GENEARAL EQUILIBRIUM WITH UNEMPLOYMENT: THEORY AND APPLICATION

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ABSTRACT
A general equilibrium model with equilibrium unemployment rate is developed to evaluate the cost of unemployment in an economy by comparing responses to policy changes in an economy with equilibrium unemployment rate to the economy with full employment. Model can assess the cost and benefits of unemployment and transfer programmes. A lower rate of frictional unemployment is not necessarily growth retarding in the long run when economy goes through the dynamic adjustment process. From studying impacts uniform capital and labour income taxes across sectors in the multi-household multi-sectoral general equilibrium model of the UK with unemployment rate it can be stated that unemployment of some workers may raise labour supply of others as the existing workers supply more labour in response to higher wage rates. Labour income, consumption and saving by households and production and accumulation of capital rise. Inequality in income and consumption can be reduced by usual tax more productive worker and transfers to job trainings and other activities transitionally for unemployed ones.

Key words: Unemployment, general equilibrium, growth, redistribution, UK

JEL Classification: C68, D63, O15

October 2007
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I. INTRODUCTION

How far is it plausible to stick to the market clearing and full employment assumptions that are central to the applied general equilibrium modelling in tradition of Shoven and Whalley (1973) and Whalley (1975)? Can the economy wide welfare analysis and the Pareto optimality conditions be evaluated even if the general equilibrium structure is modified to incorporate the unemployment or under-utilisation of resources in an economy or is the quantification of the Walrasian general equilibrium system for a realistic analyses on the efficient allocation of resources and redistribution of income in market economies impossible without full employment assumption? How far is the ignoring of the persistence of unemployment of the labour force as commonly seen in a modern economy reasonable in applied equilibrium modelling? Is it possible to incorporate disequilibrium in labour market in these models? What are the impacts of such modifications in the allocations and efficiency of the price system? Can a sensible welfare analyses be applied after including unemployment in the model? What sort of policy alternatives can be designed to address the problems of millions of unemployed job seekers (Figure 1) and workers who are who are in transition to different status in the labour market? Can very elegant analyses of individual and social welfare shrewdly enshrined in an elegant general equilibrium system be applied even with these modifications? Should the process of creating vacancies or creation of new jobs and destruction of old jobs through redundancies of obsolete workers and hiring of new workers through the matching process between the skills of job seekers and requirement of employers (job
creators) (Figure 2) be taken normally from the perspective of enhancing the dynamic adjustment process for long run growth in a modern economy?

In one form or the other, economists for more than three decades, have tried to incorporate unemployment as a special feature of the equilibrium process in the modelling of an economy. Job matching and search models developed by Phelps (1968), Mortensen and Pissarides (1984), Shapiro and Stiglitz (1984), Dixon (1988), Lindbeck and Snower (1988), Lockwood, Miller and Zhang (1989), Lockwood and Manning (1989), Blanchard et. al (1989), Layard and Nickell (1990), Manning (1990) Blanchflower and Oswald (1994), Ball and Mankiw (2002), Hall (2003), Gilles (2004), Lonquivists and Sargent (2007) have significantly contributed to the analysis of bargaining and unemployment dynamics. Despite such extensive research, there still remains enough scope in studying the general equilibrium impacts of unemployment on growth, capital accumulation among various sectors of the economy and in utility, wages and labour supply levels of households and other macroeconomic impacts of such unemployment in the long run. Few studies in applied general equilibrium literature have recently (Hutton-Ruocco (1999), Rutherford et al (2000), Bohringer, Beoters and Feil (2005),) started such evaluation though full scale impact of such analysis at the household level is still an overdue.

Unemployment, Vacancies and Redundancies in UK

Before modelling equilibrium unemployment it is relevant to present some empirical facts based on time series data from the Office of the National Statistics on unemployment, vacancies and redundancies in the UK as following:
1. In the past the rate of unemployment had gone up from around 4 percent in early 1970s up to 12 percent in 1984 and remained at fairly higher level till mid 1990s, after which it has gradually declined to about 5 percent as a result of the New Deal measures adopted for the labour market reforms in the last decade.

