Demographic change, pension reform and redistribution in Spain *

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Abstract

Recent demographic changes have spurred pension reforms aimed at restoring the financial sustainability of PAYG systems. In Spain, the most significant reforms were undertaken in 1997 and in 2002, entailing an increase in the length of the averaging period in the pension formula, an increase in the penalties for early retirement and for retirement with short contributive records, a bonus for retirement after the age of 65, and a change in the eligibility conditions. In this paper we use an Applied General Equilibrium model populated by two-earners households to evaluate the redistributive impact of the pension system and the financial and welfare consequences of these reforms on households that differ in their education, region of residence and year of birth. The initial redistribution is assessed by comparing the internal rate of return provided to different households. We find that they vary considerable depending on education and cohort. Regarding the reforms, we find an increase in the implicit debt of the pension system after the reforms, and important changes in welfare. Households up to secondary education born between 1935 and 1975 are predicted to benefit from the reform, while the welfare of younger cohorts will be hit by higher taxes and unfavorable macroeconomic changes.

JEL codes: D58, H55, J11

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1 Introduction

The aging of the population and the imminence of the retirement of the large cohorts of “Baby Boomers” has generated widespread concern about the future financial sustainability of PAYG pension systems. In response, most countries have engaged in reforms of their current systems. In the case of Spain, the most significant reforms of the pension system were undertaken in 1997 and in 2002. These reforms implied a substantial reformulation of the way the individual pension benefits were computed by: (i) increasing the length of the averaging period in the pension formula from the 8 years immediately before retirement to 15 years; (ii) changing the proportionality factors associated with penalties for early retirement and for retirement with insufficiently large contributive records; (iii) increasing the bonus for retirement after the age of 65 and, finally, (iv) changing the eligibility conditions for old-age pensions. These changes are mirrored by similar parametric changes across OECD countries over the last decade.

Spurred by this environment, academic research on the impact of pension systems and their reforms has grown dramatically. Both the financial implications and the redistributive effects have been analyzed in the literature, using a number of alternative methodologies. Roughly speaking, two main methodological approaches can be distinguished. On the one hand, financial forecast has usually been made using Aggregate Accounting. This technique involves making a set of assumptions on the evolution of demographic and economic variables and then using accounting identities to make projections for expenditures and revenues. On the other hand, Applied General Equilibrium Models (AGE) have been widely used to address a variety of academic issues. AGE models consists of artificial economies populated by households and firms that are rational. Macroeconomic variables in these models are obtained through aggregation of the intertemporal decisions of the agents. In this paper we use an AGE model populated by two-earners households of different characteristics to evaluate the financial and welfare impact of the two latest reforms of the pension system in Spain.

The paper contributes to the literature in two main directions. First, the paper explores the redistributive effects of pension systems and their reforms. Up to now, this broad field has received relatively little attention, as devising strategies to avoid the future deficits of PAYG pension systems have been on the top of the research agenda. In this paper we explore the redistributive effects of the pension system and their reforms on households that differ in their education, region of residence and year of birth (i.e. cohort). Our second main contribution is a methodological one: our representative household is a two-earners one, rather than the standard individual-based household. As a result, consumption and savings decisions take into account the present and future labor and pension income obtained by each of the household members. We pay special attention to reflect the changing pattern of the household’s pension income by age, driven by the effects of mortality and the substitution of old-age pensions for survival pensions upon the death of the spouse. Note that survival pensions represent an important share of the total pension expenditure on any modern economy (around 20% in Spain in 2006), a feature that is absent in the equilibrium models based on individual behavior. Thanks to this approach, our model simulations include the effects of (i) gender differences in life-expectancy, which have a large impact in the pension income of the household and (ii) secular changes in the employment patterns of males and females and their ensuing effects on individual eligibility and pensions. This alternative modeling framework is specially important for the evaluation of the redistributive implications of pensions reforms.

Our paper is specially related to the literature that uses AGE models to explore the intra and inter-generational redistribution properties of pension systems. Cubeddu (1996) studies the redistribution implied by the US pension system across individuals that differ by education, sex and race. He finds that the system is progressive with respect to education
and sex, but regressive with respect to race. Huggett and Ventura (1999) measure the redistributive effects of an alternative pension system for the US that combines an actuarially-fair pension formula with a minimum pension, and find that it would benefit those with high earnings against those with low earnings. The impact of the insurance provided by pension formulas against idiosyncratic income shocks has also been evaluated in eg. Storesletten et al. (1999) and ˙Imrohoroglu et al. (1995) The inter-generational impact of alternative pensions systems has been explored in articles that address questions related to population aging and the privatization of the pension system. A good example for the US economy is De Nardi et al. (1999). For the Spanish economy, Conesa and Garriga (2001) study the inter-generational properties of the pension system in a representative agent life-cycle model. Rojas (2005) extends the analysis to a framework in which productivity depends on age and Sánchez-Martín (2001) to a framework in which individuals are ex ante heterogeneous in their education. Finally, Díaz-Saavedra (2006) considers an economy in which individuals are heterogeneous due to both ex ante differences and also due to the different earnings shocks they suffer along their life. Apart from the welfare analysis, all these papers include projections of the size of pension expenditures. Our work is, then, also related to them in that dimension.

Our main findings are as follows. First, we detect substantial differences in the Internal Rate of Return (IRR) by education and cohort, and rather small differences by region of residence. In particular, our analysis indicates that the pension system is regressive at the household level, due to the differences in female employment behavior by education. Differences in the IRR across cohorts are related to increases in life-expectancy along time and to the consequences of recent pension reforms (whose costs and benefits are distributed across cohorts in a rather uneven way). Second, we find that the reforms implemented so far fail to restore the financial sustainability of the system. Finally, we detect sizable modifications in the welfare of our representative agents as a result of the reforms. The change that produces the largest effects is the extension in the length of the averaging period in the pension formula and the associated relaxation in the eligibility conditions. Households with up to secondary education born between 1935 and 1975 are predicted to benefit from the reform, mainly thanks to the increase produced in eligibility rates. The welfare of younger cohorts, however, will be hit by the higher taxes and unfavorable macroeconomic changes implied by the reform. The welfare consequences of the other legislative changes is much smaller.

The rest of the paper is organized as follows. In Section 2 we describe the model economy that serves as the basic tool for our analysis. In Section 3.1 we explain the details of the calibration of the (exogenous) demographic processes used in our simulations. Section 3.2 deals with the calibration of the unobservable parameters of the model and of the details of the pension system. Finally in Section 4 we provide the results of our analysis of the redistributive effects of the current pension system, of our projection of future pension expenditures and of the impact of the analyzed reforms.

