Efficacy of Fiscal Policy Changes in a Liquidity Trap: Does Household Heterogeneity Matter?

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Abstract

This paper aims to provide a better understanding of the efficacy of fiscal policy and distortionary-tax cuts in a zero interest rate environment. The paper uses a standard New-Keynesian model, but allows for heterogeneity in consumption behavior by including Keynesian (rule-of-thumb) households that consume their current after tax income. The paper studies how the fraction of the Keynesian households interacting with nominal rigidities, in an economy with distortionary taxes, changes the effectiveness of countercyclical fiscal policy. As a starting point, the model employs labor-income tax cuts to analyze the effectiveness of tax cuts for recovery. Further, the model employs a range of other distortionary taxes (such as income tax and sales tax changes, as has been offered by many economists during the 2008 crisis) for a richer fiscal policy setup, and the automatic stabilizers analysis as well as the financing method. I look if the estimated effects change in a more realistic taxation and household set-up where distortionary taxes interact with fraction of the Keynesian agents. The paper considers a banking shock to put the economy into a recession. Output is demand determined in a liquidity trap (as in sticky price models, output adjusts to the demand in the economy) and demand is usually not adequate. This is the main problem and policies aimed at increasing production capacity or the potential output level as in the Neoclassical theory, would not be directly relevant in a zero interest rate case. The paper is mainly concerned with the question of whether fiscal policy can reverse an output collapse in a recession such as 2008.

Keywords: Fiscal Policy, Tax Cut, DSGE, New-Keynesian model, Household Heterogeneity

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1 Introduction

Following the outbreak of the Great Recession, the newly elected Obama administration in the US initially announced a three-year fiscal stimulus package, in 2008, to stimulate the aggregate demand; though it was mostly deemed insufficient. This paper aims to better quantify the real effects of similar fiscal stimulus packages. It analyzes whether the fiscal policy itself is able to reverse the output collapse in a liquidity trap. I try to measure the quantitative effects of fiscal shocks.[2] The paper provides a critical analysis on the empirical evidence on the effectiveness of fiscal policy shocks. In particular, I ask whether the evidence based on post-WWII data (that the FP ineffective) is relevant for a liquidity trap case such as the 2008 crisis. All these questions are more relevant today than they have ever been; in particularly, after the policies implemented post the Great Recession, such as the American Recovery Act that was passed in January 2009.

Although the full tax-smoothing prescriptions, due originally to Barro (1979), have mostly been found relevant in most of the developed countries (and for the U.S. federal tax rates), which Talvi and Vegh (2004) calls irresponsive fiscal policy over business cycles, particularly after the 2008 financial crisis, severe discussions on tax-cuts has reemerged. This paper discusses the efficacy of distortionary tax cuts in a more realistic model with constrained agents and nominal rigidities. The economy has a group of agents that do not use (or do not have access to) the financial and capital markets. As will be discussed below, this is important to get a positive demand effect after a tax cut. The paper studies how the fiscal multipliers found in the literature on the liquidity trap vary by existence of the Keynesian households.[3]

This paper aims to provide a better understanding of the efficacy of changes in fiscal policy in a recessionary environment that represents the 2008 financial crisis. It considers a banking shock to put the economy into a recession. Output is demand determined in a liquidity trap (as in sticky price models, output adjusts to the demand in the economy) and demand is usually not adequate. This is the main problem and policies aimed at increasing production capacity or the potential output level as in the Neoclassical theory, would not be directly relevant in a zero interest rate case. The paper is mainly concerned with the question of whether fiscal policy can reverse an output collapse in a recession such as 2008. Labor-income tax cuts are used as an

[2]The *Fiscal Multiplier* (or just the multiplier) is a measure that shows by how much GDP (or any other measure of output) responds to a tax change or a change in government spending (a fiscal variable or a fiscal policy shock), a one percent change as a fraction of GDP. For example: How much, in dollars, does GDP change if government expenditure increases by one dollar. $multiplier = \frac{\Delta GDP}{\Delta G}$.

[3]A *liquidity trap* is a case where conventional monetary policy is ineffective in increasing demand or dealing with deflation because the standard tool, short-term nominal interest rate (i.e. the federal funds rate or the overnight nominal interest rate), is down against the zero lower bound constraint.

example of a distortionary tax cut (starting point). I then analyze some other controversial tax cuts that have recently been proposed, such as capital income taxes and consumption taxes.

This paper uses a standard New-Keynesian dynamic stochastic general equilibrium (DSGE) model, but extends it by adding households with heterogenous consumption behavior to study the effectiveness of the fiscal stabilizers in a liquidity trap.[4] I assume there are two kinds of households: a Keynesian (rule-of-thumb) household that consumes his current after tax income - due to Campbell and Mankiw (1989) - and a Ricardian household whose consumption decisions follow the permanent income hypothesis - due to Friedman (1957).[5] This model set-up is motivated by significant empirical and theoretical evidence, see e.g. Mankiw (2000) and Gali et al. (2007) among others, for heterogenous consumption-saving decisions and varying effects of fiscal policy in a liquidity trap.[6] The paper studies how the fraction of Keynesian households interacting with nominal rigidities, in an economy with distortionary taxes, changes the effectiveness of countercyclical fiscal policy. This model considers the fiscal policy effects from automatic stabilizers and those from labor-income tax cut that has been debated widely among leading economists, including Mankiw (2008), Hall and Woodward (2008), Barro (2009) and Feldstein (2009) among other.

Given a government expenditure shock or tax cut, the Ricardian agents (inter-temporally optimizing) increase their labor supply and decrease their consumption considering future taxes (needed to satisfy the inter-temporal GBC). The Keynesian agents, on the other hand, are not much concerned with future taxes. Including sticky prices (and wages), real wages (or real income) goes up (or at least a smaller decline is observed), as is discussed in section 1.2, and labor income increases. Increasing labor income, raises consumption and therefore demand in the economy. Therefore, including the Keynesian agents eliminates some of the negative wealth and substitution effect (of fiscal expansion) observed in the traditional Neoclassical models. This is why Gali et al. (2007) argue the combination of the existence of hand-to-mouth (Keynesian) agents, nominal rigidity and deficit financing is required for a positive demand effect following a fiscal shock. Having Keynesian agents causes larger multiplier effects as in the traditional Keynesian models, because in particular, the Keynesian agents' consumption goes up as it is depended upon the current income. Thus the aggregate consumption will not be crowded out.

[4]The NK DSGE models have recently (after 2000) been very common, particularly in policy analysis, in policy institutions and the academic world to find the best policy and offer (the new normative macroeconomics in Taylor 2000, and Eggertsson (2010)). They allow showing policies explicitly.

[5]I use the term 'Keynesian' because consumption of these agents is proportional to current income in the Keynesian models, while in the traditional classical DSGE models it is proportional to the wealth (hence the need to use the inter-temporal budget constraint).

[6]As argued by, among others, Feldstein (2009), Erceg and Linde (2010), Eggertsson (2010).

Theoretically, adding the Keynesian agents affects the two demand equations in my model. The fraction of the Keynesian households shows up in the aggregate households' consumption Euler equation (showing off direct demand effect) and the firms' investment Euler equation (by change in labor supply and marginal product of labor). By including nominal rigidities and hand-to-mouth agents (via the direct demand effect) into the model, I primarily focus on the positive effect from this countercyclical discretionary fiscal policy that has been controversial in recent studies. As a mater of fact, given these changes, and compared to the benchmark Eggertsson (2010) model, the paper finds significant effects for consumption and labor-income taxes.

1.1 The Case for Tax cuts

Temporary one-time tax-cuts (tax rebates) have been offered by many economists recently. They have also been used, both by the classical supply side Reagan and Bush and the demand side leader president Obama; and have the advantage of being implemented instantaneously. One problem is, they could be saved instead of being spent. Yet they are mostly found to increase the after-tax income of households and result in high demand increase, see e.g. Gali et al. (2007) for a discussion of findings in Parker (1999) and Johnson et al. (2004) and support for another tax-cut.[7] In an empirical analysis of the European economy, Forni et al. (2009) use a similar model set up and quarterly data from the Euro area and find significant effects from labor-income and consumption tax cut on consumption and output. Capital income tax cuts, on the other hand, increase investment and output in a longer period (medium-run). His consumption taxes are VAT, though, unlike sales taxes in the U.S.[8]

Feldstein (2002 and 2009) and Barro (2009), for instance, offer tax cuts on capital and labor income and taxes on firm's profits in order to stimulate the economic activity. Mankiw (2008) and Robert Hall and Susan Woodward, on the other hand, called on for labor-income tax cuts by the end of 2008. Meanwhile, Christiano et al. (2009), Eggertsson (2010), and Erceg and Linde (2010) argue that the efficacy of fiscal policy changes is changing substantially in the zero

[7] It should be noted that, although some, as Feldstein (2009), argue the May-June tax rebate in 2008, had a much lower marginal propensity to consume (MPC) compared to the others in the post-1980 period, that tax cut was permanent. President Obama promised a permanent 500 dollar tax-cut per worker per annum (total of 70 billion).

[8] Uhlig and Drautzburg (2011), on the other hand, compare effectiveness of multiplier effects of distortionary taxes and lump-sum taxes. They find similar short-run multipliers for both, while in the long-run multipliers from distortionary taxes decrease substantially (to almost -1 compared to over-1 for lump-sum transfers).

nominal interest rate case.[9] However, classicals such as Barro (2009) would say this is only an excuse for the use of old Keynesian prescriptions. Taylor (2000), on the other hand, claims discretionary fiscal policy could also be appropriate for long-term issues. For example, reducing the marginal tax rates is helpful for long-term growth and economic efficiency. This goes back to the same point stated by Barro (2009) and Feldstein (2009). For instance, permanent capital tax cuts could increase investment and capital stock and hence output level in the steady state (under normal circumstances).

In Japan, the newly elected Abe government increased its consumption taxes from %5 to %8 in April 2014.[10] Another increase is also planned in the future (maybe in Spring of 2017, to about 10 percent). In 1997, when it was raised, it was reported to negatively affect the economy. But this time, they plan to accompany this increase with a fiscal stimulus package worth \$70bn. Additionally, as Krugman (1998) and Eggertsson (2010) discuss, in Japan, traditional government expenditure increases that were used since 1992 have been ineffective and the focus has shifted to tax cut offers.

On the other hand, Romer and Romer (2007), and Mountford and Uhlig (2009) find much higher multipliers for exogenous distortionary tax cuts than the multipliers for government expenditures. Burnside, Eichenbaum and Fisher (2003) and Mountford and Uhlig (2009) find that private consumption does not change much in response to government expenditure increases. All findings in these papers, regarding response of C, I and real wage to the government expenditure changes, are not consistent with what the standard Keynesian theory would suggest.

Moreover, in explaining management of expectations regarding future policy, Eggertsson and Woodford (2003) argue, when zero bound binds, one policy that can help managing expectations is cutting taxes and financing it by issuing nominal debt. Another option is, cutting taxes and financing it by printing money (considering inflation is a tax)[11].

Bils and Klenow (2008) discuss labor tax cuts as a stabilizer in a recessionary case. As in Gali et al. (2007) and Bils and Klenow (2008), business cycle accounting theorem in Chari et al. (2007) explains the process for a decrease in employment with increase in distortions between intra-temporal consumption-leisure MRS and MPL, i.e. the Hall residual. See discussion in nominal rigidities below. Chari, Kehoe and McGrattan (2007) find dominance of labor wedges (from labor

[9]Indeed, as discussed in the monetary chapter, Del Negro et al. (2010) show even nonstandard monetary policy actions has large effects in a zero short-term nominal interest rate case with nominal rigidities in both price and wage.

[10]In October 2013, Abe decided to impose a massive tax hike on consumers (doubling of Japans consumption tax) beginning in April 2014.

[11]Walsh (2010) chapter 4 and Wickens (2008) chapter 5.

tax or sticky wages or prices, i.e Hall residual), together with the efficiency/productivity wedges, in causing most of the fluctuations in real activity. This model is therefore consistent with findings and suggestions in Chari et all. (2007). Uhlig and Drautzburg (2011), using a similar model setup with constrained agents, evaluate fiscal multipliers for the 2009 fiscal stimulus in the US (ARRA). They find fiscal multipliers around 0.52 for the short-run and −0.42 in the long-run. If the Keynesians have a very low discount rate, as in my model, the fiscal stimulus transfers some of the wealth to the Keynesians (negative welfare effects for the Ricardians).

I focus on income tax and sales tax changes, as an example of distortionary tax cuts. Cutting labor income taxes has been widely discussed recently (in the context of 2008 crisis) and offered by many economists including Barro (2009), Feldstein (2002 and 2009), Hall and Woodward (2008) and Mankiw (2008). Most of these discussions were in their blogs and were limited to policy discussions without a concrete model. The theoretical studies were missing. This paper is an attempt to understand its efficacy theoretically as there is still a limited number of papers on the issue, particularly for the case of a liquidity trap. This tax-cuts has been found contractionary in Eggertsson (2010) again for a liquidity trap environment.[12] In contrast to my paper, the labor tax in Eggertsson (2010) is a payroll-tax paid by firms. It acts more like a VAT (value-added-tax). It has been analyzed and offered by Eggertsson and Woodford (2004) and Feldstein (2002) for European countries and Japan, respectively.

1.2 Why do we need nominal rigidities?

The existence of the Keynesian agents, itself, is not enough to capture the positive demand effect according to Gali et al. (2007). This is briefly explained in the log-linearized equation below that shows the relationship between the (aggregate) marginal product of labor (MPL) and the marginal rate of substitution (MRS), that would be equal to each other, absent nominal rigidity and perfect competition as in RBC models.[13]

$$MPL = \frac{U_L}{U_C} \qquad \text{where L is leisure and C is private consumption.}$$

[12]In Eggertsson's (2010) model, a reduction in labor tax stimulates deflationary pressures through its effect on firms' marginal cost. That is, people start working more, which decreases real wages and therefore the marginal cost of production for firms. With the decreasing marginal cost, firms start producing more and prices go down. Deflationary expectations increase the real interest rate, but the Fed is not able to respond since the federal funds rate is already at the zero bound. A higher real interest rate decreases demand in the economy.

[13]As I mentioned above, consumption and labor income taxes also distort this relationship and therefore have the same function as this markup coming from monopolistic competition and sticky wage and prices.

The idea is that it is not possible to explain simultaneous changes (drops) in consumption and employment, in the above equation, during a recession by movements in productivity of labor (or wage movements) due originally to Mankiw, Rotemberg and Summers (1985).

$$M\hat{P}L_s = \hat{\mu}_s + M\hat{R}S_s, \qquad \text{with } M\hat{R}S_s = \sigma_u \hat{C}_s + \eta \hat{N}_s, \text{ and } \sigma_u > 0,\ \eta > 0$$

where in (log-linear form) $M\hat{P}L_s$ is the marginal product of labor, $\hat{\mu}_s$ is the wedge between the marginal product of labor (MPL) and the marginal rate of substitution (MRS) that comes from monopolistic competition, and wage and price rigidities (if constant, frictionless, then only from monopolistic competition), $M\hat{R}S_s$ is the marginal rate of substitution between labor and consumption, $\hat{C}_s$ is aggregate comsumption and $\hat{N}_s$ is labor supply.