2. The fall in the unemployment rate in this manner has been possible by a consistent rate of economic growth that has created more vacancies than redundancies. One finds three percent gap between the unemployment rate and vacancy rate in the monthly data since 2000. It in a way can be considered as natural rate of unemployment. The net vacancy number has remained about 400 thousand each year.

3. Most vacancies have mainly been in the distribution, finance, and education sectors. Redundancies have been higher than vacancies in the manufacturing, transportation, construction and other sectors.

4. The evidence supports a negatively sloped Beveridge curve for UK. Unemployment rate is lower when firms create more jobs and higher when firms are not able to create more vacancies. It is natural to have more vacancies than redundancies in a growing UK economy.
Monthly Unemployment Rate in the UK, 2007

\[ y = -0.0001x^2 + 0.0627x + 2.5695 \]

\[ R^2 = 0.6502 \]

Figure 1: Trends of Unemployment in the UK

Vacancies by Industry in UK

Figure 2: Vacancies by production sectors in UK
Figure 3: Trends of Unemployment and Vacancy in the UK

Evidence of Beveridge Curve in UK, 2001 to 2007
(Source: www.statistics.gov.uk)

Figure 4: Evidence for Beveridge Curve in UK
Objective of this paper is to integrate the basic results on equilibrium unemployment in the job search model (the Beveridge curve phenomena) into a standard applied dynamic general equilibrium model. It takes essential features of job search models contained in Mortensen and Pissarides (1994) and Pissarides (2000) and putting them into a dynamic equilibrium model extended from Bhattarai (2007). Thus the point of departure in this paper, apparently missing from the existing literature, is the consideration of the full impacts of the natural rate of unemployment on labour supply, consumption and saving behaviours of households, investment and capital accumulation behaviour of firms and relative prices of commodities and factors of production in the broader economy. This issue is investigated here by comparing results of the equilibrium unemployment general equilibrium model (EUGEM) to the results of the full employment general equilibrium model (FEGEM) benchmarking the micro-consistent data of multi sectoral multi household model of the UK economy that is calibrated to the reference paths of 80 years ahead from its initial year in 1995.
Preliminary results of this analysis are quite appealing. Higher rate of unemployment raises wage rates of employed workers who work more hours in response to higher wages. Thus higher unemployment can raise labour supply in aggregate when the employed workers work more hours in response to higher wages. As the aggregate output relates to the amount of active work hours, it does not necessarily result in lower output, as commonly feared. If workers are more productive, they may pay for training of the part of work force facing redundancies and let the economy adjust towards more efficient path. Work based trainings are as important as the employing unemployed workers. This result obviously is based on the assumption that economy continues to grow along its reference path - with exuberant consumers and firms.

Inclusion of multiple households in the model in the dynamic multisectoral model makes it possible to assess the redistribution issues particularly in relation to the skill and wage differences among household groups over the model horizon - a policy issues of immense interest on its own. Unemployment can be very costly if it is centred at certain groups of workers such as the low income households. Burden of unemployment can be reduced by designing optimal set of policies under its disposal. Unemployment above the natural rate however can harm economic growth and the lower level of life time utility for all categories of households. First section of the paper introduces the concepts of unemployment rate, wage rate and wage curve as commonly found in the equilibrium unemployment and job search literature. A brief description of general equilibrium structure of the economy used for this analysis is given in section II. Calibration and computation of model is presented in section III with discussion of results in section IV and conclusions and references in section V.
II. Model of Equilibrium Unemployment (Vacancies, Job Search and Matching)

Economy includes \( N \) number of firms and \( H \) number workers/households. Numerous profit maximising firms create vacancies for specific tasks and hire workers when they find suitable candidates for their jobs following the market signals of demand and relative prices of products and costs of inputs. Similarly there are many workers seeking jobs that match their skills or many others who quit jobs and join the pool of unemployed when those jobs are not suitable for them. Market specific positive or negative idiosyncratic shocks cause such entries and exits in the labour market. Unemployment and wage rates in equilibrium are result of the bargaining between workers and firms. Whether the rate of unemployment falls or rises depends on relative proportion of entry and exit into the labour market.