2 The model

The model consists of overlapping generations of agents that live up to $I$ periods. A period in the model stands for one year of real time, which we denote by $t$ when referring to calendar time and by $i$ when referring to age. The cohort the individual belongs to is denoted by $u$ ($u = t - i + 1$). After the age of entrance in the labor market (20 years) individuals are grouped in representative households and start making economic decisions. At that time, households are classified according to their educational attainment and region of residence into one of $J$ possible categories (denoted by $j \in J = \{1, \ldots, J\}$). As a general rule,

\footnote{For simplicity, we assume perfect assortative mating implying one unique educational level for each household.}
variables characterizing the household are written in lower case with a couple of subscripts and a superscript representing age, education and calendar year respectively. Aggregate variables are denoted with capital letters and have just one superscript indicating calendar time.

2.1 The demographic model

We model a one sex population were individuals are classified according to their country of birth as “Natives”, $N^t$, or “Migrants”, $M^t$. We model different demographic processes for each region, although we omit this dependence in what follows to ease notation. The number of people born at $t$ is determined by the profile of age-specific fertility rates $\{\theta^t_i\}$ (assumed to vary between the threshold ages $f_0^t$ and $f_1^t$):

$$N^t_i = \sum_{i=f_0^t}^{f_1^t} \theta^t_i (N^{t-1}_i + M^{t-1}_i)$$

(1)

After-birth population dynamics is given by:

$$N^{t+1}_i = h s^t_i N^{t-1}_i - I^{t-1}_i + F^t_i, \quad 1 < i \leq I$$

(2)

where $F^t_i$ stands for immigrants flows and $\{h s^u_i\}^I_i=1$ for the cohort-$u$ vector of age-conditional survival probabilities. Equations (1) and (2) constitute the law of motion of the population in the interval $t \in (t_0, t_1)$, a stage of demographic transition in which the fertility and mortality parameters are changing in time (see section 3.1). After $t_1$ (set to 2050 in the simulations) the fertility and mortality patterns stay constant and immigration flows progressively die out. After $t_2 = t_1 + I$ a stable population is achieved, and we assume the convergence of the entire economy to a final balance growth path. The complete time span of the simulation is represented by $T$.

2.2 The Economic model

We use a extended overlapping generations model of a close economy including life uncertainty, borrowing constraints at the end of the life cycle, flows of workers from abroad and representative households as decision makers. At the aggregate level the economy is deterministic, while at the micro level households are uncertain about the length of their life. There is no insurance market for this risk, as annuity markets are closed by assumption. Households in the model differ in their educational attainment and in the region of residence. We also take into account gender differences in labor income and survival probabilities within each household. The production side of the model is entirely neoclassical: we assume a standard technology, with a constant returns to scale production function, $F(K, L)$, no adjustment costs and exogenous labor-augmenting technological progress, $A^t$. The growth rate of labor productivity, $\rho$, is constant.

2.3 The public sector

The main role played by the Public Sector is to run a Defined-Benefit pension system, financed with the contributions made by active workers (ie, run on a PAYG basis). Contributions paid by a worker of age $i$ in calendar time $t$, are a fixed proportion ($\varsigma$) of covered

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3 The household formation process for migrants older that 20 years reflects the general distribution of households in the population at large. I.e, we group the newly arrived immigrants into biparental households in the same proportion as the weight of non-individual households of that particular age in the entire population.

4 In the simulation, the balance growth path is reached in finite time, at period $t_2$ (2220 in the current calibration). We check that the chosen $t_2$ does not affect the performance of the economy in the interval of interest $(t_0, t_1)$.
**earnings** (“bases contributivas”, $bs_t^i$) which coincide with gross labor income, $il_t^i$ up to an annually legislated maximum $C_M^t$.

Eligible workers (those with a long enough contributive record) can claim an old-age pension at any time after the *early retirement age*, $\tau_m$, an following a complete withdrawal from the labor force. The *initial* pension for an individual belonging to cohort $u$ who retires at age $\tau$ after $h$ years of contributions is:

$$b(\tau, h, u) = \alpha_E(\tau) \alpha_H(h) \left( \frac{\sum_{e=\tau-D}^{\tau-1} bs_u^{n+e-1}}{D} \right)$$  \hspace{1cm} (3)

The formula combines a moving average of *covered earnings* in the $D$ years immediately preceding retirement (called *benefit base*) and two linear replacement rates:

- The replacement rate, $\alpha_E(\tau)$ captures early retirement penalties. For each year that the individual anticipates retirement (from the Normal Retirement Age of 65), the final benefit is reduced by a $\Delta \alpha_E$ percent. This penalty depends on the length of the contributive record (see section 3.2). There is also an annual bonus $\Delta \alpha_{E_{+65}}$ for staying after 65. Formally:

$$\alpha_E(\tau) = \begin{cases} 
\alpha_E^0 & \text{if } \tau < 60 \\
\alpha_E^0 + \Delta \alpha_E(\tau - 60) & \text{if } \tau \in \{60, \ldots, 64\} \\
1.0 + \Delta \alpha_{E_{+65}}(\tau - 65) & \text{if } \tau \geq 65 
\end{cases}$$  \hspace{1cm} (4)

- The replacement rate $\alpha_H(h)$ captures penalties associated with an insufficient record of social contributions. The length of the record is deemed to be *sufficient* only after 35 years:

$$\alpha_H(h) = \begin{cases} 
0.0 & \text{if } h < 15 \\
\alpha_H^0 + \Delta \alpha_H(h - 15) & \text{if } h \in \{15, \ldots, 25\} \\
\alpha_H^1 + \Delta \alpha_H^2(h - 25) & \text{if } h \in \{25, \ldots, 35\} \\
1.0 & \text{if } h \geq 35 
\end{cases}$$  \hspace{1cm} (5)

In the current formulation, the penalty is higher for those with shorter records ($\Delta \alpha_1^H > \Delta \alpha_2^H$).

The *initial* pension is fully indexed to price inflation and subject to an annually legislated minimum, $bm_t^i$, and maximum, $bM_t^i$. The *effective* pension income in year $t$ for the individual above is then:

$$ib_t^i(\tau) = \min\{bM_t^i, \max\{bm_t^i, b(\tau, t - i + 1, h)\}\}$$  \hspace{1cm} (6)

Finally, surviving spouses may receive a fraction of the benefit base of the deceased.\(^5\) There is also an specific guaranteed minimum for survival pensions, denoted by $bm_t^V$.

