Government debt and macroeconomic effects: 
Analysis with real-financial CGE model

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Abstract
The aim of this paper is to show how choosing modalities to finance public deficit, such as, financing by bonds or monetary financing, affects the real economy and income distribution. General equilibrium analysis is most appropriate to simulate the medium and long term effects of different funding policies of government debt and to identify the different transmission channels on the real economy. 

This work lies indeed among recent attempts of modelling financial aspects in a computable general equilibrium model (CGEM). "Standard" walrasien model was extended by taking into account monetary and financial dimension.

So, our contribution consists in proposing a recursive dynamic general equilibrium model, whose real part is Walrasian type. The real part of the model allows generating savings of each agent. Allocation of savings among different assets determines the money market equilibrium.

Thus, the closure rule between savings and investment is not neoclassical since investment does not automatically adjust to savings. Moreover, investment of firms and of the government will be financed by domestic and foreign borrowing. For this reason, allocation of household savings and borrowings of firms and government will be treated in the financial part of the model.

Keywords: computable general equilibrium model, government debt, financial assets, macroeconomic policies.

JEL Classification : D58, C68, E62

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1. Introduction

Theories about the economic role of the state have always been subject to economic and political controversies. The need of state intervention in the regulation of economic activity has been justified by the fact that market economy can not spontaneously solve some economic and social problems.

This idea argued for the first time by Keynes (1936), following the overproduction 1929’s crisis, gives the state a stabilizing role during the recession period. This goal is achieved by expansionary policies which affect the effective demand and subsequently determine the level of output and employment.

Thus, Keynes suggests a short-term increase of public expenditures to stimulate economic growth and to contribute to of unemployment’s reduction, through the multiplier effect.

If the state finances additional expenditures by higher taxes, the stimulating effect on demand can be cancelled; that’s why Keynes proposes a counter-cyclical variation of budget deficit. According to him, the budget deficit is chronic or temporary (short-term analysis).

The crisis of the 1970s, characterized by the growing indebtedness of the government as well as by high unemployment and inflation (stagflation), resists to Keynesian stimulus policies which are unable to pull the economy out of durable crisis.

This situation favours a return to liberal principles, mainly with the permanent income theory of M. Friedman who demonstrated that interventionist policies are ineffective and destabilising. Furthermore, financing budget deficit by borrowing leads to an eviction effect on the private investment.

Starting from the same liberal logic, the Ricardien equivalence theory\(^3\) asserts that the budget deficit is neutral without any multiplier effect on the demand, and the effect of the budget deficit increase is independent from the way it is financed.

According to this approach, financing deficit by loans brings rational individuals to anticipate the future taxes increase, in order to save more. Also, if the financing is monetary, the individuals will rationally anticipate the erosion of their cash money.

During the 1980s, the International Monetary Fund (IMF) has established sets of policies to reduce macroeconomic imbalances through the public sector reduction in favour of private sector.

In this context, developing countries, such as Tunisia, have adopted structural adjustment programmes so as to improve the competitiveness of the economy in order to its progressive integration within the global economy.

As a consequence, the government budget has not ceased to be modified following the progressive abolition of tariffs and hence the decrease of an important part of the fiscal revenues.

The paper is organised as follows. Section 2 below discusses the issue, methodology and gives an overview of theoretical literature. Section 3 presents the model characteristics. Section 4 discusses macroeconomic closure rule of the model and, finally, some conclusions in the last section.

2. Research problem and Methodology

General equilibrium analysis is the most appropriate to simulate medium and long term effects of different funding policies of government debt and to identify the different transmission channels on the real economy.

To develop our model, we relied on the theoretical literature of financial computable general equilibrium models (FCGEM). There are two approaches to modelling. The first is neoclassical, developed by researchers at the World Bank. It is based on a combination of microeconomic modelling, with a Walrasian foundation, and a macroeconomic modelling based on the IS-LM model for endogenous determination of asset prices. In line with this model, there are models developed by Bourguignon, Branson and de Melo (1989), Fargeix and Sadoulet (1990) to analyze the structural adjustment effects on income distribution, and the work of Lewis (1992) to study the impact of financial liberalization.

