An Input-Output Model Building for Nine Regions of Japanese Economy Based on Microeconomic Foundation

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Abstract

Traditional input-output model has been built since primary work of W.Leontief; a sector production model plus a price determination model. This model is simplistic one which are aggregated by each economic element. However, in order to reflect tendency of model building in recent years, it is advocated that model should be built based on the micro economic theory, in which behaviors of all economic agents are deduced from optimization. Following this line, input-output modeling also should be done in a way of reflecting this tendency.

Thus, we intend to develop input-output modeling for the nine interregional system of the Japanese economy based on the micro foundation. We are going to endogenize intermediate demand, consumption expenditure, labor demand, and price determination in interregional system; a)consumption expenditure is explained by Almost Ideal Demand system which comes from expenditure minimization, b)intermediate and labor demand are derived from generalized Ozaki cost function, and c)sector price is determined by the Bertrand equilibrium of oligopolistic market. We next incorporate these systems into interregional input-output model of nine regions, and it is estimated from every five years data of having eight time points from 1965 to 2000. In a final step of this paper, we show their model performance.

Keywords：***Interregional Input-output model, AIDS demand function, Cost function.***

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CONTENTS

1. Introduction

A pioneer work on macroeconomic modeling can historically date back to L.R.Klein [1950], who developed Keynesian macroeconomic model composed of six simultaneous equations. Since then, following the rapid progress of personal computers, estimation methods are becoming sophisticated and complex calculation can be dealt with. Today, constructing large scale economic model, which contain hundreds or thousands simultaneous equations, has become possible.

However, in the late 1970s, the rational expectation hypothesis has raised. They have criticized the traditional Keynesian macroeconomic model. In particular, R.Lucas's critique [1976] was very influential. His critique is that, although the shift in economic policy alters agent's behavior or expectation, estimated parameters of reduced form in traditional model are still constant. That means these parameters are unstable under a change in the policy regime. As a response to this criticism, macroeconomic model admitting structural changes is required.

In empirical studies, many econometricians have begun to construct models with microeconomic foundation. Representative model is Computable General Equilibrium model (CGE), which has the micro economic foundation highly in the sense that behaviors of all agents are derived from economic optimization. It is assumed that consumer behaves to maximize their utility or satisfaction, while producer acts to minimize their costs (or maximize its profit). Each economic agent (consumer, producer, and worker) makes decisions rationally against market. Finally, price is designed to adjust to achieve balance between demand and supply in the market. Thus, as the economic actors are dealt with under micro economic theory, CGE model enables us to analyze how each agent might react to changes in policy, external and internal factors. Furthermore, while macroeconomic model seek to explain the aggregated economy, CGE can deal with that of multi-sectors, which makes us possible to analyze more real and detailed economy. This modeling approach has robustness to Lucas's critique. However, CGE has some defects on the other hand. Especially, it is criticized, on determining parameters of each equation using calibration, CGE specifies parameters by single observation. Then it has problem on reliability of estimated parameters. Whereas CGE model might carry through the micro economic theory, the model neglect statistical plausibility, and it abandons empirical verification.

In spite of the current tendency of macro-economic modeling, IO related research have never been released from Leontief's spell. In the next, we will briefly review of IO analysis. Input-output table, which is first developed for empirical analysis by W.Leontief, has nowadays become the origin of CGE model.[W.Leontief,1936 and 1941] Input-output model is formally consisted of production and price models. The production model is to determine total output by summarizing intermediate and final demand. The price model is to determine price in virtue of intermediate input and value added. Both models should interact with each other. Whereas change of price influences the production in terms of intermediate demand and final demand, change of production affects price reciprocally. Then, combing both, we could construct simultaneous model. Although production and price must be determined from demand/supply nexus, two variables are determined quite separately in Leontief model. Unfortunately, abandoning price model, researchers use production determination model, following Leontief.

a) Production model on structural analysis

Although input-output analysis should be done based on the product and price models, most studied implement structural analysis, focusing only on the production side. The first study of structural decomposition is made by Leontief [1941, 1953]. Then, H.Chenery and T.Watanabe have studied the structural decomposition analysis, dealing with national production structures of the developed countries.[H.Chenery and T.Watanabe,1958] Also H.Chenery has developed the methodology of decomposition analysis with input-output system.[H.Chenery,1960] K.V.Santhanam and R.H.Patil have conducted the national production structure on undeveloped country.[K.V.Santhanam and R.H.Patil,1972] Over the decades, there is a considerable number of literatures on the decomposition analysis.

b) Price model

We can often see price model in input-output analysis. The first version of the price model was made by Leontief; he has analyzed the interdependence of prices within inter-industry for the US economy. [W.Leontief, 1947] However the model have two problems. Namely, price model have two assumptions: i) fixed input coefficients, which cannot be affected by changes of final demand and ii) factors expected cost are not taken into accounts. These difficulties have stimulated researches on input-output analysis. However, efforts for the researches have not gone beyond original price model.

Thus, input-output analysis has scarcely stepped out from structural analyses and simple price model, originated by Leontief, for nearly half century. Does research of input-output challenge to Lucas critique? Although macroeconomic and CGE models have been developing continuously, input-output model stays in traditional level without change. Input-output modeling also should reflect recent tendency of model building.

In line with above consideration, our study aims at achieving input-output modeling based on the micro foundation. We endogenize main variables from this point of view: private consumption, intermediate demand, labor and price. First, private consumption will be derived from optimization of expenditure function in employ Almost Ideal Demand System (AIDS). Second, factor demands such as intermediate and labor demands are specified from generalized Ozaki cost function via Shephard's lemma. Finally, price will be determined from Bertrand equilibrium where the price is assumed to be a strategic variable in oligopoly market. Our attempt believes to contribute potential applicability of input-output model.

The rest of the paper will be organized in several sections. Section 2 explains the data, section 3 presents the structure of the model, section 4 shows estimated results, section 5 describes results for final test of the whole system, and finally section 6 provides conclusions.

2. DATA

2.1 Interregional Input-Output Table of Nine Region

This study utilizes the interregional input-output tables of Nine Regions. Tables of time series have been complied by Ministry of Economy, Trade and Industry in Japan, which have been published from 1960 1965 1970 1975 1980 1985 1990 1995 and 2000 at present. Our research uses eight point time series data starting from 1965 to 2000. Original tables in current price are mixture of the price effect and the quantity effect together. In order to analyze real economy, we make sharp distinction between them. Hence, we deflated these nominal input-output tables by sectoral prices. Sectoral price are computed by using the sectoral GDP deflators from SNA with based year 1990. Next, sectors and regions classification has to be consistent over all samples; from 1965 to 2000. Then, we rearranged them; they converge to eight sectors. The details are below.

Table 1 Regions Classification

Region	Prefecture
Hokkaido	Hokkaido
Tohoku	Aomori, Iwate, Miyagi, Akita, Yamagata, Fukushima
Kanto	Ibaraki, Tochigi, Gunma, Saitama, Chiba, Tokyo, Kanagawa, Niigata, Yamanashi, Nagano, Shizuoka
Chubu	Toyama, Ishikawa, Gifu, Aichi, Mie
Kinki	Fukui, Shiga, Kyoto, Osaka, Hyogo, Nara, Wakayama
Chugoku	Tottori, Shimane, Okayama, Hiroshima, Yamaguchi
Shikoku	Kagawa, Kochi, Ehime, Tokushima
Kyushu	Miyazaki, Nagasaki, Saga, Fukuoka, Kagoshima, Oita, Kumamoto
Okinawa	Okinawa

Table 2 Sectors Classification

	Sector
1	Agriculture
2	Mining
3	Manufacture of Metal product
4	Manufacture of Machinery
5	Miscellaneous manufacturing industries
6	Construction
7	Wholesale and retail trades and transportation Trade and transportation
8	Services

Table 3 The Whole Schematic Image of the Interregional Input-Output Table in Constant Prices

		Region 1		Region 2		…	Region r		Total Output
		Intermediate Demand	Final Demand	Intermediate Demand	Final Demand	…	Intermediate Demand	Final Demand	
Region 1	Intermediate Input	$\mathbf{XVR}^{11}$	$\mathbf{FDR}^{11}$	$\mathbf{XVR}^{12}$	$\mathbf{FDR}^{12}$	…	$\mathbf{XVR}^{1r}$	$\mathbf{FDR}^{1r}$	$\mathbf{XXR}^{1}$
	Value Added	$\mathbf{VAR}^{11}$		$\mathbf{VAR}^{12}$			$\mathbf{VAR}^{1r}$		
Region 2	Intermediate Input	$\mathbf{XVR}^{21}$	$\mathbf{FDR}^{21}$	$\mathbf{XVR}^{22}$	$\mathbf{FDR}^{22}$	…	$\mathbf{XVR}^{2r}$	$\mathbf{FDR}^{2r}$	$\mathbf{XXR}^{2}$
	Value Added	$\mathbf{VAR}^{21}$		$\mathbf{VAR}^{22}$			$\mathbf{VAR}^{2r}$		
…	…	…		…		…	…		…
Region r	Intermediate Input	$\mathbf{XVR}^{r1}$	$\mathbf{FDR}^{r1}$	$\mathbf{XVR}^{r2}$	$\mathbf{FDR}^{r2}$	…	$\mathbf{XVR}^{rr}$	$\mathbf{FDR}^{rr}$	$\mathbf{XXR}^{r}$
	Value Added	$\mathbf{VAR}^{r1}$		$\mathbf{VAR}^{r2}$			$\mathbf{VAR}^{rr}$		
Total Input		$\mathbf{XXR}^{1}$		$\mathbf{XXR}^{2}$		…	$\mathbf{XXR}^{r}$		

Table 4 The Details of Schematic Image of the Interregional Input-Output Table in Constant Price

			Region k									
			Intermediate Demand (XVR)				Final Demand (FDR)					
			sector 1	sector 2	…	sector j	Private Consumption	Government Consumption	Investment	Inventories	Export	Import
Region h	Intermediate Input (XVR)	Sector 1	$\mathbf{XVR}^{hk}_{11}$	$\mathbf{XVR}^{hk}_{12}$	…	$\mathbf{XVR}^{hk}_{1j}$	$\mathbf{CPR}^{hk}_{1}$	$\mathbf{CGR}^{hk}_{1}$	$\mathbf{IR}^{hk}_{1}$	$\mathbf{IVR}^{hk}_{1}$	$\mathbf{EXR}^{hk}_{1}$	$\mathbf{IMR}^{hk}_{1}$
		Sector 2	$\mathbf{XVR}^{hk}_{21}$	$\mathbf{XVR}^{hk}_{22}$	…	$\mathbf{XVR}^{hk}_{2j}$	$\mathbf{CPR}^{hk}_{2}$	$\mathbf{CGR}^{hk}_{2}$	$\mathbf{IR}^{hk}_{2}$	$\mathbf{IVR}^{hk}_{2}$	$\mathbf{EXR}^{hk}_{2}$	$\mathbf{IMR}^{hk}_{2}$
		…	…	…	…	…	…	…	…	…	…	…
		Sector i	$\mathbf{XVR}^{hk}_{i1}$	$\mathbf{XVR}^{hk}_{i2}$	…	$\mathbf{XVR}^{hk}_{ij}$	$\mathbf{CPR}^{hk}_{i}$	$\mathbf{CGR}^{hk}_{i}$	$\mathbf{IR}^{hk}_{i}$	$\mathbf{IVR}^{hk}_{i}$	$\mathbf{EXR}^{hk}_{i}$	$\mathbf{IMR}^{hk}_{i}$
	Value Added (VAR)	Consumption Expenditures Outside Households	$\mathbf{CPOR}^{hk}_{1}$	$\mathbf{CPOR}^{hk}_{2}$	…	$\mathbf{CPOR}^{hk}_{j}$						
		Wages	$\mathbf{WAGER}^{hk}_{1}$	$\mathbf{WAGER}^{hk}_{2}$	…	$\mathbf{WAGER}^{hk}_{j}$						
		Operating Surplus	$\mathbf{YCR}^{hk}_{1}$	$\mathbf{YCR}^{hk}_{2}$	…	$\mathbf{YCR}^{hk}_{j}$						
		Depreciation of Fixed Capital	$\mathbf{DEPR}^{hk}_{1}$	$\mathbf{DEPR}^{hk}_{2}$	…	$\mathbf{DEPR}^{hk}_{j}$						
		Indirect Tax	$\mathbf{TAXR}^{hk}_{1}$	$\mathbf{TAXR}^{hk}_{2}$	…	$\mathbf{TAXR}^{hk}_{j}$						
		Subsidy	$\mathbf{SUBR}^{hk}_{1}$	$\mathbf{SUBR}^{hk}_{2}$	…	$\mathbf{SUBR}^{hk}_{j}$						

2.2 The Data of Saving

Our model uses k-th regional saving data Y^k. These data are retrieved from Monthly Economic Report made by Research and Statistics Department (Economic Statistics Division), the Bank of Japan. We use saving data of the 47 prefectures of Japan starting from 1964 to 2000 (totally 37 time point data). We aggregate 47 prefectural data into 9 regional data followed by the classification of input-output data in table 1.

2.3 The Data of Employment

The data of employment of j-th industry in k-th region L_j^k is retrieved from “Establishment and Enterprise Census” made by Statistics Bureau, Ministry of Internal Affairs and Communications. “Establishment and Enterprise Census” is also utilized for making input-output table. The census was made every three years from 1948 to 1981, and thereafter every five years. From the census, we retrieve the data of employment from the item of prefectural “Total number of Establishments” and “Employees by Sex classified by Industry (Minor Groups) for Japan from 1965 to 2000. We aggregate 47 prefectural data into 9 regional data followed by the classification of input-output in table 1.

3. Model Structure

In the first we assume the following.

Assumption 1: Region and Sector

It is supposed that country's economy is decomposed into R regional economies geographically, each having *N* sectors. Multi-region and multi-sector model (MRMS model) is exposed on Table of Interregional Input Output System of Nine Regions of Japanese Economy (IIO9 in abbreviation). In the Table IIO9, included regions are Hokkaido, Tohoku, Kanto, Chubu, Kinki, Chugoku, Shikoku, Kyushu, and Okinawa (i.e. *R*=9). And, included sectors are aggregated into eight categories: 1) Agriculture; 2) Mining; 3) Manufacture of Metal product; 4) Manufacture of Machinery; 5) Miscellaneous manufacturing industries; 6) Construction; 7) Wholesale and retail trades and transportation Trade; 8) Services; (*N*=8).

Assumption 2: *N* Monopoly Markets

Each sector has unique firm producing single commodity for the monopoly market, and each firm has headquarter for planning production plus *R* production plants at each region.

Assumption 3: Block-recursive Interconnection of *N* Commodity Markets

Each commodity markets are interconnected via intermediate transactions in block-recursive way.

3.1 Monopoly Market Equilibrium-Sectoral Total Output-

Total sectoral output is assumed to determine to meet sectoral demand; i.e. intermediate plus final demands.

$$XXR_i = \sum_{l=1}^{n} xvr_{il} + \sum_{k=1}^{r} CPR_i^k + \sum_{k=1}^{r} CGR_i^k + \sum_{k=1}^{r} IR_i^k + \sum_{k=1}^{r} IVR_i^k + \sum_{k=1}^{r} EXR_i^k + \sum_{k=1}^{r} IMR_i^k \quad (3.01)$$

XXR_i	:	Total Output in *i*-th Industry Output in Constant Price
xvr_{ij}	:	Intermediate Input of *i*-th Commodity in Sector *j*-th Industry
CPR_i^k	:	Private Consumption in *i*-th Sector of *k*-th Region in Constant Price
CGR_i^k	:	Government Consumption in *i*-th ector of *k*-th Region in Constant Price
IR_i^k	:	Investment in *i*-th Sector of *k*-th Region in Constant Price
IVR_i^k	:	Inventories in *i*-th Sector of *k*-th Region in Constant Price

EXR_i^k	:	Export in *i*-th Sector of *k*-th Region in Constant Price
IMR_i^k	:	Import in *i*-th Sector of *k*-th Region in Constant Price

In our model, we assume that private consumption CPR_i^k and intermediate input xvr_{ij} are crucial variables for determinants of market equilibrium in (3.01), in which private consumption CPR_i^k and intermediate input xvr_{ij} are endogenized respectively. The variables CPR_i^k and xvr_{ij} are derived from optimization based on the micro economic decision making. Process of deriving CPR_i^k is shown in section 3.2, and that of xvr_{ij} is in section 3.3.