Matching and bargaining functions are the major elements of an equilibrium unemployment model. Matching function aggregates vacancies and unemployment with job creation as:

\[
M = M(V, U) = V^\gamma U^{1-\gamma}
\]  

(1)

where \( M \) denotes the number of matching of vacancies and job seekers, \( V \) is number of vacancies and \( U \) the number of unemployed, \( \gamma \) is the parameter between zero and one, which can be adjusted for prosperous period when there are more vacancies than job seekers or in recession when there are more unemployed than vacancies, in steady state it should be about 0.5 to reflect the balance between job creation and job destruction. Job seekers and employers bargain over expected earnings by maximising the Nash-product of the bargaining game over the difference between the earnings from work \( (W) \) rather than being unemployed \( (U) \) and earnings to firms from filled and vacant jobs.
\[(W_i - U)^\beta (J_i - V)^{1-\beta}\]  \hspace{1cm} (2)

Market imperfections in the labour market creates opportunity of gains from bargains which is divided between firms and workers as indicated by parameter $\beta$ that can assume any value between zero and one, reflecting the relative strength of unions (workers) over firms in such bargains. Symmetric solution of this satisfies joint profit maximisation condition as:

\[ (W_i - U) = \beta(J_i + W_i - V - U) \]  \hspace{1cm} (3)

A job search model is often explained using three simple equations (Pissarides (2000) or Shimer (2005); work out of the optimisation process using a value function as presented in the standard literature is presented in the appendix for reference to it):

1) Dynamics of unemployment depends on the rate of job destruction - $\lambda(1-u)$, and the rate of job creation - $\partial q(\theta)u$.

\[ \dot{u} = \lambda(1-u) - \partial q(\theta)u \] and in equilibrium \[ u = \frac{\lambda}{\lambda + \partial q(\theta)} \]  \hspace{1cm} (4)

where $\lambda$ is the rate of idiosyncratic shock of job destruction and $\theta$ is the ratio of vacancy to the unemployment and $q(\theta)$ is the probability of filling a job with a suitable candidate through the matching process explained in (4). As shown in diagram 1 an upward sloping wage curve shows that a tighter labour market results in higher wage rates. The downward sloping job creation curve shows the possibility of job creation at lower wage rates and creation of fewer jobs at higher wage rates.
2) Optimal job creation or (demand for labour curve) shows how firms balance the marginal revenue product of labour to wage and hiring and firing costs.

\[ p - w - (r + \lambda) \frac{pc}{q(\theta)} = 0 \]  \hspace{1cm} (5)

where \( p \) is the price of product, \( w \) the wage rate, and \( (r + \lambda) \frac{pc}{q(\theta)} \) is the cost of hiring and firing.

3) the wage curve shows positive links between the reservation wage (\( z \)) the price of product \( p \) and costing of hiring (\( \theta \)).
Derivation of these equations from value functions of employed and unemployed workers and from occupied and non-occupied vacancies along with the union-firm wage bargaining is presented in the appendix.

The major point of departure of this paper, from the equilibrium unemployment model presented above, remains in evaluating how much these features of equilibrium unemployment impact on the model economy. What are the likely impacts of vacancies, redundancies and unemployment in the growth of output, employment, relative prices and allocations of factors between private and public sectors? How do they impact on the welfare of various categories of households?

The frictional unemployment literature suggests that some degree of unemployment can make an economy more flexible and would allow the dynamic process of adjustment smoother while the rigid and structural unemployment may result in the lower level of output and living standards. How do they relate to fundamental properties of preferences of households between consumption and leisure as well as among various commodities? How much do they influence the extent of flexibility of markets and tax transfer systems? How much they affect the accumulation of capital, decisions regarding working and non-working and overall process of growth of output and productivity? These questions can be evaluated using a general equilibrium model in the next three sections.