The second is the structuralist approach, inspired by the work of Rosensweig and Taylor (1984-1990), which considers the structural characteristics of economies and their specificities. Thus, the CGE structuralist type views that markets do not work perfectly and assumes the possibility of disequilibrium in one or more markets. Yeldman (1997) introduces in his model the portfolio choice of agents and shows that savings generated from the real

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4 Bourguignon, F., Bransonb, W. et De Melo, J.(1989): "Macroeconomic adjustment and income distribution: a macro- micro simulation model". For these models the emphasis is on money: the intersection between money supply and money demand determine the general level of prices.
side is the deposits of banks that will finance investment, which connects the real and financial economies.

From their side, Nasstepad (2002), Easterly (1990) and Vos (1998) focus on the impact of budget deficit on the availability of credits, output and prices. They introduce credit constraints considering that the rate of interest is administered by the State, reflecting a situation of financial repression. In these models, the link between real and financial sub-models lies in demand and supply of credits that depend on production and investment policies. If the supply of credit is sufficient, investment is exogenous. Otherwise, the investment is endogenous and the excess of demand is absorbed by credit rationing.

These theoretical lessons will help us build a financial computable general equilibrium model for Tunisia which should answer questions about the effect of alternative budget deficit financing policies.

3. Specification of the theoretical model

3.1 The real side of the model

It is a computing general equilibrium model with financial assets whose real part is based on the walrasien spirit and financial part inspired from Rosensweig and Taylor’s (1990) model. Specificities of the behaviour of both financial institutions and economic agents form the main difference from the latter. Additionally, our model is dynamic recursive without any anticipation mechanism, which means that it is solved for static equilibrium sequence of one period, linked by dynamic equations of capital and assets.

The model includes six agents: households (including individual enterprises), private firms, government, commercial banks, Central Bank and the rest of the world.

3.1. Production and Employment

The economy consists of three sectors of production indexed by i: agriculture, industry and public services. For each sector i, production technology is nested, characterized by an input-output production function, à la Leontief, and a CES function for production factors: capital (K) and labor (L), which satisfies the standard conditions of constant returns to scale neoclassical production functions with.

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5 In these models the credit channel links the real and financial side of the economy.
The total production per sector i is the ratio of the added value and the respective fixed technical coefficient $a_i$.

$$X_{i,t} = \frac{V_{A_{i,t}}}{a_i}$$

With $0 \leq a_i \leq 1$

Intermediate consumption by production sector i:

$$CI_{j,t} = a_{j,i}X_{i,t}$$

Total intermediate consumption demand per sector i:

$$CI_{i,s} = \sum_j CI_{j,s}$$

Producers are assumed to maximize profits given their technology and the prices of inputs and outputs. In the short-term the capital is supposed to be fixed and production depends only on labor.

Labor market is competitive; labour supply is assumed to be exogenous and the demand for labor is given by the first order conditions of profit maximization, where the wage rate equals the marginal productivity of labor.

$$L_{i,t}^D = \left( \frac{r}{w} \times \frac{\alpha}{1 - \alpha} \right)^\sigma K_{i,t}$$

$\sigma = \frac{1}{\lambda \rho}$ is the elasticity of substitution between capital and labor; $r$ and $w$ are respectively the capital and labor prices.

The gross operating surplus (returns to capital by producing sector i):

$$RK_{i,t} = P_{i,t}^{VA}V_{A_{i,t}} - wL_{i,t}^D$$
3.1.2 Foreign trade

On the demand side, imports and domestic outputs are treated as imperfect substitutes, by the specification of Armington (1969), using CES functions.

\[ Q_{n,t} = A_m \left[ \beta M_{n,t}^{-\sigma_m} + (1 - \beta)D_{n,t}^{-\sigma_m} \right]^{-\frac{1}{\sigma_m}} \]  \hspace{1cm} \text{2t equations}

With \( Q \) being the composite good, \( M \) the imported and \( D \) the produced locally, \( \delta \) the share of imports in demand of good \( i \) and \( \rho_m = \frac{1}{1+\sigma_m} \) the elasticity of substitution (or elasticity aggregate) of the CES function.

Optimal mixes between imports and domestic outputs are achieved through expenditure minimization, given prices of imports and of domestic sales and subject to the CES function.