In addition to running the pension system, the Public Sector performs two additional functions: it runs a fiscal system and incurs in a certain amount of public expenditure, $CP_t$. Both tasks are extremely simplified in our model. We assume that fiscal revenue comes from the confiscation of involuntary bequests and from a system of lump sum taxes, while public expenditure is a fixed proportion of the annual GDP.

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\(^5\)We simplify the complex eligibility conditions by assuming that all surviving spouses older than a (advanced enough) age qualify for the survival pension. The age requirement is a proxy for the real requirement in terms of number of years of contributions.
2.4 The representative household

We assume that pairs of individuals of identical age of birth, region of residence and education form new households at the age of entrance in the labor market. These representative households are thereafter dissolved only by the effects of mortality. These households are the economic agents of the model: they choose the optimal life cycle consumption and savings, given the expected streams of labor, pension and capital income of each of the two individuals that belong to it. Formally, they solve the following problem (we omit the type of the household to simplify notation):

\[
\max_{\{c_{i}^{u+i-1}, a_{i}^{u+i-1}\}} \sum_{i=1}^{I} \beta^{i-1} s_{i} u(c_{i}^{u+i-1}, l_{i})
\]

\[
c_{i}^{u+i-1} + a_{i}^{u+i-1} = inc_{i}^{u+i-1} + (1 + r^{u+i-2})a_{i}^{u+i-1} - \varphi^{u+i-1}
\]

\[
a_{i}^{u} = 0 \quad a_{i}^{u+i-1} = 0 \quad a_{i}^{u+i-1} \geq 0 \quad \forall i \geq \tau
\]

where \(inc_{i}^{t}, c_{i}^{t}\) and \(a_{i}^{t}\) stand for, respectively, the total disposable income, consumption and accumulated assets of the household in year \(t = u + i - 1\). \(\varphi\) stands for the income tax paid by the household and \(\beta\) is a discount factor. Note that we assume exogenous labor supply, borrowing constraints after retirement and no initial wealth.

The life-cycle profile of household income reflects the differences in earnings, employment rates and survival probabilities of each spouse. Formally, the labor earnings for a household of type \(j\) is:

\[
in{c}_{i,j}^{t} = (\omega_{i,j,3} + \omega_{i,j,1}) E_{i,j,1}^{t} i_{i,j,1}^{t} + (\omega_{i,j,3} + \omega_{i,j,2}) E_{i,j,2}^{t} i_{i,j,2}^{t}
\]

where \(E_{i,j,g}^{t}\) and \(i_{i,j,g}^{t}\) stand for the employment rate and net labor income of spouse \(g = \{1, 2\}\) at age \(i\) in year \(t\). Weights \(\omega_{i,j,g}\) represents the proportion of single households, ie of households where, due to the effect of mortality, just the spouse \(g\) survives at age \(i\). \(\omega_{i,j,3}\) is the proportion of household with two surviving spouses. The household pension income is constructed in a similar manner:

\[
in{c}_{i,j}^{t} = \sum_{g=1,2} (\omega_{i,j,3} + \omega_{i,j,g}) C_{j,g} i_{i,j,g}^{t} + \sum_{g=1,2} \omega_{i,j,g} C_{j,g} i_{i,j,g}^{t}
\]

where \(i_{i,j,g}^{t}\) is the old-age pension of spouse \(g\), computed according with (6); \(iv_{i,j,g}^{t}\) is the associated survivor pension, and \(C_{j,g}\) is the share of spouses of gender \(g\) and type \(j\) that are entitled to get an old-age pension according with the legal requirements in (5). More details about the construction of all the components of income can be found in section 3.2.3.

2.5 Equilibrium

An equilibrium path over the time interval \(T\) is a set of time series of population aggregates and distributions, household decisions (consumption and savings), aggregate inputs, prices and public policies (taxes, public consumption and minimum and maximum pensions and contributions) with the standard properties: households are rational, factor markets clear, prices are competitive, the public budget balances and an aggregate feasibility constraint is observed. Appendix A provides a formal definition of the equilibrium of the model economy. As in the standard Auerbach and Kotlikoff (1987) methodology, we assume the convergence of the equilibrium path to a final balance growth path in finite time.
3 The model calibration

We calibrate our model to mimic the observable economic and demographic characteristics of the Spanish economy, and in accordance with standard projections for future demographic and productivity trends. The next two sections review the details of the demographic and economic calibrations.

3.1 The calibration of the demography

A period in the model stands for one year of calendar time and we assume a maximum lifespan $I$ of 100 years. The simulated equilibrium path starts in $t_0 = 1998$. For each region, we reproduce the observed population distribution and the age profiles of fertility and survival probabilities in that age. From $t_0$ to $t_1$ (2050) we simulate a changing pattern of fertility and mortality, according with the main scenario (“hipótesis 1”) by the Spanish Statistical Institute (INE). The total fertility rate is assumed to recover from the extremely low values observed during the nineties (in 1995, 1.2 children per women nationwide) to a final stationary value of 1.53 in 2050. We also reproduce the trend towards lower mortality rates by assuming that life expectancy rises from the 76.6 years observed in 2000 to 81.0 years in 2050 for males and from 83.4 to 87 for females. In both cases, the bulk of the recovery is concentrated in the first decades of the simulation. For the immigration flows, we reproduce the observed data between 2000 and 2005 (which registered flows of unprecedented size for the Spanish historical experience) and follow INE (“hipótesis 1”) for the projection in the interval 2005/2050.

Demographic projections

Figure 1 summarize the main demographic developments during the first half of the 21st century. Among the different patterns, we highlight three outstanding phenomena:

- The intensity of the immigration flows is large enough to fuel a rather dramatic increase in the absolute size of the Spanish population. This can be appreciated in the top-left panel of figure 1. In the process, the share of first-generation immigrants grows to an astonishing 25% of the total population (bottom-right panel of figure 1).

- As the distribution by age of the immigration flows is younger than that at population at large, immigration flows alleviate the aging process. The bottom-left panel of figure 1 makes clear, however, that they cannot stop it altogether. The dependency ratio (defined as the ratio of the number of people older than 65 to those between 20 and 64) almost doubles in the interval 2000/2050 (from 26.9 to 51.2%). The intensity of the aging process can also be appreciated in the change of the shape of the population pyramid in the top-right panel of figure 1.