3.2 Consumer's Behavior

In this sub-section, we show the whole picture of household behavior consumption such as consumption CPR_i^k and related variables, following the theory of micro economics.

3.2.1 Definition of Nominal Disposable Income

Firstly, we consider household's disposable income, namely, household's budget set. Household can earn wage from work in *j*-th sector firm of *k*-th region. And, they have property income. This aspect is shown following equation:

$$M^k = \sum_{j=1}^{n} w_j^k \cdot L_j^k + Y^k \tag{3.02}$$

M^k	:	Budget Constraints in *k*-th Region in Current Price
w_j^k	:	Wage Rate in *j*-th Industry of *k*-th Region in Current Price
L_j^k	:	Employment in *j*-th Industry of *k*-th Region in Current Price
Y^k	:	Property Income in *k*-th Region in Current Price

Our budget set is shown by the total of wage and property income. We assume that household attempts to consume commodities under the budget set of (3.02).

The analyses on the consumption of commodities are old topic in econometrics. There are numerous studied of demand system. At first, Stone [1954] developed the linear expenditure system (LES). Since then, study of demand system models has flourished and many models are developed. The Rotterdam model is developed by Barten [1964] and Theil [1965], and then the translog model by Christensen, Jorgenson and Lau [1975], and Almost Ideal Demand System (AIDS) by Deaton and Muelbauer [1980].

Their functions have much information on commodity demand, where the matter of concern is mainly to explain the price and income. Yet, by shading more light on other aspects of household behavior, we can discuss the economic implications of consumption mechanism except the effect of price and wage. Here, we want to introduce six issues as follows:

a) Cohort and Life Cycle

We will focus on the question of how household behavior changes over his life-cycle. Consumption behavior is assumed to be influenced by two effects: life-cycle effect and cohort effect. The life-cycle effect is measured by the age (biological data). The cohort effect can be explained by a group of people (generation) who share a common characteristic, conscience or experience. A.S.Deaton and C.Paxson [1994] analyze that inequality in consumption changes with these effects.

b) Habit Formation - Dynamics in Consumption –

In traditional economic modeling, it is assumed that consumers optimally purchase of commodities to current changes in prices and income. This kinds of model neglect persistence of consumption for particular commodities, namely, habit formation. In reality, current preference depends on the past consumption pattern. In order to analyze the habit formation, we need to take into consideration consumption in the past, which lead to develop dynamic model. Blanciforti and Green [1983] try to incorporate the habit formation into demand system.

c) Family

The size or composition of family impacts economic activity and consumption of commodity. As Engel's important work [1895], equivalence scales is developed to identify household characteristics. Barten [1964] incorporated Engel's equivalence scale into expenditure function, which analyze that household expenditure (utility) patterns differ across households according to family size.

d) Advertisement

In real world, the factors to accelerate household's consumption depend on not only price or income, but also on advertisement or sales promotion. These effects cannot be negligible. Thus, from a viewpoint of empirical studies, it is necessary to take into consideration. For example, E.A.Selvanathan and K.W.Clements [1995] focus on the relation between alcohol advertising and alcohol consumption, and examine how advertising have an impact on people's consumption behavior.

e) Population

We cannot overlook the effects of population. The decrease of population impacts on the amount of the commodity demand. In fact, Japan faces this problem. There are several analyses about demographic change. (Faruqee and Mühleisen [2003]. McKibbin and Nguyen [2004].)

f) Consumption of Leisure - Labor Supply -

As a special commodity, Ballard et al introduced leisure in household utility, then they deduced labor supply. [C.L.Ballard, D.Fullerton, J.B.Shoven and J.Whalley,1985]

g) Recent Tendency for Food Consumption

In most developed countries, consumers tend to modify their commodity choices in reaction to the diet and health information. We need to incorporate these impacts of health information on the demand for commodities into demand system. D.J.Brown and L.Schrader [1990], and Capps and Schmits [1991] analyze that information on the health has induced significant changes in the consumption of certain food products.

h) Future Consumption - Household Saving -

As a final extension, we are going to introduce future consumption in the last commodity, namely household saving [C.L.Ballard, D.Fullerton, J.B.Shoven and J.Whalley,1985].

Thus, in order to capture the complex consumer behavior in real economy, it is necessary to extend demand model from various aspects as above. In this study, we focus on the future consumption. Traditional model like linear expenditure system (LES) deal with saving, but doesn't have the concept of future consumption. In assumption of this model, saving decision is predetermined, and afterwards, household spend all budget on commodity. In this approach, saving is explicit. However, this saving assumption is too simple. In reality, we face with selection between present commodity and future commodity (saving). So, we consider endogenizing household future consumption (saving), following C.L.Ballard et al.

3.2.2 Definition of Future Price

The model allocates household's wealth into the current and future consumptions. For this purpose, in advance, we need to define price of future consumption. Here, price of future consumption $p_f(t)$ is defined as weighted average of consumption for N goods at recent year.

$$p_f(t) = \sum_{i=1}^{N} \frac{CPR_i(2000)}{CPR(2000)} p_i(t) \tag{3.03}$$

p_f : Future Price

t : Eight Points Time Series Data Starting from 1965 to 2000

$CPR_i(2000)$	:	Consumption in Real Value of *i*-th Commodity in 2000
$CPR(2000)$	:	Consumption in Real Value in 2000

We state that disposable income goes to consumptions of *N* commodities and increase of saving (future consumption) which is treated as (*N+1*)-th commodity.

3.2.3 Definition of Future Consumption (Saving)

Household optimally chooses consumption decision between current consumption and future consumption. In line with our IIO9 model, household in *k*-th region is supposed to decide optimal allocation of disposable income over *i*-th commodity (CPR_i^k) and future consumption (CFR^k) by maximizing household utility under its budget constraint (3.04).

$$M^k = p_f CFR^k + \sum_{i=1}^{N} p_i CPR_i^k \quad (3.04)$$

CFR^k : Future Consumption

Since future consumption implies an increase of saving, we can define as follows:

$$\Delta S^k = p_f CFR^k \quad (3.05)$$

$$S^k = S_{-1}^k + \Delta S^k \quad (3.06)$$

where S^k means saving in *k*-th region.

3.2.4 Determination of Property Income

The property income is determined as follows:

$$Y^k = d^k + e^k\left(RGB * S_{-1}^k\right) \quad (3.07)$$

RGB	:	Long-Term Interest Rate
S^k	:	*k*-th Region's Saving
d^k, e^k	:	Estimation Parameters

Yet, we do not touch contents of saving, namely, its portfolio. Total saving will become non-derivative saving, then we may have financial model. Yet, we do not explain it.

3.2.5 Almost Ideal Demand System for Multi-Region Multi-Sector (MRMS)

In order to analyze this mechanism, we employ Almost Ideal Demand System (AIDS) by Deaton and Muellbaur [1980] in our IIO9 model. This demand model is a widely used approach for analyzing consumer behavior by goods. There are also several studies that have analyzed Japanese consumption structure by AIDS model for example, Matsuda [2001], Ganga-Nobuko [2002] and Hashimoto [2004]. Hashimoto [2004] empirically has examined that the expenditure patterns of households in Japanese vary by attributes as annual income class, using the micro data. The object of these studies is to investigate AIDS model in isolation, ignoring supply side. However, in order to represent the interdependence among economic actors, it would be very crucial to apply AIDS in a whole system. Thus, we are going to put an AIDS into final demand of IIO9.

Almost Ideal Demand system (AIDS) is derived from solving expenditure minimization as the dual problem. In duality of demand theory, prime problem of demand theory is basically assumed that consumer demand is determined on the base the utility maximization under constrained budget. In contrast to prime approach, the dual problem is assumed that consumer demand is determined on the base the expenditure minimization subject to a certain utility level. These Optimization processes are shown in Appendix A1. Optimization process for IIO9 is explained in Appendix A2. And, modification for empirical analysis is noted in Appendix A3. Thus, in this section, we will explain how to incorporate AIDS model for IIO9 (MRMS).

$$\omega_i^k = \frac{p_i CPR_i^k}{M^k} = \alpha_i^k + \beta_i^k \left(\frac{\log M^k}{\sum_{l=1}^{n} \alpha_l^k \log p_l + \log \alpha_{n+1}^k p_f} \right) . \tag{3.08}$$

ω_i^k	:	Budget Share for *i*-th Commodity of *k*-th Region
α_i^k, β_i^k	:	Estimation Parameters
p_i	:	Sectoral Price of *i*-th Commodity

In this model, other factors of shifting consumer demand such as advertisement, sale promotion activities, product innovation and transportation ((a)-(h) in section 3.2.1) are assumed to be all expressed in α_i^k.

$$\alpha_i^k = \alpha_i^k \left(sm_i^k \right) \tag{3.09}$$

Above equation for *i*=*N*+1=9 is future consumption.

$$\omega_f^k = \frac{p_f CFR^k}{M^k} = \alpha_f^k + \beta_f^k \log \left(\frac{M^k}{\sum_{l=1}^{n} \alpha_l^k \log p_l + \alpha_{n+1}^k p_f} \right). \tag{3.10}$$

And, AIDS model for IIO9 have three condition as follows:

$$\sum_{i=1}^{n} \omega_i^k = 1$$

$$\sum_{i=1}^{n} \alpha_i^k = 1$$

$$\sum_{i=1}^{n} \beta_i^k = 0$$

Elasticity of Demand

The income elasticity of demand and Marshallian price elasticity of demand is defined as follows:

a) Income Elasticity of Demand

The income elasticity of demand can measure the percentage change of demand in proportion to 1 percentage change of income. The definition is as follows:

$$\eta_i^k = \frac{\partial \log q_i^k}{\partial \log M_i^k} = 1 + \frac{\beta_i^k}{w_i^k} \tag{3.11}$$

$\eta_i^k > 1$: luxury goods

$\eta_i^k < 1$: necessary goods

where η_i^k is income (expenditure) elasticity. This result is presented in figure 1 of the section 4.

b) Marshallian Price Elasticity of Demand

Price elasticity can measure the percentage change in quantity of the demanded goods in response to 1 percent change in its price. The definition is as follows:

$$\varepsilon_{ij}^k = \frac{\partial\left(\log\omega_j^k\right)}{\partial(\log p_i)} = -\delta_i^k - \beta_i^k \frac{\alpha_j^k}{\omega_i^k} \tag{3.12}$$

where ε_{ij}^k represent Marshallian price elasticity of demand. From this equation, two types of price elasticity are derived under condition *i=j* or *i≠j*.

i) Self Price Elasticity of Demand

$$\varepsilon_{ii}^k = -1 - \beta_i^k \frac{\alpha_i^k}{\omega_i^k} \qquad \delta_i^k = 1 \tag{3.13}$$

$|\varepsilon_{ii}^k| > 1$: inelastic

$|\varepsilon_{ii}^k| < 1$: elastic

ii) Cross Price Elasticity of Demand

$$\varepsilon_{ij}^k = -\beta_i^k \frac{\alpha_j^k}{\omega_i^k} \qquad \delta_i^k = 0 \tag{3.14}$$

$\varepsilon_{ij}^k > 0$: substitute

$\varepsilon_{ij}^k < 0$: complement

We evaluate these two types of the price elasticity by applying estimated parameters. These calculated results are presented in Figure.2-Figure.3 of the section 4.

3.3 Producer's Behavior

In this sub-section, we will explain the whole picture of producer's behavior.

3.3.1 Profit Maximization in Monopoly Firm

Monopoly firm, producing *j*-th commodity, determines sectoral price p_j, intermediate demand xvr_{ij} and employent L_j^k by profit maximization. We use argument by W.E.Diewert and K.J.Fox [2004][1]. Now, we consider the profit of *j*-th firm. Profit function of firm to produce *j*-th commodity is given as:

$$\max_{p_j} \max_{xvr_{ij},L_j^k} \pi_j = \max_{p_j} \max_{xvr_{ij},L_j^k} \left\{ p_j(XXR_j)XXR_j + \left(-\sum_{j=1}^{n} p_i xvr_{ij} - w_j^k L_j^k \right) \right\} \tag{3.15}$$

where π_j is total profit in *j*-th firm.

$$\max_{p_j} \pi_j = \max_{p_j} \left\{ p_j(XXR_j)XXR_j - \min_{xvr_{ij},L_j^k} \left(\sum_{j=1}^{n} p_i xvr_{ij} + w_j^k L_j^k \right) \right\} \tag{3.16}$$

And,

$$\max_{p_j} \min_{xvr_{ij},L_j^k} \pi_j = \max_{p_j} p_j(XXR_j)XXR_j - C_j\left(XXR_j, p, w\right) \tag{3.17}$$

where C_j is optimal cost which is derived by cost minimization of equation in section 3.3.2.

3.3.2 Cost Function

Concerning monopoly firm's behavior, the primal problem is profit maximization subject to resource constraints. The dual problem is cost minimization under production function. In the latter approach, we derive intermediate and labor demands by Shephard's lemma under cost function given a prior.

We will consider M.A.Fuss type cost function, which is a generalization of Leontief cost function by Fuss [1977]. For a special case of M.A.Fuss, we will take Generalized Ozaki cost function which is named by Nakamura [1990]. Ozaki cost function is nonlinear and has many parameters to be estimated for empirical studies. For overcoming this difficulty, we omit the off-diagonal term of $i \neq j$ in Fuss type cost function (See Appendix A4). Generalized Ozaki cost function in our MRMS for IIO9 is as follows:

[1] Although W.E.Diewert and K.J.Fox have treated domestic input output system, we are going to treat MRMS system.

$$C_j = \sum_{l}^{n} b_{jl}(p) \cdot p_l \cdot {XXR_j}^{b_{jl}(X)} \cdot e^{b_{jl}(t)t} + \sum_{m=1}^{r} b_j^m(w) \cdot w_j^m \cdot {XXR_j}^{b_j^m(X)} \cdot e^{b_j^m(t)t} \tag{3.18}$$

C_j	:	Cost function of *j*-th Industry
$b_{jl}(p), b_{jl}(X), b_{jl}(t)$	:	Estimation Parameters
$b_j^m(w), b_j^m(X), b_j^m(t)$	:	Estimation Parameters
XXR_j	:	Output of *j*-th Industry
t	:	Progress of Technology
w_j^m	:	Factor price (Wage Rate) of *j*-th Industry in *m*-th Region
p_l	:	*l*-th Factor Price (Sectoral Price)

where the first term stands for intermediate cost and the second term for labor cost. And, t shows Hicks neutral technological progress.

Applying the Shephard's lemma, we differentiate it partially with respect to factor price p_i and w_j^m, to obtain the intermediate and labor demands respectively:

$$xvr_{ji} = \frac{\partial C_j}{\partial p_i} = b_{ji}(p) \cdot \left(XXR_j\right)^{b_{ji}(X)} \cdot e^{b_{ji}(t)t} \tag{3.19}$$

$$L_j^k = \frac{\partial C_j}{\partial w_j^k} = b_j^k(w) \cdot \left(XXR_j\right)^{b_j^k(X)} \cdot e^{b_j^k(t)t} \tag{3.20}$$

These equations have alternative expressions for estimation.

$$\log xvr_{ji} = \log b_{ji}(p) + b_{ji}(X) \log XXR_j + b_{ji}(t)t \tag{3.21}$$

$$\log L_j^k = \log b_j^k(w) + b_j^k(X) \log XXR_j + b_j^k(t)t \tag{3.22}$$

$\log b_{jl}(p)$and $\log b_j^k(w)$ express the shift factor of intermediate demand by product innovation and transportation distance et al. Equation (3.19)-(3.20) or (3.21)-(3.22) are estimated in use of IIO9 data. By connecting estimated results, we could specify the cost function (3.18).