The first order solution determines the import function of relative prices of foreign goods and goods produced in the domestic market:

\[ M_{n,t} = \left[ \left( \frac{P_{n,t}^D}{P_{n,t}^M} \right)^{\mu_m} \left( \frac{\beta}{1 - \beta} \right)^{\mu_m} \right]^{\frac{1}{\mu_m}} D_{n,t} \]  \hspace{1cm} \text{2t equations}

The price of the composite is a weighted average of domestic prices and prices of imported goods:

\[ P_{n,t}^c = \frac{P_n^M M_{n,t} + P_n^D D_{n,t}}{Q_{n,t}} \]  \hspace{1cm} \text{2t equations}

with

\[ P_n^M = P_t^{WM} e (1 + t_{m,t})(1 + tt_t) \]  \hspace{1cm} \text{2t equations}

The domestic currency price of imported goods is obtained by adjusting the world price by the exchange rate \( e \), the import tariff rate, \( t_{m,t} \) and indirect taxation \( tt_t \).
On the offer side, tradable goods are allocated either to exports or to domestic sales, and imperfect transformability between these two, are reflected through Constant Elasticity of Transformation (CET) functions that assume the following form:

$$X_{n,t}^s = A_e \left[ \delta \frac{EX_{n,t}^{s}}{\sigma_e} + (1 - \delta)XD_{n,t}^{s} \right]^{-\frac{1}{\sigma_e}}$$  \hspace{1cm} \text{2t equations}

Domestic production is totally divided with imperfect substitution among products sold in the domestic market and products intended to the foreign market. Thus, it is assumed that the producers do not specialize in only one market. Optimal mixes between exports and domestic sales are achieved through the profit maximization of firms which behave as perfect competitors in the goods markets, taking prices of exports and of domestic sales as given and subject to the CET functions.

The export volume is given by the first order condition:

$$EX_{n,t} = \left[ \frac{P_i^D}{P_i^E} \times \left( \frac{\delta}{1-\delta} \right) \right]^{\mu_e} \times XD_{n,t}$$  \hspace{1cm} \text{1t equation}

With $\mu_e = \frac{1}{1+\sigma_e}$ the CET elasticity of transformation.

We assume that the country is price-taker on the international market, thus the world prices of imported and exported goods are exogenous. The domestic currency prices of these goods are obtained by adjusting the world prices with the exchange rate and export tariffs:

$$p_{n,t}^E = \frac{e_t p_{n,t}^{WE}}{(1 + t e_{n,t})}$$  \hspace{1cm} \text{1t equation}

Trade balance is written as:

$$BCOM_t = \sum_n p_{n,t}^{WE} EX_{n,t} - p_{n,t}^{WM} M_{n,t}$$  \hspace{1cm} \text{1t equation}

Therefore, the current account balance is:

$$BOC_t = BCOM_t + T_{RG,t} + T_{RM,t} - i_{EE,t} CE_{E,t} - i_{EG,t} CE_{G,t}$$  \hspace{1cm} \text{1t equation}

3.1.3 Income and savings
Household receives income from labour, capital, transfers and its portfolio $YPm_t$.

$$ Ym_t = w_t \times L_t^D + \text{div} \sum_{N} RK_t + T_{gm,t} + \epsilon \times T_{rm,t} - T_{mr,t} + i_D D_{M,t} + i_B BT_{M,t} $$ \hspace{1cm} 1t equation

The model assumes that the household is a net lender, and that his decision to savings is independent of its consumption decision. This representative household first determines the share of disposable income (YDM) to be allocated to savings. Then he assigns the remaining income to consumption:

$$ YDm_t = (1 - t_{ym,t})Ym_t $$ \hspace{1cm} 1t equation

$$ Sm_t = pms_t \times YDm_t $$ \hspace{1cm} 1t equation

$$ Cm_t = YDm_t - Sm_t $$ \hspace{1cm} 1t equation

The marginal propensity to save (pms) is endogenous, related to average yield of household portfolio.

Private firms’ income consists of capital income and capital transfers. What remains from their income after paying taxes and interest payments on loans for investment purposes, constitutes firms’ saving.

$$ Ye_t = RK_t + T_{GE,t} $$ \hspace{1cm} 1t equation

$$ Se_t = Yt_t - (t_{RK,t} + \text{div}_t) \sum_{N} RK_t - i_{C,t} CD_{E,t} - T_{ER,t} - e_{i_{EE,t}} CE_{E,t} $$ \hspace{1cm} 1t equation

### 3.1.4 Public Budget

The review of financing modalities of the public deficit requires beforehand a description of Tunisian public finance.