- There are striking disparities in the demographic prospects of the Spanish regions. Roughly speaking, we can identify two broad areas with quite opposite demographic tendencies: the North-west (NW) versus the South-East (SE). Regions in the NW area share low fertility patterns, high longevity, almost non-existing secular tradition of immigration and, specifically, the inability to attract any significant part of the large recent incoming flows.

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6 All these information is publicly available from INE (Instituto Nacional de Estadísticas) website.
7 This is achieved through parallel shifts in the initial age-profiles of fertility of the different regions.
8 We use the 1997-2004 Encuesta de Variaciones Residenciales microdata to compute the net aggregate flows of incoming population and its breakdown by age and region of destination. This breakdown is assumed constant throughout the simulation path 2005-2050.
9 The detailed results are available from the authors upon request.
Figure 1: Demographic simulation: total population; age distribution of the population in 2000 and 2050; Dependency ratio and immigrant share of the total population.

3.2 Calibration of the economic model

To assign specific values to the parameters of our economic model we proceed in three different ways: (1) the unobservable preference and technology parameters are calibrated in the way that is standard in the Applied General Equilibrium literature\(^{10}\); (2) the parameters of the pension system are taken from their direct observable counterparts; (3) finally, the life-cycle profiles of earnings, employment rates and hours worked by the members of our representative households (differing by generation and type), along with the educational distributions by generation and region, are all estimated from large samples of microdata. We review the three processes in turn.

3.2.1 Preference and technology parameters

The household period utility is a separable CES function, with unitary intertemporal elasticity of substitution (IES): \( u(c_i, l_i) = \log(c_i) + \sigma \log(l_i) \).\(^{11}\) Individual preferences are, therefore, fully specified by choosing \( \sigma \) and \( \beta \). We assume a Cobb-Douglas production function to generate the aggregate output ie, \( Y = K^\zeta L^{1-\zeta} \). This part of the model is completely specified by choosing the capital share in aggregate income \( \zeta \), the rate of capital depreciation, \( \delta \), and the constant productivity growth rate, \( \rho \).

The finally implemented parameter values, presented in table 1, are selected to mimic the performance of some basic macroeconomic indicators of the Spanish economy. We choose \( \beta \) and \( \delta \) to target the average capital/output and investment/output ratios respectively. \( \zeta \) in

\(^{10}\)I.e., a set of properties of the data is chosen, and the parameter space is then searched to find the set of values that make the model predictions closest to the selected properties.

\(^{11}\)The logarithm is adopted in accordance with econometric evidence (Hurd (1989) for US or Jiménez-Martín and Sánchez-Martín (2003) for Spain).
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Observation</th>
<th>Model</th>
<th>Parameter value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$rK/Y%$</td>
<td>34.7</td>
<td>34.7</td>
<td>$\zeta = 0.347$</td>
</tr>
<tr>
<td>$K/Y$</td>
<td>2.57</td>
<td>2.62</td>
<td>$\beta = 0.970$</td>
</tr>
<tr>
<td>$I/Y%$</td>
<td>23.6</td>
<td>24.0</td>
<td>$\delta = 0.060$</td>
</tr>
<tr>
<td>$CP/Y%$</td>
<td>13.3</td>
<td>13.3</td>
<td>$c_p = 0.133$</td>
</tr>
<tr>
<td>$\Delta \ln C%$</td>
<td>2.12</td>
<td>2.12</td>
<td>$\rho = 2.12$</td>
</tr>
</tbody>
</table>

Table 1: Macroeconomic calibration targets and parameter choices: Average 1970-2000 basic macroeconomic ratios according with the Spanish National Accounts and their calibrated counterparts in the model, along with the implemented parameter values.

<table>
<thead>
<tr>
<th>Minimum</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Maximum</th>
<th>Pay-roll</th>
<th>Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>O-E pension</td>
<td>Surv pension</td>
<td>Pension</td>
<td>Contribution</td>
<td>rate</td>
<td>rate %</td>
</tr>
<tr>
<td>0.57</td>
<td>0.48</td>
<td>2.48</td>
<td>3.3</td>
<td>20.9</td>
<td>1.75</td>
</tr>
</tbody>
</table>

Table 2: Pension system calibration. All minimums and maximums are ratios with respect to 2004 average pension. Abbreviations: O-E= Old age pension; Surv=Survivors; The growth rate applies to all floor and ceilings of the system.

the production function is set to reproduce the average capital income share (as measured in Puch and Licandro (1997)) while the exogenous productivity growth rate, $\rho$, is set to the average growth rate of per capita consumption. The future value of productivity growth, however, is set to a lower figure (1.7%), following the long term convergence scenario in EPC (2006). Finally, the government expenditure to output ratio is directly reflected in the parameter $c_p$. All empirical values are 1970/2000 averages from the Spanish National Accounts (CNA86) with the exception of the capital stock (obtained from BBV (2001)).

### 3.2.2 Pension System parameters

The institutional parameters are set to reproduce the General Regime (RGSS) of Spanish social security system.\(^{12}\) Our benchmark case correspond to the structure in place after the latest reforms in 2002. More precisely, we reproduce the values observed in 2004 (table 2).\(^{13}\) Proceeding in this way we achieve a good reproduction of the levels of the system at the beginning of the simulation.\(^{14}\) To project the discretionary components of the system (the values of the floors and ceiling on benefits) we apply the exogenous growth rate of productivity. Finally, note that the time series of the tax on households and the inheritance tax are endogenously determined in the solution of the model.

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\(^{12}\)At the moment, 76% of the registered workers and 57% of the pensioners belong to the General Regime. Besides, there is a political agreement (Pacto de Toledo) to eliminate most Special Regimes. See (Boldrin, Jiménez, and Peracchi 2004) for a complete description of the Spanish pension system.

\(^{13}\)The directly observable pay-roll tax rate cannot be applied, as it covers a larger range of contingencies that those included in the model. We assign this value following Jimeno and Licandro (1999): we scale the real-world value down by multiplying it by the share that old-age and survivors pensions represent of the total Social Security expenditure in the selected year.