In the case of a labor tax cut (as in an increase in G), the nominal wage from work goes up and that increases labor supply (theory and evidence supports, both SVAR and NE models above, Gali et al. (2007) and Eggertsson (2010)) and the latter is followed by a marginal product of labor (MPL) fall.[14] If we had a constant wedge (or zero as in standard RBC models) then consumption would have to go down to have equality in the above equation. However, by assuming imperfect competition and nominal rigidities in both goods and labor markets I allow the wedge to go down, such that consumption does not have to fall. This means nominal rigidities are necessary in this case.

It is crucial to get a positive co-movement of consumption and real wages, due originally to a 1992 paper by Rotemberg and Woodford, in a theoretical model. This is because high real wages are an empirical reality. Although some papers using the standard RBC models and some empirical papers using the narrative approach find decreasing real wages as a response to a fiscal expansion (as in Ramey and Shapiro (1998)), most of the empirical papers using the SVAR method find an increasing real wage. Examples include Fatas and Mihov (2001) and Gali et al. (2007).

1.3 Heterogeneity in consumption behavior

As discussed above, since standard models with inter-temporally optimizing agents alone are not able to capture the positive response of consumption to a fiscal shock, there is clearly a

[14] Why does labor demand increase? Here, I assume, as in the standard NK models, firms are committed to supply any amount of good demanded at the price they set. They have to increase their demand for labor, thus, in order to increase their production. They set a price of their goods and supply any amount that is demanded at that price. Households are also supplying any labor that is demanded since real wage will always be higher than the MRS as I will assume later.

gap between the empirical evidence and the (NK or RBC) literature (especially for C and real wage responses).[15] Hence, heterogeneity in consumption behavior is needed (is necessary along with nominal rigidities) in the standard New-Keynesian models. This paper allows for existence of constrained agents, households in particular, in addition to the basic setup in a standard New-Keynesian model.

The problem with a basic set-up that includes only the inter-temporally optimizing agents is that we are ignoring a significant positive direct effect on demand and making a pretty strong assumption (simplification) about the consumption behavior of the agents. This, as was discussed earlier, causes discrepancies between forecasts of standard DSGE models (that are widely used in policy analysis) and findings in empirical evidence (Gali et al., 2007). Meanwhile, because of the decreased participation in the financial markets after the crisis, if the argument of Gali et al. (2007) and Bilbiie et al. (2005) about the declining fiscal multipliers in empirical studies caused by the rising fraction of Ricardians is true, then the financial crisis in 2008 may be of particular importance for analyzing the effect of the fraction of Keynesian agents that might have increased. What I mean is that the fraction of people having access to the financial markets to smooth their consumption most probably have fallen. Ilzetzki et al. (2010) consider this change more in terms of monetary policy change, where my argument is that fraction of the Keynesian agents might also be changing.

The idea of the rule-of-thumb consumers is from a 1989 paper by Campbell and Mankiw, Campbell and Mankiw (1989). They employ some households that follow the permanent income hypothesis of Friedman (1957) and some others that consume their current disposable income only (which they call the rule-of-thumb of consumption) and find that half of the income goes to the rule-of-thumb consumers. They provide evidence for the importance of the Keynesian (rule-of-thumb) households and heterogeneity in consumption-saving decisions in major economies. As for why they behave in the Keynesian fashion: Gali et al. (2007) and Campbell and Mankiw (1989) claim a fraction of agents do not smooth consumption in response to labor-income changes or do inter-temporal substitution for interest rate movements; while Uhlig and Drautzburg (2011) argue either because their discount factor is very small such that they do not want to smooth consumption by capital or bond accumulation or because they are not able to borrow due to high risks of default.

By adding the rule-of-thumb agents, I look at the direct effects from the income and substitution effects of the fiscal policy changes on spending in addition to the indirect effects revealed in the Eggertsson (2010) model. For instance, the Keynesian agents eliminate some of the neg-

[15] See Gali et al. (2007) and references therein.

ative wealth and substitution effects from future lump-sum taxes. The case for direct effects of tax-cuts on aggregate demand is motivated by Gali et al. (2007) who show that when Keynesian households are added to the model, consumption and therefore demand (key issue in the short-run for a deflationary situation) increase in response to fiscal shocks.

Eggertsson and Krugman (2010), on the other hand, use a model where some agents are debt constrained (as the Keynesians in my model) and a deleveraging shock to the economy, and show that they both results in depression (fall in aggregate demand) since agents are not able to consume due to high debt payments. They show that making some agents debt constrained is very helpful in understanding most of the disputed propositions from mostly the Keynesian economics including effective expansionary fiscal policy and very high multipliers for fiscal shocks. They find the fiscal multipliers positively related to share of the debtor agents.[16] Consumption of debtors in Eggertsson and Krugman (2010) model is also depended on current income, at the margin. This causes larger multiplier effects as in the traditional Keynesian models.

As rightly pointed out by Stiglitz (2002) the fact that individuals don't behave rationally is well known in practice and even in some theory (experimental, imperfect information). The rational expectations theory (which assumes all agents have the same information, act rationally, markets are perfectly efficient, unemployment never exists, credit rationing (crunch) never happens) is not applicable anymore (or in practice). Adding the Keynesian aspect with the constrained households in Uhlig and Drautzburg (2011) shows that; very high negative long-run multipliers in Uhlig (2010b), due to distortionary tax increases in the long-run to finance short-run debt financed expenditures, are going down to slightly negative numbers.

1.4 Relation to the Literature

This paper analyzes the effective role of the countercyclical fiscal policy argued in the standard Keynesian models in the special case that conventional monetary policy is not effective. The paper deviates from the ad-hoc nature of lump-sum taxes and focuses on distortionary tax cuts that have been popular in policy discussions lately. The model includes a variety of taxes that distort choices of households with heterogenous consumption behavior. It considers the income and substitution effects of fiscal changes for different households. I expect to have better estimates for the effective role of the countercyclical fiscal policy, particularly for the case that policy-makers have only the fiscal policy instruments.

This paper studies a liquidity trap case where the conventional monetary policy is not effec-

[16] For a horizontal SR-AS curve: With a share of 1/3, they find multiplier equal to 1.5; and with share of debt-constrained agents being 1/2, they get a multiplier equal to 2.

tive, demand is low and the economy experiences deflation. I study how effective fiscal policy is in a heterogenous agents model with distortionary taxes in a zero-interest rate environment. The paper aims to contribute to the existing literature in several ways. First, it is related to Eggertsson and Krugman (2010), Forni et al. (2009), Gali et al. (2007), Carroll (1997), Mankiw (2000) and Campbell and Mankiw (1989) in that it considers heterogenous households (including some NonRicardian and some other Ricardian households) with different consumption-saving behaviors. I use this household setup to particularly consider distortionary taxes and study a liquidity trap case to look at the efficacy of discretionary fiscal policy. This is why I include heterogeneity in consumption behavior and allow for the existence of Keynesian agents with the direct demand effect (direct spending effect from tax-cuts).

Secondly, this paper is related to literature on the effectiveness of fiscal policy and the quantitative measures of this effect. Christiano (2004), Christiano et al. (2009), Eggertsson (2010), and Erceg and Linde (2010), as well as Romer and Bernstein (2009) argue that the multipliers are changing substantially in the zero nominal interest rate case, there are many problems related to use of public spending that causes inefficiencies (as has been discussed at the beginning). Christiano, Eichenbaum and Rebelo (2011), Eggertsson (2011) Woodford (2011) Carlstrom, Fuerst, and Paustian (2012) all find large fiscal multipliers, and multipliers that increase with the duration of fiscal expansion, in new-Keynesian zero-bound models. However, Barro (2009) says this is only an excuse for use of the old Keynesian prescriptions. Barro (2009) and Feldstein (2002 and 2009) are in favor of stimulating the economic activity via private sector support, by some tax changes for instance. This paper argues that instead of inefficient government expenditure increases, the government should use its tax policy to substitute for the interest rate instrument. It is consistent with the idea of Barro (2009), Feldstein (2002), Hall and Woodward (2008) and Correia et al. (2010) in that sense.

Third, the paper is related to a literature studying distortinary taxes. Correia et al. (2010) and Feldstein (2009) argue that tax policy is very flexible in a recession, such as 2008, due to the need for the use of fiscal tools. Tax cuts on labor income and capital are found to be leading to further deterioration (contraction) according to some NK analysis, such as Eggertsson (2010). Feldstein (2002), Feldstein (2009) and Barro (2009) offered capital income tax cuts for the U.S. economy and Japan economy. Mankiw (2008) and Woodward and Hall (2008) offered labor tax cuts again for the U.S. Adding the Keynesian households, as in Mankiw (2000) and Gali et al (2007), the tax cuts such as those on wage or capital may have positive direct effects on spending and aggregate demand. Moreover, most of the papers discussed above were policy discussions more than theoretical analysis. This paper is an attempt to see the theoretical validity of these offers.

Lastly, this paper is related to a line of papers that show the economy is in a liquidity trap such that open-market-economies are irrelevant. Wallace (1981), Krugman (1998), Curdia and Woodford (2011) and Eggertsson and Woodford (2003) - if expectations are not changed. This paper is most closely related to Eggertsson and Woodford (2003 and 2004), Christiano (2004), Eggertsson (2010), Christiano et al. (2009), Erceg and Linde (2010), Feldstein (2002), and Romer and Bernstein (2009) who analyze effective policy in a zero-interest rate case. In contrast to my paper, all of these papers consider only the Ricardian households. They find an increasing multiplier effect for mainly government expenditure shocks. I use a range of distortionary and lump-sum taxes.

The New-Keynesian (NK) dynamic stochastic general equilibrium (DSGE) model is built on Chari et al. (2000), Christiano (2004), Woodford (2003), Smets and Wouters (2007), Gali et al. (2007) and Eggertsson (2010). The paper puts the heterogeneity idea of Gali et al. (2007) into the zero-interest rate framework of Eggertsson and Woodford (2003).

2 The Model

I use a real business cyclye (RBC) model with a Dixit-Stiglitz (1977) monopolistic competition framework among firms and workers in the goods and labor markets respectively, and Calvo (1983) type nominal rigidities in firms' price setting and fixed wages[17]. Given these nominal frictions and markups, (actual) labor and output will be demand determined. In other words, firms and workers commit to supply any amount of goods and labor demanded at the prices set by the corresponding agents. All agents in the model, except the CB that follows the Taylor rule, are optimizing. Up to this point, this is a standard New-Keynesian DSGE model. However, I add some Keynesian households to the model. The economy has two types of households therefore: Keynesian (rule-of-thumb) households and Ricardian (inter-temporally optimizing) households. The households are accompanied by a continuum of intermediate good producers and a representative final good producer. Additionally, there is a central bank conducting monetary policy and a government as the fiscal authority. Time is discrete and the only uncertainty comes from an aggregate banking shock (ξ). I assume a complete asset market and the economy is cashless (as is common in the NK literature. Therefore, I ignore the costs of inflation associated with the inflation tax resulting from deviations from the Friedman rule).

2.1 Households' problem

I assume a continuum of households, indexed by $j \in [0, 1]$, who are monopolistically competitive in their labor supply as in Erceg et al. (2000). The growth rate of population is zero and the population is normalized to one. Each household is infinitely-lived and provides a differentiated labor service $l_t(j)$ to the (single, economy-wide) factor market.[18]. $'f'$ fraction of households are Keynesian and do not have access to the capital and financial markets.[19] They consume only their current after tax (disposable) income. The remaining '$1 - f$' fraction are Ricardian. They buy and sell assets and use capital to smooth their consumption inter-temporally. The two type of households differ in their rate of time preference (most basically with $\beta^r >> \beta^k$). The Keynesian and the Ricardian households are uniformly distributed across labor types.[20] I assume

[17]I make these changes to a RBC model because, as Gali et al. (2007) show, the standard RBC models are not able to capture the positive response of private consumption to a fiscal shock that exists in the empirical analysis.

[18]Whereas in Woodford (2003), Christiano (2004) and alike - Eggertsson (2010) etc, each household provides every type of labor.

[19]See footnote 5 for the reason.

[20]Or both Ricardian and Keynesian households supply labor of any type j and demand for a differentiated labor type j is uniformly distributed among these households, R and K.

real wages are always higher than the mrs, therefore agents always provide any amount of labor demanded by firms, as in Gali et al. (2007).

Following Erceg et al. (2000) and Forni et al. (2009), I assume an (zero-profit) employment agency (labor aggregator agency) combines all the imperfectly substitutable labor supply provided by different household in accordance with firms' demand and creates homogenous labor inputs for firms.[21] The employment agency's demand for each particular labor type will be equal to the total demand for that labor type by all firms. The aggregate labor index has the Dixit-Stiglitz (1977) form as in equation (1).

$$L_s = \left[\int_0^1 L_s(j)^{\theta_l} dj\right]^{\frac{1}{\theta_l}}, \qquad \text{with } 0< \theta_l <1 \tag{1}$$

where $L_s(j)$ is labor supply of type j and unit cost of labor demand is W_s. I implicitly assume that firms allocate their labor demand uniformly across the continuum of labor types.

The employment agency takes $W_s(j)$ - the nominal wage chosen by the Ricardian households (consistent with labor market in the US) - and W_s as given and maximizes profit (or minimizes the cost of producing the aggregate labor index) in the same way as firms demanding labor, below, subject to L_s equation above and with respect $L_s(j)$, as in Forni et al. (2009) or the labor packers in Smets and Wouters (2007). In other words, it minimizes the cost of producing the aggregate-labor-index L_s demanded by firms. The Aggregator's problem is below.

$$\max_{L_s(j)} \Pi_s = W_s L_s - \int_0^1 W_s(j) L_s(j) dj = W_s \int_0^1 l_s(i) di - \int_0^1 W_s(j) L_s(j) dj$$

[21] This is in contrast with Gali et al. (2007), and Chari, Kehoe and McGrattan (2007) that assume a continuum of labor unions (as a continuum of hhs and represented by j) that are monopolistically competitive (consistent with labor markets where labor-unions are powerful - as in Europe). Each of these unions set their own wages and each of them represents all households who supply a specific type of labor. Gali et al. (2007) and Forni at al. (2009) consider an alternative labor market structure for robustness check. In this setup the Keynesians do not necessarily change their labor supply in the same way as the Ricardians and a union representing both the Ricardians and the Keynesians sets wages in a monopolistically competitive labor market, by maximizing weighted average of utility of the two type of households. This means a common wage and labor supply for both types. However, the differences in their results are insignificant. It therefore makes sense to stick to the this structure and assume the same labor supply and same average wages. Moreover, Gali et al. (2007) shows in the log-linearized form both labor market structures give even the same linear equation (Appendix A of Gali et al. (2007), under the assumption of the same SS consumptions).

The aggregator's labor demand in my model, L_t, will be equal to the firms' labor demand, N_t.

See also Uhlig and Drautzburg (2011) that also assume Calvo type differentiated sticky wages set by unions. They assume wages are set by maximizing utility of the unconstrained households, however, under the assumption that they represent the majority in unions. Firms hire labor from both types randomly again and labor supply will be the same for both types in the equilibrium.