The state revenues consist in direct taxes on labor income, corporate profits taxes and indirect taxes on consumption; another part of its income consists of non-tax revenue mainly from privatization (transfers from private companies) and foreign grants (transfers incoming).
\[ YG_t = t_{m,t} \times Ym_t + t_{rk,t} \times \sum_{N} RK_t + \sum_{I} TAXX_t + \]
\[ \sum_{N} TAXM_t + \sum_{N} TAXE_t + e_t \times T_{rg,t} + T_{eg,t} \]

We assume a balanced public budget in the basic situation. Therefore, the State finances these expenditures \( G_t \) (these are all purchases of durable goods and unsustainable nature of current record without public investment) and transfers through the tax revenue.

Thus, public investment is a decision variable in the model which will be financed by the use of debt; this means that the budgetary savings at time \( t \) must take into account the interest payments of the debt:

\[ S_{G,t} = Y_{G,t} - G_t - T_{pc,t} - T_{gm,t} - e_t \times i_{EE,t} CE_{G,t} - i_{B,t} BT_t \]

However, the closure rule\(^6\) between savings and investment is not neoclassical because the investment does not automatically adjust to savings. Indeed, the investment of both firms and the State will be financed by domestic and foreign borrowing.

For this reason, the allocation of household savings and firms and the State borrowings will be treated in the financial model.

\subsection*{3.2. The model dynamic equations}

Dynamics are based on the equations of transitions which link the present period to the future one.

\textbf{3.2.1 Investment}

The investment function adopted is proposed by Bourguignon, Branson and de Melo (1989)\(^7\), where the investment demand of the firms depends on the performance of capital return and its use cost within a quadratic form

\[ \frac{I_{d,t}}{K_{E(i,t)}} = \gamma_{11} \left( \frac{R_{t}}{U_{t}} \right)^2 + \gamma_{12} \left( \frac{R_{t}}{U_{t}} \right) \]

\(^6\) The closure rule depends on the choice of endogenous and exogenous variables of the model.

\(^7\) The investment demand function is based on the theoretical basis of Nickell (1978), where investment demand derived from the microeconomic behavior of the firm.
With $R_{it}$ the value of the marginal productivity of capital and $U_t$ the use cost of capital. Under this form, investment is profitable if the return on capital exceeds its use cost.

The total investment of the private sector is obtained by multiplying the price of capital goods - which is determined as the weighted average of prices of various goods used for investment in each sector - by the demand for private investment:

$$I_{E,t} = P K_{E,t} \sum_n I_{dn,t}$$  

The public sector investment is exogenous.

$$I_{G,t} = P K_{G,t} \sum_n I_{G,t}$$  

As it’s said, prices of public and private capital are determined successively as the weighted average price of various assets used to invest in private and public sectors.

Thus, the dynamics of private and public capital accumulation is successively:

$$K E_{n(t+1)} = (1 - \gamma) K E_{n,t} - I E_{n,t}$$  

$$K G_{(t+1)} = (1 - \gamma) K G_t - I G_t$$

For the model dynamics, we also assume that population growth rate is exogenous.

$$L S_{(t+1)} = (1 + \eta) L S_t$$

3.3. The financial part of the model

3.3.1 Household portfolio allocation

Household behavior to determine its demand functions of assets is based on the approach proposed by Tobin (1969). The household’s gross financial wealth consists of: liquid currency $\Delta LL$, bank deposits $\Delta D$ and treasury bonds $\Delta B$.

$$RF_{m(t)} = \Delta LL_{m(t)} + \Delta D_{m(t)} + \Delta BT_{m(t)}$$
Initially, household requires liquid currency\(^8\) for reasons of transaction with a fixed proportion of their nominal consumption.

\[
\Delta LL_{m,t}^D = v_t Cm_{i,t} 
\]

Remaining households’ gross financial wealth (financial wealth minus currency required for transaction purposes \(\Delta LL_{m,t}^r\)) is the portion of household gross financial wealth available for allocation among the remaining two assets (\(RFN_t\)), to which can apply a CES utility function type.

\[
RFN_t = \Delta BT_{m,t} + \Delta D_{m,t} 
\]

To decide optimal flow of assets, household maximizes utility function returns on these assets subject to the wealth constraint.