\(^{14}\)Total pension expenditure in 2005 was 8.47% of the GDP. The equivalent figure in our model is 8.58%. This mostly reflects the well known fact that, in average, the General Regime imposes a stricter proportionality between benefits and income than other regimes (eg. self-employed). Our initial stationary structure also overstates the eligibility rates of the model (1.07 pensions per person aged 65 or more versus 1.04 in the data).
3.2.3 Household life-cycle earnings and employment rates

We build the life-cycle earnings profiles of each representative household using individual data on the employment, hours worked and wages of Spanish males and females. The household earnings (recall equation (8)) are calculated as a weighted sum of the earnings of each spouse. The labor earnings of each spouse, in turn, is the product of the estimated employment rate and the average wage of employees with the same household characteristics.\(^\text{(15)}\)

We reproduce the observed dynamics of the life-cycle employment rates by cohorts born before 1975. For the future, we assume a progressive deceleration in the improvement in employment rates by gender and education. We control the speed of this process to reproduce the aggregate projection for the Spanish economy in 2005 in EPC (2006) (71.4%). Figure 11 in appendix B reproduces the resulting time series of aggregate employment rates (by gender). The estimated life-cycle profiles of employment rates are also used to compute the average retirement age for each cohort, educational level, sex and region of residence.\(^\text{(16)}\)

3.2.4 Eligibility rates

To be entitled to an old-age pension, individuals are required to (i) have contributed to the system for at least fifteen years, (ii) with at least two of them taking place within the fifteen years immediately preceding retirement. The eligibility rate at each age is, them, the proportion of people of that age that fulfill these conditions. Given that is not possible to calculate the eligibility rates for each of the cohorts in our simulation, we proceed with an approximation based on the life-cycle employment rates described above. We assume that the fraction of individuals of a certain type who comply with the eligibility requirement (ii) is equal to the employment rate fifteen years before the average retirement age of that type. For the total number of years contributed by an individual we use the sum of his/her life-cycle employment rates.\(^\text{(17)}\) Figure 13 in appendix B shows the eligibility rates obtained when proceeding in this way, for males and females of different education levels. We find a strongly increasing pattern for females belonging to cohorts before 1980.

3.2.5 Education

Individuals are classified in three educational groups. The lowest education level correspond to those who fail to complete secondary education, the highest level is made of tertiary graduates and the medium level includes all the rest.\(^\text{(18)}\) The general view obtained from historical data is coherent with the hypothesis of convergence among regions. For instance, the proportion of graduates in regions with low initial levels of highly educated people have grown faster than in regions with higher initial levels. We use this information to project the distribution of cohorts born after 1975. The resulting time series of the evolution of the distribution by education is displayed in figure 14 in appendix B.

\(^\text{15}\)The detailed results of the estimation of labor earnings and employment rates can be obtained from the authors upon request.
\(^\text{16}\)The retirement age is related to the employment rates by applying \(\tau = \sum_{t=56}^{70} \left( \frac{P^{t-1} - P^t}{P^t} \right) \) where \(P^t\) is the employment rate at age \(t\). With this procedure the retirement rate increases with the cohorts born before 1975 and decreases thereafter. See figure 12 in appendix B.
\(^\text{17}\)For example, if the employment rate of one particular type of individual is one at all ages from 25 to 69, his/her total number of years contributed to the pension system would be 45. This procedure only give us and imperfect measure of the eligibility rate, but still it has the important property of "endogenously" accounting for the effect of changes in employment on it and on the size of individual pensions.
\(^\text{18}\)We choose this classification to mimic as closely as possible the distribution implemented in the European Community Household Panel (ECHP), from where we estimate the household earnings. The historical evolution of earnings, however, was taken from the Labor Force Survey (EPA) whose large sample size permits a disaggregate
<table>
<thead>
<tr>
<th>Año</th>
<th>K/Y</th>
<th>PP/Y</th>
<th>Tax ratio</th>
<th>Gen ratio</th>
<th>E-Dep ratio</th>
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<tr>
<td>2005</td>
<td>2.641</td>
<td>8.587</td>
<td>1.000</td>
<td>18.580</td>
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<tr>
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<td>8.369</td>
<td>0.856</td>
<td>18.844</td>
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<tr>
<td>2015</td>
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<td>8.643</td>
<td>0.863</td>
<td>19.237</td>
<td>44.926</td>
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<tr>
<td>2020</td>
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<td>9.478</td>
<td>0.980</td>
<td>20.544</td>
<td>46.132</td>
</tr>
<tr>
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<td>10.799</td>
<td>1.143</td>
<td>21.877</td>
<td>49.361</td>
</tr>
<tr>
<td>2030</td>
<td>2.874</td>
<td>12.557</td>
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<td>23.330</td>
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<tr>
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<td>14.684</td>
<td>1.577</td>
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<tr>
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<td>1.881</td>
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<td>21.383</td>
<td>2.118</td>
<td>29.617</td>
<td>72.200</td>
</tr>
</tbody>
</table>

Table 3: Base simulation: main macroeconomic indicators.
PP= Aggregate pension expenditure; Tax ratio= $\frac{\varphi_t}{\varphi_{2005}}$ = ratio of the income tax levied on households in t to its value in 2005; Gen ratio = Generosity ratio (ratio of the average pension (per person of more than 64 years of age) to average productivity per employee; E-Dep ratio= Effective dependency ratio (number of people older than 64 to number of employees).

4 Simulation Results

4.1 Benchmark simulation

Our benchmark simulation is a projection of the performance of the Spanish economy assuming the continuity of the present institutional environment (ie, after the changes introduced in 2003 in the pension legislation). Results under other institutional settings are explored in section 4.2.

4.1.1 Macroeconomic performance

The aggregate performance of the economy is characterized by a mild process of capital deepening (top panel of figure 2 and second column of table 3), and a progressive reduction in the economy growth rate (from an initial value slightly below 3.5% to a figure barely above 1% in 2050). The impact of population aging is even more evident in the finances of the pension system (third column of table 3 and figure 3). The share of pension expenditure on the GDP more than doubles between 2010 and 2055, from a relatively modest initial value (8.4%) to a figure well beyond 20% after 2045. That process is not monotonous: the initial decade is favorable (thanks to large immigration flows), but the condition of the system experiences a continuous deterioration thereafter. To cope with this imbalance, the fiscal burden placed on the households evolves in lockstep with the financial condition of the system (forth column of table 3 and bottom panel of figure 2). Consequently, the fiscal burden is projected to be below the value in 2005 for almost two decades. This situation, however, eventually reverses, and rather extreme tax hikes follows. By the year 2050, the fiscal burden suffered by families is 130% higher than that at the start of the simulation.