III. General Equilibrium Model with Unemployment

Unemployment is underutilisation of labour force available for production and results in the loss of output to the economy and loss of utility to households. This is not necessarily true. It cannot be stated definitely without considering the productivity

\[
w_i = z(1 - \beta) + \beta p(1 + \theta c)
\]
of existing workers. This is the reason why it is necessary to integrate the equilibrium unemployment into the general equilibrium model for evaluating impacts of equilibrium unemployment in the economy.

**Household Preferences and Demand for Goods**

Households receive utility from consumption of goods and leisure. Their objective is to maximise lifetime utility against their lifetime budget constraints. They receive income supplying labour and capital services to firm which pay them according to the marginal productivity. Lower income households receive transfers from the government which collects revenue by taxing high income households.

$$\sum_{t=0}^{\infty} \beta^t U(C_t^h, L_t^h)$$  \hspace{1cm} (7)

subject to

$$\sum_{t=0}^{\infty} P_t (1 + t_e) C_t^h = \sum_{t=0}^{\infty} (1 - t_w) r(1 - t_k) K_t^h + R_t^h$$ for employed  \hspace{1cm} (8)

$$\sum_{t=0}^{\infty} P_t (1 + t_u) C_t^h = \sum_{t=0}^{\infty} (r(1 - t_k) K_t^h + R_t^h)$$ for unemployed  \hspace{1cm} (9)

A fraction of households are unemployed in each period as shown above in the equilibrium unemployment model. Unemployed households do not contribute in production but take non-labour income and transfer for their consumption. There is a transition from employment to unemployment and from unemployment to employment. This process is imposed exogenously in the model.

**Production Technology and Supply of Goods**

Firms in the economy have usual CES (Cobb-Douglas) production technology. They hire workers and capital stocks from households. The objective of a firm in the
$j$th sector of the economy is to maximise the present value of profits subject to production technology constraints. Sectoral profits are given by the differences between the revenue from sales and the cost of production/supply. The unit revenue function is a constant elasticity transformation (CET) composite of the unit price of domestic sales and the unit price of exports. The unit costs are divided between value-added, i.e. payments to labour and capital, and domestic and imported intermediate inputs as in Bhattarai (2007):

$$
\Pi_{jt} = \left[ ((1-\delta_j^e)PD_{jt}^{\sigma_j} + \delta_j^e PE_{jt}^{\sigma_j}) \right]^{-\frac{1}{\sigma_j}} - \theta_j^e PY_{jt}^{\sigma_j} - \theta_j^d \sum_i a_{ij}^d P_{ij} - \theta_j^m \sum_i a_{im}^m PM_{ij} \tag{10}
$$

where: $\Pi_{jt}$ is the unit profit of activity in sector $j$; $PE_{jt}$ is the export price of good $j$; $PD_{jt}$ is the domestic price of good $j$; $PY_{jt}$ is the price of value added per unit of output in activity $j$; $\sigma_j$ is a transformation elasticity parameter; $P_{jt}$ is the price of final goods used as intermediate goods; $\delta_j^e$ is the share parameter for exports in total production; $\theta_j^e$ is the share parameter for imports in total production; $\theta_j^d$ is the share of costs paid to labour and capital; $\theta_j^m$ is the cost share of imported intermediate inputs; $a_{ij}^d$ are input-output coefficients for domestic supply of intermediate goods; $a_{ij}^m$ are input-output coefficients for imported supply of intermediate goods. Their investment activities, which depend on sector specific profitability conditions result in accumulation of sector specific capital net of depreciation. Greater amount of capital enhances productivity of labour and raises the wage rate.

**Trade arrangements**

Economy is open. Exports and imports are guided by the ratio of domestic to foreign prices and balanced over years. Trade takes place between the EU and the ROW and given by the standard Armington functions.

$$
A_{jt} = \Phi \left( \delta_j^e D_{jt}^{\sigma_j} + \delta_j^m M_{jt}^{\sigma_j} \right) \Phi^{-1} \tag{11}
$$
where $A_{i,t}$ is the Armington CES aggregate of domestic supplies $D_{i,t}$ and import supplies $M_{i,t}$ for each sector, $\delta^d_i$ is the share of domestically produced goods, $\delta^m_i$ is the share of good $i$ imports, $\sigma_m$ is the elasticity of substitution in the aggregate supply function, and $\Phi$ is the shift parameter of the aggregate supply function.