The cost minimization (or profit maximization) problem for the labor aggregator agency results in the following overall demand, across all firms, for household j's labor (the employment agency's demand for that particular labor type $L_s(j)$).

$$L_s(j) = \left[\frac{W_s(j)}{W_s}\right]^{\frac{-1}{1-\theta_l}} L_s \tag{2}$$

The wage-aggregator for the labor index L_s, by using equation (2) in (1) is below.

$$W_s = \left[\int_0^1 W_s(j)^{\frac{\theta_l}{(\theta_l-1)}} dj\right]^{\frac{(\theta_l-1)}{\theta_l}} \tag{3}$$

Households' decision problem is two stages. The first step is cost minimization and the second step is utility maximization. The final-good producer, we will see later, deals with the first step by minimizing the cost of producing a composite consumption good $C_s^h(j)$, $h = r, k$ ('k' for Keynesian and 'r' for Ricardians). I show that part below for illustration only. This means households face the following cost minimization problem (given that $c_s(i)$ is consumption of goods of type i).

$$\min_{c_s(i)} \int_0^1 P_s(i)c_s(i)di$$

subject to achieving a Dixit-Stiglitz aggregate consumption level C_s^h, where

$$\int_0^1 N_s(j)C_s^h(j)dj = \left[\int_0^1 (c_s(i))^\theta di\right]^{\frac{1}{\theta}}, \qquad \text{with } 0< \theta <1 \tag{4}$$

And their total consumption, C, provided by all of the firms, is a Dixit-Stiglitz composite consumption index.

$$P_s = \left[\int_0^1 (P_s(i))^{\frac{\theta}{(\theta-1)}} di\right]^{\frac{(\theta-1)}{\theta}}$$

is the corresponding Dixit-Stiglitz composite price index (the same as equation 22 below).

A fraction, $'1-f'$, of households hold bonds and own a share of firms. They buy composite consumption goods, and supply labor to the single economy-wide labor market. All of the households earn an after-tax labor income $'(1-\tau_s^w)W_s(j)L_s(j)'$. The agents decide $W_s(j)$ - the nominal wage - endogenously. I assume all capital is owned and accumulated by firms and therefore the Ricardian agents in particular. Hence, capital does not show up in the household problem.[22] Firms are owned by the Ricardian households, hence, all the after-tax profits goes to these households as $(1-\tau_s^P)Z_s(j)$. The utility maximization problem of a Ricardian agent j is

[22] I follow Christiano et al. (2005) - footnote 8 - and assume that it does not matter whether capital is endogenously accumulated by households or firms.

below. They maximize the expected present discounted value of their inter-temporal utility with respect to (wrt) $C_s^r(j)$, $B_{s+1}(j)$ (and $l_s^r(j)$, if needed).

$$\max_{\{C_s^r(j),L_s^r(j),B_{s+1}(j)\}_{s=t}^{\infty}} E_t \sum_{s=t}^{\infty} \beta^{s-t}\xi_s \left[U(C_s^r(j), G_s^{not}, l_s^r(j))\right]^{23} \tag{5}$$

I assume a separable period utility function (in C, L and G) for agents. This is both for simplicity and to account for importance of the fraction of rule-of-tumb behavior as in Campbell and Mankiw (1989) - as they use separable utility. As discussed by Gali et al. (2007), a non-separable utility function might have some other implications that do not guarantee a significant fraction of Keynesian behavior in consumption (See Basu and Kimball (2002) in their references). The utility function is a common labor-leisure decision utility function consistent with stylized facts (balanced growth path). All agents have the following identical period utility function.

$$U(C_s^r(j), G_s, l_s^r(j)) = \frac{(C_s^r(j))^{1-\sigma_u}}{1-\sigma_u} + b_1\frac{(G_s)^{1-b}}{1-b} - \frac{(l_s^r(j))^{1+\eta}}{1+\eta}^{24,\,25}$$

The Ricardian agents maximize the utility function (5) subject to the inter-temporal version of the following period budget constraint for each household,

$$(1+\tau_s^c)P_sC_s^r(j) + R_s^{-1}B_{s+1}^r(j) \tag{6}$$

$$= (1-\tau_s^a)B_s(j) + (1-\tau_s^p)Z_s(j) + (1-\tau_s^w)W_s(j)l_s^r(j) - P_sT_s^r(j)$$

where $R_s = (1+i_s)$ is the gross nominal interest rate (R_s^{-1} is price of the riskless nominal bond at time s). Households take the tax rates, prices, transfers (or taxes) from government and all the aggregates as given and maximize utility subject to the budget constraint (and the demand function for their labor, if needed). In other words, they maximize inter-temporal utility function (5), subject to equations (6) (the period budget constraint) - and (2) (demand for its labor supply). Each household has monopoly power over his/her nominal wage $'W_t(j)'$ such that he/she resets his wage at the end of the contract periods (which have random durations).

[23]The preference shock functions as in Christiano et al. (2009), Eggertsson (2010) and Correia et al. (2011). It affects the consumption Euler equation, but not the MRS between C_s and L_s.

[24]Or simply: u is increasing and concave in C and G; v is increasing and convex in L.

[25]The exogenous government expenditure is separable from private consumption to make sure it does not distort inter-temporal decision of households, as in Eggertsson and Krugman (2010) and Eggertsson (2010). I assume, all the government expenditure consists of expenditures non-substitutable with private consumption (such as infrastructure and military spending), i.e. $G_s = G_s^n$. It's a Dixit-Stiglitz aggregator analogous to C_s: $G_s = \left[\int_0^1 (g_s(i))^\theta di\right]^{\frac{1}{\theta}}$. Government expenditure perfectly substitutable with private consumption, G_s^p, has proven to have no effect on equilibrium outcomes, See, e.g., discussion in Eggertsson (2010).

This process is analogous to the price setting process for firms for their output, which we will see in the next section. $0 < \beta < 1$ is the discount factor, and $L_s(j)$ is amount of household-specific labor supplied. Agents get wage $W_s(j)$ for the labor supplied. ξ_s is a preference shock representing the banking crisis.[26] B_s, the beginning of period bond holding, is a one period risk-less bond issued by the government. $Z_s(j)$ ($\int_0^1 Z_s(i)di = \int_0^1 Z_s(j)dj$) is lump-sum profit distributed between households (households have the same share of firms). T_s^h, for $h = k, r$ are lump-sum taxes (or transfers if negative) from the government. The distortionary taxes will be: a sales tax τ_s^c, a labor-income (paid-by household) tax τ_s^w, financial asset (savings) tax τ_s^a, investment tax credit τ_s^I and profit tax τ_s^p. Because I am particularly interested in how tax cuts affect households' behavior in the two models, I use the same differentiation between capital (that affects households' consumption/saving behavior) and profit taxes (that affects firms' investment/hiring and pricing behavior) as in Eggertsson (2010). Capital taxes, τ_s^a, in my model follow the similar set up in Eggertsson (2010).[27] E_s is conditional expectation based on information available at time s.

The first-order necessary conditions for the Ricardian households' optimality (for the rational expectations equilibrium) are derived below by maximizing the utility function subject to the households inter-temporal budget constraint with respect to their choice variables $C_s^r(j)$, $l_s^r(j)$, B_{s+1}, and $Z_s(j)$. Note that I assume wages are a markup over the mrs due to the monopolistic competition.

An Euler equation (EE) for inter-temporal consumption allocation for the Ricardian household (the Keynesian households don't have inter-temporal consumption/saving decisions), which links the marginal cost of consumption today to the expected marginal benefit of consumption in the future period, is below.[28]

$$U_{c,s} = R_s(1 - \tau_{s+1}^A)\beta E_t U_{c,s+1} \frac{\xi_{s+1}}{\xi_s} \frac{P_s}{P_{s+1}} \frac{1 + \tau_s^c}{1 + \tau_{s+1}^c} \tag{7}$$

(Assuming wages are not set by households, there is also) an optimality condition that sets marginal rate of substitution between leisure and consumption to real wage with taxes (an intra-

[26] If it was a simple taste shock (basic demand shock), it would only show up with C_t (Erceg and Linde (2010), Walsh (2010)); here it is a (general) preference shock.

[27] Eggertsson (2010) uses a tax on the stock of savings instead of taxing nominal capital income, which we observe in practice. So, τ_s^a is a tax on the capital/financial stock of households in his model. This is because the tax on nominal capital income, τ_s^k, is zero in a zero-interest rate environment. He rescales τ_s^a such that 1 percent variation in τ_s^a equals to a change in tax equivalent to a 1 percent variation in tax on capital income in steady state. In other words, a tax cut that is equal to a 1 percent fall in capital income tax in steady state.

[28] Sacrificing one unit of consumption, c_t, today to buy $\frac{1}{p_t}$ units of bond/money and $\frac{1}{q_t}$ units equity, in order to consume one unit of consumption good at time 't+1', c_{t+1}.

temporal EE for labor):

$$\frac{1-\tau_s^w}{1+\tau_s^c}\frac{W_s(j)}{P_s} = \mu_L \frac{U_{l,s}}{U_{c,s}}{}^{29} \tag{8}$$

which corresponds to the $\gamma_l \to 0$ case that will be shown below, in the Calvo type nominal wage setting. More broadly, it should be as:

$$\frac{1-\tau_s^w}{1+\tau_s^c}\frac{W_s(j)}{P_s} = H(C_s, N_s) > \frac{U_{l,s}}{U_{c,s}} \tag{9}$$

and the Transversality condition (TVC) - no-ponzi condition (that hhs use all of their inter-temporal BC).

$$\lim_{s\to\infty} E_t \frac{B_{s+1}}{P_s(1+\tau_s^c)} U_{c,s} = 0 \left(\lim_{s\to\infty} E_s \lambda_s b_{s+1} = 0\right) \tag{10}$$

I assume, as in Gali et al. (2007), that μ_l over mrs is sufficiently high and fluctuations in μ_{ls} (due to stickiness) are small enough such that real wage is always higher than mrs ($\frac{U_{l,s}}{U_{c,s}}$). This makes sure both types of households are always willing to (they promise) supply any amount of labor demanded by firms, at this high wage set by the union. Meanwhile, balanced-growth path requires that $H(C_s, N_s)$ could be written as $C_s h(N_s)$. We assumed wage is set by households (or by market, with a markup), and labor supply is determined by demand of firms (given the wage, employment is demand-determined as output is demand determined) and households are supplying any amount of labor demanded by assuming wage is always above mrs. Demand for a differentiated labor type j is uniformly distributed among these households, r and k. Therefore, $N_s^r = N_s^k$, ∀ s.

If we assume households are setting their wage, then labor is not a choice variable any more. An alternative labor setting is that I assume wages are set at the beginning of period, before government cuts taxes. And hence fixed for one period.[30] Then, we have the above intra-temporal Euler equation for labor consumption decisions. Assumption of fixed wages for one period has the advantage of not causing any inefficiency or distortions, since Calvo type nominal wage setting causes dispersions in wages across households. This means distortion in the employment allocation - (since demand for each labor is depended on its price).

A fraction $'f'$ of the households are assumed to be Keynesian ($\beta = 0$ or very low) in their consumption behavior. While the Ricardian households have no limits on borrowing against all their future income, the Keynesian are constrained fully. Keynesian households have the same labor supply $'L_s^r(j) = L_s^k(j)'$ as the Ricardians (they face the same labor demand), the same

[29] Where the markup $\mu_L = \frac{1}{\theta_l}$.

[30] Another sticky wage model such as a one period model, as suggested by Prof. Walsh and Prof. Aizenman, might also be considered.

wage and the same utility function. The only difference is that they don't have inter-temporal decisions, and capital and assets are removed from their budget constraints. As pointed out by Forni et al. (2009), this also means labor supply of both agents responds to a tax cuts or other fiscal changes in the same way. The Keynesian households do not have access to the capital and financial market and only consume their current after tax income. I assume (since they cannot inter-temporally optimize to set an optimal wage) they set their wage as the average of the optimizing/Ricardian households (W_s) (as in Gali et al. (2007), Forni et al. (2009) and Erceg et al. (2005)). Their utility function and the budget constraint will be as below.

$$\max_{\{C_s^r(j), L_s^r(j)\}} \left[U(C_s^k, G_s, l_s^k(j))\right]$$

$$(1+\tau_s^c)P_s C_s^k = (1-\tau_s^w)W_s l_s^k - P_s T_s^k(j) \tag{11}$$

consumption in real terms is as follows.

$$C_s^k = \frac{(1-\tau_s^w)w_s l_s - T_s^k(j)}{(1+\tau_s^c)}$$

And they have an optimality condition that sets marginal rate of substitution between leisure and consumption to the marginal product of labor (MPL) - an intra-temporal EE for labor, as in the Ricardians case.

$$\frac{1-\tau_s^w}{1+\tau_s^c}\frac{W_s(j)}{P_s} = \frac{U_{l^k,s}}{U_{c^k,s}}\mu_L \tag{12}$$

I first assume $f = 1/2$ of population are Keynesian households. Then, I change it to see the effect of the fraction of Keynesian households on fiscal multipliers.[31]

Aggregation among the Keynesian and the Ricardian households is as below. 'f' is the fraction of the aggregate consumption coming from the Keynesian households, while '1-f' is the fraction of the Ricardian households.

$$C_s = fC_s^k + (1-f)C_s^r \tag{13}$$

where $C_s^r = \int_0^1 C_s^r(j)dj$ and $C_s^k = \int_0^1 C_s^k(j)dj$.

In the labor market, on the other hand, we have the following aggregation.

$$L_s = fl_s^k + (1-f)l_s^{r} \text{[32]} \tag{14}$$

[31] Consistent with estimates in Campbell and Mankiw (1989). Erceg and Linde (2010) take it as 1/3 (1/3 of consumption goes to Keynesians). Gali et al. (2007) start with 1/2 as well.

[32] $L_s^k = L_s^r = L_s$, for all t, in aggregate. See, for example, Appendix A of Gali et al. (2007). Forni et al. (2007): Given that Keynesians set their wage at average of the Ricardians and since all agents face the same labor demand, their labor supply and wages should be (and will be) the same. Ricardian and Keynesian's labor supply responds to a fiscal shock in the same way (in case of a G change).

In the goods market,

$$Y_s = C_s + I_s + G_s \tag{15}$$

2.1.1 Calvo type nominal rigidity in wage setting

Assuming each household has monopoly power over his/her nominal wage $'W_t(j)'$ such that he/she resets his wage at the end of the contract periods (which have random durations). This process is analogous to the price setting process for firms for their output, which we will see in the next section. Households take the tax rates, prices, transfers (or taxes) from government and all the aggregates as given and maximize utility subject to the budget constraint. In other words, households maximize inter-temporal utility function (5), subject to equations (6) (the period budget constraint) and (2) (demand for its labor supply). Each period a fraction of agents (from the law of large numbers) are able to reset their wages. It could also be read as the probability of each household resetting its wage in each period is the same. I will call this probability as $'1-\gamma_l'$.[33]

Given all the information available, when an agent is allowed to set the optimal wage W_s^*, he/she chooses the wage $W_s(j)$ that maximizes his utility - equation (5) - subject to the period budget condition and demand for his demand, considering that he might not even have another chance ever to reoptimize. Households take L_s and W_s as given.[34] E_s is again a conditional expectation based on information available at time 's'. I assume wages are not changing at all when a household type is not allowed to reset its wage for a period.[35] First order conditions (FOC) of the above utility maximization problem (inter-temporal decision/problem), subject to the the two conditions and, with respect to W_s^*, then, are as follows ($\max_{W_s^*}$).
Here, we will need to assume that, since all the households face the same problem, we are looking for a 'symmetric equilibrium' where all the households choose the same wage $W_s^*(j) = W_s^*$.

$$E_t \left\{ \sum_{s=t}^{\infty} (\gamma_l \beta)^{s-t} \left[\frac{W_t^*}{P_s} L_s (\frac{W_t^*}{W_s})^{-\theta_l} U_{C,s} - \frac{1}{\theta_l}(-U_{L_s(j)}) L_s (\frac{W_t^*}{W_s})^{-\theta_l} \right] \right\} = 0 \tag{16}$$

or

$$E_t \left\{ \sum_{s=t}^{\infty} (\gamma_l \beta)^{s-t} \left[\frac{W_t^*}{P_s} L_s (\frac{W_t^*}{W_s})^{-\theta_l} + \frac{1}{\theta_l} \frac{U_{L_s(j)}}{U_{C,s}} L_s (\frac{W_t^*}{W_s})^{-\theta_l} \right] \right\} = 0$$

[36]

[33] γ_l fraction of agents are <u>not able</u> to reset their wages.
[34] Christiano et al.(2005)
[35] Christiano et al. (2005) and Erceg and Linde (2010)
[36] Where $MRS_s = -\frac{U_{L_s}}{U_{C_s}}$ and the markup $\mu_L = \frac{1}{\theta_l}$.