\[
\max_{\Delta B, \Delta D} U = [A_1^m (i_B \Delta B)^\alpha + A_2^m (i_D \Delta D)^\alpha]^{\frac{1}{\alpha}} 
\]

Subject to the wealth

\[
RFN_{m,t} = \Delta B_{m,t} + \Delta D_{m,t} 
\]

The first order conditions yield the demand of assets functions of the average return of portfolio, called harmonic average return.

\[
rhm_{m,t} = A_1^m i_{1, B, t}^{(\delta_m - 1)} + A_2^m i_{1, D, t}^{(\delta_m - 1)} 
\]

### 3.3.2 Firms portfolio allocation

Firms finance their borrowing through the domestic and foreign loans.

\[
BEF_{E,t} = P_{E, t}^k \times I_{E, t} - S_{E, t} 
\]

\[
\Delta CE_{E,t} + \Delta CD_{E,t} = P_{E, t}^k \times I_{E, t} - S_{E, t} 
\]

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\(8\) The demand for money can depend negatively on the inflation rate and positively on income as in the model of Bourguignon, Branson and de Melo (1989).
3.3.3 Government portfolio allocation

The budget deficit of the government can be funded by monetary creation, by loans from the banking system or by foreign loans.

In our model we consider only the last financing modalities, as in Tunisia the public financing deficit is less and less done by the use of monetary creation due to its inflationary implications.

Consequently, we will assume that there is no monetization of public deficit and that financing deficit is achieved by external loans in the form of foreign assets and domestic loans expressed by emission of Treasury bonds.

\[ p_{G,t}^k I_{G,t} = S_{G,t} + \Delta BT_t + e\Delta CE_{G,t} \]

1t equation

3.3.4 Banks portfolio allocation

Banking system is composed of Central Bank and commercial banks.

In addition, because of the instability of the institutions financial behaviors, their portfolio allocation is presented only under an accountant identity.

**Commercial banks account**

Commercial banks collect deposits from households paid at credit interest rate \((i_d)\). Reserves requirement of the commercial banks constitutes a regulatory constraint for deposit banks, which binds them to have credit notes in central monetary proportional to the amount of the collected deposits.

\[ \Delta RES_t = \theta_{R,t} \Delta D_t \]

1t equation

After deducting these reserves, the remaining deposits and the refinancing of banks from the Central Bank will be used to finance part of the firms’ credit.

\[ \Delta CD_{E,t}^S = \Delta D_{M,t} + \Delta REF_t - \Delta RES_t \]

1t equation

In return, banks pay interest on their refinancing and on deposits collected from households.
\[ i_c CD_{E,t} = iD_{M,t} + i_{REF,t} REF_t \]

**Central Bank account**

The role of the Central Bank can be summarized mainly in the creation of the monetary base, the control of bank liquidity and the management of foreign reserves.

The only control mechanism is based on direct monetary setting reserve requirements, which can cause side effects of crowding-out private investment for the benefit of public investment. The monetary base and the reserves of the Central Bank are used to refinance commercial banks and to cover the foreign exchange reserves.

\[ \Delta REF_t = \Delta RES_t + \Delta LL_t + e \Delta REX_t \]

**1t equation**

**4. Macroeconomic closure of the model**

Specification of the closure rules of the model should reflect the structure of the economy and depends on the objective of the study. Therefore, we consider the three macroeconomic closure rules corresponding respectively to the current account, public finance and the savings-investment balance. These closure rules are as follows:

- The current account is balanced by the change in exchange rate, foreign savings being fixed.
- The State budget is balanced by the public savings (deficit) and public debt, assumed endogenous in the model so that public expenditure and tax rates are the decision variables. This is justified by the fact that the State has not only a redistributive role but also contributes to the capital accumulation through investments in non-market goods and services.
- The balance between savings and investment is not the neoclassical one, investment decision being independent of saving decision, and the equilibrium is realized by the flows of loanable funds.
5. Conclusion remarks

In this paper, we presented a model dealing with the issue of public debt and its economic consequences. Financing is essentially undertaken through the emission of treasury bonds on national and foreign market. Therefore, we excluded financing public debt by money creation. The specificity of the model consists, on one hand, in modelling portfolio choice of the economic agents, and on the other hand in considering credit market and links between real and financial spheres of the economy. Consequently, this model will allow simulating the economic effect of the State debt through the public expenditure, taxes and public investment.
References