The value of exports balances to the value of imports.

$$\sum_i P E_{i,t} E_{i,t} = \sum_i P M_{i,t} M_{i,t}$$

**Drivers of the Dynamic in the Economy**

Dynamics of the economy are driven by the accumulation of capital and fluctuations in labour supply because of fluctuations in the supply of labour. Capital stock evolves naturally with its initial and boundary conditions:

$$K_{i,t} = (1 - \delta) K_{i,t-1} + I_{i,t} ; K_{i,0} = K_{i,T} = (g_i + \delta) K_{i,T-1}$$

Similarly there labour supply equations for each household with some transition probability between employment and unemployment

$$L_{i,t}^e + L_{i,t}^u = \bar{L}_{i}^e ; \bar{L}_{i}^u = \bar{L}_0 e^{\alpha t}$$

Labour supply of people who are unemployed equals zero. In aggregate the link between employment, unemployment and the total labour force of the economy takes the following form: $E_t + U_t = \sum_{h=1}^{H} \bar{L}_i^e$, where $E_t$ denotes the number of total employed, $U_t$ number of unemployed out of total labour force $\sum_{h=1}^{H} \bar{L}_i^u$. Both the unemployment rate and labour supply converge to the steady state path in the long run.

**Public sector**

At every period government provides public services to households and pays for them using taxes. Tax transfer system influences choices of households and creates
distortions in the system. Government provides public services - education, health, security, law and order - to households. It is given by government consumption:

\[ G_t = \sum_{i=1}^{N} g_{i,t} \]  

(15)

It collects revenue from direct and indirect taxes as:

\[ RV_t = \sum_{h=1}^{H} t^h_w W_t L^h + \sum_{h=1}^{H} t^i_x r^h_{x} K^h_{i,t} + \sum_{h=1}^{H} t^h_{i,j} p^h_{i,j} C^h_{i,j} \]  

(16)

Revenue is balanced over the model horizon:

\[ \sum_{t=1}^{\infty} G_t = \sum_{t=1}^{\infty} \left( RV_t + \sum_{h=1}^{H} R^h_t \right) \]  

(17)

Optimal level of public sector balances benefits and costs from the public sector activities.

IV. SOLUTION PROCEDURE

Demand and supply functions for the applied model are derived for heterogeneous households and firms in the economy solved numerically using relative prices to link markets for goods and factors. All markets clear except the labour market which is subject to unemployment restrictions. Model is benchmarked to the reference path of the evolving economy with initial values aggregated from the 123 sector input output table of the UK from the ONS and solved using GAMS/MPSGE software. Impact of unemployment into the general equilibrium can be modelled by modifying the Walrasian demand for and supplies of labour. Unemployed part of the total labour force does not contribute in production but receives transfers and capital expenditure to pay for consumption. Results of the model with equilibrium unemployment rate are compared to the full employment equilibrium model in order
to assess the cost of unemployment and benefits of unemployment reduction programmes.

**General Equilibrium**

Relative prices of commodities and factors of production keep adjusting until the demand and supply balance. This model allows existence of unemployment in the steady state. It is possible to analyse creation of endogenous vacancies and redundancies within the model.