Rearranging this FOC, I get the following simpler form:

$$W_t^* = \frac{1}{\theta_l} \frac{E_t \sum_{s=t}^{\infty} (\gamma\beta)^{s-t} ln(C_s) MC_s(i) (\frac{p_t^*}{P_s})^{\frac{1}{1-\theta}}}{E_t \sum_{s=t}^{\infty} (\gamma\beta)^{s-t} ln(C_s) (\frac{p_t^*}{P_s})^{\frac{\theta}{1-\theta}}}. \tag{17}$$

If wages are sticky here (with the monopoly power over labor) and set by households or unions, then households promise to supply any labor that is demanded at that wage. Therefore there is no FOC wrt labor ($L_s(j) = L_s^d(j)$). See the discussion in the footnote 32 above about findings in Forni et al. (2009) and Gali et al. (2007).[37]

In other words, wages are set such that the expected discounted marginal benefits are equal to the expected discounted marginal disutility from working.
If I assume a perfectly flexible case, then $\gamma_l \to 0$ (meaning there is no agent that is not allowed to reoptimize) and the above equation reduces to,

$$W_t^* = \frac{1}{\theta_l} MRS_s P_s \text{}^{38} \tag{18}$$

By taking γ_l as the fraction of agents that keep their wages fixed each period, I get the following wage index.

$$W_s = [\gamma_l W_{s-1}^{1-\theta_l} + (1-\gamma_l)(W_s^*)^{1-\theta_l}]^{\frac{1}{1-\theta_l}} \tag{19}$$

Wage inflation π^w depends on the real marginal cost, which is equal to the gap between real wage and marginal rate of substitution between consumption and leisure.

Given sticky wages, in the steady state, we get the following equality that I used above a lot.

$$\frac{1-\tau^w}{1+\tau^c} \frac{W}{P} = \frac{1}{\theta_l} \frac{U_l}{U_c}$$

2.2 Firms' problem

I assume there is only one final good produced by a representative final-good-firm and a continuum of intermediate goods, indexed by $i \in [0,1]$, produced by a continuum of firms indexed by the good they produce. Final-good firm buys intermediate goods, assembles them in the same proportions as consumers (households, firms and the government) demand and sells it to the private sector and the government in a competitive market.[39] They do not use any labor, and

[37] If we have a competitive labor market, then given the wage set by market, households choose amount of labor supplied. $w = mrs$.

[38] This is (real) marginal rate of substitution.

[39] Following findings in Eggertsson (2010), I assume all government expenditures are imperfectly substitutable with private consumption. These include military expenditures and infrastructure spendings. Eggerstsson (2010) show that the perfectly substitutable government expenditures (with C_t) are not changing output or inflation.

therefore pay only for the intermediate good they use. Following Chari, Kehoe and McGrattan (2000), I use the following Dixit-Stiglitz form production function for the final good produced by a perfectly competitive firm.

$$Y_s = \left[\int_0^1 Y_s^d(i)^\theta di\right]^{\frac{1}{\theta}}, \qquad \text{with } 0< \theta <1 \tag{20}$$

Final-good firms choose Y_s (given) and $Y_s^d(i)$ (demand for good 'i') to maximize profit (or minimize cost) below:

$$\max_{Y_s(i)} \Pi_s = P_s Y_s - \int_0^1 P_s(i) Y_s(i) di$$

subject to equation (20), where $Y_s(i)$ is an intermediate good produced by firm i.[40] Final good producers face a perfectly competitive market for both their output and the intermediate goods they need for production. They take the prices of final good P_s and intermediate good $P_s(i)$ as given, and choose final good Y_s and intermediate good $Y_s(i)$ for production. Profit maximization of the final good producers results in the following demand function for each intermediate good. This is the sum of the demand for a particular intermediate good, by consumers in the economy, since final-good producers assemble intermediate goods according to the demand in the market.[41]

$$Y_s^d(i) = \left[\frac{P_s(i)}{P_s}\right]^{\frac{-1}{1-\theta}} Y_s \tag{21}$$

The zero-profit condition (due to perfect competition in final goods market) for the final good producers gives the price of final good (or the aggregate price index), which is equal to the marginal cost of production.

$$P_s = \left[\int_0^1 P_s(i)^{\frac{\theta}{(\theta-1)}} di\right]^{\frac{(\theta-1)}{\theta}} \tag{22}$$

The intermediate goods, on the other hand, are produced by monopolistically competitive firms. Each intermediate-good firm uses firm-specific capital and (composite) labor rent from labor aggregator agency (thus households), and produces a differentiated good $Y_s(i)$. All producers hire the same kind of labor (homogenous labor input) and face the same wages, W_s. In a way, all types of labor are used in producing a differentiated good. They face the demand function in equation (21) for their output (they all have the same constant demand elasticity). All of the

[40] Constant elasticity of substitution (CES) between the intermediate goods is $\frac{1}{1-\theta}$ - this is also price elasticity. As $\theta \to 0$, ES goes to 1 (unit elastic). Constant markup over the marginal cost for monopolistically competitive firms is $\frac{1}{\theta}$.

[41] Government expenditure and therefore its demand for each differentiated good is analogous to that of households and firms. The same aggregation as consumption and investment.

firms are owned by the Ricardian households (each household own an equal share of all firms and capital stock as in Erceg at al. (2000)). Therefore, all the profit goes to households as the dividend payment. The production function for intermediate goods has a usual CRS Cobb-Douglass form. All intermediate-good firms have the same following production function.[42]

$$Y_s(i) = F(K_s(i), N_s(i)) = K_s(i)^{\alpha} N_s(i)^{1-\alpha} \tag{23}$$

where $N_s(i)$ and $K_s(i)$ are labor and firm specific capital inputs for production of the intermediate goods. Capital is assumed to accumulate endogenously by firms. The total factor productivity (TFP) is normalized to 1. And the intermediate-good firms maximize profit (or minimize cost) below.

$$\max_{\{N_s(i),K_s(i)\}} \Pi_s(i) = P_s(i)Y_s(i) - W_s N_s(i) - (1+\tau_s^I)(1+\tau_s^c)P_s I_s(i)^{43}$$

The monopolistically competitive intermediate-goods firms face a perfectly competitive factor market in contrast to the imperfect goods market for their output. They take the purchasing price for the investment P_s, all taxes and the aggregate wage W_s as given and choose $Y_s(i)$ (*function given), $N_s(i)$ and $I_s(i)$ to maximize profit function w.r.t. $P_s(i)$, $K_{s+1}(i)$ subject to the demand function (21). They set a price of their goods and supply any amount that is demanded at that price. They have to hire labor to produce goods demanded.

Firms need to consider their capital accumulation process. I include endogenous capital variations into the model because access to the capital markets by a fraction of households is a key point of my paper. I assume, in order to increase the capital stock from $K_s(i)$ to $K_{s+1}(i)$, a firm must invest (one period in advance) according to the rule at (21).[44] I first don't include a preference shock in the cost of adjustment function $I(.)$, as in Christiano (2004), and then add it as in Eggertsson (2010) to compare the two cases. ξ_t is a banking crisis shock which increases the default risk in a crisis period and hence cost of loans. It raises cost of borrowing for firms as well as consumers. Therefore, I include the shock into the cost of adjustment function of investment for the firms.

$$I_s(i) = I(\frac{K_{s+1}(i)}{K_s(i)}, \xi_s)K_s(i) \tag{24}$$

where $I_s(i)$ is again a Dixit-Stiglitz composite analogous to C_s^h or G_s. The function $I(.)$, in the steady state, satisfies $I(1,\xi) = \zeta$ (the depreciation rate of capital), $I_{I\xi}(1,\xi) \neq 0$ and $I_\xi(1,\xi) = 0$

[42] Since the production function is constant returns to scale, $Y_s(i) = F(K_s(i), N_s(i)) = F_k(K_s(i), N_s(i))K_s(i) + Fn(K_s(i), N_s(i))N_s(i)$ and $\frac{K_s(i)}{N_s(i)}$ is the same across firms, $\frac{K_s(i)}{N_s(i)} = \frac{K_s}{N_s}$.

[43] Where $Y_s(i) = \left[\frac{P_s(i)}{P_s}\right]^{\frac{-1}{1-\theta}} Y_s$. The union sells units of labor index L_s, $L_s(i)$, at the cost W_s to the intermediate-goods sector. $w_s = \frac{W_s}{P_s}$ is real wage.

[44] The investment adjustment cost is as in Woodford (2003), Christiano (2004) and Eggertsson (2010).

(to make it comparable with Eggertsson (2010)), $I_I(1,\xi) = 1$, and $I_{II}(1,\xi) = \epsilon_x$ (the degree of adjustment cost in log-linear approximation or curvature on the investment adjustment cost function[45]) with the following conditions for parameters: $0 < \zeta < 1$ and $\epsilon_x > 0$. I_s in the resource constraint is the sum over all the firms' investment.

$$\int_0^1 I_s(i)di = I_s \tag{25}$$

Firms pay wages for the labor they hire from households and buy investment goods from the final good producers. They also pay consumption tax τ_s^c for the investment good they buy, a profit tax τ_s^P and an investment tax credit τ_s^I. The profit maximization problem of firm i is below.

$$\max_{\{P_s^*, K_{s+1}(i)\}} \Pi_s(i) = \left[P_s(i)Y_s(i) - W_s N_s(i) - (1+\tau_s^I)(1+\tau_s^c)P_s I_s(i)\right] \tag{26}$$

and adding the profit taxes,

$$\max_{\{P_s^*, K_{s+1}(i)\}} E_t \left\{ \sum_{s=t}^{\infty} (\beta)^{s-t} Q_s (1-\tau_s^P) \left[P_s(i)Y_s(i) - W_s N_s(i) - (1+\tau_s^I)(1+\tau_s^c)P_s I_s(i)\right] \right\}$$

where as in Correia et al. (2011) and Eggertsson and Woodford (2003) the following variables (excluding the β) will be used, $Q_{s+1} = \beta\frac{\lambda_{s+1}}{\lambda_s} = \frac{R_s^{-1}}{(1-\tau_{s+1}^A)} = \frac{1}{(1+i_s)(1-\tau_{s+1}^A)} = \frac{\beta U_{c,s+j}\xi_{s+j}}{U_{c,s}\xi_s}\frac{(1+\tau_s^c)P_s}{(1+\tau_{s+1}^c)P_{s+j}}$ is the nominal price at time s of a unit of money at a state in period $s+1$ (I use real Q_{s+1} though: $Q_{s+1}^{real} = Q_{s+1}\frac{P_{t+1}}{P_t}$) - stochastic discount factor for Ricardians (share owners of firms) and $\lambda_s = \frac{U_{c,s}\xi_s}{(1+\tau_s^c)P_s}$ is the Lagrange multiplier on the households' BC. It is the shadow value of a dollar to the hh. The FOC with respect to the capital stock chosen for time $'s+1'$, $K_{s+1}(i)$, is below.

$$I'(I_s^N(i), \xi_s)(1+\tau_s^c)(1+\tau_s^I)(1-\tau_s^P) = \tag{27}$$

$$E_s Q_{s+1}\Pi_{s+1}(1-\tau_{s+1}^P)\left[r_{s+1}^k(i) + I'(I_{s+1}^N(i), \xi_{s+1})I_{s+1}^N(i)(1+\tau_{s+1}^c)(1+\tau_{s+1}^I) - I(I_{s+1}^N(i), \xi_{s+1})(1+\tau_{s+1}^c)(1+\tau_{s+}^I\right.$$

where

$$I_s^N(i) = \frac{K_{s+1}(i)}{K_s(i)} \tag{28}$$

is the net increase in the capital stock per period, $\Pi_{s+1} = (1 + E_s\pi_{s+1})$ and

$$R_s^k(i) = \frac{\alpha}{1-\alpha}\frac{N_s(i)}{K_s(i)}W_s(j) = \frac{\alpha}{1-\alpha}\frac{N_s(i)}{K_s(i)}MRS_s P_s\frac{(1+\tau_s^c)}{(1-\tau_s^w)}.\text{[46]} \tag{29}$$

[45] When ϵ_x is large (the more concave), capital stock is constant. When it is small, then investment is elastic, or changing (Christiano (2004)).

[46] where $\frac{\alpha}{1-\alpha}\frac{N_s(i)}{K_s(i)} = \frac{MPK_s}{MPL_s}$.

$\frac{R_s^k}{P_s} = r_s^k$ is the real shadow value of a marginal unit of additional capital (functions as the 'rental cost of capital' in models where capital is rent from hhs).[47] From the production function, it is possible to derive the following equality (firm choices must satisfy this according to Correia et al. (2011) and Chari, Kehoe and McGrattan (2007)).[48]

$$\frac{F_N(K_s(i), N_s(i))}{F_K(K_s(i), N_s(i))} = \frac{(1-\alpha)K_s(i)}{\alpha N_s(i)} = \frac{W_s}{R_s^k} = \frac{w_s}{r_s^k}(= \frac{F_N(K_s, N_s)}{F_K(K_s, N_s)} = \frac{W_s(i)}{R_s^k(i)})$$

The nominal marginal cost for all firms (common, because of the CRS property of the prodution function - also average cost) and per output is below.

$$MC_s{}^{49} = \frac{w_s}{MPL_s} = \frac{r_s^k}{MPK_s}$$

This implies

$$MC_s(i) = \frac{r_s^k(i)}{MPK_s(i)} = (\frac{K_s(i)}{\alpha Y_s(i)})\frac{\alpha}{1-\alpha}\frac{N_s(i)}{K_s(i)}W_s(i) = \frac{N_s(i)}{(1-\alpha)Y_s(i)}\frac{1+\tau_s^c}{1-\tau_s^w}MRS_s\frac{1}{\theta_l}$$

$$MC_s = \frac{N_s(i)}{(1-\alpha)Y_s(i)}MRS_s\frac{1+\tau_s^c}{1-\tau_s^w}\frac{1}{\theta_l} = \frac{N_s(i)}{(1-\alpha)Y_s(i)}\frac{(l_s)^\eta}{(C_s)^{-\sigma_u}}\frac{1+\tau_s^c}{1-\tau_s^w}\frac{1}{\theta_l}$$

where

$$MPK_s = \alpha K_s^{\alpha-1}N_s^{1-\alpha} = \alpha Y_s/K_s$$

with K_t and L_t are aggregate capital and labor stocks.