Appendix 1

List of equations

I- Equations for the Real part

sets $i \in I= \{N, S\}$ All sectors
n $\in N=\{\text{agr,ind}\}$ market sectors
s=S : non-market sector (public sector services)

*Production and Employment

$VA_{i,t} = A_i \left[ \alpha L_{i,t}^{-\rho} + (1 - \alpha)K_{i,t}^{-\rho} \right]^{\frac{1}{\rho}}$

$X_{i,t} = \frac{VA_{i,t}}{a_i}$

$CI_{j,t} = a_{j,t} X_{i,t}$

$L_{i,t}^D = \left( \frac{r}{w} \times \frac{\alpha}{1 - \alpha} \right)^\sigma K_{i,t}$

$RK_{i,t} = P_{i,t} VA_{i,t} - wL_{i,t}^D$

*Income and savings

$YM_t = W_t \times L_t^D + \text{div} \sum_{i} RK_{i} + T_{gm,t} + e \times T_{rm,t} - T_{mr,t} + i_D D_{M,t} + i_B BT_{M,t}$

$YDm_t = (1 - t_{ym,t})YM_t$

$Sm_t = pms_t \times YDm_t$

$Cm_t = YDm_t - Sm_t$

$Ye_t = RK_t + Tge_t$

$Se_t = Ye_t - (t_{RK,t} + \text{div}_t) \sum_n RK_t - c_t CD_{E,t} - T_{ER,t}$

$\sum_n TAGE_t + c_t \times Trg_t$

$SG_{t} = YG_t \times Tge_t - G_t - c_t \times i_{EG,t} CE_{G,t} - i_{B,t} BT_t$

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\[ SCCX_{i,t} = tX_{i,t} P_{i,t} X_{i,t} \]  
\[ TAXM_{M,t} = t_m e_t P_{t,WM} M_{n,t} \]  
\[ TAXE_{n,t} = t e_t P_e t,EX_{n,t} \]  
\[ TYm_{i} = t Ym_{i} \]  
\[ TDe_{i} = t r, Y_{i} \]  
\[ *External \ trade \]  
\[ Q_{n,t} = A_m \left[ \beta M_{n,t}^{-\sigma_m} + (1 - \beta) D_{n,t}^{-\sigma_d} \right] \]  
\[ M_{n,t} = \left( \frac{P_{t, n}^{D}}{P_{t, n}^{M}} \right)^{\frac{\mu_{m}}{\mu_{d}}} \left( \frac{\beta}{(1 - \beta)} \right) D_{n,t} \]  
\[ X_{n,t}^{S} = A_s [\delta EX_{n,t}^{-\sigma_e} + (1 - \delta) XD_{n,t}^{-\sigma_d}] \]  
\[ EX_{n,t} = \left[ \frac{P_{t, n}^{D}}{P_{t}^{E}} \right] \left( \frac{\delta}{1 - \delta} \right) XD_{n,t} \]  
\[ BCOM_{t} = \sum_{R} p_{n,t}^{WF} EX_{n,t} - P_{n,t}^{WM} M_{n,t} \]  
\[ BOC_{t} = BCOM_{t} + T_{RGT_{t}} + T_{RM_{t}} - i_{EE_{t}} CE_{E_{t}} - i_{EG_{t}} CE_{G_{t}} \]  
\[ BK_{t} = \Delta CEE_{t} + \Delta CEG_{t} \]  
\[ *Demand \]  
\[ L_{E_{t}} = P K_{E_{t}} \sum_{n} I_{d_{n,t}} \]  
\[ L_{G_{t}} = P K_{G_{t}} I_{dG} \]  
\[ INV_{t} = \beta^{IE} I_{E_{t}} + \beta^{IG} I_{G_{t}} \]  
\[ G_{t} = P S_{t} X S_{t} \]  
\[ P_{t} C_{i,t} = \beta_{i} C_{m_{t}} + \beta_{i} C_{G} \]  
\[ *Prices \]  
\[ P_{n,t}^{VA} = \frac{P_{n,t}^{D}}{P_{n,t}^{C}} \frac{C I_{t}}{VA_{t,n}} \]  
\[ P_{n,t}^{C} = \frac{P_{n,t}^{M}}{Q_{n,t}} + P_{n,t}^{D} D_{n,t} \]  
\[ P_{n,t}^{E} = \frac{D_{n,t} P_{n,t}^{D} + EX_{n,t} P_{n,t}^{E}}{XX_{n,t}} \]  
\[ P_{n,t}^{M} = P_{t}^{WM} \epsilon_{1} + t t_{t} \]
\[
p_{n,t}^E = \frac{e_t^t p_{n,t}^{WE}}{(1+te_{n,t})}
\]