Model is computed for reference path of 80 years ahead. Theoretically a general equilibrium in an economic system like this is described by a system of $T.n(n-1)$ relative prices that clear all goods and factor markets. It is stated in terms of vectors of prices, demand and supply and excess demand functions for inputs and outputs. Given the vector of prices $p = (p_1, p_2, \ldots, p_j, \ldots, p_n)$, demand for commodities are expressed in terms of the price vector $X^d_j = X^d_j(p) = X^d_j(p_1, p_2, \ldots, p_j, \ldots, p_n)$ and supply functions defined similarly $X^s_j = X^s_j(p) = X^s_j(p_1, p_2, \ldots, p_j, \ldots, p_n)$ and the excess demand functions reflect the gap between demand and supply for each commodity $E_j(p) = X^d_j(p) - X^s_j(p)$ for $j = 1, 2, \ldots, n$. Economy has $n$ excess demand functions $E(p) = (E_1(p), E_2(p), \ldots, E_j(p), \ldots, E_n(p))$. The general equilibrium is a price vector, $p^*$, such that $p^* \geq 0$, $E(p^*) \leq 0$ if $E(p^*) < 0$, $p^* = 0$. The excess demand functions are single valued continuous functions, bounded from below $E(p) \geq b$ for all $p$ and it is homogenous of degree zero in all prices $E(\alpha p) = E(p)$ for all $\alpha$; only relative prices that satisfy the Walras’ law matter; $p.E(p) = \sum_{i=1}^{n} p_i E_i(p) = 0$ for all
$p \geq 0$. If the excess demand functions satisfy above properties then, the existence of the general equilibrium is guaranteed by fixed point theorems. The fixed equilibrium point is found by continuous transformation of the nonempty convex set onto itself $p^* \rightarrow E(p^*) \rightarrow p^*$. Given the properties of demand and supply functions equilibrium is stable and unique.

V. ANALYSIS OF RESULTS

A multisectoral multi-household dynamic model of the economy looking 80 years forward generates massive information regarding equilibrium prices and quantities of the economy. Equilibrium unemployment reduces amount of labour supplied to the production process resulting in lower output, income and demand. Higher unemployment reduces growth, investment and capital accumulation. In a growing economy one would expect utility to be rising over time, $U^h_T < U^h_{T-1} < \ldots < U^h_1$. This is exactly what happens in the current model as shown by the levels of utilities of households over the model horizon in Table 1.

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Utilities of households do not fall in underemployment equilibrium compared to full employment equilibrium. Theoretically the labour supply patterns is unpredictable because of the rise in income as it involves both income and substitution effects. Model results show substitution effect to dominate the income effects as given in
sensitive to the inter-temporal elasticity of substitution.

Consumption inequality does not decline unless specific policies are designed to

\[\text{Table 2: Ratios of labour supply under unemployment and full employment}\]

<table>
<thead>
<tr>
<th>Year</th>
<th>H1</th>
<th>H2</th>
<th>H3</th>
<th>H4</th>
<th>H5</th>
<th>H6</th>
<th>H7</th>
<th>H8</th>
<th>H9</th>
<th>H10</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.0301</td>
<td>0.9989</td>
<td>0.9968</td>
<td>0.1144</td>
<td>0.2336</td>
<td>0.9968</td>
<td>0.9968</td>
<td>0.9968</td>
<td>0.9968</td>
<td>0.9968</td>
</tr>
</tbody>
</table>

\[\text{Table 3: Ratios of personal consumption under unemployment and full employment}\]

<table>
<thead>
<tr>
<th>Year</th>
<th>H1</th>
<th>H2</th>
<th>H3</th>
<th>H4</th>
<th>H5</th>
<th>H6</th>
<th>H7</th>
<th>H8</th>
<th>H9</th>
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<td>0.9968</td>
<td>0.9968</td>
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</tr>
</tbody>
</table>

Impact of labour supply is felt in the level of consumption of the households.

Household consumptions rise over the model horizon.

\[\text{Table 4: Index of consumption inequality under unemployment and full employment}\]

<table>
<thead>
<tr>
<th>Year</th>
<th>H1</th>
<th>H2</th>
<th>H3</th>
<th>H4</th>
<th>H5</th>
<th>H6</th>
<th>H7</th>
<th>H8</th>
<th>H9</th>
<th>H10</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.9989</td>
<td>0.9968</td>
<td>0.1144</td>
<td>0.2336</td>
<td>0.9968</td>
<td>0.9968</td>
<td>0.9968</td>
<td>0.9968</td>
<td>0.9968</td>
</tr>
</tbody>
</table>

The pattern of consumption inequality persists over the model horizon in absence of specific programmes that redistribute incomes from low income to higher income households over the year.

Consumption inequality does not decline unless specific policies are designed to transfer resources between household categories.