Firms also consider profit maximizing price setting P_t^* in addition to their capital accumulation process. Intermediate-good producers have a monopoly power over the price of the differentiated good they produce. Prices for intermediate goods are set in a staggered fashion (at random durations) *à* la Calvo (1983). Each period a fraction ($'1-\gamma'$, since we have a continuum of intermediate good firms) of firms are able to reset their prices and all firms that reset their prices set the same price for their goods in equilibrium. It could also be read as the probability of each firm resetting its price in each period is the same. I will call this probability, $'1-\gamma'$.[50] Intermediate-good firms set their price and then decide the amount of labor they need for production, as they are obliged to supply any amount that is demanded at that price. All the after-tax profit of intermediate-good firms is going to the households, since they own all firms.

[47]Steady state analysis show that $r^k = (\beta^{-1} - 1 + \zeta)(1 + \bar{\tau}^c)(1 + \bar{\tau}^I)$ and $\gamma_k = \frac{K}{Y} = \frac{\alpha}{r^k}\frac{1}{\theta}$ where $\frac{1}{\theta} = \mu_{ss}$ is the pricing markup.

[48]Firms choose capital, investment and labor supply to maximize profit function. Given production function and capital accumulation law.

[49]MC_s is real marginal cost and w_s (small) is real wage.

[50]γ fraction of firms are not able to reset their prices.

The profit maximization problem for a typical intermediate-good firm resetting its price $p_t^* = p_t(i)$ is below. When a firm is allowed to set the optimal price, given all the information available, it chooses the price that would maximize its profit even if it never has another chance to re-optimize. E_t is conditional expectation based on the information available at time t.

$$\max_{\{P_t^*\}} E_t \left\{ \sum_{s=t}^{\infty} (\gamma\beta)^{s-t} Q_{s+1} (1-\tau_s^P) \left[(\frac{p_t^*}{P_s}) Y_s(i) - (\frac{MC_s(i)}{P_s}) Y_s(i) \right] \right\}^{51}$$

subject to demand determined output $Y_s(i) = \left[\frac{P_s(i)}{P_s}\right]^{\frac{-1}{1-\theta}} Y_s$. This equals to the following,

$$\max_{\{P_t^*\}} E_t \left\{ \sum_{s=t}^{\infty} (\gamma\beta)^{s-t} Q_{s+1} (1-\tau_s^P) \left[(\frac{p_t^*}{P_s})^{\frac{\theta}{\theta-1}} - (\frac{MC_s(i)}{P_s})(\frac{p_t^*}{P_s})^{\frac{-1}{1-\theta}} \right] Y_s \right\} \tag{30}$$

The first order condition (FOC) of the above profit maximization problem wrt to p_t^*, is then

$$E_t \left\{ \sum_{s=t}^{\infty} (\gamma\beta)^{s-t} Q_{s+1} (1-\tau_s^P) \left[\frac{\theta}{\theta-1} (\frac{p_t^*}{P_s})^{\frac{1}{\theta-1}} (\frac{1}{P_s}) + \frac{1}{1-\theta} (\frac{MC_s(i)}{P_s})(\frac{1}{P_s})(\frac{p_t^*}{P_s})^{\frac{\theta-2}{1-\theta}} \right] Y_s \right\} = 0 \tag{31}$$

which equals

$$E_t \left\{ \sum_{s=t}^{\infty} (\gamma\beta)^{s-t} Q_{s+1} (1-\tau_s^P) \left[\frac{\theta}{\theta-1} (\frac{p_t^*}{P_s}) + \frac{1}{1-\theta} (\frac{MC_s(i)}{P_s}) \right] (\frac{1}{P_t^*})(\frac{p_t^*}{P_s})^{\frac{1}{\theta-1}} Y_s \right\} = 0.$$

Rearranging this FOC and substituting $Q_{s+1} = \frac{\beta\lambda_{s+1}}{\lambda_s}$ we get the following simpler form: and Q_{s+1} is the real (that is why we multiply it by inflation - $(1+E_s\pi_{s+1})$- to get stochastic discount factor $Q_{s+1}\Pi_{s+1}$) stochastic discount factor ($\frac{\beta\lambda_{s+1}}{\lambda_s}$ for some lagrange multiplier $\lambda_s = \frac{U_{c,s}}{(1+\tau_s^c)P_s}$)

$$P_t^* = \frac{1}{\theta} \frac{E_t \sum_{s=t}^{\infty} (\gamma\beta)^{s-t} ln(C_s) MC_s(i) (\frac{p_t^*}{P_s})^{\frac{1}{1-\theta}}}{E_t \sum_{s=t}^{\infty} (\gamma\beta)^{s-t} ln(C_s) (\frac{p_t^*}{P_s})^{\frac{\theta}{1-\theta}}}. \tag{32}$$

If I assume a perfectly flexible case, then $\gamma \to 0$ (meaning there is no firm that is not allowed to reoptimize) and the above equation reduces to,

$$P_t^* = \frac{1}{\theta} MC_s^{52} \tag{33}$$

By taking γ as the fraction of firms that keep their prices fixed each period. We will get the following price index. I assume prices are not changing at all when a firm is not allowed to reset its price. [53]

$$P_t = [\gamma P_{t-1}^{\frac{\theta}{\theta-1}} + (1-\gamma)(p_t^*)^{\frac{\theta}{\theta-1}}]^{\frac{\theta-1}{\theta}} \tag{34}$$

[51] Where $\lambda_s = \frac{U_{c,s}\xi_s}{(1+\tau_s^c)P_s}$ And, as in Forni et al. (2009) and Correia et al. (2011), $Q_{s+1} = \frac{\beta U_{c,s+j}\xi_{s+j}}{U_{c,s}\xi_s} \frac{(1+\tau_s^c)P_s}{(1+\tau_{s+j}^c)P_{s+j}}$ (nominal price at time s of a unit of money at a state in period $s+j$ - stochastic discount factor for Ricardians (share owners of firms)). Eggertsson and Krugman (2010): Q or λ does not play any role in log-linear economy.

[52] This is nominal marginal cost again.

[53] See, Christiano et al. (2005) and Erceg et al. (2006) for an alternative specification.

2.3 The Monetary Authority - Central Bank

The central bank controls the short-term nominal interest rate i_s. I assume the central bank (CB) follows a Taylor rule (Taylor, 1993) to implement monetary policy and that a zero lower-bound for i_s holds.

$$\hat{i}_s = \max\{0, \hat{r}_s^n + \phi_p \pi_s + \phi_y \hat{Y}_s\}$$

where $\phi_\pi > 1$ and $\phi_y > 0$. $\hat{r}_s^n = log\beta^{-1} + \hat{\xi}_s - \hat{\xi}_{s+1}$ is the efficient real interest rate or the natural rate of interest (goes to $\hat{r}_s^n < 0$, if there is a large enough negative shock, $\hat{\xi}_{s+1}$). r^n is the steady state interest rate ($i = r^n = log\beta^{-1}$ with zero inflation and no shock).

The objective of the central bank is to achieve zero inflation under normal circumstances. The Central Bank, with a Taylor rule, sets i_s to achieve zero inflation. Given a positive interest rate, it means CB supplies any base money that is demanded at that nominal rate of interest. However, if interest rates are down to zero, then it sets i_s at zero and lets π_s be determined by the equilibrium conditions, which usually puts it down below zero. Central Bank's commitment to a higher future inflation could be an alternative to temporary government expenditure shocks or temporary cuts in taxes in shifting the aggregate demand, therefore.[54] Commitment policy does not require any change in G or tax cut, but it has a credibility problem (Kydland and Prescot (1977), Walsh (2010) chapter 6). I assume the central bank is not able to commit to future policy.

Absent the Keynesian households and given the tax policy, the Taylor principle is a necessary and sufficient condition for uniqueness of equilibrium for the linear system of equations for long-run with $\pi = 0$ and $i = r^n = \bar{r}$.[55]

As pointed out by Christiano (2004), if $\hat{i}_s = \hat{r}_s^n$ holds for all periods, from the IS equation, then there is an equilibrium where C stays at its steady-state level and $\hat{\pi}_s = 0$ for all periods. Yet if $\hat{r}_s^n < \beta^{-1} - 1$ - there is a shock to $\hat{r}_s^n$, then $\hat{i}_s = \hat{r}_s^n$ does not hold (it violates ZLB condition).

2.4 The Fiscal Authority - Government

Governments basically have three financing methods: taxation, borrowing (issuing debt) and printing money (seigniorage revenue). At its core, all of these methods are varying forms of

[54] See e.g. Krugman (1998), Eggertsson and Woodford (2003) and Eggertsson (2010)

[55] The Taylor principle is a property of the interest rate rule that tells when there is a change in inflation, the nominal interest rate responds more than one for one (more than proportional) to that change in inflation. See, among others, the discussion in Walsh (2010) chapter 8, Gali et al. (2007), Eggertsson and Krugman (2010) and Correia et al. (2011).

taxation. While printing money is taxation of money holdings of the public, borrowing or bond issuance is taxation in the future. I will consider the first two here.

This paper studies the efficacy of the distortionary taxes in eliminating the recession. The government sets different taxes and issues bonds to balance its budget every period. B_s is a government bond. Because the Ricardian equivalence does not hold, the timing of taxation matters. Therefore, we need the government budget constraint as another equilibrium condition.[56] I break Ricardian equivalence by adding distortionary taxes and the Keynesian households to the model. Gali et al. (2007) and Erceg and Linde (2010) argue the short-run fiscal multiplier is greater than one if hand-to-mouth consumers are added to the model and private consumption goes up.

Because the cost of stimulus packages is low in a zero interest rate case, Keynesian economists argue it is beneficial for the government to issue bonds to finance its deficit in short-run. I assume the government finances its deficit (from tax cut / expenditure increase) with some bonds in one case and also consider another case where government expenditure is financed by simultaneously increasing another tax.

Government expenditures in the model economy are financed by means of a variety of distortionary taxes such as labor income tax, capital income taxes (both the asset and profit taxes), sales tax, investment tax credit, a lump-sum tax and nominal debt, risk-less one period bonds B_t. Having bonds to finance the government budget constraint means the government does not need to balance its budget each period. Having distortionary taxes to finance the government spending may change the effect of policy instruments, especially with different income and substitution effects on different households (in their consumption behaviors). I will also assume fiscal rules for discretionary fiscal policy, in the case of income-tax changes as in Gali and Perotti (2003). There are many hand-to-mouth agents who consume their current disposable income. If the government decides to have a budget deficit to finance with bonds, it will have significant effect on the aggregate demand due to existence of the Keynesian households who optimize per period. The government budget constraint will be as follows.

$$R_s^{-1}B_{s+1} + P_sT_s + \tau_s^c P_s C_s + \tau_s^w W_s L_s + \tau_s^p P_s Z_s + \tau_s^A B_s = P_s G_s + B_s \tag{35}$$

[56] If we had only Ricardian households and government expenditures were financed by lump-sum taxes then from the Ricardian equivalence, the timing of taxation would not matter. Fiscal policy would be ineffective. Meaning government expenditure increases or lump-sum tax decreases would be ineffective in raising private comsumption and GDP. This is because, the public would perfectly anticipate future taxes to finance current increases in expenditures.

or in real terms,

$$R_s^{-1}B_{s+1}/P_s + T_s + \tau_s^c C_s + \tau_s^w w_s L_s + \tau_s^p Z_s + \tau_s^A B_s/P_s = G_s + B_s/P_s$$

where $R_s^{-1} = \frac{1}{1+i_s}$ is the gross nominal interest rate and $(1 + i_s) = (1 + \pi_{s+1})(1 + r_s)$. Also $T_s = fT_s^k + (1 - f)T_s^r$.

I also include lump-sum taxes which are standard in the New-Keynesian models and change endogenously (as a residual) to keep government budget in balance. We need a fiscal policy rule that shows how lump-sum taxes change (for financing of tax cuts) to keep the government budget in balance and ensure non-explosive debt dynamics.

$$\hat{t}_s = \phi_b \hat{b}_s + \phi_t \hat{\tau}_s^x, \qquad \text{where } \phi_b > 0 \text{ and } \phi_t > 0 \text{ and } x = c, w, p, A \text{ or } I. \tag{36}$$

Following Eggertsson (2010) and Gali et al. (2007), I use $\hat{g}_s = (G_s - G)/Y$, $\hat{t}_s = (T_s - T)/Y$, $\hat{c}_s = (C_s - C)/Y$ and $\hat{b}_s = [(B_s/P_s) - (B/P)]/Y$.

I assume a two-state markov process for resetting the labor income taxes exogenously (as is the banking shock to the economy), which is the same idea as having a stochastic process such as the following AR(1) process $\hat{\tau}_s^w = \phi_{tax}\hat{\tau}_{s-1}^w + \epsilon_s$ where ϵ_s would be a normally distributed i.i.d. process. I assume a tax cut rule as,

$$\hat{\tau}_s^w = \phi_t r_s^n, \qquad \text{with } \phi_t > 0. \tag{37}$$

Once we have a tax cut, with probability $(1-\omega)$ the tax cuts converges to steady state $\hat{\tau}_s^w = 0$ and with probability (ω) it stays at the short run level $\hat{\tau}_s^w < 0$.

3 The Market Clearing

Factor market clearing (for all s):
The market for capital is in equilibrium since firm-specific capital is accumulated by the firm itself. Demand for capital by the intermediate good firm equals the supply of capital by the intermediate good firm.

At the wage rate set by the households, $W_s(j)$, the labor market will be in equilibrium. Since we assumed the real wage was always higher than the mrs, and labor demand was uniformly distributed among both type of households, labor demand by intermediate good firms equals labor supply by households.

$$\int_0^1 N_s(i)di = N_s = L_s = \left[\int_0^1 L_s(j)^{\theta_l}\right]^{\frac{1}{\theta_l}} \tag{38}$$

Market clearing for the dividend payments requires total dividends from all intermediate firms equal total dividend payment to the households.

$$\int_0^1 Z_s(i)di = \int_0^1 Z_s(j)dj \tag{39}$$

Bond market clearing[57]:

$$B_s = \left[\int_0^1 B_s(j)\right] = 0 \tag{40}$$

Market clearing in the intermediate goods sector:

$$c_s^r(i) + c_s^k(i) + g_s(i) + I_s(i) = Y_s^d(i) = Y_s(i) \tag{41}$$

Resource constraint:

$$P_s Y_s = P_s C_s + P_s I_s + P_s G_s \tag{42}$$

and in real terms.

$$Y_s = C_s + I_s + G_s$$

where

$$I_s = I(\frac{K_{s+1}}{K_s}, \xi_s) K_s$$

and $\frac{K_{s+1}}{K_s} = I_s^N$ is the per period net increase in the capital stock, which implies the following equality.

$$Y_s = C_s + I(I_s^N, \xi_s) K_s + G_s$$

4 The Steady State

For simplicity, it is assumed that the steady-state consumption levels are the same across household types, $C^k = C^r = C$. It can be made sure by right choice of T^r and T^k.[58] This paper is not focused on the steady-state differences, we are interested in responses to shocks, thus this assumption (while simplifies analysis a lot) is not affecting results.

I assumed labor market are not perfectly competitive and wages are set by households (or by union), and labor supply is determined by demand of firms (given the wage) and households are supplying any amount of labor demanded by assuming wage is always above the mrs between C and L. Demand for a differentiated labor type j is uniformly distributed among these households,

[57] In a closed economy, if the government has no bond, then in equilibrium, $B_t = 0$. But in my model, since government has bonds, then $B_t \neq 0$

[58] See e.g. Gali et al. (2007) and Eggertsson and Krugman (2010).

r and k. Therefore, $N_s^r = N_s^k$, $\forall$ s and $N^r = N^k = N$ in the steady state. Labor supplies might diverge because there are distortionary taxes, Walsh (2010).