The gloomy financial prospect of the pension system is not just the result of the expected unfavorable demographic changes. It is also the result of the inner workings of the system analysis by cohort, gender, education and region.
Figure 2: Base simulation: time series of capital to output ratio (K/Y) and of family fiscal burden index (ratio of the current income tax levied on households to the value in 2005) and its interaction with the changes in the labor market. To show this, we use the following accounting identity:

\[
P P_t^t = \left( \frac{\bar{y}^t}{\bar{Y}^t} \right) \left( \frac{P_{t+65}^t}{E^t} \right) = \left( \frac{\bar{y}^t}{\bar{Y}^t} \right) \left( \frac{P_{t+65}^t}{P_{20-64}^t} \right) \left( \frac{P_{20-64}^t}{E^t} \right) = \left( \frac{\bar{y}^t}{\bar{Y}^t} \right) \left( \frac{d^t}{e^t} \right)
\]

We decompose total pension expenditure as the product of the number of people age 65 or more, \(P_{t+65}^t\), and the average pension per person older than 65, \(\bar{y}^t\). Similarly, we express the GDP as the product of the number of employees, \(E^t\), times average productivity per employee, \(\bar{Y}^t\). The pension expenditure to GDP ratio is, then, the product of two factors: a *generosity* ratio relating the average pension to the average productivity; and an *effective* dependency ratio that combines the *demographic* dependency ratio, \(P_{t+65}^t/P_{20-64}^t\), and the employment rate, \(E^t/P_{20-64}^t\). Figure 4 and the two rightmost columns of table 3 show the evolution of these new variables in our simulation. Two aspects become immediately evident:

- The impact of demographic aging is substantially softened by the changes in the labor market. The growth rate of the effective dependency ratio between 2005 and 2050 is close to 60%, far less than the variation associated with purely demographic changes (recall that the *demographic* dependency ratio is expected to double in the same period). The reason for the difference lies in the increase in the employment rates, derived from higher labor participation (especially of females and, indirectly, also due to the change in the distribution by education).
- The generosity of the system is bound to increase significantly. This generosity has two dimensions: the increase in the value of pensions and the increase in the eligibility...
Figure 3: Base simulation: time series of the aggregate pension expenditure to GDP ratio (PP/Y).

Figure 4: Base simulation: Decomposition of PP/Y. Generosity index and effective dependency ratio.
rates. The increase in value is associated to the changes in the labor market (higher participation rates imply lower penalties for early retirement and for insufficient contributive records) and to the generous updating of minimum pensions. The change in the eligibility rates is, once again, the product of the increases in the employment rates. Note that both changes extend from old-age pensions to survivors pensions.

4.1.2 Redistributive impact of the pension system

A traditional approach to analyze the degree of redistribution achieved by a pension system is the comparison of the internal rates of return (IRR) of the system across different types of individuals. The IRR is the interest rate that makes the present value of the contributions to the pension system along the life-cycle equal to the present value of the pensions received by the individual. A formal definition of the IRR, along with a graphical presentation of our computed values by cohort and region is provided in appendix C.

We find important differences by education and cohort. According to our analysis the pension system is regressive at the household level. The average IRR is 3.58% for households with less than secondary education, 3.97% for those who completed secondary education and, finally, 4.40% for those with tertiary education. To interpret this result it is important to keep in mind the strong dependence of the household retirement income on the employment life-cycle profiles of both husband and wife. As the attachment to the labor market is inversely related to the educational achievement (specially for the females), we find that household of low education have lower entitlement rates and also lower final pension benefits.

The variation in the IRR of different cohorts owes to several factors. First, improvements in life expectancy imply, ceteris paribus, increases in the generosity of the system. This phenomenon underlies the overly positive trend followed by the IRR in figures 15 and 16. A second relevant aspect derives from reforms that changes the eligibility conditions or the way the benefits are computed. As we explain in section 4.2.2 these reforms affect different cohorts in different ways. Note that older cohorts can be subject to transitory pension rules for part of their life-cycle. As we abstract from those initial historical dispositions, the IRR obtained for cohorts of advanced age most certainly underestimate the real transfer obtained from the system.19

Finally, although there are some differences in the internal rates of return across regions, the differences are small20. The existence of a unique, countrywide pension system for the whole regions of Spain is crucial for this result, as there are substantial differences in the demographic scenarios across regions. There is, then, a large degree of interregional sharing of the cost of aging.

4.2 Reforms

In this section we analyze the effects on welfare and on pension expenditure of two recent reforms undertaken in Spain in 1997 and 2002. The welfare impact of each reform is evaluated by computing its associated equivalent variation (EV) for each of the representative households of our model. The equivalent variation is formally defined as the percentage change in life-cycle consumption that a household has to receive in the initial economy (without reform) to achieve the same welfare as after the implementation of the reform. A positive equivalent variation, then, implies a welfare gain for the household. The reforms have altered the two main components of the pension formula: the penalties for early retirement and for short

---

19Our simulations model the performance of the economy before 1998 as a steady state. The initial cohorts of pensioners, however, got their pensions with less contributions that those implied by the current form of the system. Our results are precise only for relatively young cohorts, whose lifespan belongs to our simulation path.

20The IRR tends to be higher for regions with larger per capita income. The detail results are available upon request.
contributive records, and the *Benefit Base* (moving average of previous covered earnings). We review them in turn in the next two sections.

### 4.2.1 Changes in the age and contribution penalties

In this section we analyze three recent changes in the Spanish pension formula:

(i) In 1997 the penalty for those who retire with short contributive records was increased. The annual penalty for insufficient contributions was raised from 2 to 3% for workers retiring with less than 25 years of contributions. This means that individuals enrolled for the shortest possible period (15 years) receive 50% of the benefit with no penalties (after 35 years). Before the reform this figure was 60%.

(ii) Penalties for early retirement were reduced for workers with long contributive records. Before the reform, an annual 8% reduction was applied to all workers independently of the length of their previous contributions. After the 1997 and 2002 reforms, this penalty was reduced to 6% for those with more than 40 years in the system and to 7% for those with more than 30 years of enrollment.

(iii) A premium for delaying retirement after the normal retirement age of 65 was established in 2002. For the first time, staying in the labor force after 65 (with more than 35 years of previous contributions) was granted an annual 2% premium in the final benefit.

Figure 5 shows the impact of the reform on individual pensions, conditional on gender and the year of birth. The changes imply pension cuts for workers with very short contributive records, mostly women, specially those with low education, for which the drops can be as high as a 15% reduction. In contrast, pension increases for some cohorts of workers with high attachment to the labor force, specially men of advanced age thanks to the reduction in early retirement penalties, and young men of high education, thanks to the bonus obtained from staying after 65. Overall, we find effects that work in opposite directions on pension expenditure and of similar quantitative importance. As a result, the macroeconomic impact of the reform is very small, with taxes, capital to output ratios and the overall pension expenditure (as a share of the GDP) being largely unaffected by the changes.