Investment and capital stock also rise across sector.
Above results are representative samples of larger general equilibrium models and number may vary according to the underlying intra-temporal and inter-temporal elasticities of substitution. Despite this the results seem to suggest that a lower rate of equilibrium unemployment does not necessarily reduce the growth prospects of the economy.

VI. CONCLUSION

A general equilibrium model is developed to evaluate the cost of unemployment in an economy. Results of model with equilibrium unemployment are compared to the full employment equilibrium model in order to assess the cost of unemployment and...
benefits of unemployment reduction programmes. Analysis of impacts of tax reforms in the multi-household multi-sectoral general equilibrium model with unemployment shows that such unemployment is not necessarily growth retarding in the long run when economy runs through the dynamic adjustment process. Lower labour supply raises wage rates, labour supply, consumption and saving by households. It raises investment, capital accumulation and production by firms. In the base as usual scenario the inequality of income and consumption persists over periods unless policy measures are taken to reduce such inequality.
VII. References


Appendix  
Derivations of the search model

The starting point of the equilibrium unemployment model, as presented in Pissarides (2000), is parameter $\theta$ which is the ratio of vacancy to job seeking workers $\theta = \frac{V}{U}$.

The probability filling a vacancy is given then by $f(\theta)$ and not filling it by $1 - f(\theta)$.

In each period the probability of finding a job by an unemployed worker is $\theta f(\theta)\delta t$ and the not finding is $1 - \theta f(\theta)\delta t$; job creation occurs when matching takes place between firms with vacancies and workers seeking the job. With labour force $L$ and the unemployment rate $u$, the number of workers who enter unemployment is $\lambda (1 - u)L \delta t$. There is a balance between job creation, $mL \delta t = u \partial q(\theta) L \delta t$ and job destruction, $\lambda (1 - u)L \delta t$ in the steady state. The term $\partial q(\theta)$ measures the transition probability from unemployed to employed. Normalising $L$ to 1 the dynamics of unemployment is explained by transition dynamics between the job destruction and job creation $\dot{u} = \lambda (1 - u) - \partial q(\theta) u$ and in equilibrium, $u = \frac{\lambda}{\lambda + \partial q(\theta)}$. Thus the equilibrium unemployment rate is determined by the parameters of employment shocks and the probability of job finding ratio.

Pissarides (2000) brings returns on vacancy and occupancy of jobs, expected income of being unemployed and employed in determining the demand and supply functions of labour instead of marginal productivity theory of labour and utility maximising behaviour of labour supply. Return from vacancy is measured by $rV = -pc + q(\theta)(J - V)$ where $V$ denotes the value of vacancy and $J$ the expected value for occupied jobs, $pc$ the cost of vacancy. In equilibrium $V = 0$ and thus $J = \frac{pc}{q(\theta)}$. 
Similarly the returns from an occupied job is given by \( rJ = p - w - \lambda J \), where a job generates revenue \( p \) against the cost of wage rate \( w \) and loss due to the stochastic job termination \( \lambda J \), \( \lambda \) being the ratio of idiosyncratic shocks. Thus the optimal condition for employment is given by equality between price of the product, wage rate and the hiring cost of the job, \( p - w - (r + \lambda) \frac{pc}{q(\theta)} = 0 \).

Price of a product should cover wage payment and the expected hiring costs. Firms take price and interest rate as given in the market, parameters \( \lambda \) and \( \theta \) are set exogenously. Value of unemployment and wage rate \( rU = z + \partial q(\theta)(W - U) \) or \( rU + \partial q(\theta)U = z + \partial q(\theta)W \). Return for employed worker is \( rW = w + \lambda(U - W) \) or \( (r + \lambda)W = w + \lambda U \) or \( W = \frac{w}{(r + \lambda)} + \frac{\lambda}{(r + \lambda)} U \). Putting this in unemployment equation

\[
 rU + \partial q(\theta)U = z + \partial q(\theta) \left[ \frac{w}{(r + \lambda)} + \frac{\lambda}{(r + \lambda)} U \right] = z + \partial q(\theta) \left[ \frac{w}{(r + \lambda)} + \partial q(\theta) \frac{\lambda}{(r + \lambda)} U \right]
\]