Indeed, if the labor market was perfectly competitive, then $N_s^r = N_s^k$, $\forall$ s without the need for any other assumption. Since the MRS would have to be equal to the real wage for both type of households.

The steady state has zero inflation ($\pi = 0$), constant taxes (τ^c, τ^w, τ^P, τ^a and τ^I) and the nominal interest rate equals to real rate of interest, and hence to natural rate of interest ($\hat{i} = r^n = ln\beta^{-1}$) since there is no shock and natural rate of interest is positive[59].

I call the steady state ratios $K/Y = \gamma_k$, $C/Y = \gamma_c$, $G/Y = \gamma_g$, and $I/Y = \gamma_i$.[60]

The FOC from the endogenous capital accumulation is again given below.

$$I'(I_s^N(i), \xi_s)(1+\tau_s^c)(1+\tau_s^I)(1-\tau_s^P) = \tag{43}$$

$$E_s Q_{s+1}\Pi_{s+1}(1-\tau_{s+1}^P)\left[r_s^k(i) + I'(I_{s+1}^N(i), \xi_{s+1})I_{s+1}^N(i)(1+\tau_{s+1}^c)(1+\tau_{s+1}^I) - I(I_{s+1}^N(i), \xi_{s+1})(1+\tau_{s+1}^c)(1+\tau_{s+1}^I)\right]$$

In the steady state, given that $I(1,\xi) = \zeta$, $I_I(1,\xi) = 1$, and $I_{II}(1,\xi) = \epsilon_x$, I get,

$$(1+\bar{\tau}^c)(1+\bar{\tau}^I)(1-\bar{\tau}^P) = Q(1-\bar{\tau}^P)[r^k + (1+\bar{\tau}^c)(1+\bar{\tau}^I) - \zeta(1+\bar{\tau}^c)(1+\tau^I)]$$

Rewriting the equation (and using the steady state value $Q = \beta$), I get

$$r^k = (\beta^{-1} - 1 + \zeta)(1+\bar{\tau}^c)(1+\bar{\tau}^I) \tag{44}$$

The firm's pricing equation in the SS is: $MC = \frac{1}{\mu_{ss}} = \theta$, since $P_t(i) = P_t^* = P_t$ in steady state.

We know that $\frac{R_s^k/P_s}{MPK_s} = MC_s = \frac{R_s^k/P_s}{(\alpha)Y_s/K_s} = \frac{R_s^k K_s}{(\alpha)Y_s P_s}$. In steady state: $MC = \frac{1}{\mu_{ss}} = \frac{r^k K}{\alpha Y}$. Therefore, $(\frac{K}{Y}) = \gamma_k = (\frac{\alpha}{\mu_{ss} r^k}) = (\frac{\alpha\theta}{r^k})$.

$$\frac{K}{Y} = \gamma_k = \frac{(\alpha)}{\mu_{ss} r^k} = \frac{(\alpha)}{\mu_{ss}[(\beta^{-1} - 1 + \zeta)(1+\bar{\tau}^c)(1+\bar{\tau}^I)]} \tag{45}$$

And from the same idea,

$$\frac{wN}{Y} = \frac{wL}{Y} = \frac{(1-\alpha)}{\mu_{ss}}{}^{61} \tag{46}$$

The resource constraint:

$$Y_s = C_s + I_s + G_s$$

[59] See the footnote below to see why $\pi = 0$ is optimal in the steady-state.

[60] As in Christiano (2004).

[61] We know that $\frac{W/P}{MPL} = \frac{w}{MPL} = MC = \frac{w}{(1-\alpha)Y/L} = \frac{wL}{(1-\alpha)Y}$. In steady state: $MC = \frac{1}{\mu_{ss}} = \frac{wL}{(1-\alpha)Y}$. Therefore, $(\frac{wL_s^k}{Y}) = (\frac{(1-\alpha)Y}{\mu_{ss}Y}) = (\frac{(1-\alpha)}{\mu_{ss}})$.

$$Y = C + I + G$$

since $I_s = I(\frac{K_{s+1}}{K_s}, \xi_s)K_s$ and $I = I(1, \bar{\xi})K = \zeta K$,

$$Y = C + \zeta K + G$$

therefore, and since we know that $(\frac{K}{Y}) = (\frac{(\alpha\theta)}{r^k}) = (\frac{(\alpha)}{r^k}\mu_{ss})$.

$$\frac{C}{Y} = \gamma_c = 1 - \zeta\gamma_k - \gamma_g = 1 - \gamma_g - \frac{(\zeta\alpha)}{\mu_{ss}r^k} = 1 - \gamma_g - \frac{(\zeta\alpha\theta)}{(\beta^{-1} - 1 + \zeta)(1 + \bar{\tau}^c)(1 + \bar{\tau}^I)} \tag{47}$$

Which shows that γ_c does not depend on the fraction of the rule-of-thumb (Keynesian) agents, given γ_g that is exogenous.

5 Log-Linearization: For Illustration

I use basic FOCs with their steady state versions to work on Dynare, to get the impulse responses. Yet for the sake of illustration and to use it for future works; I will discuss the log-linearization process as well. This section analyzes the log-linear approximation to the structural equations of the model (market clearing and optimality conditions used to analyze equilibrium dynamics). I log-linearize the structural equations of the model around paths of inflation, interest rate and output related to zero-inflation steady-state (which is optimal policy absent any shock), without any shock ($\xi_t = 0$).[62]

Before we start the log linearization, I list some parameters of the model. $\sigma_u = -\frac{u''C}{u'} > 0$ and I call $\sigma = -\frac{u''C}{u'}\frac{Y}{C} = \sigma_u(\gamma_c)^{-1} > 0$, $b = -\frac{u''G}{u'} > 0$, $\eta = \frac{u''L}{u'} > 0$ and $\alpha = -\frac{f''L}{f'}$, for $'$ and $''$ standing for the first and second derivatives.

I use the Hansen method $\hat{x}_s = \frac{X_s - X}{X}$ and $log X_s$/X for log-linearization. A 'hat' over a variable, $\hat{x}_t$, means deviation of a variable (x_t) from its steady state value (x) as a fraction of its steady-state value again. All the aggregate variables that show up in the resource constraint will be linearized as deviation from the steady state value over steady state output level. Which basically means $\hat{C}_t = \frac{C_t - C}{Y}$, $\hat{G}_t = \frac{G_t - G}{Y}$, and $\hat{I}_t = \frac{I_t - I}{Y}$.

[62] A long-run inflation-target of zero is optimal, Woodford (2003) chapter 7. Zero-inflation is needed to get to the efficient production level. A positive inflation means price dispersion (relative price differences among firms), since we have a staggered price setting. This means distortion in the economic activity - allocation of resources - (since demand for each good is depended on its price, as in equation 20. Policy-makers choose (zero) inflation to maximize utility of the representative hh). Zero inflation, $\forall$ firms $P_s(i) = P_s = P_{-1}$, can only be achieved if all firms start initially at the same price (such as P_{-1}) and all firms that have the chance to make a change in their prices choose the same initial price (P_{-1}).

5.1 Log-linearization of the households' optimality conditions

Log-linearizing the Ricardian agent's inter-temporal Euler Equation, I get (from the Appendix),

$$\hat{C}^r_s = E_s\left\{\hat{C}^r_{s+1}\right\} - \frac{1}{\sigma}\left[\hat{i}_s - E_s\pi_{s+1} - \hat{r}^n_s\right] - \frac{\chi^c}{\sigma}E_s\left\{\hat{\tau}^c_s - \hat{\tau}^c_{s+1}\right\} + \frac{\chi^A}{\sigma}(\hat{\tau}^A_s)$$

and the log-linearized consumption equation for the Keynesian agents is below

$$(\chi^c\hat{\tau}^c_s + \hat{c}^k_s)\frac{(1+\bar{\tau}^c)C^k}{Y} = (-\chi^w\hat{\tau}^w_s + \hat{w}_s + \hat{l}^k_s)(\frac{(1-\bar{\tau}^w)WL^k}{Y}) - \hat{t}^k_s$$

where $t^k_s = (\frac{(T^k_s - T^k)}{Y})$ as in Gali et al. (2007). I assume steady state consumption levels are the same across heterogenous households. See, e.g., discussion in Gali et al. (2007). This implies $\hat{l}^k = \hat{l}^r = \hat{l}$, since the MRS between consumption and labor supply is equalized among the heterogenous agents (due to the same wages for both types of households in the labor market). Also $\frac{WL^k_s}{Y} = \frac{(1-\alpha)}{\mu_{ss}}$. See the appendix, for details. then the above log-linearized equation turns to a simpler form.

$$(\chi^c\hat{\tau}^c_s + \hat{c}^k_s)\frac{(1+\bar{\tau}^c)C^k}{Y} = (-\chi^w\hat{\tau}^w_s + \hat{w}_s + \hat{l}^k_s)[\frac{(1-\alpha)}{\chi^w\mu_{ss}}] - \hat{t}^k_s$$

The wage schedule is given below, which is derived from the (in perfect competition labor market) intra-temporal Euler equations (both) combined with the aggregation equations (for L and C). Gali et al. (2007) shows how we get a log-linear approximation of the form below, from an intra-temporal equation as $W_s = H(C_s, N_s)$ (in an imperfect labor market).

$$\hat{w}_s - \chi^c\hat{\tau}^c_s - \chi^w\hat{\tau}^w_s = \sigma\hat{c}_s + \eta\hat{l}_s$$

The inter-temporal equilibrium condition for the aggregate consumption is below then.

$$\hat{c}_s = E_s\hat{c}_{s+1} + \frac{DD}{BB}E_s\Delta\hat{\tau}^c_{s+1} - \frac{f(1+\eta)(1-\alpha)\chi^c}{BB}E_s\Delta\hat{l}_{s+1} + \frac{AA}{BB}f\frac{\chi^c}{\gamma_c}E_s\Delta\hat{t}^k_{s+1} \tag{48}$$

$$-\frac{AA.CC}{BB}(\hat{i}_s - E_s\pi_{s+1} - \hat{r}^n_s) + \frac{AA.CC}{BB}\chi^A(\hat{\tau}^A_s)$$

Where

$$AA = \chi^w\mu_{ss}\gamma_c$$

$$BB = \frac{\chi^w\mu_{ss}\gamma_c - f\sigma(1-\alpha)\chi^c}{\chi^w\mu_{ss}\gamma_c} = \frac{AA - f\sigma(1-\alpha)\chi^c}{AA}$$

$$CC = \frac{(1-f)}{\sigma}$$

$$DD = -f\left(\chi^c(1-\alpha)\chi^c + \chi^c AA\right) + (1-f)\frac{\chi^c AA}{\sigma}$$

5.2 Log-linearization of the firms' optimality conditions

Appendix D shows that the real MC in log-linearized form is below.

$$\hat{MC}_s^{real} = (\frac{\eta}{1-\alpha} + \frac{\alpha}{1-\alpha})(\hat{Y}_s - \hat{K}_s) + \eta\hat{K}_s + \sigma\hat{C}_s + \chi^c\hat{\tau}_s^c$$

Appendix C shows how to derive and log-linearize $\hat{r}^k$,

$$\hat{r}_s^k = \mu_y\hat{Y}_s - \mu_y\hat{K}_s + \sigma\hat{C}_s + \chi^c\hat{\tau}_s^c + \chi^w\hat{\tau}_s^w$$

$$\hat{r}_s^k = \mu_y(\hat{Y}_s - \hat{K}_s) + \eta\hat{K}_s + \sigma\hat{C}_s + \chi^c\hat{\tau}_s^c + \chi^w\hat{\tau}_s^w \tag{49}$$

where

$$\mu_y = \frac{\eta}{1-\alpha} + \frac{1}{1-\alpha}$$

$$\mu_k = \frac{\eta}{1-\alpha} + \frac{1}{1-\alpha} - \eta$$

Appendix C shows that the FOC from the endogenous capital accumulation by firms in the log-linearized form is below.

$$\hat{I}_s^N = \beta E_s\hat{I}_{s+1}^N - \sigma_I(i_s - E_s\pi_{s+1} - \hat{r}_s^n - \chi^A\hat{\tau}_{s+1}^A) + \chi E_s\hat{r}_{s+1}^k \tag{50}$$

$$-\chi^c[\hat{\tau}_s^c - \beta(1-\zeta)E_s\hat{\tau}_{s+1}^c] + \chi^P[\hat{\tau}_s^P - \beta(1-\zeta)E_s\hat{\tau}_{s+1}^P] - \chi^I[\hat{\tau}_s^I - \beta(1-\zeta)E_s\hat{\tau}_{s+1}^I]$$

for some,

$$\chi = \frac{[1-(1-\zeta)\beta]}{\epsilon_x} = \frac{\beta r^k}{(1+\bar{\tau}^c)(1+\bar{\tau}^I)\epsilon_x}$$

$$\sigma_I = \frac{1}{\epsilon_x}$$

Appendix D shows step-by-step derivation of the following NK phillips curve (to shows how firms set prices),

$$\pi_t = \kappa(\psi + \sigma)\hat{Y}_t - \kappa\sigma\hat{G}_t - \kappa\sigma\gamma_k\hat{I}_t^N + \kappa(\eta - (\psi) - \sigma\gamma_k\zeta)\hat{K}_t + \kappa\chi^c\hat{\tau}_t^c + \beta E_t\pi_{t+1} \tag{51}$$

Log-linearization of the production function is shown in the appendix,

$$Y_s = (N_s)^{1-\alpha}(K_s)^\alpha \tag{52}$$

It equals to,

$$\hat{Y}_s = (1-\alpha)\hat{N}_s + (\alpha)\hat{K}_s \tag{53}$$

5.3 Log-linearization of the Government Budget Constraint

Following Eggertsson (2010) and Gali et al. (2007), I assume, $g_s = (G_s - G)/Y$, $t_s = (T_s - T)/Y$, $c_s = (C_s - C)/Y$ and $b_s = [(B_s/P_s) - (B/P)]/Y$. Then the GBC in log-linearized form is below. Appendix shows that the government budget constraint in log-linearized form is as below.

$$\hat{b}_{s+1} = (1+r)[\hat{g}_s - \hat{t}_s - [\hat{c}_s + \frac{\hat{\tau}^c_s}{\bar{\tau}^c}](\bar{\tau}^c \gamma_c) - [\hat{w}_s + \hat{L}_s + \frac{\hat{\tau}^w_s}{\bar{\tau}^w}](\bar{\tau}^w \frac{(1-\alpha)}{\mu_{ss}}) - [\hat{Z}_s + \frac{\hat{\tau}^P_s}{\bar{\tau}^P}](\frac{\bar{\tau}^P Z}{Y}) + (-\chi^A \hat{\tau}^a + \hat{b}_s)(\frac{(1-\bar{\tau}^a)b}{Y})] \tag{54}$$

where

$$\hat{b}_{s+1} = \frac{(b_{s+1} - b)}{Y}$$

$$\hat{g}_s = \frac{(G_s - G)}{Y}$$

$$\hat{t}_s = \frac{(T_s - T)}{Y}$$

It also equals to,

$$\hat{b}_{s+1} = (1+r)[\hat{g}_s - \hat{t}_s - [\hat{c}_s + \frac{\hat{\tau}^c_s}{\bar{\tau}^c}](\frac{\bar{\tau}^c C}{Y}) - [\hat{w}_s + \hat{L}_s + \frac{\hat{\tau}^w_s}{\bar{\tau}^w}](\frac{\bar{\tau}^w wL}{Y}) - [\hat{Z}_s + \frac{\hat{\tau}^P_s}{\bar{\tau}^P}](\frac{\bar{\tau}^P Z}{Y}) + (-\chi^A \hat{\tau}^a + \hat{b}_s)(\frac{(1-\bar{\tau}^a)b}{Y})]$$

where all variables are aggregated over households and firms.