3.3.3 Innovation Attached to Cost Function

The cost function provides information on the firm's technology. In order to explain technological progress in more detail, we intend to refine cost function by giving additional information to (3.21) and (3.22). S.Shishido [1990] has pointed out that there are three kinds of factors in accelerating technology innovation, extending the concept of RAS method as follows:

(a) Price Independent Technical Progress

(b) Technical Progress Depending on Input Price

(c) Technical Progress Depending on Output Price

First, technical progress in (a) corresponds to S in RAS method. The progress of this type implies bottom-up of technology progress, which does not save particular production factors such as labor, but the total factor production overall (independent of price of output or input). For example, it is IT innovation, which enables the cost reduction overall. Secondly, technical progress in (b) corresponds to R in RAS method. This is the technology progress accelerated by the factor of rising input price (material, labor or capital). For example, when material price rises, firms try to reduce using it. Thirdly, technical progress in (c) implies the cost cutting by cost management efforts of producers. Producers are sensible of the change of prevalent price (final production price, market price). If market price (output price) is low, they make efforts to perform efficiently. These producer's behaviors stands out especially in manufacturing industries in Japan after 1990s.

Technical progress of type (a) is described by the term $e^{b_{jl}(t)t}$ and $e^{b_j^k(t)t}$ on the right side of Ozaki Cost Function, and that of type (b) by the term $b_{jl}(p) \cdot p_l$ and $b_j^k(w) \cdot w_j^k$. However, technical progress of type (c) is not reflected in Ozaki cost function. Therefore, we are going to have new Ozaki cost function by attaching expected output price. Then, Ozaki cost function is modified as follows:

$$C_j\left(XXR_j, p, w\right) = \sum_{l}^{n} b_{jl}(p) \cdot p_l \cdot {XXR_j}^{b_{jl}(X)} \cdot e^{b_{jl}(t)t} \cdot {p_j^e}^{b_{jl}(pe)} + \sum_{m=1}^{r} b_j^m(w) \cdot w_j^m \cdot {XXR_j}^{b_j^m(X)} \cdot e^{b_j^m(t)t} \cdot {p_j^e}^{b_j^m(pe)} \tag{3.23}$$

where p_j^e is the *i*-th expectation of output price. Expectation price is expressed by arithmetic mean of total output with base year 1990 in the following.

$$p_e(t) = \sum_{i=1}^{N} \frac{XXR_i(1990)}{XXR(1990)} p_i(t) \tag{3.24}$$

p_e	:	Expectation Price
t	:	Time from 1965 to 2000
$XXR_i(1990)$	:	Output *i*-th Aggregated with Regions Based on Price in 1990
$XXR(1990)$	:	Total Output Based on Price in 1990

Applying the Shephard's lemma yields the derived intermediate and labor demands:

$$xvr_{ji} = \frac{\partial C_j}{\partial p_i} = b_{ji}(p) \cdot \left(XXR_j\right)^{b_{ji}(X)} \cdot e^{b_{ji}(t)t} \cdot \left(p_j^e\right)^{b_{ji}(pe)} \quad (3.25)$$

$$L_j^k = \frac{\partial C_j}{\partial w_j^k} = b_j^k(w) \cdot \left(XXR_j\right)^{b_j^k(X)} \cdot e^{b_j^k(t)t} \cdot \left(p_j^e\right)^{b_j^k(pe)} \quad (3.26)$$

Estimated coefficients of (3.25) and (3.26) enable us to have coefficients of corresponding cost function. Equations (3.25) and (3.26) also take logarithm form.

$$\log xvr_{ji} = \log b_{ji}(p) + b_{ji}(X) \log XXR_j + b_{ji}\,(pe) \log p_j^e + b_{ji}(t)t \quad (3.27)$$

$$\log L_j^k = \log b_j^k(w) + b_j^k(X) \log XXR_j + b_j^k\,(pe) \log p_j^e + b_j^k(t)t \quad (3.28)$$

Economy of Scale

$$SE_j = \frac{AC_j}{MC_j} = \frac{C_j/XXR_j}{MC_j} \quad (3.29)$$

SE_j >1 : increasing returns to scale

SE_j =1 : constant returns to scale

SE_j <1 : decreasing returns to scale

Rate of Technical Progress

Formula of technical progress of *j*-th sector in *k*-th country in virtue of cost function is product of two elements associated with cost function where left hand side is unknown, but elements of right hand side are computed in ease.

$$\frac{\partial \log f_j(x,t)}{\partial t} = -\left(\frac{\partial \log C_j\left(XXR_j, p, w, t\right)}{\partial t}\right) \times \frac{\partial \log XXR_j}{\partial \log C_j\left(XXR_j, p, w, t\right)} \quad (3.30)$$

Total Factor Productivity (TFP)

TFP of *j*-th sector in *k*-th country is straightforward.

$$\frac{d \log TFP_j}{dt} = \frac{d \log XXR_j}{dt} - \frac{d \log C_j\left(XXR_j, p, w, R_j(t)\right)}{dt} \quad (3.31)$$

The producing process TFP in IIO9 is explained in Appendix A5.

3.3.4 Pricing Monopoly Firm

Profit maximization (3.17) forces monopoly firm to determine monopoly price.

$$\frac{\partial \pi_j}{\partial p_j} = \frac{\partial\left[p_j XXR_j - C_j\left(XXR_j, p, w\right)\right]}{\partial p_j} = 0 \tag{3.32}$$

$$XXR_j + p_j \frac{\partial XXR_j}{\partial p_j} = \frac{\partial C_j\left(XXR_j, p, w\right)}{\partial p_j} \tag{3.33}$$

As we have $\frac{\partial XXR_j}{\partial p_j}$ in (3.33), we evaluate it in referring (3.01) with replacing *i* by *j*.

$$\frac{\partial XXR_j}{\partial p_j} = \frac{\partial\left(\sum_{l=1}^{n} xvr_{il} + \sum_{k=1}^{r} CPR_j^k + \sum_{k=1}^{r} CGR_j^k + \sum_{k=1}^{r} IR_j^k + \sum_{h=1}^{r} IVR_j^k + \sum_{h=1}^{r} EXR_j^k + \sum_{h=1}^{r} IMR_j^k\right)}{\partial p_j} \tag{3.34}$$

$$= \frac{\partial\left(\sum_{l=1}^{n} xvr_{il}\right)}{\partial p_j} + \frac{\partial\left(\sum_{k=1}^{r} CPR_j^k\right)}{\partial p_j} + \frac{\partial\left(\sum_{k=1}^{r} CGR_j^k + \sum_{k=1}^{r} IR_j^k + \sum_{k=1}^{r} IVR_j^k + \sum_{k=1}^{r} EXR_j^k + \sum_{h=1}^{r} IMR_j^k\right)}{\partial p_j} \tag{3.35}$$

$$= 0 + \frac{\partial\left(\sum_{k=1}^{r} CPR_j^k\right)}{\partial p_j} + \frac{\partial\left(\sum_{k=1}^{r} CGR_j^k + \sum_{k=1}^{r} IR_j^k + \sum_{k=1}^{r} IVR_j^k + \sum_{k=1}^{r} EXR_j^k + \sum_{h=1}^{r} IMR_j^k\right)}{\partial p_j} \tag{3.36}$$

where we do not set partial differentiation of third term to zero. We introduce conjectural variation λ_j [Iwata, 1974][2] as:

$$\lambda_j = \frac{\partial\left(\sum_{k=1}^{r} CGR_j^k + \sum_{k=1}^{r} IR_j^k + \sum_{k=1}^{r} IVR_j^k + \sum_{k=1}^{r} EXR_j^k + \sum_{h=1}^{r} IMR_j^k\right)}{\partial p_j} \tag{3.37}$$

We can rewrite (3.36) as follow

$$\frac{\partial XXR_j}{\partial p_j} = 0 + \frac{\partial\left(\sum_{k=1}^{r} CPR_j^k\right)}{\partial p_j} + \lambda_j \tag{3.38}$$

Conjectural Variation

The sectoral production does not go to infinity by profit maximization in case of increasing return of scale, meanwhile, they does not go to zero in case of decreasing return of scale. Their behavior has constraint of capital. The optimal prices are assumed to adjust to finite value between zero and infinity.

Conjectural variations describe variation factors that firm reacts to competitive rivals. In addition, λ_j enable to satisfy optimization condition within a time series context, which absorbs

[2] G.Iwata has treated homogeneous commodity market of oligopolistic flat glass in Japanese economy.

residue from overall model.

$$\lambda_j = -\frac{\partial\left(\sum_{k=1}^{r} CPR_j^k\right)}{\partial p_j} \tag{3.39}$$

By letting notations of marginal cost and price elasticity of demand be as follows:

Marginal Cost $$MC_j = \frac{\partial C_j}{\partial XXR_j} \tag{3.40}$$

Price Elasticity of Demand $$\varepsilon_j = -\frac{\partial XXR_j}{\partial p_j} \cdot \frac{p_j}{XXR_j} \tag{3.41}$$

Finally, using equation (3.40) and (3.41), we lead to price determination equation as:

$$\frac{p_j - MC_j}{p_j} = \left(\frac{1}{\varepsilon_j}\right) \tag{3.42}$$

$$p_j = \left(\frac{\varepsilon_j}{\varepsilon_j - 1}\right) MC_j \tag{3.43}$$

Equation (3.43) is embodied as:

$$p_j = \left\{ \frac{\left[\frac{\partial\left(\sum_{k=1}^{r} CPR_j^k\right)}{\partial p_j} + \lambda_j\right] \frac{p_j}{XXR_j}}{\left[\frac{\partial\left(\sum_{k=1}^{r} CPR_j^k\right)}{\partial p_j} + \lambda_j\right] \frac{p_j}{XXR_j} + 1} \right\} MC_j \tag{3.44}$$

System of equations (3.44) determine R sector prices simultaneously given R exogenous conjectural variations λ_j.

3.4 Other Variables

3.4.1 Wage Rate

The wage rate equation is formulated by W.J.McKibbin and J.Nguyen [2004].

$$w_j^k = \alpha_j^k (p_e)^{\beta_j^k} \left(\frac{XXR_j^k}{L_j^k}\right)^{\gamma_j^k} \tag{3.45}$$

w_i^k : Wage Rate in *j*-th Industry of *k*-th Region

p_e : Expected Price

where is explained by expected price and employment.

4. Estimated Results

We execute Ordinary Least Squares for the sample of input-output data covering eight time points from 1965 to 2000 (every five years). Tables below are estimation results.

4.1 AID System Model

Although we deal with some parameters are not necessarily satisfactory, we accepted.

Table 5 Estimation Results in Equation (3.08)

Panel A: Parameters α_i^k in equation (3.08)

i \ k	Hokkaido	Tohoku	Kanto	Chubu	Kinki	Chugoku	Shikoku	Kyushu	Okinawa
1	0.252	0.263	0.172	0.185	0.187	0.208	0.201	0.228	0.191
	(0.005)	(0.003)	(0.003)	(0.002)	(0.003)	(0.003)	(0.003)	(0.002)	(0.006)
	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.108]
	{0.926}	{0.971}	{0.949}	{0.98}	{0.96}	{0.955}	{0.947}	{0.978}	{0.452}
2	0.022	0.006	0.001	0.001	0.001	0.001	0.001	0.003	0.000
	(0.001)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
	[0.011]	[0.011]	[0.014]	[0.044]	[0.092]	[0.048]	[0.062]	[0.022]	[0.278]
	{0.67}	{0.67}	{0.632}	{0.512}	{0.398}	{0.502}	{0.467}	{0.594}	{0.286}
3	0.028	0.034	0.029	0.028	0.027	0.026	0.028	0.031	0.046
	(0.002)	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)	(0.001)	(0.001)	(0.001)
	[0.048]	[0.001]	[0.001]	[0.003]	[0.005]	[0.037]	[0.007]	[0.001]	[0.005]
	{0.417}	{0.822}	{0.811}	{0.738}	{0.699}	{0.446}	{0.66}	{0.824}	{0.876}
4	0.061	0.012	-0.018	-0.012	-0.020	0.007	0.027	0.005	0.025
	(0.007)	(0.003)	(0.005)	(0.005)	(0.004)	(0.005)	(0.006)	(0.004)	(0.008)
	[0.136]	[0.544]	[0.502]	[0.664]	[0.364]	[0.796]	[0.42]	[0.842]	[0.833]
	{0.059}	{0.287}	{0.457}	{0.403}	{0.538}	{0.233}	{0.023}	{0.27}	{0.004}
5	0.899	1.056	0.792	0.918	0.852	0.952	0.944	0.948	1.021
	(0.014)	(0.013)	(0.012)	(0.012)	(0.013)	(0.016)	(0.013)	(0.013)	(0.007)
	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.001]
	{0.928}	{0.956}	{0.943}	{0.961}	{0.936}	{0.931}	{0.958}	{0.95}	{0.942}
6	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.007]
	{0.952}	{0.937}	{0.957}	{0.942}	{0.934}	{0.927}	{0.91}	{0.935}	{0.827}
7	-0.823	-0.820	-0.688	-0.734	-0.756	-0.672	-0.672	-0.688	-1.666
	(0.064)	(0.064)	(0.051)	(0.055)	(0.057)	(0.061)	(0.062)	(0.059)	(0.045)
	[0.051]	[0.055]	[0.041]	[0.037]	[0.048]	[0.076]	[0.07]	[0.066]	[0.065]
	{0.567}	{0.552}	{0.601}	{0.611}	{0.578}	{0.508}	{0.524}	{0.537}	{0.653}

8	0.229	0.121	0.180	0.254	0.060	0.327	0.247	0.286	-0.048
	(0.051)	(0.047)	(0.045)	(0.040)	(0.042)	(0.039)	(0.037)	(0.043)	(0.027)
	[0.431]	[0.653]	[0.467]	[0.249]	[0.799]	[0.151]	[0.227]	[0.250]	[0.908]
	{0.113}	{0.237}	{0.207}	{0.147}	{0.343}	{0.077}	{0.187}	{0.117}	{0.318}
9	0.333	0.328	0.531	0.361	0.649	0.151	0.225	0.187	1.431
	(0.026)	(0.032)	(0.044)	(0.033)	(0.050)	(0.032)	(0.042)	(0.032)	(0.036)
	[0.052]	[0.111]	[0.06]	[0.07]	[0.051]	[0.400]	[0.317]	[0.311]	[0.052]
	{0.298}	{0.200}	{0.297}	{0.195}	{0.358}	{0.001}	{0.024}	{0.031}	{0.607}

Panel B: Parameters β_i^k in equation (3.08)

i \ k	Hokkaido	Tohoku	Kanto	Chubu	Kinki	Chugoku	Shikoku	Kyushu	Okinawa
1	-0.014	-0.015	-0.008	-0.01	-0.01	-0.011	-0.011	-0.012	-0.012
	(0.005)	(0.003)	(0.003)	(0.002)	(0.003)	(0.003)	(0.003)	(0.002)	(0.006)
	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.143]
	{0.926}	{0.971}	{0.949}	{0.98}	{0.96}	{0.955}	{0.947}	{0.978}	{0.452}
2	-0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	(0.001)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
	[0.013]	[0.013]	[0.018]	[0.046]	[0.093]	[0.049]	[0.062]	[0.025]	[0.275]
	{0.67}	{0.67}	{0.632}	{0.512}	{0.398}	{0.502}	{0.467}	{0.594}	{0.286}
3	-0.001	-0.002	-0.001	-0.001	-0.001	-0.001	-0.002	-0.002	-0.003
	(0.002)	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)	(0.001)	(0.001)	(0.001)
	[0.084]	[0.002]	[0.002]	[0.006]	[0.01]	[0.07]	[0.014]	[0.002]	[0.006]
	{0.417}	{0.822}	{0.811}	{0.738}	{0.699}	{0.446}	{0.66}	{0.824}	{0.876}
4	-0.001	0.002	0.003	0.003	0.003	0.002	0.001	0.002	0.001
	(0.007)	(0.003)	(0.005)	(0.005)	(0.004)	(0.005)	(0.006)	(0.004)	(0.008)
	[0.562]	[0.171]	[0.066]	[0.091]	[0.038]	[0.226]	[0.718]	[0.187]	[0.91]
	{0.059}	{0.287}	{0.457}	{0.403}	{0.538}	{0.233}	{0.023}	{0.27}	{0.004}
5	-0.043	-0.052	-0.034	-0.044	-0.038	-0.046	-0.048	-0.045	-0.059
	(0.014)	(0.013)	(0.012)	(0.012)	(0.013)	(0.016)	(0.013)	(0.013)	(0.007)
	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.001]
	{0.928}	{0.956}	{0.943}	{0.961}	{0.936}	{0.931}	{0.958}	{0.95}	{0.942}
6	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.012]
	{0.952}	{0.937}	{0.957}	{0.942}	{0.934}	{0.927}	{0.91}	{0.935}	{0.827}
7	0.061	0.058	0.045	0.051	0.051	0.049	0.052	0.049	0.126
	(0.064)	(0.064)	(0.051)	(0.055)	(0.057)	(0.061)	(0.062)	(0.059)	(0.045)
	[0.031]	[0.035]	[0.024]	[0.022]	[0.029]	[0.047]	[0.042]	[0.039]	[0.052]
	{0.567}	{0.552}	{0.601}	{0.611}	{0.578}	{0.508}	{0.524}	{0.537}	{0.653}
8	0.015	0.022	0.016	0.012	0.024	0.009	0.014	0.012	0.038
	(0.051)	(0.047)	(0.045)	(0.04)	(0.042)	(0.039)	(0.037)	(0.043)	(0.027)
	[0.416]	[0.221]	[0.257]	[0.348]	[0.127]	[0.507]	[0.285]	[0.407]	[0.244]
	{0.113}	{0.237}	{0.207}	{0.147}	{0.343}	{0.077}	{0.187}	{0.117}	{0.318}