\[
 rU - \partial q(\theta) \frac{\lambda}{(r + \lambda)} U + \partial q(\theta)U = z + \partial q(\theta) \frac{w}{(r + \lambda)}
\]

\[
rU(r + \lambda) - \partial q(\theta)\lambda U + \partial q(\theta)U(r + \lambda) = z(r + \lambda) + \partial q(\theta)w
\]

\[
rU[(r + \lambda) + \partial q(\theta)] = z(r + \lambda) + \partial q(\theta)w
\]

\[
rU = \frac{z(r + \lambda) + \partial q(\theta)w}{r + \lambda + \partial q(\theta)}
\]

Similarly \( (r + \lambda)W = w + \lambda U \) or \( (r + \lambda)W = w + \frac{\lambda}{r} \left[ \frac{z(r + \lambda) + \partial q(\theta)w}{r + \lambda + \partial q(\theta)} \right] \)

\[
 (r + \lambda)W = \frac{wr(r + \lambda + \partial q(\theta)) + \lambda(z(r + \lambda) + \partial q(\theta)w)}{r(r + \lambda + \partial q(\theta))} \quad \text{or} \quad rW = \frac{wr + \partial q(\theta)w + \lambda z}{r + \lambda + \partial q(\theta)}
\]

Wage bargaining between firms and workers
\[ rJ_i = p - w_i - \lambda J_i \quad \text{or} \quad J_i = \frac{p - w_i}{r + \lambda} \]
\[ rW_i = w_i - \lambda(W_i - U) \quad \text{or} \quad W_i = \frac{w_i}{r + \lambda} + \frac{w_i - U}{r + \lambda} \]

Nash-product of the bargaining game \((W_i - U)^\beta (J_i - U)^{1-\beta}\)

Symmetric solution of this satisfies value maximisation jointly by firms and workers

\[
(W_i - U) = \beta(J_i + W_i - V - U) \quad \text{with} \quad V = 0 \quad W_i(1 - \beta) = \beta J_i + (1 - \beta)U
\]

\[
\left[ \frac{w_i}{r + \lambda} + \frac{\lambda}{r + \lambda} U \right] (1 - \beta) = \beta \left[ \frac{p - w_i}{r + \lambda} \right] + (1 - \beta)U
\]

\[ w_i(1 - \beta) + (1 - \beta) \Delta U = \beta(p - w_i) + (1 - \beta)(r + \lambda)U \]

\[ w_i(1 - \beta) = \beta(p - w_i) + (1 - \beta)rU \]

\[ w_i = \beta p + (1 - \beta)rU \quad \text{or} \quad w_i = \beta p + (1 - \beta)(z + \theta q(\theta)(W_i - U)) \]

\[
\text{From} \quad (W_i - U) = \beta(J_i + W_i - V - U); \quad (W_i - U) = \frac{\beta}{1 - \beta} J_i; \quad (W_i - U) = \frac{\beta}{1 - \beta} \frac{pc}{q(\theta)} \]

Therefore

\[ w_i = \beta p + (1 - \beta) \left( z + \theta q(\theta) \frac{\beta}{1 - \beta} \frac{pc}{q(\theta)} \right) \quad \text{or} \quad w_i = \beta p + z(1 - \beta) + \beta \theta pc \]

\[ w_i = z(1 - \beta) + \beta p (1 + \theta c) \]

Thus wage rate includes reservation wage \((z)\) and average hiring costs \(\theta pc\). Putting the wage curve in job creation curve

\[ p - w - (r + \lambda) \frac{pc}{q(\theta)} = 0 \quad \text{or} \quad p - z(1 - \beta) - \beta p (1 + \theta c) - (r + \lambda) \frac{pc}{q(\theta)} = 0 \]

\[ (1 - \beta)(p - z) - \frac{r + \lambda + \beta \theta q(\theta)}{q(\theta)} pc = 0 \]

This analysis is based on constant labour supply assumption though could be extended to a growing economy. Adding sectoral and structural features of the economy makes equilibrium unemployment theory even closer to the real economy as presented in this paper.