Using the fiscal policy rule assumed earlier and the log-linearized government budget constraint, we get the following equilibrium condition.

$$\hat{b}_{s+1} = (1+r)[\hat{g}_s - (\phi_b \hat{b}_s + \phi_t \hat{\tau}^x_s) - [\hat{c}_s + \frac{\hat{\tau}^c_s}{\bar{\tau}^c}](\frac{\bar{\tau}^c C}{Y}) - [\hat{w}_s + \hat{L}_s + \frac{\hat{\tau}^w_s}{\bar{\tau}^w}](\frac{\bar{\tau}^w wL}{Y}) - [\hat{Z}_s + \frac{\hat{\tau}^P_s}{\bar{\tau}^P}](\frac{\bar{\tau}^P Z}{Y}) + (-\chi^A \hat{\tau}^a + \hat{b}_s)(\frac{(1-\bar{\tau}^a)b}{Y}$$

5.3.1 Log-linearization of the Resource Constraint

Log-linearizing the resource constraint, we get the following result.

$$\hat{Y}_t = \hat{C}_t + \hat{I}_t + \hat{G}_t \tag{55}$$

since $\hat{X}_t = \frac{X_t - X}{Y}$ for all X = C, G, and I. We also know that[63],

$$\hat{I}_t = \gamma_k[\hat{K}_{t+1} - (1-\zeta)\hat{K}_t] = \gamma_k[\hat{K}_{t+1} - \hat{K}_t] + \gamma_k \zeta \hat{K}_t$$

using this equation, the above log-linearized resource constraint turns into,

$$\hat{Y}_t = \hat{C}_t + \hat{G}_t + \gamma_k \hat{I}^N_{t+1} + \gamma_k \zeta \hat{K}_t \tag{56}$$

[63] From the Appendix Ea

6 Equilibrium

An equilibrium of model is characterized by stochastic processes for a long list of endogenous variables, including $C_s, L_s, K_s, Y_s,\ P_s^*, P_s, W_s, R_s^k,\ B_s,\ {}_s,\ \tilde{\rho}_s,\ \pi_s,\ mc_s$, and policy variables $i_s, \tau_s^c, \tau_s^w, \tau_s^A, \tau_s^p, T_s, G_s^N, \tau_s^I$; as well as an initial value for price p_{-1}. The economy have an exogenous sequence ξ_s. All these variables along with earlier created model equations satisfy the household optimality conditions (inter-temporal and intra-temporal Euler-Equations), firm price setting equations (the aggregate price index and optimal price setting equation), firms' endogenous capital accumulation equation, a market clearing equation (the resource constraint) and the government budget constraint. There is no need to keep track of the other budget constraints, because lump-sum taxes adjust to keep the other budget constraints satisfied.

Meanwhile following the shock, $i_s \geq 0$. Also assume that all those firms that haven't yet set their prices (α^{s+1} share of firms) and those that have set it j periods ago ($\alpha^j(1-\alpha)$ share of firms) all have an exogenous price p_{-1}.

An estimation process, that we don't contemplate on here directly, checks for uniqueness of the equilibria, and conditions for a unique equilibrium. Yet, I stick to parameter values that give a unique equilibrium (as in Gali et al. (2007)). Gali et al. (2007) show having rule-of-thumb agents change equilibrium properties a lot, and that high degree of price stickiness and large share of the Keynesian agents together cause indeterminacy (even in case of interest rule satisfying the Taylor principle). Low and average values of 'share of Keynesians' and 'share of firms not able to change their prices' will still give a unique equilibrium (such as share of Keynesians = 1/2 and price stickiness = 0.75, baseline calibration values in Gali et al. (2007)).

6.1 Approximate Equilibrium

Because I look at multiplier effect of tax cuts, not the optimal fiscal policy, I can get a closed form solution as in Eggertsson (2010).[64] I also assume a short and long-run in the economy which makes it easier to get a closed form solution, even-though we have an infinite horizon problem.

I combine all equilibrium conditions finally and get log-linear equations that characterize the equilibrium dynamics, accompanied by a monetary policy rule. They can be summarized by aggregate demand and aggregate supply equations. The equilibrium equations will include two Euler equations for the AD (the optimal inter-temporal consumption decision of households

[64] As pointed out by Uhlig and Drautzburg (2011), policy-makers usually care about welfare and thus the optimal policies. Although this paper is not directly focusing on optimal fiscal policy, its findings may be used in that direction.

and optimal investment decision of firms) and one equation for the AS (the firm pricing EE). Equilibrium of the model will be reduced to stochastic processes for the endogenous variables $\hat{Y}_s, \pi_s, \hat{r}^n_s, \hat{i}_s$ and fiscal policy rules for $\tau^c_s, \tau^w_s, \tau^A_s, \tau^p_s, \tau^p_I, G^N_s$ that solve the following equations.

When I assume government is using sales tax-cut, for instance, to stimulate the economy, government expenditure and the other taxes do not change and $\hat{G}_s = \hat{G}_{s+1} = 0$ and $\hat{\tau}^w_s = \hat{\tau}^w_{s+1} = 0$.

The Aggregate Demand (AD) is from the optimal inter-temporal consumption decision of households and optimal investment decision of firms. The aggregate demand equations are as follows, then.

$$\hat{c}_s = E_s\hat{c}_{s+1} + \frac{DD}{BB}E_s\Delta\hat{\tau}^c_{s+1} - \frac{f(1+\eta)(1-\alpha)\chi^c}{BB}E_s\Delta\hat{l}_{s+1} + \frac{AA}{BB}f\frac{\chi^c}{\gamma_c}E_s\left(\phi_b(\hat{b}_{s+1}-\hat{b}_s) + \phi_t(\hat{\tau}^x_{s+1}-\hat{\tau}^x_s)\right)$$
$$-\frac{AA.CC}{BB}(\phi_y\hat{Y}_s + \phi_\pi\hat{\pi}_s + \hat{r}^n_s - E_s\pi_{s+1} - \hat{r}^n_s) + \frac{AA.CC}{BB}\chi^A(\hat{\tau}^A_s) \quad (57)$$

Where

$$AA = \chi^w\mu_{ss}\gamma_c$$
$$BB = \frac{\chi^w\mu_{ss}\gamma_c - f\sigma(1-\alpha)\chi^c}{\chi^w\mu_{ss}\gamma_c} = \frac{AA - f\sigma(1-\alpha)\chi^c}{AA}$$
$$CC = \frac{(1-f)}{\sigma}$$
$$DD = -f\left(\chi^c(1-\alpha)\chi^c + \chi^c AA\right) + (1-f)\frac{\chi^c AA}{\sigma}$$

$$\hat{I}^N_s = \beta E_s\hat{I}^N_{s+1} - \sigma_I(\phi_y\hat{Y}_s + \phi_\pi\hat{\pi}_s + \hat{r}^n_s - E_s\pi_{s+1} - \hat{r}^n_s - \chi^A\hat{\tau}^A_{s+1}) \quad (58)$$
$$+\chi E_s[\mu_y\hat{Y}_s - \mu_y\hat{K}_s + \sigma\hat{C}_s + \chi^c\hat{\tau}^c_s + \chi^w\hat{\tau}^w_s]$$
$$-\chi^c[\hat{\tau}^c_s - \beta(1-\zeta)E_s\hat{\tau}^c_{s+1}] + \chi^P[\hat{\tau}^P_s - \beta(1-\zeta)E_s\hat{\tau}^P_{s+1}] - \chi^I[\hat{\tau}^I_s - \beta(1-\zeta)E_s\hat{\tau}^I_{s+1}]$$

for some,

$$\chi = \frac{\beta r^k}{(1+\bar{\tau}^c)(1+\bar{\tau}^I)\epsilon_x}$$
$$\sigma_I = \frac{1}{\epsilon_x}$$

and

$$\mu_y = \frac{\eta}{1-\alpha} + \frac{1}{1-\alpha}$$
$$\mu_k = \frac{\eta}{1-\alpha} + \frac{1}{1-\alpha} - \eta$$

Aggregate Supply (AS), from the optimal pricing and consumption decision of firms, in log linearized form is below.

$$\pi_t = \kappa(\psi + \sigma)\hat{Y}_t - \kappa\sigma\hat{G}_t - \kappa\sigma\gamma_k\hat{I}_t^N + \kappa(\eta - (\psi) - \sigma\gamma_k\zeta)\hat{K}_t + \kappa\chi^c\hat{\tau}_t^c + \kappa\chi^w\hat{\tau}_t^w + \beta E_t\pi_{t+1} \quad (59)$$

where,

$$\kappa = \frac{(1-\gamma\beta)(1-\gamma)}{(\gamma)}$$

$$\psi = \frac{\alpha+\eta}{1-\alpha}$$

I also need the Monetary policy rule to close the model, which is zero in this specific environment.

$$i_t = 0 \quad (60)$$

6.2 Discussion of the log-linearized equations above

The aggregate EE for households' consumption decisions is the only log-linear equation involving 'f' coefficient, fraction of the Keynesian households. Including the Keynesian households into the model generates 'direct demand effects' of distortionary tax and employment changes on C and demand. For instance, as a sales-tax is cut temporarily, it increases households purchasing power; and also makes consumption in current period cheaper than that in the future, once taxes go back to their steady-state level. As consumption increases, the aggregate-demand (AD) goes up. The AD increase, expands the output and that increases employment and real wage further. Hence the bigger multiplier effect as in the old-Keynesian theory.

The effect of tax cut on consumption and output will be maximized, if the response of interest rate and change in other taxes is muted. This would be done by appropriate fiscal and monetary policies, Gali et al. (2007).

It should also be noted that, I assumed real wages are always higher than the MRS (between C and L) and hence households are willing to supply any amount of labor demanded. I also assumed both households are supplying every type of labor (both types of households are uniformly distributed among labor types), and firms' demand is uniformly distributed among households. This means their labor supply is responding to tax changes in the same way. Thus, the firm's Euler Equation is not affected through labor supply differences.

7 Calibration of the Model

Theoretical background of the calibration of the model economy to the data and selection of the parameter values follows Eggertsson (2010), Christiano (2004), Woodford (2003), Gali et al. (2007) and Walsh (2010). I follow Eggertsson (2010) and use parameter and shock values to match an output contraction of 30 percent and a deflation of 10 percent, both of which are statistics from the first quarter of 1933 in the U.S. economy. This was trough of the Great Depression with a zero nominal interest rate. This benchmark is to strengthen the argument that the fiscal stimulus in 2009 was, as argued by many economists, more like a reaction to avoid another great depression due from the banking shock in 2008. The magnitude of the crash due to the 2008 crisis was comparable to the Great Depression according to Reis (2010). Great depression is the main example for any liquidity trap analysis according to Krugman (1998).

I use parameter values close to the benchmark Eggertsson (2010) model, as much as possible, to make sure I have a good comparison between my and his model. Each time period in the model is a quarter of a year. $\beta = 0.995$ is the discount factor for the Ricardian households and imply a long-run real-rate of interest $\overline{r} = r_{ss}$ equal to 2 percent.[65] κ is consistent with the empirical estimates from Rotemberg and Woodford (1997). σ_u, coefficient of relative risk aversion is 1.1599. α and ζ are standard in the literature.

$\eta = 1.5692$ is the inverse Frisch elasticity of labor supply. It is very high in Woodford (2003) - ZLB does not bind in small-shock case according to Christiano (2004), and he keeps it at a standard value (for 1) and shows that with a relatively smaller value, even a big shock is not causing ZLB to bind (never binds). Gali et al. (2007) sets it to $\eta = 0.2$. If $\eta = 1$ then in no-investment case same results as in Eggertsson and Woodford are obtained, yet for investment case, results are very sensitive to the value of η. Christiano (2004) argues if parameters in Woodford (2003) are used, in a model with investment, then probability of output collapse and negative inflation is reduced a lot. An elasticity of 100 (very high compared with literature) is needed for the worst case scenario to happen. Christiano (2004) sets $G/Y = \gamma_g = 0.18$, the average government expenditure - output ratio in the post-WWII US economy. While Gali et al. (2007) take it as (0.2).

$'1-\omega_w'$ shows the probability that the economy converges to its steady-state equilibrium each period. γ (fraction of firms that keep their prices fixed) imply prices are fixed for $\frac{1}{1-\gamma}$ periods on average. Depreciation rate ζ and β are standard as in RBC models. Markup $\mu = \frac{1}{\theta_p}$ means a markup of price on marginal cost in the steady state.

Eggertsson and Woodford (2003) has no capital accumulation (no investment), therefore $\epsilon_x = \infty$. Woodford (2003), on the other hand, suggests $\epsilon_x = 3$ s.t. for a small shock ($r_t^n = -2$ from SS value 4 percent) ZLB is not binding. Eggertsson (2010) chooses ϵ_x such that output contraction in the fourth quarter the depression is -30 percent (he assumes consumption and

[65] $\beta = (1 + r_{ss})^{-\frac{1}{4}}$ and $(1 + 0.02)^{-\frac{1}{4}} = 0.995$

investment decline in the same proportion). Gali et al. (2007) choose a value equivalent to 1 in baseline calibration.

Calibration takes place in accordance with the following target values for the benchmark economy:

Targets	Values
$\hat{Y}_s$	-30 percent
π_s	-10 percent

(61)

Parameter values of policy rules are as follows. $\phi_p = 1.5$, the Taylor principle, as commonly assumed and $\phi_y = 0.5/4$ as in Taylor (1993). Choice of fiscal policy (rule) parameters, ϕ_b and ϕ_t, affect aggregate consumption and hence demand in the economy. These parameters will also be changed or sensitivity analysis.

Gali et al. (2007) - estimated averages from VAR: $\phi_b = 0.33$ and $\phi_t = 0.1$, their ϕ_g.

Table 2: Parameter values of the model economy

Parameters	Description	Values
σ_u	coefficient of relative risk aversion	$1.1599\gamma_c$
β	subjective discount factor	0.995
η	inverse elasticity of labor supply	1.5692
γ_l	Calvo hazard rate	0.75
γ_p	Calvo hazard rate	0.75
θ_p	degree of MC in goods market	0.1
θ_l	degree of MC in labor market	0.1
ζ	depreciation rate of capital	0.025
α	capital share of output	0.4
ϵ_x	degree of adjustment cost parameter	71.9
ϕ_p	coefficient on $\hat{\pi}_s$	1.5
ϕ_y	coefficient on $\hat{Y}_s$	0.5/4
ϕ_b	coefficient on $\hat{b}_s$ in fiscal rule	0.33
ϕ_t	coefficient on $\hat{t}_s$	0.1
r_s^n	natural rate of interest rate	-0.0104
$\bar{\tau}^c$	SS value of consumption tax	0.05
$\bar{\tau}^w$	SS value of labor-income tax	0.2
$\bar{\tau}^A$	SS value of capital tax	0
$\bar{\tau}^P$	SS value of profit tax	0.3
$\bar{\tau}^I$	SS value of investment-tax credit	0
f	measure of Keynesian households	between 0.5 and 0
ω_w	prob. of $\hat{r}_t^n$ not returning to its SS value	0.9030
γ_g	average share of G in GDP	0.2
γ_i	average share of I in GDP	0.2

8 Discussion of the Model Calibration

A key assumption here, as is usually assumed in all DSGE models, is that the economy faces a shock that takes the economy into a liquidity trap with zero nominal interest rate. I assume, given all the fiscal variables stay at their steady state values (no intervention) the economy faces an output collapse comparable to the Great Depression after a shock. The Great Depression is a useful benchmark for my model for two main reasons. The first is the trivial case that it is a good benchmark for any liquidity trap analysis, as pointed out by Krugman (1998); and secondly, as pointed out by Eggertsson (2010) among others, economic models show if the government did not use its fiscal tools to intervene into the economy in 2008; the US would have faced another Great Depression.