9	-0.014	-0.013	-0.021	-0.012	-0.029	-0.001	-0.005	-0.005	-0.091
	(0.026)	(0.032)	(0.044)	(0.033)	(0.05)	(0.032)	(0.042)	(0.032)	(0.036)
	[0.161]	[0.267]	[0.162]	[0.273]	[0.117]	[0.928]	[0.716]	[0.677]	[0.068]
	{0.298}	{0.2}	{0.297}	{0.195}	{0.358}	{0.001}	{0.024}	{0.031}	{0.607}

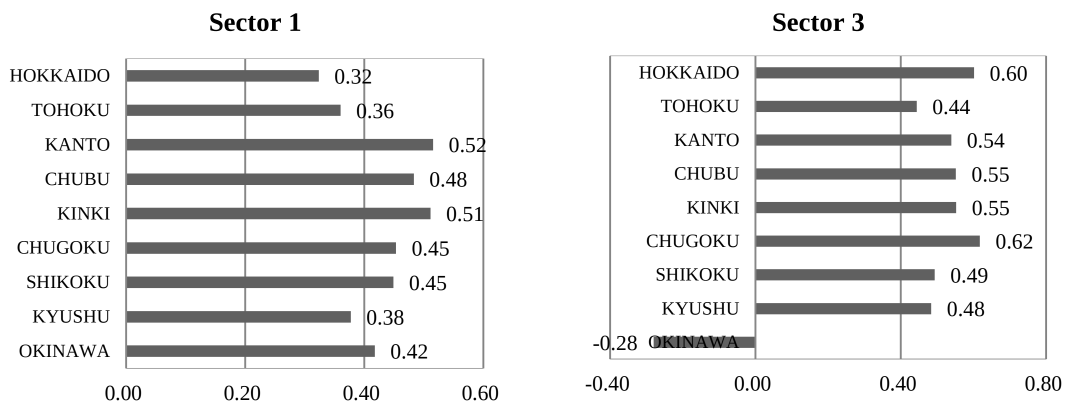
* The number of observations is 8 for each estimation (in the case of Okinawa, 6 observations, from 1975 to 2000).
* Adj. R^2 is adjusted R-squared. Standard errors, p-values and Adj. R^2 are in parentheses, brackets and curly brackets, respectively.
* Sector numbers (i) Sectors 1, 2, 3, 4, 5, 6, 7 and 8 denote agriculture, mining, manufacture of metal product, manufacture of machinery, miscellaneous manufacturing industries, construction, wholesale and retail trades and transportation trade and transportation, services, respectively. The Consumption has 9th commodity which means future consumption (saving).

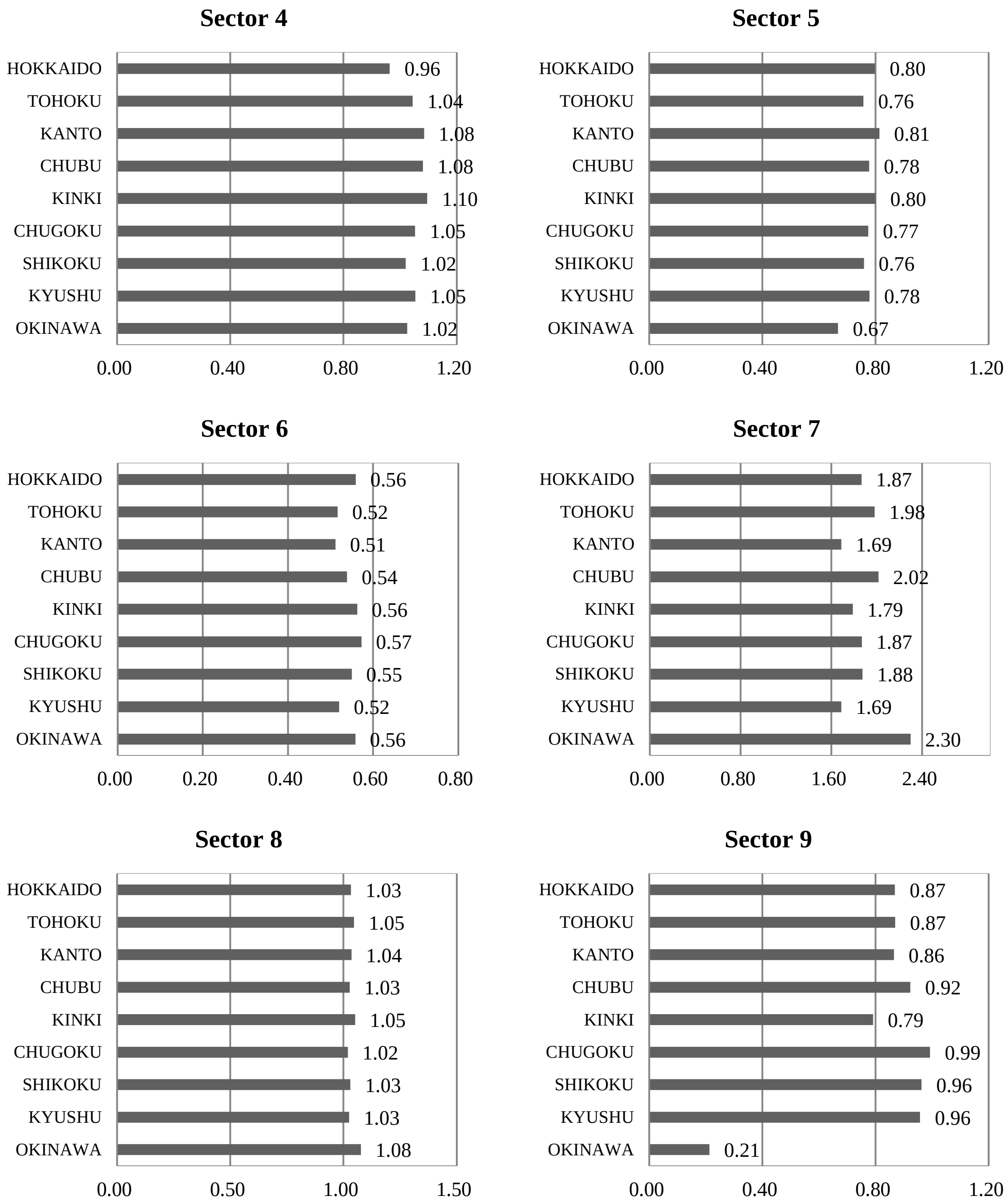
4.1.1 Income Elasticity of Demand

By using estimated results of AIDS model (3.08)-(3.10), we can compute sectoral elasticity of income from (3.11). Thus, we calculated an arithmetic mean of time series from 1965 to 2000 every five years (eight points). If elasticity of income is greater than one in absolute value, it implies the luxury goods. If it is less than one, it is necessary goods.

Results are reported in Figure 1. From results, the 4th sector (machinery industry), the 7th sector (wholesale and retail trades and transportation Trade and transportation), and the 8th sector (services) have a tendency as the luxury goods on all regions. These sectors are highly value-added. On the other hand, the 9th sector (future consumption, namely, saving) appears regional characteristics. Chubu, Chugoku, Shikoku and Kyushu are close to unity showing 0.92, 0.99, 0.96, 0.96 respectively. Final, Okinawa is very low with value 0.21.

Figure 1 Sectoral Income Elasticity of Demand



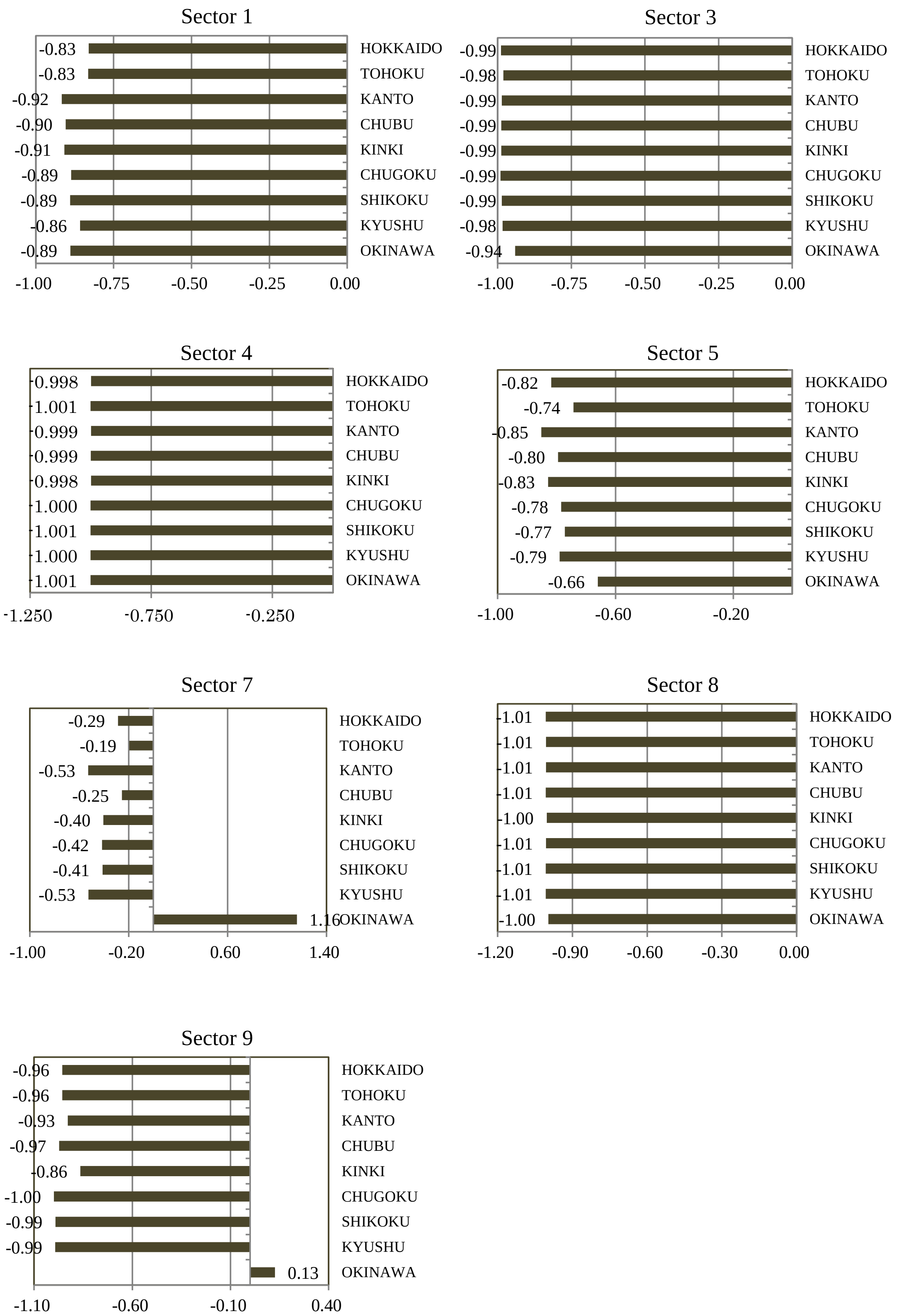


4.1.2 Price Elasticity of Demand

(i) *Marshallian Self Price Elasticity of Demand*

Next we calculate arithmetic mean of Marshallian self price elasticity of demand from 1965 to 2000 (time series of eight points), following equation (3.13). If value are greater than unity in absolute, then demand is meant to be sensitive to price change. From estimated results, the 3th sector (metal industry), the 4th sector (machinery industry), and the 8th sector (services) is close to unity in absolute. These commodities are said to be sensitive to price.

Figure 2　Self price elasticity of the demand

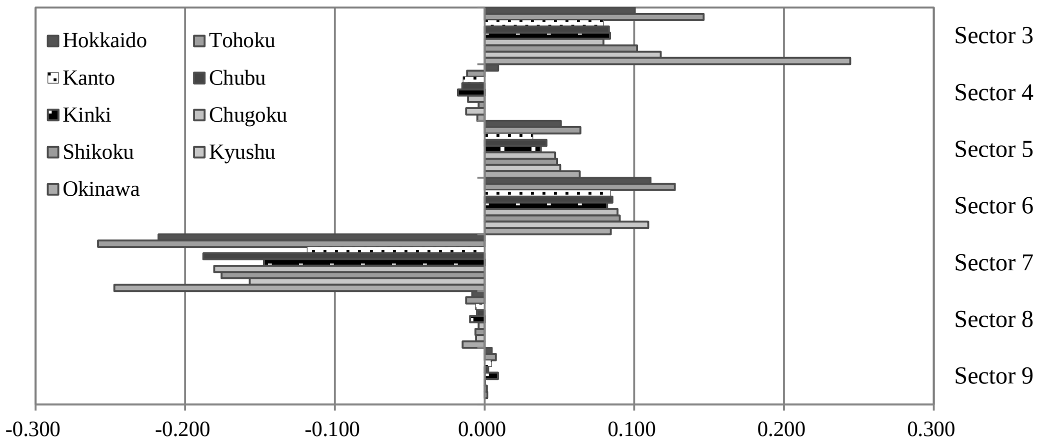


(ii) *Marshallian Cross Price Elasticity of Demand*

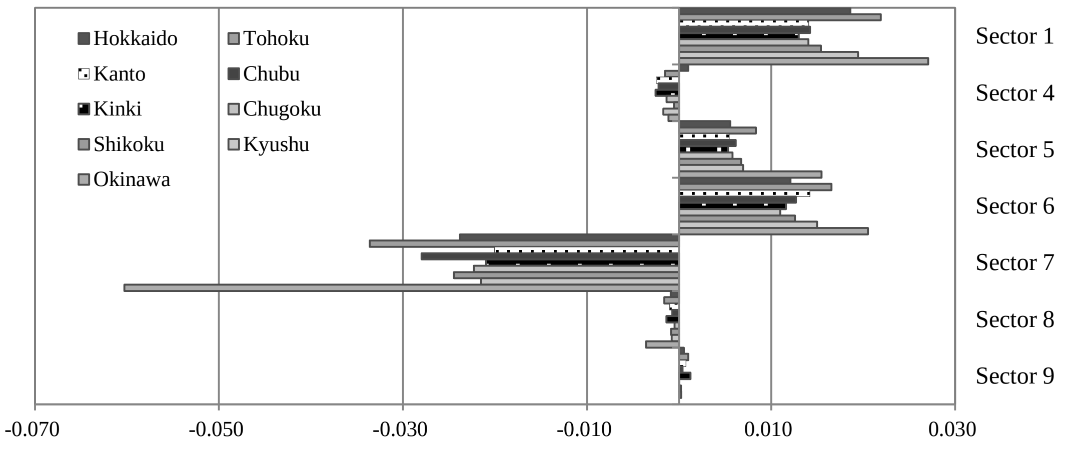
The cross price elasticity of demand measures how current demand reacts to the change of the other price. If it shows positive value, two goods are said to be substitute. On the other hand, if it shows negative value, two goods are complementary. If it is zero, each goods are not relation. From the estimated values of AIDS, we could compute cross price elasticity of demand. Results are presented in Figure 3. It should be noted that it seems to be less worthwhile to examine rigorously the estimated parameters of highly aggregated sectors. Yet, it is possible to analyze the results in formal way. The substitute of Sector 1 is Sector 3, Sector 5, and Sector 6. And, the complement of Sector 1 is Sector 7. Other sector is nearly zero value. These sensitive are so different each region that we can see the characteristics of regions respectively.