Figure 6 shows the welfare impact of the reform. We can observe three broad patterns. Firstly, women suffer disproportionately with the increase in the penalties for insufficient contributions, due to their low initial participation rates. The impact, however, varies a great deal with education and the year of birth. In general, women belonging to older cohorts are not affected by the policy as their employment rates are so low that they do not qualify for a pension. But as younger cohorts of women increase their participation rate, they cannot escape the burden of the higher penalties. As a result, there is always a range of cohorts who suffer welfare losses. The educational level determines when these losses are experienced and the number of affected cohorts.21 A second broad pattern is the existence of sizable welfare gains for cohorts of low educated workers born in the period 1945-1955. This is observed in regions with relatively large labor participation, whose older men early retire in significant numbers and simultaneously have large enough contributive records. Finally, there are rather widespread welfare gains for households with secondary and tertiary education belonging to cohorts born after 1950. These gains are related to the premium given to workers who delay retirement after 65. In accordance with the lower intensity of their labor attachment, the welfare gains for those with secondary education start later in time and are smaller.

21For women with tertiary education the losses affect only a few cohorts born around 1935; for women of lower education the losses start later and are more intense and widespread. Eventually, the increase in participation is large enough for younger women to escape the penalties. It is worth noting that minimum pensions clearly moderate the welfare impact of the reform. This can be appreciated in the difference in the number of cohorts suffering drops in the individual pensions benefits (before applying minimum pensions) versus those suffering welfare losses.
4.2.2 The overall reform

In this section we study the impact of changing the length of the averaging period in the pension formula (recall expression (3)) from 8 to 15 years. This modification is added to the legislative changes explored in the previous section. The reform also relaxed the pension eligibility conditions: to be entitled, workers must had made contributions for at least two periods in the 15 years immediately preceding retirement (up from 8 before the reform). In contrast, the impact of the change in the size of the pension formula cannot be predicted \textit{a priori}, as it depends on the shape of the life-cycle earnings profile of the individuals.

The two dominant consequences of the reform can be appreciated in Figures 8 to 9. On the one hand, the reform produces a sizable decrease (between 5-7\%) in the size of the individual pension benefit, independently of gender or education. This reflects the prevalence of concave life-cycle earning profiles. On the other hand, the reform substantially increases the eligibility rate, especially in the case of males. The macroeconomic consequences are presented in figure 7: total pension expenditure increases by around 8\%, indicating that prevalence of the increase in eligibility over the cuts in individual pensions. The overall generosity of the system increases, and larger contributions rates are needed to keep the system balanced. Higher pension income also reduces the incentives of households to save, leading to a decline in the capital-output ratio and wages.

The welfare consequences are shown in Figure 10 (note the change in the scale of the graphs with respect to Figure 6). We find substantial quantitative changes, whose distribution varies significantly with the year of birth and the educational level. Households with secondary education or less born between 1935 and 1975 benefit most. The welfare gains reach values of around 1\% of the life cycle consumption of these agents. Households with tertiary education belonging to the same cohorts see smaller welfare gains -and even losses for the younger cohorts in some regions. Cohorts born after 1975 experience reductions in
Figure 6: Changes in the age and contribution penalties: Equivalent Variation by education, year of birth and region of residence. Low education (−), Medium education (−−) and high education (− · −).
welfare, with independence of their educational level. These welfare changes result both from the direct impact of the reform of individual benefits and from the sizable aggregate effects it generates. The welfare losses of younger cohorts stem from the pension cuts and, specially, from the increase in taxes and the decrease in wages, which they suffer during most of their lifetime. Older cohorts fare better because they partially escape from the unfavorable macroeconomic consequences of the reform. The relative improvement for those with secondary education or less is due to the big increases in the eligibility rates of these two educational groups. This increase is smaller for those with tertiary education, what explains why a negative overall impact of the reform is observed for the older cohorts of this group.

5 Conclusions

This paper is a contribution to the literature that analyzes pension reforms with Applied General Equilibrium models. We contribute in two main directions. On the one hand we explore the redistributive effects of the Spanish pension system and of its latest legislative reforms. On the other hand our approach introduces an improvement in the evaluation of pension reforms by modeling a two-earners household as the basic representative agent of the model, rather than the standard individual-based household. We consider households that differ in their education, region of residence and year of birth. We pay special attention to reflect the changing pattern of the household pension income by age, driven by the effects of mortality and the substitution of old-age pensions for survival pensions.

We find that the PAYG pensions have important redistributive effects depending on the education and cohort of the household. The system is regressive by education, due to the differences in female employment behavior on this dimension. Across cohorts, younger cohorts tend to enjoy larger rates of return thanks to secular increases in life-expectancy and to the extra generosity introduced by recent pension reforms (although this result abstracts from the asymmetric treatment of cohorts at the introduction of the system). Regarding the latest pension reforms (Laws 24/1997 and 35/2002) we find modest macroeconomic and welfare consequences. Extending the averaging period in the pension formula, for instance, reduces the generosity of the pensions; but relaxing the eligibility conditions in the same
Figure 8: The overall reform: Ratio of pension Post vs Pre Reform, by sex and year of birth.

Figure 9: The overall reform: Female eligibility rates after and before the reform, by year of birth.
Figure 10: The overall reform: Equivalent Variation by education, year of birth and region of residence. Low education (−), Average education (−−) and high education (− · −).
reform works in the opposite direction. Overall, our calculations confirm the widely held view that further changes would be needed to restore the financial sustainability of the system. In welfare terms, households with up to secondary education and born between 1935 and 1975 benefit most from the reforms (mainly due to the increase in their eligibility rates), whereas younger cohorts are hit by the higher taxes and unfavorable macroeconomic changes implied by the reforms.

