
</tr>
<tr>
<td>$\sigma_{1/2}$</td>
<td>$\sigma_{1/2}$</td>
<td>$\sigma_{1/2}$</td>
</tr>
<tr>
<td>1.67 / 0.67</td>
<td>1.67 / 0.67</td>
<td>0.50 / 0.21</td>
</tr>
</tbody>
</table>

Labour Demand / Wage Rates

- Low-Skilled: 2.84 / 5.22
- Medium-Skilled: 5.97 / 9.19
- High-Skilled: 6.20 / 8.84

Skill Premium (in %)

- High- vs. Low-Skilled: 3.44
- Medium- vs. Low-Skilled: 3.77
- High- vs. Medium-Skilled: -0.32

Source: Own calculations.

The results reported under Scenarios 1 and 2 are quite similar, even though the computed skill premia under Scenario 2 are slightly higher compared to the results under Scenario 1. Accordingly, if we simultaneously consider both the demand and the supply side effect, an increase in the capital stock by 21.8 per cent - which corresponds to the accumulation of capital in Germany between 1994 and 2004 - results in an increase in the demand for medium- and high-skilled labour by about 6 per cent, and by roughly 3 per cent for low-skilled labour. Given the supply behaviour of the different skill types, the wage rates for low-, medium-, and high-skilled workers change by 5.2, 9.2 and 8.8 per cent, respectively. Consequently, the model predicts an increase in the skill premium for medium- and high-skilled labour by 3.8 and 3.4 per cent, respectively.

If we use the estimates for the long-run labour supply elasticities by Fehr (1999), the resulting change in the respective labour demands is slightly smaller and thus the increase...
in the corresponding wage rates is more pronounced. The latter also explains the larger values for the increase in the skill premium for medium- and high-skilled labour under Scenario 2.

A different picture arises under Scenario 3 where the estimates for the substitution elasticities by FitzRoy and Funke (1995) are applied. Given these low values for the substitution elasticities, the model predicts an increase in the demand for low- and medium-skilled labour by about 11 per cent and an increase by 7.3 per cent for high-skilled labour. This change in the demand for the different labour skill types and the respective wage rates imply that the wage inequality between high- and low-skilled labour as well as between medium- and low-skilled labour increases by 3.8 and 5.1 per cent respectively.

4 Further Considerations and Conclusion

In this paper we estimate the quantitative importance of capital-skill complementarity and endogenous labour supply for wage inequality in Germany. Our simulation results are derived using a dynamic computable general equilibrium (CGE) model in the spirit of Auerbach-Kotlikoff (1987).

According to our first conjecture, the demand side effect following from capital-skill complementarity leads to a significant increase in the skill premium for medium- and high-skilled labour if additional capital is accumulated. Moreover, we find that the increase in the skill premium under capital-skill complementarity depends crucially on (1) the substitution elasticities between medium- and high-skilled labour and capital and the elasticity between low-skilled labour and the capital-skill input, as well as on (2) the employed labour supply elasticity. More specifically, we showed that the skill premium depends positively on the elasticity of substitution between low-skilled labour and the capital-skill input and negatively on the substitution elasticity between medium- and high-skilled labour and capital. With regard to the labour supply elasticities we find that the rise in the skill premium will be more pronounced the more inelastic labour supply is.

Our second conjecture, namely, that the more elastic supply of medium- and high-skilled labour will have an alleviating effect on the skill premium is also confirmed — even though the supply side effect has only a very small quantitative impact.
By using a value of 0.50 for the elasticity of substitution between low-skilled labour and capital and a value of 0.21 for the one between high-skilled labour and capital, as estimated by FitzRoy and Funke (1995) for the Germany economy, we compute that the rise in the German skill premium over the decade 1994-2004 would have been by 1.71 percentage points higher\textsuperscript{18} without any differentiated labour supply among the different skill-types. In case we apply the estimates by Krusel et al. (2000), who present an estimate of 1.67 and 0.67 for the respective substitution elasticities, the rise in the German skill premium would have been only 0.4 percentage points higher\textsuperscript{19}. Moreover, in the absence of capital-skill complementarity, the actual rise in the relative supply of medium- and high-skilled labour would have induced a decrease in the German skill premium by about 0.03 percent – or by 0.1 percent, if we assume that low-skilled labour is inelastically supplied.

References


\textsuperscript{18}See the skill premium reported under Scenario 3 in Table 4 and the one under Scenario 3 in Table 6.

\textsuperscript{19}Compare the skill premium reported in the third column of Scenario 1 in Table 4 and the on of Scenario 2 in Table 6.