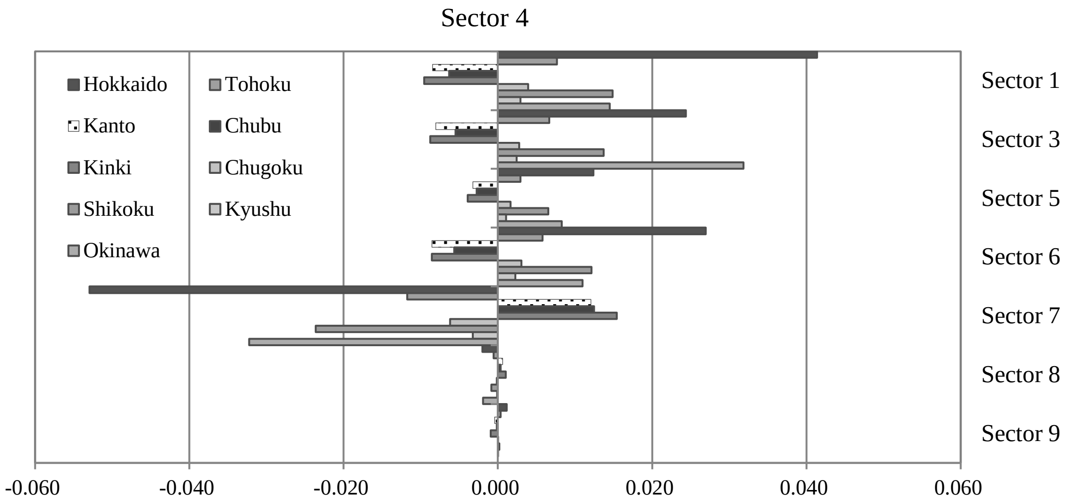
Figure 3　Cross Price Elasticity of Demand

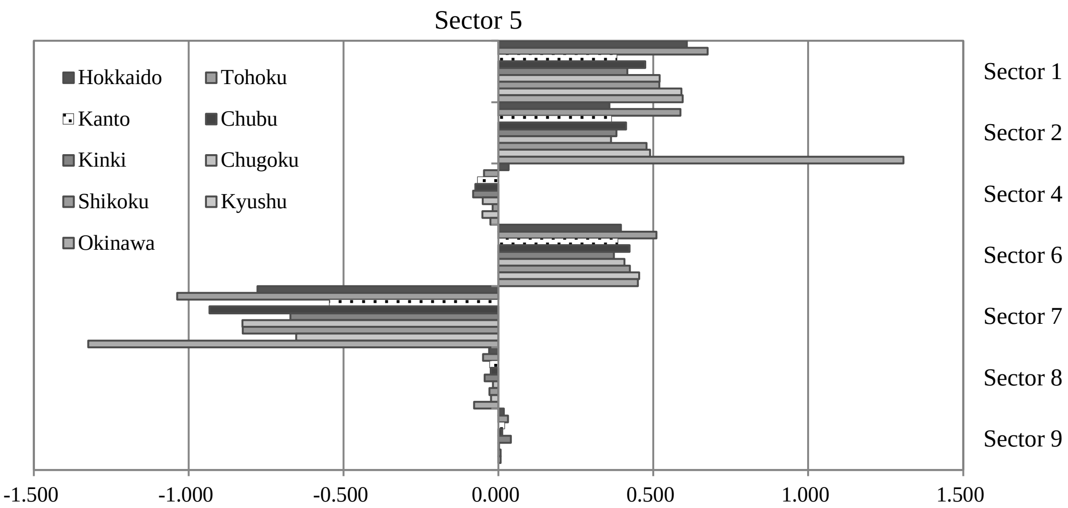
Sector 1

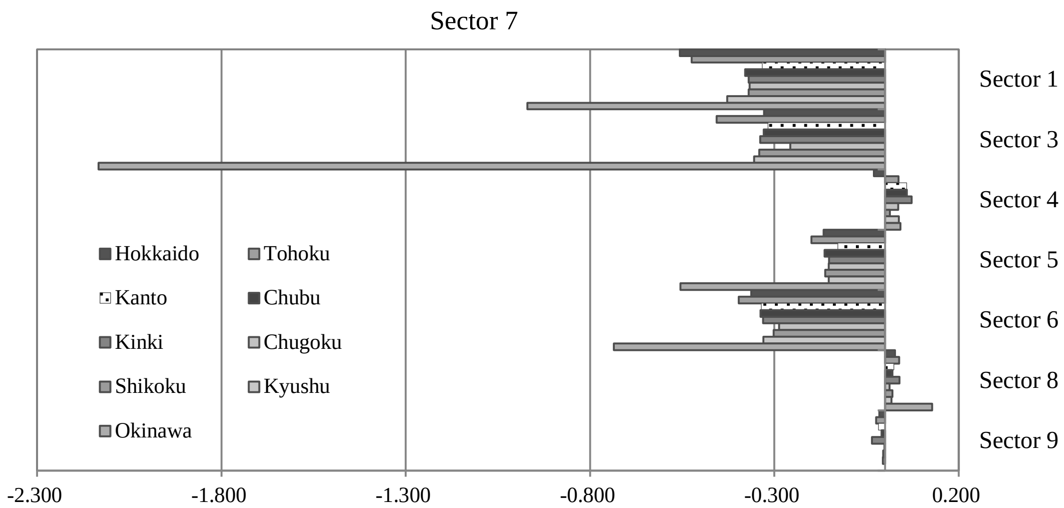


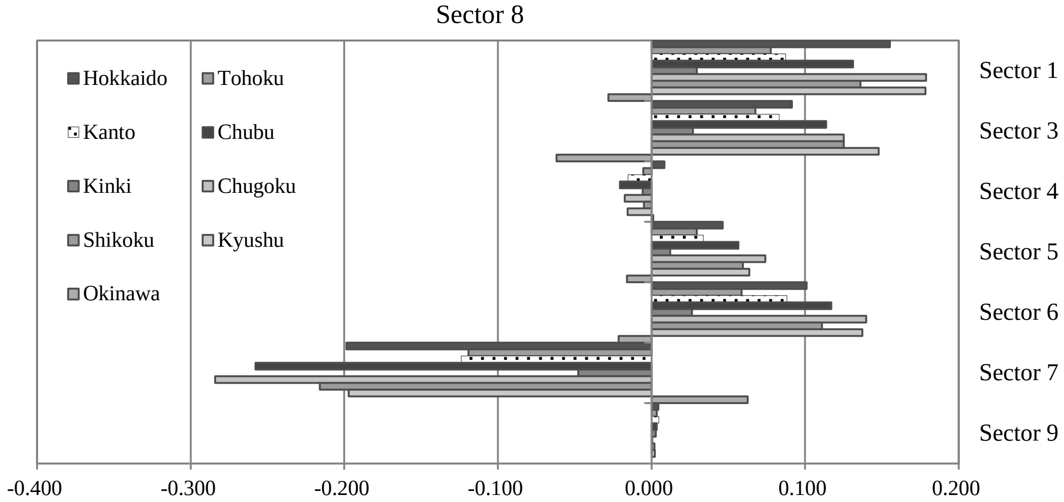
Sector 3

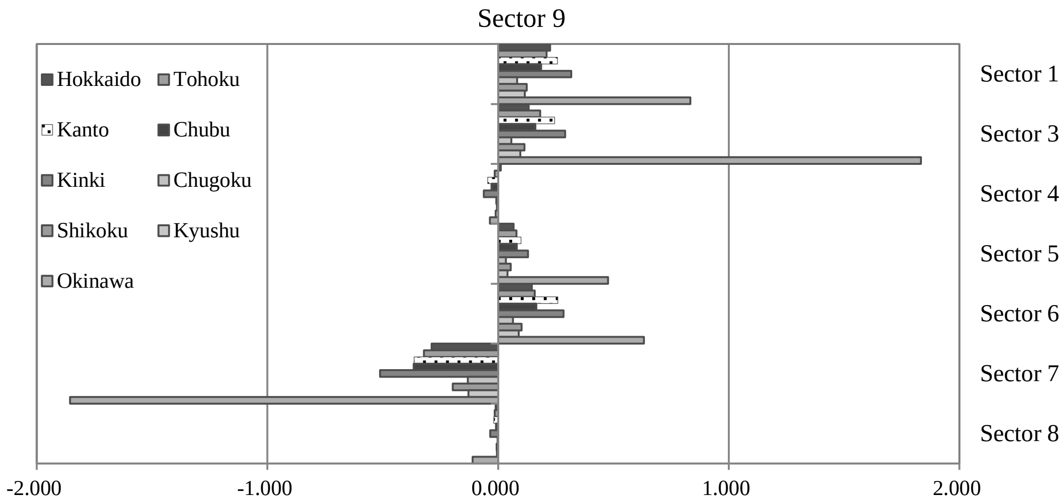



Sector 4
Hokkaido
Tohoku
Kanto
Chubu
Kinki
Chugoku
Shikoku
Kyushu
Okinawa
Sector 1
Sector 3
Sector 5
Sector 6
Sector 7
Sector 8
Sector 9
-0.060
-0.040
-0.020
0.000
0.020
0.040
0.060


Sector 5
Hokkaido
Tohoku
Kanto
Chubu
Kinki
Chugoku
Shikoku
Kyushu
Okinawa
Sector 1
Sector 2
Sector 4
Sector 6
Sector 7
Sector 8
Sector 9
-1.500
-1.000
-0.500
0.000
0.500
1.000
1.500


Sector 7
Hokkaido
Tohoku
Kanto
Chubu
Kinki
Chugoku
Shikoku
Kyushu
Okinawa
Sector 1
Sector 3
Sector 4
Sector 5
Sector 6
Sector 8
Sector 9
-2.300
-1.800
-1.300
-0.800
-0.300
0.200



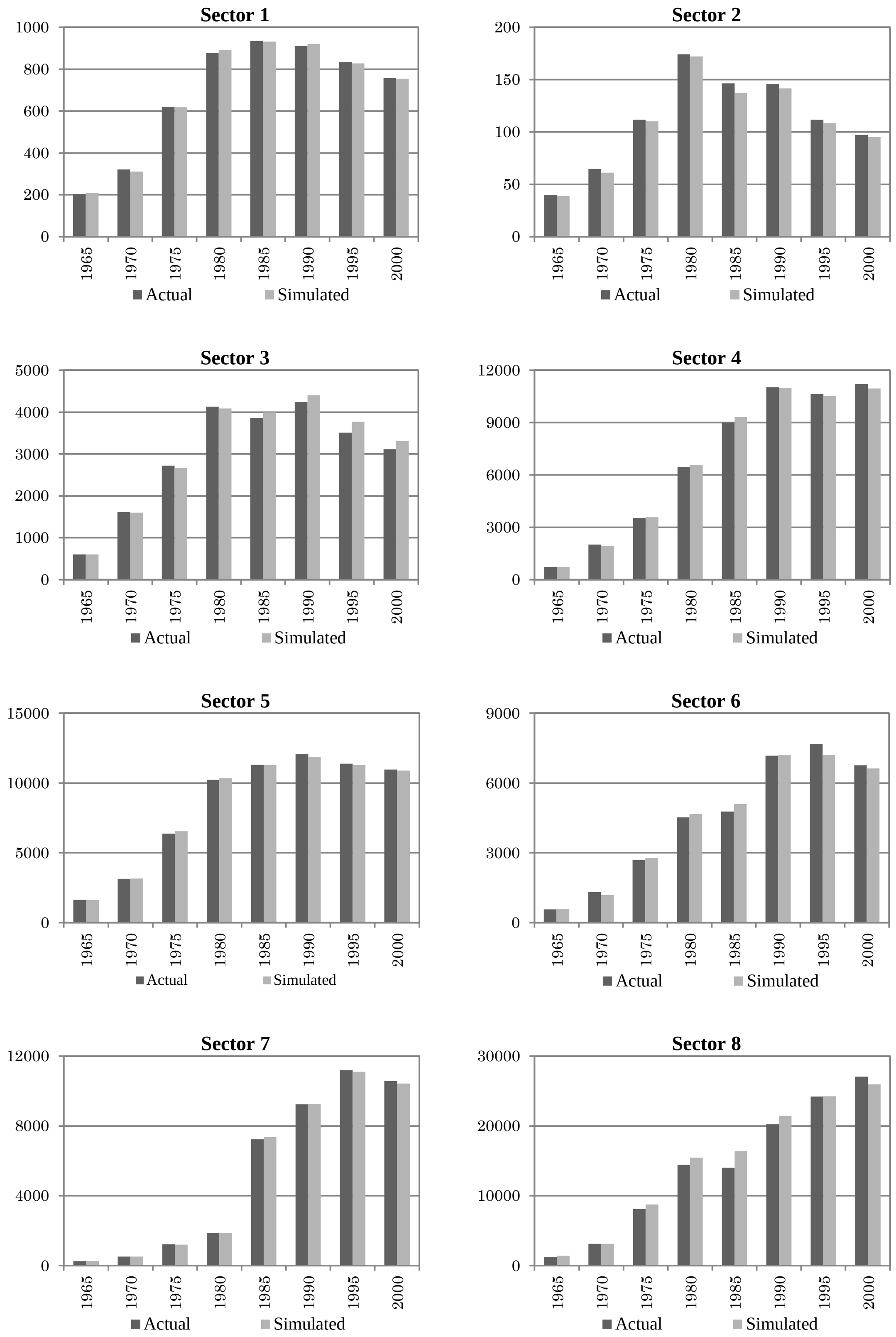


4.2 Ozaki Cost Function

4.2.1 Specification of Cost Function

By putting estimated parameters of equation (3.27) and (3.28) into Ozaki Function (3.23), we can compute empirical cost function (theoretical value). Empirical cost function is shown in Figure 4. The Figure 4 also shows actual total cost value in order to show the gap where actual cost value is calculated by adding intermediate input and wage in IIO9. The estimated result is acceptable.