\[
PKE_t = \sum \beta^t \beta_{i,t} P_{i,t}^c
\]

\[
PKG_t = \sum \beta^t \beta^t_{i,t} P_{i,t}^c
\]

\[
PINDEX_t = \sum \beta^t \beta^t_{i,t} P_{i,t}^c
\]

\[
rhm_{m,t} = A^1_{m,t} (\delta_{m-1}) + A^2_{m,t} (\delta_{m-1})
\]

\[
U_t = PKE_t (i + \gamma)
\]

**II- The dynamic equations of the model**

\[
\frac{I_d,t}{K_{E,(t)}} = \gamma \left[ \left( \frac{R_d}{U_t} \right)^2 + \gamma \frac{R_d}{U_t} \right]
\]

\[
KE_{n(t+1)} = (1 - \gamma) KE_{n,t} - I E_{n,t}
\]

\[
KG_{t(t+1)} = (1 - \gamma) KG_t - I G_t
\]

\[
LS_{t+1} = \left( 1 + \eta \right) LS_t
\]

**III- Equations for The financial side**

* **Household portfolio**

\[
\Delta LL_{m,t} = v_t C_m^t
\]

\[
RFN_t = \Delta BT_{m,t} + \Delta D_{m,t}
\]

\[
\Delta B_{m,t} = A^1_{m,t} (i_b,t)^{\sigma m-1} \frac{\sigma m-1}{rhm} RFN_t
\]

\[
\Delta D_{m,t} = A^2_{m,t} (i_d,t)^{\sigma m-1} \frac{\sigma m-1}{rhm} RFN_t
\]

* **Entreprise portfolios**

\[
\Delta CE_{E,t} + \Delta CD_{E,t} = P_{E,t}^k \times I_{E,t} - S_{E,t}
\]

* **Government portfolio**

\[
P_{G,t}^k I_{G,t} = S_{G,t} + \Delta BT_t + e \Delta CE_{G,t}
\]
* Commercial bank’s portfolio

\[ \Delta RES_t^f = \theta_{R,t} \Delta D_t \]
\[ \Delta CD_{E,t}^S = \Delta DM_t + \Delta REF_t - \Delta RES_t \]
\[ \Delta REF_t = \Delta RES_t + \Delta LL_t + \varepsilon \Delta REX_t \]

* Asset accumulation

\[ BT_{t+1} = BT_t + \Delta BT_t \]
\[ D_{m,t+1} = D_{m,t} + \Delta D_{m,t} \]
\[ CD_{E,t+1} = CD_{E,t} + \Delta CD_{E,t} \]
\[ CE_{E,t+1} = CE_{E,t} + \Delta CE_{E,t} \]
\[ CE_{G,t+1} = CE_{G,t} + \Delta CE_{G,t} \]

* Balance market

\[ LS_t = \sum_i LD_{i,t} \]
\[ BK_t = e_t SR_t \]
\[ \Delta CD_{E,t}^S = \Delta CD_{E,t} \]
## Appendix 2

### List of variables

**Exogenous variables**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_t$</td>
<td>Government expenditure</td>
</tr>
<tr>
<td>$T_{GMT}$</td>
<td>Transfers for government to household</td>
</tr>
<tr>
<td>$I_{GI}$</td>
<td>Government investment</td>
</tr>
<tr>
<td>$P_{GM}$</td>
<td>World price of imported</td>
</tr>
<tr>
<td>$P_{we}$</td>
<td>World price of exported</td>
</tr>
<tr>
<td>$\theta_{R,i}$</td>
<td>Reserve requirement ratio</td>
</tr>
<tr>
<td>$V_t$</td>
<td>Currency to consumption ratio</td>
</tr>
<tr>
<td>$i_{EG,i}$</td>
<td>Rate of interest on foreign loans to government</td>
</tr>
<tr>
<td>$i_{EE,i}$</td>
<td>Rate of interest on foreign loans to firms</td>
</tr>
<tr>
<td>$i_{B,i}$</td>
<td>Rate of return on treasury bonds</td>
</tr>
<tr>
<td>$i_{D,i}$</td>
<td>Rate of return on bank deposits</td>
</tr>
<tr>
<td>$i_{C,i}$</td>
<td>Rate return on bank credit</td>
</tr>
<tr>
<td>$SR_t$</td>
<td>foreign saving</td>
</tr>
<tr>
<td>$PINDEX_t$</td>
<td>Price index</td>
</tr>
<tr>
<td>$LS_t$</td>
<td>Labor supply</td>
</tr>
</tbody>
</table>