Our current framework may be extended in several ways. A more thorough analysis of the intergenerational welfare consequences of the reforms demands the explicit consideration of the use of a Trust Fund to spread the fiscal burden across generations (although a Fund of this type has been setup in Spain only very recently). A second avenue for improvement derives from the Close Economy assumption of our simulations. An Open Economy setting seems to suit better the case of a relatively small economy like Spain. However, reflecting the impact of widespread ageing on international prices is an important modeling challenge in the Open setting. Finally, it would be good to disaggregate within our representative household to allow for more realistic heterogeneity in the labor patterns of different households (as a result of different households been hit by different sequences of shocks during their life-cycles). This improvement, however, would demand a total change in the nature of the equilibrium of the model and on the algorithm used to solve it. It would take us away from the general strategy of achieving the maximum possible detail in the reproduction of the institutional framework, in the context of general equilibrium models. We leave these extensions for future research.
References


Jimeno, JF and O. Licandro (1999). La tasa interna de rentabilidad y el equilibrio financiero del sistema español de pensiones de jubilación. Investigaciones Económicas, XXIII.


APPENDIX

A Formal definition of equilibrium

An equilibrium path over the time interval $T$ consists of the following objects:

- Time series of the aggregate number of households $\{P^t\}$ and their distribution by age and type, $\mu^t_{ij}$ for all $i \in I$, $j \in J$, $t \in T$.
- Assignments of consumption and assets holdings $\{c^t_{ij}, a^t_{ij}\}$ for all cohorts alive in $t \in T$ and all types $j \in J$.
- Time series of the inputs employed by the competitive firms of the economy $(K^t, L^t)$, $t \in T$.
- A Public Policy consisting of the time series of taxes, public expenditure, minimum pensions (old-age and survivors), maximum old-age pensions and maximum contributions:
  \[
  \{\varphi^t, CP^t, bm^t, bmV^t, bM^t, C_M^t\} \quad t \in T
  \]
- A price system: $\{r^t, w^t\}$, $t \in T$ such that the following properties apply:

  1. Endogenous population dynamics
     Population aggregates and distributions are coherent with our demographic model, given the exogenous patterns specified for fertility, mortality, immigration flows and education distribution.
  
  2. Rational behaviour by the households.
     Household decisions are optimal (ie. solve problem (7)) given the price system and the public policy.
  
  3. Clearance of the markets for capital and labor.
     The capital and labor effectively employed by firms come from the aggregation of the household savings and labor supply:
     \[
     L^t = A^t H^t = \sum_{j=1}^{J} \sum_{i=1}^{I-1} P^t_{ij} e_{ij} \quad K^t = \sum_{j=1}^{J} \sum_{i=1}^{I-1} P^t_{ij} a_{ij} \quad t \in T
     \]
     Where $e_{ij}$ is the household labor supply, obtained by weighting both spouses contributions as in equation (8). Obviously, the number of households of age $i$ and type $j$ is $P^t_{ij} = \mu^t_{ij} P^t$.
  
     \[
     r + \delta = \frac{\partial F}{\partial K}(K^t, L^t) \quad w^t = \frac{\partial F}{\partial H}(K^t, L^t)
     \]
  
  5. Balanced Public budget.
     \[
     FI^t(\varphi^t) + PSB^t = CP^t
     \]
     where the public expenditure is a fixed proportion of aggregate output $CP^t = c_p Y^t$; the fiscal income, $FI^t$, and the income from bequest, $BI^t$, take the form:
\[ FI^t(\varphi^t) = \varphi^t P^t + BI^t \]
\[ BI^t = \sum_{j=1}^{J} \sum_{i=1}^{l-1} (1 - h_{s_{i,j}}^t) P_{i,j}^{t-1} a_{i+1,j} \]

where \( h_{s_{i,j}} \) stands for the household survival probability (i.e., the probability of survival of at least one of its members). The pension system balance is given by
\[ PSB^t = COT^t - PP^t \]

where \( PP^t \) and \( COT^t \) stand for the aggregate pension expenditures and the aggregate social contributions:
\[ PP^t = \sum_{j=1}^{J} \sum_{i=1}^{l} P_{ij}^t ib_{ij}(\tau_j) \]
\[ COT = \sum_{j=1}^{J} \sum_{i=1}^{l} P_{ij}^t cot_{ij}^t \quad t \in T \]

6. Aggregate feasibility
\[ Y^t + (1 - \delta) K^t + BI^t = K^{t+1} + BI^{t+1} + \sum_{j=1}^{J} \sum_{i=20}^{l} P_{ij}^t cot_{ij}^t + CP^t \]

B Life cycle profiles of earnings and employment rates

The life-cycle profiles of earnings and participation of our representative households are constructed combining estimation and projection techniques. The employment rate of the older cohorts conditional on sex, educational level and region of residence is estimated using data from EPA 1977-2002. Female labor participation has increased systematically for younger cohorts, while that of men has decreased slightly. These tendencies are projected to continue and progressively disappear along our non-stationary simulation path (for men we assume a small recovery associated with a delay in retirement ages). The resulting participation rates by gender are shown in figure 11. The associated average retirement age and pension eligibility rates are presented in figures 12 and 13. The individual life-cycle profiles of earnings for employees are estimated using data in the European Community Household Panel (ECHP) 1994-2004. The profiles are assumed to be invariable across cohorts. These profiles are combined with the life-cycle employment rates to generate the labor income profiles of our representative households.

C Internal Rates of return

Formally, we define the IRR of the pension system for a household of type \( j \) belonging to cohort \( u \) as the interest rate \( r_{j}^u \) that satisfies the following condition:
\[ \sum_{i=1}^{l} E_u \left[ \frac{cot_{u,i+1}^t - ib_{u,i}^t}{(1 + r_{j}^u)^t} \right] \]
\[ = \sum_{i=1}^{l} E_u \left[ \frac{ib_{u,i}^t}{(1 + r_{j}^u)^t} \right] \]
\[ \Leftrightarrow \sum_{i=1}^{l} S_{i,j}^u P_{i,j}^t cot_{ij}^t + \sum_{i=1}^{l} \frac{S_{i,j}^u}{(1 + r_{j}^u)^t} ib_{ij}^t \]

where \( cot_{u,i+1}^t \) stands for the social contributions paid by the household, \( ib_{u,i}^t \) stands for the final pension received by the household and \( S_{i,j}^u \) is the probability of survival to age \( i \). Figures 15 and 16 represents the IRR generated by the Spanish pension system after the latest legislative changes.
Figure 11: Time series of the aggregate employment rate by gender

Figure 12: Average retirement age by cohort
Figure 13: Eligibility rates by cohort, gender and education

Figure 14: Time series of the population distribution by educational attainment.
Figure 15: Internal Rate of Return by education, year of birth and region of residence. Low education (−), average education (−−) and high education (− −).)

Figure 16: Internal Rate of Return by education, year of birth and region of residence (continuation).