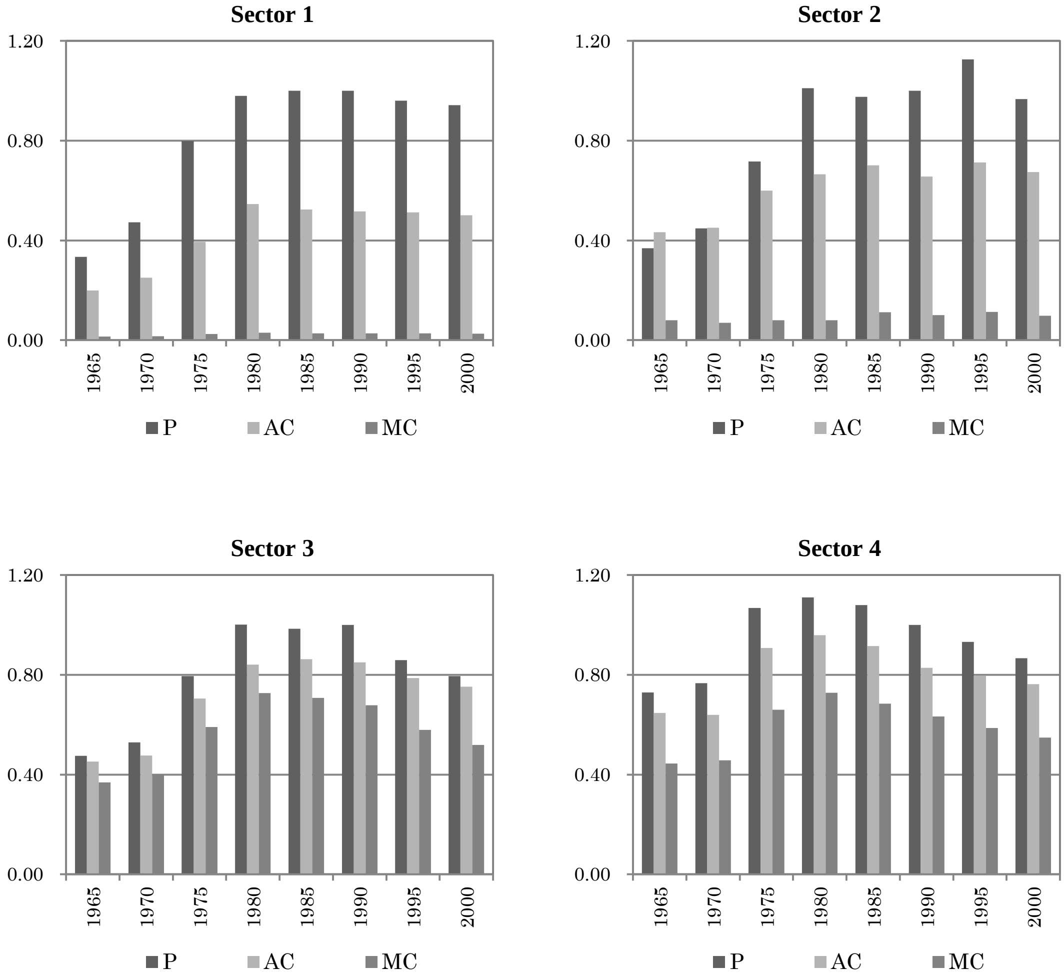
Figure 4 Empirical Cost Function

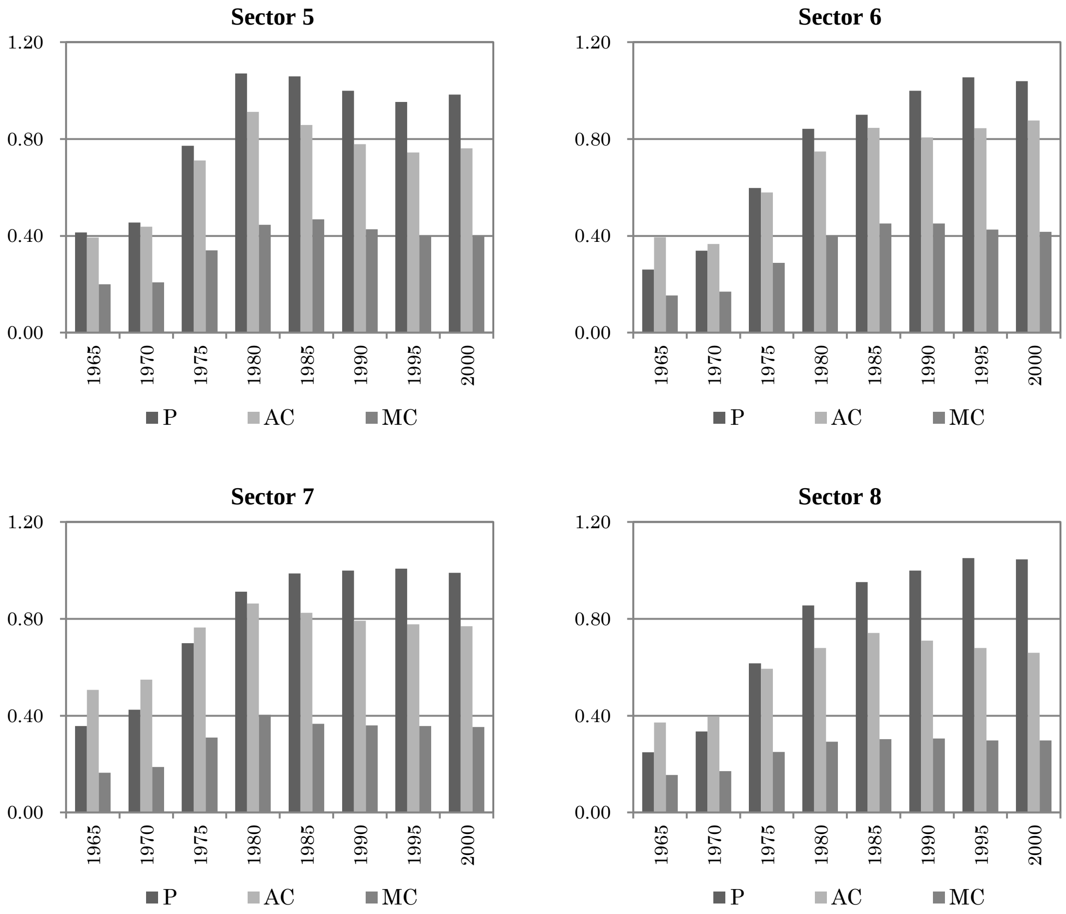


4.2.2 Specification of Average Cost and Marginal Cost

Following empirical cost function, we become to know empirical average and marginal cost. These costs are shown in Figure 5. As is well known in imperfect market, industry is interpreted to set price higher than marginal cost and to be eager to sell more at the current market price. Surplus of $p_j - MC_j$ is industry's profit of monopoly market. To uncover these relations, we insert price P (actual data p_j) , showing P, AC, and MC in Figure 5. As a whole, Figure 5 demonstrates that sectoral prices are set higher than marginal cost. In particular, primary industries (sector 1and secto2) and service industries (sector 7, and sector 8) show big surplus of $p_j - MC_j$. However, in the manufacturing industry, market power is relatively small. This result reflect severe competition of manufacturing industries in Japan in recent years.

Figure 5 Average Cost (AC) and Marginal Cost (MC)





<u>Return to Scale, Rate of Technical Progress, and Total Factor Productivity</u>

Following empirical cost function, we can know empirical return to scale, rate of technical progress and economy and TFP.

Table 6 Estimates of Cost Function

	Panel A: Return to Scale in Equation (3.29)							
Economy	Sector 1	Sector 2	Sector 3	Sector 4	Sector 5	Sector 6	Sector 7	Sector 8
1965	18.369	6.738	1.107	1.353	1.728	1.662	2.329	2.167
1970	18.963	7.551	1.092	1.321	1.843	1.562	2.272	2.121
1975	16.919	8.042	1.153	1.397	2.024	1.738	2.226	2.319
1980	17.859	8.271	1.157	1.345	2.059	1.799	2.085	2.309
1985	19.017	6.226	1.217	1.358	1.846	1.823	2.244	2.441
1990	19.020	6.532	1.252	1.309	1.820	1.788	2.200	2.327
1995	18.875	6.341	1.320	1.341	1.852	2.014	2.183	2.291
2000	19.541	6.884	1.379	1.353	1.907	2.130	2.175	2.219

	Panel B: Rate of Technical Progress in Equation (3.30)							
Economy	Sector 1	Sector 2	Sector 3	Sector 4	Sector 5	Sector 6	Sector 7	Sector 8
1965	0.305	3.039	0.051	0.063	0.239	0.060	0.019	0.003
1970	0.327	3.154	0.049	0.055	0.223	0.047	0.005	0.004
1975	0.342	3.219	0.057	0.060	0.227	0.049	0.005	0.006
1980	0.398	2.899	0.058	0.062	0.208	0.047	0.006	0.008
1985	0.405	1.754	0.066	0.052	0.165	0.038	0.006	0.008
1990	0.382	1.570	0.067	0.050	0.151	0.033	0.005	0.008
1995	0.361	1.575	0.073	0.050	0.144	0.037	0.005	0.007
2000	0.377	1.665	0.077	0.050	0.141	0.033	0.004	0.007

	Panel C: Total Factor Productivity (TFP) in Equation (3.31)							
Economy	Sector 1	Sector 2	Sector 3	Sector 4	Sector 5	Sector 6	Sector 7	Sector 8
1965	-0.204	0.383	0.040	0.043	0.132	0.034	0.005	-0.004
1970	-0.201	0.368	0.039	0.039	0.115	0.028	-0.001	-0.006
1975	-0.197	0.372	0.041	0.039	0.105	0.025	-0.001	-0.005
1980	-0.192	0.327	0.040	0.043	0.092	0.023	0.000	-0.005
1985	-0.194	0.259	0.043	0.037	0.078	0.017	-0.001	-0.005
1990	-0.186	0.203	0.043	0.037	0.073	0.015	-0.001	-0.005
1995	-0.184	0.201	0.046	0.036	0.069	0.015	-0.001	-0.004
2000	-0.183	0.217	0.046	0.036	0.065	0.012	-0.001	-0.005

4.3 Conjectural Variation

The estimated value of conjectural variation is shown in Figure 6. In an economics viewpoint, the conjectural variation is considered to be the reaction to the rival in the market. In another aspect, the conjectural variation can be viewed as the gap between theoretical and actual prices. Left hand side $\tilde{p}_j$of (3.43) or (3.44) with zero conjectural variation is considered to be price of monopoly.

a) $p_j > \tilde{p}_j$: conjectural variation positive

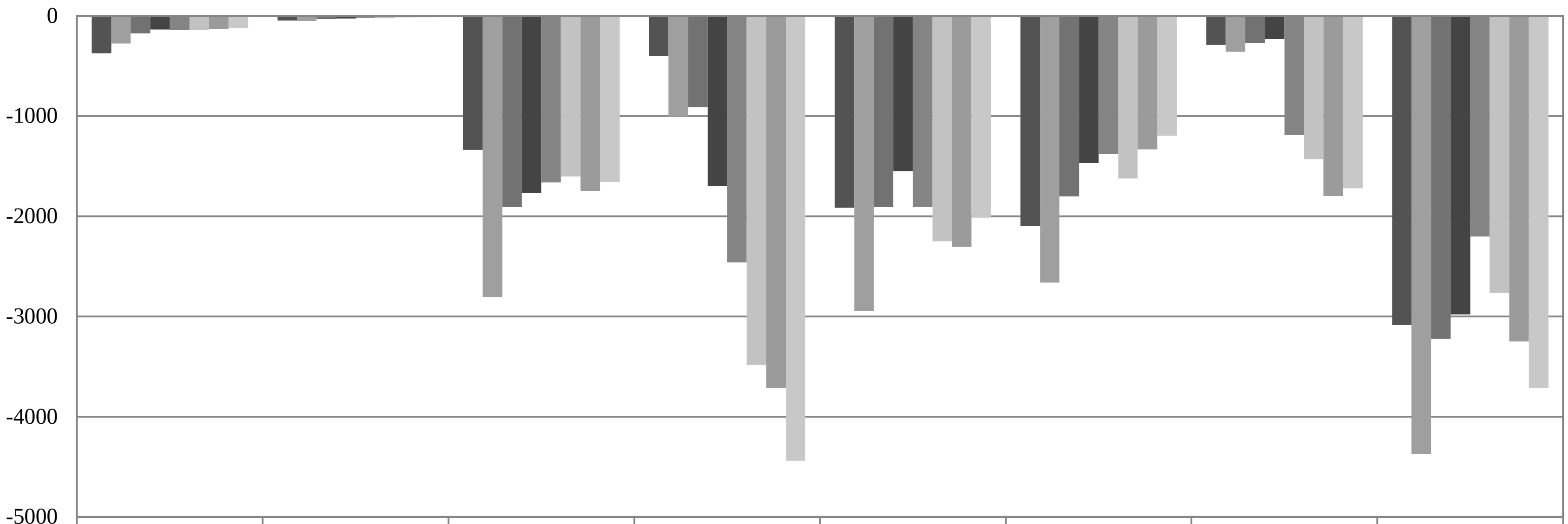
b) $p_j < \tilde{p}_j$: conjectural variation negative (more competitive than monopoly)

If the numerical value is small in b), it is considered that conjecture to the rival is low, in other words, less competitive. On the other hand, if the numerical value is large in b), it can be thought that the amount of the conjecture to the rival is large, viz. more competitive. It means that market is distant from monopoly.

Turning to our simulated results, it is demonstrated that the sector 1, the sector 2 and the sector

7 have small value, which means these industries are close to monopoly. On the other hand, as the rest of sector (the sector 3, the sector 4, the sector 5, the sector 6, and sector 8) show large value, it could be suggested that these industries are far from monopoly. It is noteworthy that sector 4 (machinery industry) is an leading industry in the Japanese economy, showing extraordinary value in conjectural variation.

Figure 6 Conjectural Variation



	Sector 1	Sector 2	Sector 3	Sector 4	Sector 5	Sector 6	Sector 7	Sector 8
1965	-373.4502781	-47.80044887	-1339.746608	-402.8831403	-1913.406628	-2095.351093	-292.1479653	-3085.512364
1970	-275.5506142	-51.94596386	-2806.446862	-1002.953696	-2946.018748	-2661.822083	-360.0908881	-4372.060226
1975	-174.9209895	-32.19401405	-1910.142361	-912.5002819	-1909.428276	-1802.61238	-274.9345777	-3223.880297
1980	-137.1939065	-27.64105264	-1766.428781	-1698.505091	-1550.001222	-1468.586496	-231.2503922	-2980.896138
1985	-144.8530453	-22.80159329	-1662.816946	-2460.792286	-1908.497156	-1379.735235	-1190.573468	-2202.754525
1990	-142.8786051	-24.08286494	-1604.249947	-3483.061586	-2248.659792	-1624.288181	-1431.242596	-2766.963314
1995	-134.4546402	-14.69754509	-1747.090791	-3711.379556	-2305.062879	-1332.153587	-1799.107777	-3249.577963
2000	-121.4039499	-16.51942187	-1659.164436	-4440.857111	-2014.069081	-1197.190366	-1721.862425	-3713.393848

[unit:1000]

5. The Results of Final Test

In order to evaluate model traceability, we further compute Root Mean Square Error (RMSE)[3] for selected variables in Table 7. Although some variables have room to be improved, the calculated values in final test are considered to trace the actual values sufficiently. Thus, we could accept interregional input-output model for simulation analysis.

Table 7 The Result of Final Test

Panel A: RMSE of Price								
Economy	Sector 1	Sector 2	Sector 3	Sector 4	Sector 5	Sector 6	Sector 7	Sector 8
	0.097	0.105	0.082	0.052	0.072	0.050	0.407	0.112

Panel B: RMSE of Output								
Economy	Sector 1	Sector 2	Sector 3	Sector 4	Sector 5	Sector 6	Sector 7	Sector 8
	0.059	0.109	0.065	0.032	0.052	0.010	0.167	0.100

Panel C: RMSE of Intermediate Demand								
Economy	Sector 1	Sector 2	Sector 3	Sector 4	Sector 5	Sector 6	Sector 7	Sector 8
Sector 1	—	0.303	0.990	—	0.078	0.346	—	0.212
Sector 2	—	0.104	0.138	0.509	—	0.076	0.810	—
Sector 3	—	0.247	0.075	0.096	—	0.081	0.240	—
Sector 4	—	—	0.513	0.044	—	0.167	—	—
Sector 5	0.111	—	0.221	0.044	0.063	0.052	0.425	0.189
Sector 6	—	—	—	0.528	0.532	—	0.345	0.131
Sector 7	0.070	0.217	0.072	0.069	0.066	0.139	0.250	0.219
Sector 8	0.056	—	0.090	0.070	0.059	0.236	0.830	0.361

Panel D: RMSE of Cost Function								
Economy	Sector 1	Sector 2	Sector 3	Sector 4	Sector 5	Sector 6	Sector 7	Sector 8
	0.121	0.105	0.148	0.091	0.127	0.132	0.578	0.158

---

[3] $RMSEP = \sqrt{\frac{1}{T}\sum_{t=1}^{T}\left(\frac{P_t - A_t}{A_t}\right)^2}$. P_t is theoretical value. A_t is actual value. T is sample size. As our interregional input-output model of nine regions is composed of eight times, $T = 8$.