**Stocks initiaux**

<table>
<thead>
<tr>
<th>Stock</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$LS$</td>
<td>Initial labor supply</td>
</tr>
<tr>
<td>$K_i$</td>
<td>Initial capital stock</td>
</tr>
<tr>
<td>$BT$</td>
<td>Initial stock of treasury bonds</td>
</tr>
<tr>
<td>$D_m$</td>
<td>Initial stock of deposit</td>
</tr>
<tr>
<td>$CE_E$</td>
<td>Initial stock of foreign loans to firms</td>
</tr>
<tr>
<td>$CE_G$</td>
<td>Initial stock of foreign loans to government</td>
</tr>
<tr>
<td>$CD_E$</td>
<td>Initial stock of domestic borrowing to firms</td>
</tr>
<tr>
<td>REF</td>
<td>Refinancing from Central Bank</td>
</tr>
<tr>
<td>RES</td>
<td>Initial stock of foreign reserves</td>
</tr>
</tbody>
</table>
Endogenous Variables

Production
VA_{i,t} \quad \text{Value added by sectors}
X_{i,t} \quad \text{output}
CI_{j,t} \quad \text{Intermediate consumption j for sector I}
CI_{i,t} \quad \text{Total Intermediate consumption per sector i}
L_{i,t}^D \quad \text{Demand for labor by sector}
RK_{i,t} \quad \text{The return of capital by sector}

Y_{m,t} \quad \text{Household revenue}
YP_{m,t} \quad \text{Household portfolio revenue}
YD_{m,t} \quad \text{Disposable income of household}
S_{m,t} \quad \text{Household saving}
p_{m,s,t} \quad \text{Saving rate for household}
C_{m,t} \quad \text{Household consumption}
Y_{e,t} \quad \text{Firms revenue}
S_{e,t} \quad \text{Firms saving}
Y_{G,t} \quad \text{Government revenue}
S_{G,t} \quad \text{Government saving}
TAXX_{i,t} \quad \text{Production tax}
TAXM_{n,t} \quad \text{Import tax}
TAXE_{n,t} \quad \text{Export tax}
T_{D_{m,t}} \quad \text{Import Tariffs}
T_{D_{e,t}} \quad \text{export Tariffs}

External trade
Q_{n,t} \quad \text{Composite good}
M_{n,t} \quad \text{Import of good}
EX_{n,t} \quad \text{Export of good}
B_{COM,t} \quad \text{Trade balance}
B_{OC,t} \quad \text{Current account balance}
B_{K,t} \quad \text{Capital account balance}

Demand
I_{E,t} \quad \text{Firms investment}
INV_{t} \quad \text{Total investment}
C_{i,t} \quad \text{Total consumption}
I_{d_{it}} \quad \text{investment demand}

Prices
$w_t$ Real wage rate

$P_{VA_{n,t}}$ Value added price

$P_{n,t}^D$ Local price

$P_{n,t}^C$ Composite good price

$P_{n,t}^M$ Price of imported goods

$P_{n,t}^E$ Price of exported goods

$P_{n,t}^f$ Price of firms capital

$P_{n,t}^g$ Price of government capital

$rhm_{m,t}$ Average return of portfolio

$U_t$ User cost of capital

$e_t$ Exchange rate

**Financial variables**

$RFN_{m(t)}$ Household financial wealth

$\Delta LL^D_{m,t}$ Within period change in liquid currency

$\Delta BT_{m,t}$ Within period change in treasury Bonds

$\Delta D_{m,t}$ Within period change in deposits

$\Delta CE_{E,t}$ Within period change in foreign loans by firms

$\Delta CD_{E,t}$ Within period change in domestic borrowing from banks

$\Delta RES_{t}$ Within period change in reserve requirement

$\Delta REF_{t}$ Within period change in refinancing from Central Bank