Panel E: RMSE of Labor

Economy	Sector 1	Sector 2	Sector 3	Sector 4	Sector 5	Sector 6	Sector 7	Sector 8
Hokkaido	0.051	0.172	0.029	—	0.035	0.041	0.122	0.043
Tohoku	0.057	0.038	—	0.113	0.042	0.045	0.138	0.057
Kanto	—	0.078	0.048	—	0.037	0.042	0.123	0.074
Chubu	—	0.057	—	0.089	0.047	0.038	0.102	0.067
Kinki	0.030	—	0.038	—	—	—	0.103	0.064
Chugoku	—	0.039	0.028	—	0.034	0.028	0.097	0.055
Shikoku	0.023	0.036	0.061	0.045	0.038	0.033	0.097	0.053
Kyushu	0.044	0.099	0.135	0.090	0.033	0.037	0.142	0.060
Okinawa	—	0.150	0.114	—	—	0.098	0.285	0.098

Panel F: RMSE of Wage Rate

Economy	Sector 1	Sector 2	Sector 3	Sector 4	Sector 5	Sector 6	Sector 7	Sector 8
Hokkaido	0.131	0.132	0.154	0.254	0.118	0.142	0.178	0.046
Tohoku	0.110	0.119	0.194	0.155	0.163	0.167	0.264	0.074
Kanto	0.076	0.147	0.144	0.119	0.115	0.159	0.311	0.062
Chubu	0.087	0.066	0.128	0.138	0.117	0.159	0.308	0.066
Kinki	0.114	0.198	0.109	0.111	0.106	0.210	0.353	0.063
Chugoku	0.123	0.134	0.141	0.118	0.137	0.160	0.178	0.078
Shikoku	0.104	0.136	0.166	0.165	0.134	0.207	0.245	0.081
Kyushu	0.089	0.131	0.208	0.154	0.118	0.157	0.241	0.060
Okinawa	0.600	0.361	0.646	0.842	0.894	0.481	0.528	0.334

Panel G: RMSE of Consumption

	Sector 1	Sector 2	Sector 3	Sector 4	Sector 5	Sector 6	Sector 7	Sector 8	Sector 9
Hokkaido	0.155	—	0.404	0.239	0.138	0.153	0.531	0.232	0.271
Tohoku	0.083	—	0.295	0.135	0.058	0.094	0.433	0.143	0.302
Kanto	0.110	—	0.285	0.171	0.051	0.096	0.363	0.100	0.234
Chubu	0.077	—	0.304	0.135	0.055	0.078	0.489	0.065	0.166
Kinki	0.099	—	0.348	0.188	0.077	0.116	0.395	0.135	0.354
Chugoku	0.140	—	0.448	0.130	0.057	0.081	0.504	0.087	0.245
Shikoku	0.161	—	0.381	0.230	0.086	0.116	0.554	0.160	0.296
Kyushu	0.082	—	0.296	0.167	0.074	0.108	0.364	0.140	0.254
Okinawa	0.177	—	0.443	0.437	0.215	0.201	0.878	0.297	0.238

	Panel H: RMSE of Property Income								
Economy	Hokkaido	Tohoku	Kanto	Chubu	Kinki	Chugoku	Shikoku	Kyushu	Okinawa
	0.249	0.119	0.148	0.248	0.170	0.133	0.145	0.139	0.076

* Sectors 1, 2, 3, 4, 5, 6, 7 and 8 denote agriculture, mining, manufacture of metal product, manufacture of machinery, miscellaneous manufacturing industries, construction, wholesale and retail trades and transportation trade and transportation, services, respectively. The 9-th commodity implies future consumption (increase of saving).

* RMSE shows the root mean squared error.

6. Conclusion

This paper tried to design the microeconomic oriented model for multi-region and multi-sector in a framework of Japanese interregional input-output system from 1965 to 2000. The main variables (i.e. private consumption, intermediate demand, labor, and price) are endogenized, which are derived from the microeconomic optimization theory. Then, we simulated the model and made some indeces of expressing structure of Japanese economy such as economy scale, elasticity of price or demand, technical progress, and total factor productivity.

To close this article the special remarks on modeling are pointed out as follows.

1) Contrast to CGE

The most of empirical studies on the microeconomic foundation models takes Computable General Equilibrium model (CGE) nowadays. On CGE modeling, the parameters of model equations are estimated by calibration. This modeling approach may have plausible aspect in a sense of seeking “normative” direction, but lacks empirical validity of model. In contrast, our modeling approach attempts to make models within historical data of multi period.

2) Price in Demand/Supply Nexus

Generally, the price has numerous affects on the whole economic system. It can be said that price dominates the reliability of model without exaggeration. However, in the price model of primary input-output system by Leontief, it is determined mainly on the cost, which has no interaction with final demand. Unfortunately it is too simple to reflect the mechanism of price determination in real economy.

On the contrary, this study poses realism of price determination. The determination of price in our approach not only has supply side through cost structure, but also takes demand side. In detail, the demand side is subject to various demand items such as household and government expenditures, and supply side through cost function comprehensively.

3) Future Issue

The econometric model is product of abstraction of reality. Then, the model needs to be extended so as to explain real world in depth. This research still has something to be challenged for future study. It is relating to conjectural variation. The conjectural variation is, in empirical sense, the gap between theoretical and empirical price. Another avenue for future research is to explore theory of market competition other than monopoly in input-output system to reduce the gap.

Again, our aim of building the input-output model based on microeconomics, could be achieved in some sense. The result can assure potential use of input-output model for practical policy recommendation. We would believe our attempt will contribute to potential applicability of input-output model.

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Appendix A1: Almost Ideal Demand System Model

Dual Problem

The demand system can be generated from dual structure: prime problem and dual problem. Prime problem of demand theory is basically assumed that consumer demand is determined on the base of the utility maximization under constrained budget. In contrast, the dual problem is assumed that consumer demand is determined on the base of expenditure minimization subject to a certain utility level. Almost Ideal Demand system (AIDS) is also deduced from solving expenditure minimization as the dual structure. The producer processes from expenditure function to AIDS model follows two approaches; via shepherd's lemma and Roy's identify. These producer processes are shown in this section.

A. Approach via Shephard's Lemma

First, we will explain the approach via Shephard's lemma. Deaton and Muellbaur [1980] have proposed consumer's expenditure function as follows:

$$C(u,p) = e^{a(p)+ub(p)} \tag{A1.01}$$

$$a(p) = \sum_{i=1}^{n} \alpha_i \log p_i + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \gamma_{ij} \log p_i \log p_j \tag{A1.02}$$

$$b(p) = \beta_0 \prod_{i=1}^{n} p_i^{\beta_i} \tag{A1.03}$$

$C(u,p)$	:	Consumer Cost function
u	:	Utility
$\alpha_i, \gamma_{ij}, \beta_i, \beta_0$	:	Parameters
p_i	:	Price in *i*-th commodity

Taking a logarithmic of both side of equation (A1.01), we can obtain following equation.

$$\log C(u,p) = \log M = a(p) + ub(p) \tag{A1.04}$$

This function is composed of two parts: the effect of the given price and the utility level. M means the expenditure function. Substituting equation (A1.02) and (A1.03) into (A1.04), we yield equation (A1.05).

$$\log M = a(p) + ub(p) = \sum_{i=1}^{n} \alpha_i \log p_i + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \gamma_{ij} \log p_i \log p_j + u\beta_0 \prod_{i=1}^{n} p_i^{\beta_i} \tag{A1.05}$$

Applying Shepheard's Lemma leads to the equation (A1.06).

$$\frac{\partial M}{\partial p_i} = \frac{\partial\{a(p) + ub(p)\}}{\partial p_i} e^{a(p)+ub(p)} \tag{A1.06}$$

By solving equation and rearranging equation (A1.06), Hicksian compensated demand function is derived as follows:

$$CPR_i = \left\{\frac{\alpha_i}{p_i} + \frac{1}{p_i}\left(\sum_{j=1}^{n} \gamma_{ij} \log p_j\right) + u\frac{\beta_0\beta_i}{p_i}\prod_{i=1}^{n} p_i^{\beta_i}\right\} M \tag{A1.07}$$

Here, the utility u of equation (A1.07) needs to be explained in more detail. As the expenditure function and the indirect utility function are the inverse relation (duality) each other as a result of duality, we can derive the indirect utility function by inverting expenditure function (A1.05).

$$u = V(p_i, M) = \frac{\log M - \sum_{i=1}^{n} \alpha_i \log p_i - \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \gamma_{ij} \log p_i \log p_j}{\beta_0 \prod_{i=1}^{n} p_i^{\beta_i}} \tag{A1.08}$$

in which $V(p_i, M)$ is the indirect utility function. We substitute the indirect utility function (A1.08) into demand function (A1.07), which leads to derive Marshallian demand function as:

$$\begin{aligned} CPR_i = \frac{M}{p_i}\Bigg\{\alpha_i + \left(\sum_{j=1}^{n} \gamma_{ij} \log p_j\right) \\ + \beta_i\left(\log M - \sum_{i=1}^{n} \alpha_i \log p_i - \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \gamma_{ij} \log p_i \log p_j\right)\Bigg\} \end{aligned} \tag{A1.09}$$

$$\log P = \sum_{i=1}^{n} \alpha_i \log p_i + \sum_{i=1}^{n}\sum_{j=1}^{n} \gamma_{ij} \log p_i \log p_j \tag{A1.10}$$

We rearrange equation (A1.09) by using equation (A1.10), then we can obtain AIDS model as:

$$\begin{aligned} \omega_i = \frac{p_i CPR_i}{M} = \alpha_i + \beta_i \log\frac{M}{P} + \sum_{j=1}^{n} \gamma_{ij} \log p_j \\ \log P = \sum_{i=1}^{n} \alpha_i \log p_i + \sum_{i=1}^{n}\sum_{j=1}^{n} \gamma_{ij} \log p_i \log p_j \end{aligned} \tag{A1.11}$$

where ω_i means budget share (the proportion of income) purchased for the *i*-th good with $\sum_{i=1}^{n} \omega_i$=1. Parameters have to have three restrictions in AIDS model as follows:

(i) Additivity $\sum_{i=1}^{n} \alpha_i = 1$ $\sum_{i=1}^{n} \beta_i = 0$ $\sum_{i=1}^{n} \gamma_{ij} = 0$

(ii) Homogeneity $\sum_{i=1}^{n} \gamma_{ij} = 0$

(iii) Symmetry $\gamma_{ij} = \gamma_{ji}$ $\forall ij(i \neq j)$

B. Approach from Roy' Identity

Hereafter, we show approach from Roy's Identity. Roy' identity is the means to derive a demand function from the indirect utility function, which states that the demand for a good is equal to the derivative of the indirect utility.

$$CPR_i = -\left\{\frac{\partial V(p_i, M)}{\partial p_i} \bigg/ \frac{\partial V(p_i, M)}{\partial M}\right\} \quad \text{(A1.12)}$$

where CPR_i is the Marshallian demand function of *i*-th good. $V(p_i, M)$ is equation (A1.08).We solve equation (A1.12) and rearrange equation. The demand function can be obtained as:

$$CPR_i = \frac{M}{p_i}\left(\sum_{j=1}^{n} \gamma_{ij} \log p_j + \beta_i \log M + \alpha_i - \beta_i \sum_{i=1}^{n} \alpha_i \log p_i - \beta_i \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij} \log p_i \log p_j\right) \quad \text{(A1.13)}$$

we can obtain AIDS model which is same model of equation (A1.11)

Appendix A2: Almost Ideal Demand System Model for MRMS

<u>Almost Ideal Demand System Model for MRMS (IIO9)</u>

AIDS model is used for MRMS system. We show optimization of AIDS for MRMS system, following approaches in Appendix A1.

A'. Input-Output Version Approach from Roy' identity

Here, this model has to be extended from consumer's expenditure function (A1.01) for MRMS system (IIO9). We assume consumer expenditure system for each region. The expenditure function of the *k*-th region is rewritten as:

$$C^k(u^k, p) = e^{a^k(p) + u^k b^k(p)} \qquad k = 1 \dots .9 \quad \text{(A2.01)}$$

$$a^k(p) = \sum_{i=1}^{n} \alpha_i^k \log p_i + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij}^k \log p_i \log p_j \quad \text{(A2.02)}$$

$$b^k(p) = \beta_0^k \prod_{i=1}^{n} p_i^{\beta_i^k} \tag{A2.03}$$

$C^k(u^k, p)$	:	Cost Function of *k*-th region in current price
u^k	:	Utility of *i*-th commodity *k*-th region
$\alpha_i^k, \gamma_{ij}^k, \beta_i^k, \beta_0^k$	:	Parameters of *i*-th commodity of *k*-th region
p_i	:	*i*-th sectoral price

Taking a logarithmic of both side of equation (A2.01), we can obtain following equation.

$$\log C^k(u^k, p) = \log M^k = a^k(p) + u^k b^k(p) \qquad k = 1 \ldots .9 \tag{A2.04}$$

where M^k means expenditure function of *k*-th region in current price. We substitute equations (A2.02) and (A2.03) into equation (A2.04) as:

$$\log M^k = \sum_{i=1}^{n} \alpha_i^k \log p_i + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij}^k \log p_i \log p_j + u^k \beta_0^k \prod_{i=1}^{n} p_i^{\beta_i^k} \tag{A2.05}$$

Applying the Shephard's lemma and rearranging the terms, we obtain demand function of *k*-th region for *i*-th commodity as:

$$CPR_i^k = \frac{\partial M^k}{\partial p_i} \tag{A2.06}$$

We can obtain Hicksian demand function as:

$$CPR_i^k = \left\{ \frac{\alpha_i^k}{p_i} + \frac{1}{p_i} \left(\sum_{j=1}^{n} \gamma_{ij}^k \log p_j \right) + u_i^k \frac{\beta_0^k \beta_i^k}{p_i} \prod_{i=1}^{n} p_i^{\beta_i^k} \right\} M^k \tag{A2.07}$$

Here, the utility u of equation (A2.07) needs to be explained in more detail. As the expenditure function and the indirect utility function are the inverse relations each other as a result of duality, we can derive the indirect utility function by inverting expenditure function (A2.05) as:

$$u^k = V^k(p_i, M^k) = \frac{M^k - \sum_{i=1}^{n} \alpha_i^k \log p_i - \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij}^k \log p_i \log p_j}{\beta_0^k \prod_{i=1}^{n} p_i^{\beta_i^k}} \tag{A2.08}$$

in which $V(p_i, M)$ is the indirect utility function. We substitute the indirect utility function (A2.08) into demand function (A2.07), which leads to derive Marshallian demand function as:

$$CPR_i^k = \frac{M^k}{p_i}\left\{\alpha_i^k + \left(\sum_{j=1}^{n}\gamma_{ij}^k \log p_j\right)\right.$$

$$\left. + \beta_i^k\left(\log M^k - \sum_{i=1}^{n}\alpha_i^k \log p_i - \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\gamma_{ij}^k \log p_i \log p_j\right)\right\} \quad \text{(A2.09)}$$

$$\log P^k = \sum_{i=1}^{n}\alpha_i^k \log p_i + \sum_{i=1}^{n}\sum_{j=1}^{n}\gamma_{ij}^k \log p_i \log p_j \quad \text{(A2.10)}$$

We rearrange equation (A2.09) by using equation (A2.10), then we can obtain AIDS model as:

$$\omega_i^k = \frac{CPR_i^k p_i}{M^k} = \alpha_i^k + \beta_i^k \log\frac{M^k}{P^k} + \sum_{j=1}^{n}\gamma_{ij}^k \log p_j$$

$$\log P^k = \sum_{i=1}^{n}\alpha_i^k \log p_i + \sum_{i=1}^{n}\sum_{j=1}^{n}\gamma_{ij}^k \log p_i \log p_j \quad \text{(A2.11)}$$

where ω_i is budget share (the proportion of income) purchased for the *i*-th good with $\sum_{i=1}^{n}\omega_i$=1.

B'. Input-Output Version Approach from Roy' identity:

Here, we show approach from Roy's Identity in same way of Appendix A1. Roy's identity for IIO9 model is as follows:

$$CPR_i^k = -\left\{\frac{\partial V(p_i, M^k)}{\partial p_i}\Big/\frac{\partial V(p_i, M^k)}{\partial M^k}\right\} \quad \text{(A2.12)}$$

where CPR_i^k is the Marshallian demand function of *i*-th good. $V(p_i, M^k)$ is equation (A2.08).We solve equation (A2.12) and rearrange equation. The demand function can be obtained as:

$$CPR_i^k = \frac{M^k}{p_i}\left(\sum_{j=1}^{n}\gamma_{ij}^k \log p_j + \alpha_i^k + \beta_i^k \log M^k\right.$$

$$\left. - \beta_i^k \sum_{i=1}^{n}\alpha_i^k \log p_i - \beta_i^k \sum_{i=1}^{n}\sum_{j=1}^{n}\gamma_{ij}^k \log p_i \log p_j\right) \quad \text{(A2.13)}$$

$$\log P^k = \sum_{i=1}^{n}\alpha_i^k \log p_i + \sum_{i=1}^{n}\sum_{j=1}^{n}\gamma_{ij}^k \log p_i \log p_j \quad \text{(A2.14)}$$

We rearrange equation (A2.13) by using equation (A2.14), then we can obtain AIDS model which is same model in (A2.11)

Appendix A3: Modifying AIDS for Empirical Analysis

Modifying AIDS for Empirical Analysis

We modify AIDS model (A2.11) in two directions below for empirical analysis.

a) Assuming $\gamma_{ij}^{k} = 0$.

Original AIDS model (A1.11) or (A2.11) is not only nonlinear model, but also has many parameters to be estimated, which make estimation impossible. Therefore, in order to overcome this difficulty, we don't impose restriction of homogeneity γ_{ij}^{k}.

b) Introducing the future consumption.

As for treatment of goods, we deal with future consumption as well as current consumption. Consumers confront the decision both current consumption and saving for future consumption. In order to take into consideration consumer's savings behavior, we define future consumption as 9th goods. We denote i=1 to 9. Accordingly, the indirect utility function and AIDS model are rewritten as:

$$u^{k} = V^{k}(p_i, M) = \frac{\log M^{k} - \sum_{i=1}^{n} \alpha_i^{k} \log p_i}{\beta_0^{k} \prod_{i=1}^{n} p_i^{\beta_i^{k}}} = \log\left(\frac{M^{h}}{\prod_{i=1}^{n} p_i^{\alpha_i^{k}}} e^{-\beta_0^{h} \prod_{i=1}^{n} p_i^{\beta_i}} \right) \qquad \text{(A3.01)}$$

$$\omega_i^{k} = \frac{p_i CPR_i^{k}}{M^{k}} = \alpha_i^{k} + \beta_i^{k} \log\left(\frac{M^{k}}{\sum_{j=1}^{n} \alpha_j^{k} \log p_j} \right). \qquad \text{(A3.02)}$$

where $\sum_{i=1}^{n} \alpha_i^{k} = 1$ and $\sum_{i=1}^{n} \beta_i^{k} = 1$ as parameter restrictions. We utilize AIDS model of equation (A3.02) into IIO9.

Appendix A4: Cost Function

<u>Cost Function by M.A.Fuss</u>

M.A.Fuss proposed a generalized Leontief cost function which comprises Leontief cost function as a special case. [M.A.Fuss,1977]

$$C(p, y, t) = \sum_i \sum_j h_{ij}(y, t) \sqrt{p_i} \sqrt{p_j} \qquad \text{(A4.01)}$$

y : Output Price

p : Input Price

$h_{ij}(y,t)$: Symmetric and Concave

Since M.A.Fuss developed this equation, many geometricians have specified $h_{ij}(y,t)$ in various way. There are pioniaring works such as E.Berndt&M.S.Khaled [1979], W.E.Diewert&T.Wales [1987] and S.Nakamura [1990].

Ozaki Cost Function

S.Nakamura [1990] exposed $h_{ij}(y,t)$ in Fuss cost function, and named Generalized Ozaki cost function.

$$C(p,y,t) = \sum_i b_{ii} y^{b_{yi}} e^{b_{ti}t} p_i + \sum_{i \neq j} b_{ij} \sqrt{p_i} \sqrt{p_j} y^{b_y} e^{b_t t} \quad \text{(A4.02)}$$

$$h_{ii}(y,t) = b_{ii} y^{b_{yi}} e^{b_{ti}t} \quad \text{for } i = j$$
$$h_{ij}(y,t) = b_{ij} y^{b_y} e^{b_t t} \quad \text{for } i \neq j \quad \text{(A4.03)}$$

$C(p,y,\mathrm{t})$: The unit cost function

$b_{ii}, b_{yi}, b_{ti}, b_{ij}, b_y, b_t$: Parameters.

y : Output

t : Time index to capture the effect of technical change

p_i : *i*-th factor price

Appling Ozaki Cost Function to Interregional Input-Output System of Nine Regions

Generalized Ozaki cost function (A4.02) has many parameters to be estimated. Input-output model has not so sufficient sample sizes. In order to meet degree of freedom, we omit the diagonal term of $i \neq j$ in (A4.03) as:

$$C(p,y,t) = \left[\sum_i b_{ii} y^{b_{yi}} e^{b_{ti}t} p_i \right] \quad \text{(A4.04)}$$

To adjust function (A4.04) into cost function for input-output model, we consider cost function for each sector, which leads to amend every subscript and index of variables for input-output model. Factors of production are considered mainly two factors; intermediate material and labor. We can specify these factors as:

$$C_j = \sum_l^n b_{jl}(p) \cdot p_l \cdot XXR_j^{\,b_{jl}(X)} \cdot e^{b_{jl}(t)t} + \sum_{m=1}^r b_j^m(w) \cdot w_j^m \cdot XXR_j^{\,b_j^m(X)} \cdot e^{b_j^m(t)t} \quad \text{(A4.05)}$$

C_j : Cost function of *j*-th industy.

$b_{jl}(p), b_{jl}(X), b_{jl}(t)$: Estimation parameters.

$b_j^m(w), b_j^m(X), b_j^m(t)$: Estimation parameters.

XXR_j : Output of *j*-th industry.

t	:	Progress of technology.
w_j^m	:	Factor price (wage rate) of *j*-th industry in *m*-th region.
p_l	:	*l*-th factor price (sectoral price).

Appendix A5: Total Factor Productivity

We show calculation process of TFP for MRMS system.

Total Factor Productivity (TFP)

TFP of *j*-th sector in *k*-th country is defined below.

$$\frac{d\log TFP_j}{dt} = \frac{d\log XXR_j}{dt} - \frac{d\log C_j\left(XXR_j, p, w, R_j(t)\right)}{dt} \quad \text{(A5.01)}$$

where

$$\begin{aligned}\frac{d\log C_j\left(XXR_j, p, w, R_j(t)\right)}{dt} &= \left(\frac{d\log C_j}{d\log XXR_j}\right)\left(\frac{d\log XXR_j}{dt}\right) \\ &+ \sum_{l=1}^{N}\left(\frac{d\log C_j}{d\log p_l}\right)\left(\frac{d\log p_l}{dt}\right) \\ &+ \sum_{m=1}^{M}\left(\frac{d\log C_j}{d\log w_j^m}\right)\left(\frac{d\log w_j^m}{dt}\right) \\ &+ \sum_{k=1}^{N+M}\left(\frac{d\log C_j}{d\log R_{jk}(t)}\right)\left(\frac{d\log R_{jk}(t)}{dt}\right)\end{aligned} \quad \text{(A5.02)}$$

We have the following frame TFP by the above two equations (A5.01)-(A5.02).

$$\begin{aligned}\frac{d\,log\,TFP_j}{dt} = &\left(1 - \frac{dC_j}{dXXR_j}\frac{XXR_j}{C_j}\right)\left(\frac{d\log XXR_j}{dt}\right) - \sum_{l=1}^{N}\left(\frac{d\log C_j}{d\log p_l}\right)\left(\frac{d\log p_l}{dt}\right) \\ &- \sum_{m=1}^{M}\left(\frac{d\log C_j}{d\log w_j^m}\right)\left(\frac{d\log w_j^m}{dt}\right) \\ &- \sum_{k=1}^{N+M}\left(\frac{d\log C_j}{d\log R_{jk}(t)}\right)\left(\frac{d\log R_{jk}(t)}{dt}\right)\end{aligned} \quad \text{(A5.03)}$$

where $\frac{dC_j}{dXXR_j}\frac{XXR_j}{C_j} = MC_j/AC_j$, namely, economy of scale $1/SE_j$. The determination of w_j^m is defined in (3.45) which is not dependent on time *t*. Thus, this term is zero and not related on calculation of TFP. Hence, we arrange (A5.03) and derive (A5.04).

$$\frac{d\,log\,TFP_j}{dt} = \left(1-\frac{1}{SE_j}\right)\left(\frac{d\log XXR_j}{dt}\right) - \sum_{l=1}^{N}\left(\frac{d\log C_j}{d\log p_l}\right)\left(\frac{d\log p_l}{dt}\right) - \sum_{k=1}^{N+M}\left(\frac{d\log C_j}{d\log R_{jk}(t)}\right)\left(\frac{d\log R_{jk}(t)}{dt}\right) \quad \text{(A5.04)}$$

To simplify this calculation, each process of differentiation is replaced with other notation.

$$\frac{d\,log\,TFP_j}{dt} = \left(1-\frac{1}{SE_j}\right)\left(DLXXR_j\right) - \sum_{l=1}^{N}\left(DCP_{jl}\right)\left(DLP_l\right) - \sum_{k=1}^{N+M}\left(DCR_{jk}\right)\left(DLR_{jk}\right) \quad \text{(A5.05)}$$

Each differentiation is explained below.

1) The calculation: $DLXXR_j$

$$DLXXR_j = \frac{d\,log\,XXR_j}{dt} \quad \text{(A5.06)}$$

where we have XXR_j in equation (3.01). $DLXXR_j$ is rewritten as follows:

$$DLXXR_j = \frac{d\log\left(\sum_{l=1}^{n} xvr_{jl} + \sum_{k=1}^{r} CPR_j^k + \sum_{k=1}^{r} CGR_j^k + \sum_{k=1}^{r} IR_j^k + \sum_{k=1}^{r} IVR_j^k + \sum_{k=1}^{r} EXR_j^k + \sum_{k=1}^{r} IMR_j^k\right)}{dt} \quad \text{(A5.07)}$$

In (A5.07), the term related to time *t* is intermediate demand which is derived from Ozaki cost function (3.23). Then, (A5.07) is shown simply as follows:

$$DLXXR_j = \frac{d\log\left(\sum_{l=1}^{n} xvr_{jl}\right)}{dt}$$

$$\text{where} \quad xvr_{jl} = b_{jl}(p)\cdot\left(XXR_j\right)^{b_{jl}(X)}\cdot\left(p_j^e\right)^{b_{jl}(pe)}\cdot e^{b_{jl}(t)t} \quad \text{(A5.08)}$$

Hence,

$$DLXXR_j = \frac{\sum_{l=1}^{n} b_{jl}(p)\cdot\left(XXR_j\right)^{b_{jl}(X)}\cdot\left(p_j^e\right)^{b_{jl}(pe)}\cdot b_{jl}(t)\cdot e^{b_{jl}(t)t}}{\sum_{l=1}^{n} b_{jl}(p)\cdot\left(XXR_j\right)^{b_{jl}(X)}\cdot\left(p_j^e\right)^{b_{jl}(pe)}\cdot e^{b_{jl}(t)t}} \quad \text{(A5.09)}$$

2) The calculation: DLP_l

$$DLP_l = \frac{d\log p_l}{dt}$$

$$\text{where} \quad p_l = \left(\frac{\varepsilon_l}{\varepsilon_l - 1}\right)MC_l \quad \text{(A5.10)}$$

The determination of price in DLP_l is defined in equation (3.43).We transform this equation to logarithmic-differential form on both sides, and then consider partial differentiation by *t*.

$$\frac{d\log p_l}{dt} + \frac{d\log(\varepsilon_l - 1)}{dt} = \frac{d\log \varepsilon_l}{dt} + \frac{d\log MC_l}{dt} \tag{A5.11}$$

Here, from (3.43) - (3.44), the definition of ε_l then becomes

$$\varepsilon_l = -\frac{\partial XXR_l}{\partial p_l} \cdot \frac{p_l}{XXR_l} = -\left(\frac{\partial CPR_l}{\partial p_l} + \lambda_l\right) \cdot \frac{p_l}{XXR_l} \tag{A5.12}$$

where CPR_l is defined in AIDS model of (3.08) and (3.10), which does not depend on time *t* at all. Thus, the differentiation of ε_l is,

$$\frac{d\log \varepsilon_l}{dt} = 0 \tag{A5.13}$$

Accordingly, (A5.11) is restated by simpler form.

$$\begin{aligned} DLP_l &= \frac{d\log p_l}{dt} = \frac{d\log MC_l}{dt} \\ &= \frac{1}{MC_l}\frac{dMC_l}{dt} \end{aligned} \tag{A5.14}$$

3) The calculation: $\boldsymbol{DLR_{jk}}$

$$DLR_{jk} = \left(\frac{d\log R_{jk}(t)}{dt}\right) \tag{A5.15}$$

ote that $R(t)$ is as follows:

$$R_j(t) = \left[R_{j1}, R_{j2}, , , , , R_{jn}, R_{j,n+1}, , , , R_{j,n+r}\right] \quad \text{where} \quad R_{jk}(t) = e^{b_{jk}(t)t} \tag{A5.16}$$

Therefore, the partial differentiation of DLR_{jk} is shown as,

$$\begin{aligned} DLR_{jk} &= \frac{d\log R_{jk}(t)}{dt} \\ &= \frac{1}{R_{jk}(t)}\frac{dR_{jk}(t)}{dt} \\ &= b_{jk}(t) \end{aligned} \tag{A5.17}$$

4) The calculation: $\boldsymbol{DCP_{jl}}$

$$DCP_{jl} = \frac{d\log C_j}{d\log p_l}$$

$$= \frac{dC_j}{dp_l}\frac{p_l}{C_j} \tag{A5.18}$$

From C_j in (3.23), we can rewrite this as follows:

$$DCP_{jl} = \frac{d\begin{bmatrix} \sum_l^n b_{jl}(p)\cdot XXR_j{}^{b_{jl}(X)}\cdot e^{b_{jl}(t)t}\cdot p_l\cdot p_j^{e\,b_{jl}(pe)} \\ +\sum_{m=1}^r b_j^m(w)\cdot XXR_j{}^{b_j^m(X)}\cdot e^{b_j^m(t)t}\cdot w_j^m\cdot p_j^{e\,b_j^m(pe)} \end{bmatrix}}{dp_l}\left(\frac{p_l}{C_j}\right) \tag{A5.19}$$

$$= b_{jl}(p)\cdot XXR_j{}^{b_{jl}(X)}\cdot e^{b_{jl}(t)t}\cdot p_j^{e\,b_{jl}(pe)}\left(\frac{p_l}{C_j}\right)$$

5) The calculation: $\boldsymbol{DCR_{jk}}$

$$DCR_{jk} = \frac{d\log C_j}{d\log R_{jk}(t)}$$

$$= \frac{dC_j}{dR_{jk}(t)}\left(\frac{R_{jk}(t)}{C_j}\right) \tag{A5.20}$$

By cost function of (3.23), we could rewrite this equation.

$$DCR_{jk} = \frac{d\begin{bmatrix} \sum_l^n b_{jl}(p)\cdot XXR_j{}^{b_{jl}(X)}\cdot e^{b_{jl}(t)t}\cdot p_l\cdot p_j^{e\,b_{jl}(pe)} \\ +\sum_{m=1}^r b_j^m(w)\cdot XXR_j{}^{b_j^m(X)}\cdot e^{b_j^m(t)t}\cdot w_j^m\cdot p_j^{e\,b_j^m(pe)} \end{bmatrix}}{dR_{jk}(t)}\left(\frac{R_{jk}(t)}{C_j}\right) \tag{A5.21}$$

$$= \frac{d\begin{bmatrix} \sum_l^n b_{jl}(p)\cdot XXR_j{}^{b_{jl}(X)}\cdot R_{jl}(t)\cdot p_l\cdot p_j^{e\,b_{jl}(pe)} \\ +\sum_{m=1}^r b_j^m(w)\cdot XXR_j{}^{b_j^m(X)}\cdot R_{jm}(t)\cdot w_j^m\cdot p_j^{e\,b_j^m(pe)} \end{bmatrix}}{dR_{jk}(t)}\left(\frac{R_{jk}(t)}{C_j}\right)$$

In view of (A5.06)-(A5.21), TFP is embodied.