The paper analyzes the question that; if we cut taxes by 1 percent (from their steady-state values), how much does the output change. Since all the aggregate variables including G are log-linearized as a fraction of steady state output level, a one percent change in taxes on the output and hence consumption and labor income will be of the same effect as a G effect. Therefore, comparison of multipliers makes sense as in Eggertsson (2010).

In a crisis period, such as that studied here, what we observe is that, initially the (log) natural rate of interest goes down to the negative territory unexpectedly; and then it goes back to its long-run (SS) value ($\overline{r} > 0$) with some fixed probability, $'1 - \omega_w'$ each period.[66] If inflation target is zero, as assumed here, even when the central-bank decreases nominal interest rate to zero, the real interest rate is positive because of expectations of deflation. However, in a negative natural rate of interest case, if the central-bank has a large enough inflation target to get zero interest (for instance $r_s^n = -2$ percent and $\pi^* = 2$ percent), then it is enough to close the output gap and keep inflation on target. However, as discussed earlier, I assume the central bank is not able to commit to a future positive inflation rate.

I assume the economy is initially in a deterministic steady state (with no shock) until period (t-1). At period $'t'$ there is, unexpectedly, a shock hits r_t^n and decreases it to maybe even negative. Each period after that, $\hat{r}_t^n$ stays low with a probability $'\omega'$ and increases to its steady-state value with probability $'1 - \omega'$.[67]

I also assume 'T' is the period, when r_s^n goes back to its steady state value with no exogenous shock. Then, the short-run is the period where we observe a preference shock s.t. $r_s^n = \frac{1}{\beta}\frac{\xi_s}{\xi_{s+1}} < 1$

[66]If $a < 1$, then $log(a) < 0$.

[67]Higher $\hat{r}_s^n$ means lower β, and thus higher consumption since agents value future consumption less. A lower $\hat{r}_t^n$, means higher β and therefore less current consumption relative to future. The shock in this economy makes $\hat{r}_t^n$ negative (in log form).

for $s < T$; and the long-run is defined by $r^n_s = \frac{1}{\beta} > 1$ for $s \geq T$ (This is because $\frac{\xi_s}{\xi_{s+1}} < \beta$ for $t < T$, and $\frac{\xi_s}{\xi_{s+1}} = 1$ for $t \geq T$ holds exogenously). And $1 + i_s = 1$ for $s < T$, and $i_s = r^n_s = \bar{r} = \frac{1}{\beta}$ for $s > T$.

In the log-linearized form, it will be as $r^n_s = log\beta^{-1} + \hat{\xi}_s - \hat{\xi}_{s+1}$. Where $log\beta^{-1} \cong r^n$, when there is no shock, the steady state real risk-free interest rate. $\hat{\xi}_s - \hat{\xi}_{s+1}$ is a measure of risk from the exogenous shock and ξ is the shock that enters the utility function. In a demand shock case r^n_s is a function of the shock (ξ) that enters the utility function. Where a lower demand (negative demand shock) would increase savings and then the real interest rate would go up. ξ is a vector of disturbances that covers an external shock such as a changing technology (A) or preference.

I assume, as in Correia et al. (2011), that the nominal interest rates are always set to zero whenever (log) natural rate of interest is negative ($r^n_s < 0$). And they start increasing when natural rate of interest becomes positive. This means, when we have a negative natural rate of interest, deflation and thus positive real interest rate at the ZLB. This is a Liquidity trap case. And deflation means an output contraction as shown in a NK IS curve.

In order to increase spending and demand in the economy, we need negative real interest rate. If we keep taxes constant, the only way to decrease the real interest rates (to negative) is to generate positive inflation as offered by Krugman (1998) and Eggertsson and Woodford (2003). Yet we know that positive inflation means relative price dispersions in economy; and given staggered price setting decision of firms, it creates distortions in real economy. We therefore, make future inflation announcements not-credible.

However, by changing taxes, as in Correia et al. (2011), it is possible to get back to an efficient outcome level even when the ZLB binds. However, to get the efficient outcome level, it is necessary that all distortionary taxes adjust simultaneously. So we can set nominal interest rate equal to natural rate of interest whenever the latter is positive, and set nominal rate to zero whenever the natural rate of interest is zero. Then we use the inter-temporal and intra-temporal Euler-Equations to set the taxes such that all the distortions are eliminated. Price level will be kept constant of course.

We assume a Markovian process for the natural rate of interest. In the short run if the shock occur, $r^n_s < 0$, then recession occurs and $i_s = 0$ (zero lower bound binding). I assume the government uses distortionary tax cuts to stimulate the economy, e.g. $\hat{\tau}^w_s < 0$ in the short-run. $\hat{\tau}^w_s < 0$ is reversed to the steady-state value $\hat{\tau}^w_s = 0$ with probability $(1 - \omega)$ each period in the short-run, once we have a cut. The two state Markov process for the shock (the assumption that the shock goes back to its long-run value each period in short-run with probability $(1 - \omega)$ each period) means inflation and output goes back back to their long-run values with the same probability $(1-\omega)$. Which basically means,

I only consider the case where condition $C1$ and $C2$ in the Eggertsson (2010) model holds and we have a unique and bounded equilibrium. Including the Keynesian households, we need to make sure the parameter values are again consistent with a uniques equilibrium. That is the case that we have zero interest rates in the short-run, due to the banking shock. In the long-run there is a unique bounded solution (if only Ricardians existed) with $\pi_s = 0$, $\hat{Y}_s = 0$ and $i_s = r_s^n = \bar{r}$.

$$i_s = r_s^n = \bar{r} \qquad s \geq T i_s = 0 \qquad s < T$$

Assuming $s < T$ and there is a banking shock, in the next period, $'s+1'$, we have the following case.

$$E_s\hat{Y}_{s+1} = \omega\hat{Y}_s + (1-\omega)0 \qquad , \qquad E_s\pi_{s+1} = \omega\pi_s + (1-\omega)0$$

and the tax cut rule is,

$$(\hat{\tau}_s^c, \hat{\tau}_s^w, \hat{\tau}_s^A, \hat{\tau}_s^P, \hat{\tau}_s^I, \hat{\tau}^{GN}_s) = (1,1,1,1,1,1) \qquad s < T$$

$$(\hat{\tau}_s^c, \hat{\tau}_s^w, \hat{\tau}_s^A, \hat{\tau}_s^P, \hat{\tau}_s^I, \hat{\tau}^{GN}_s) = (0,0,0,0,0,0) \qquad s \geq T \qquad r_s^n = \bar{r}$$

Long-run (or steady state) in this paper is the case that the shock r_s^n goes back to the steady state $r^n = \bar{r}$. Short run, on the other, is the case that the economy faces a shock, $r_s^n < 0$. In the short-run, given that the shock has occured, the shock goes back to its steady state value with probability $(1-\omega)$.

Because I am looking at the special case where the zero bound binds (due to existence of the shock), all i_s are equal to zero. $E_s\hat{Y}_{s+1} = \omega\hat{Y}_s + (1-\omega)0 = \omega\hat{Y}_s$ and $E_s\pi_{s+1} = \omega\pi_s + (1-\omega)0 = \omega\pi_s$. $E_s\hat{\tau}_{s+1}^c = \omega\hat{\tau}_s^c + (1-\omega)0 = \omega\hat{\tau}_s^c$ and $r_s^e < 0$.

I weigh the two multipliers from the EE from firm's investment decision and the EE from household's consumption decisions. Steady state (or long-run) share of investment in total GDP γ_i will be used for the multiplier from equation 128 that comes from firms' investment decision problem; and steady state share of consumption in total GDP γ_c will be used to weigh the multiplier from households' consumption EE.

The aggregate multiplier is,

$$\gamma_c M_{hh} + \gamma_i M_{firm} = M_{total} \tag{62}$$

where γ_c and γ_k were given earlier as $\gamma_c = 1 - \gamma_g - \frac{\zeta\alpha\theta}{r^k}$ and $\gamma_k = \frac{\alpha\theta}{r^k}$. From the resource constraint in steady state,

$$Y = C + I + G \qquad \Rightarrow \qquad \frac{Y}{Y} = \frac{C}{Y} + \frac{I}{Y} + \frac{G}{Y} \qquad \Rightarrow \qquad 1 = \gamma_c + \gamma_i + \gamma_g$$

Then $\gamma_i = 1 - \gamma_g - \gamma_c$.

8.1 Labor Tax Cuts

We are interested in a fiscal policy tool that increases demand and hence output in the economy such that it brings an end to the recession caused by missing demand. In the short-run, $s < T$, $\hat{\tau}_s^w < 0$ and in the long-run, $s \geq T$, $\hat{\tau}_s^w = 0$. As before, we assume, each period tax cut goes back to steady-state with probability $'1 - \omega'$. This Markov process has the same implications as a stochastic process, $\hat{\tau}_s = \mu_s \hat{\tau}_{s-1} + \epsilon_s$ with ϵ_s iid and normally distributed, assumed in Gali et al. (2007) and Eggertsson (2010).

As discussed earlier, labor tax cuts in this model are initially the labor-income taxes paid by workers rather than payroll taxes paid by firms. Thus workers are directly affected by this cut. This is both because wages are fixed in this model and nominal wages on contracts exclude any taxes.

The labor-income tax cut, $\hat{\tau}_s^w < 0$, impact and long-run multipliers will be as follows.

$$\frac{\Delta \hat{Y}_s}{-\Delta \hat{\tau}_s^w} > 0$$

and

$$\frac{\Delta \hat{Y}_{s+k}}{-\Delta \hat{\tau}_s^w} > 0$$

where Δ means change relative to steady-state of no variation.

Fscal policy is as beow,

$$\hat{\tau}_s^w = \phi_s r_s^n \qquad s < T \hat{\tau}_s^w = 0 \qquad s \geq T$$

9 Results

By including nominal rigidities and hand-to-mouth agents (via the direct demand effect) into the model, I primarily focus on and expect to see the positive effect from this countercyclical discretionary fiscal policy that has been controversial in recent studies. As a matter of fact, compared to the benchmark Eggertsson (2010) model, the paper finds significant effects for consumption and / or labor-income taxes.

Table 4: Benchmark Model Outcomes

Targets	Values initially	Value after the ZLB binds
$\frac{\Delta \hat{Y}_{s+k}}{-\Delta \hat{\tau}_s^w}$	$-1,2$ percent	- 2,1percent

I find $\frac{\Delta \hat{Y}_{s+k}}{-\Delta \hat{\tau}_s^w} = 2,1$, which means: if the fiscal authority cuts tax rate $\hat{\tau}_s^w$ by 1 percent, output increases by 2,1 percent. In dollar terms, it means in the steady state, when government cuts taxes by 1 dollar, it increases output by 2,1 dollars.

The idea is that, when there is a tax cut, nominal income from work goes up and that increases willingness to work more, to get more money for each unit of labor supply. Increasing labor supply, decreases real wages (marginal cots down). Lower real wages means lower input cost which increases supply and decreases prices. Therefore, we observe a deflationary pressure. Deflationary expectations, in return, increase the real interest rate which decreases demand and spending in the current period.

If we had positive interest rate, under normal circumstances - absent any shock, the monetary authority would cut taxes aggressively (more than proportional) in order to decrease real interest rates, and thus increase the demand in economy (CB following the Taylor principle). However, if the ZLB binds, the monetary authority is not able to cut the nominal rates to change the real rate of interest. Therefore, the AD curve becomes upward sloping. This means a low inflation will always imply a higher real rate of interest and thus lower demand, and a high inflation will give low real rate because central bank is not able to respond.

Eggertsson (2010), in a model with endogenous investment, finds a multiplier equal to 0.16 in positive interest rate, and another equal to -1.2706 for the case that the ZLB binds.[68] My multiplier is much higher since I am including the direct demand effect via the the Keynesian agent setup. Based on the multipliers he estimates, Eggertsson (2010) suggests a balanced budget (GBC) stimulus package with temporary sales-tax cuts and/or investment tax credits; financed by again temporary labor-tax and/or capital income tax increases. But, don't increase labor taxes!, because some agents consume all of their current after-tax income.

Meanwhile, Eggertsson (2010) finds adding capital does not change results a lot. Which contradicts with findings here and those in Christiano (2004). One reason could be that the same shock is included in both utility function (and thus C EE) and investment adjustment cost (hence in the I EE). Another reason is that Eggertsson (2010) does not estimate model parameters again when he adds capital to the model. Instead, he uses his same estimates from the model without endogenous capital accumulation. Yet, his paper claims, change in multipliers would be even smaller if he did reestimation of the model parameters after he adds capital.

Eggertsson and Woodford (2003) has no capital accumulation (no investment), therefore $\epsilon_x = \infty$. Woodford (2003), on the other hand, suggests $\epsilon_x = 3$ s.t. for a small shock ($r_t^n = -2$ from SS value 4 percent) ZLB is not binding. Eggertsson (2010) chooses ϵ_x such that output contraction in the fourth quarter the depression is -30 percent. Gali et al. (2007) choose (they call it η) 1 in baseline calibration.

[68] $\Delta\hat{\tau}_s^w = -1$ percent $\Rightarrow \Delta\hat{Y}_s = -1.2706$ percent (because everything is in logs).

9.1 Robustness check of fiscal multipliers - Sensitivity analysis

In an effort to check for robustness of the fiscal multipliers, I follow the literature and make a few adjustments. For instance, Uhlig and Drautzburg (2011), change capital share to 0.35. He would estimate median estimates for the Calvo parameter for prices and wages at 0.81 and 0.83 respectively, in order to increase the price stickiness. The fiscal multipliers are also sensitive to the duration of the ZLB as discussed in the literature review. $'1-\omega_w'$ shows the probability that the economy converges to its steady-state equilibrium each period. γ (fraction of firms that keep their prices fixed) imply prices are fixed for $\frac{1}{1-\gamma}$ periods on average. Depreciation rate ζ and β are standard as in RBC models. Markup $\mu = \frac{1}{\theta_p}$ means a markup of price on marginal cost in the steady state.The benchmark duration is initially set at 8 quarters and then change it to 12 quarters and in another case endogenize it. A longer duration decreases the fiscal multiplier to -0.03 or -0.19 respectively.

Results are very sensitive to the value of η. For high values, as it is not possible to observe the ZLB, the fiscal multiplier will be very slow; whereas, for very small eta values, as it is very easy to get into a liquidity trap case, the fiscal multipliers get very high. η is very high in Woodford (2003) - ZLB does not bind in small-shock case according to Christiano (2004), and he keeps it at a standard value (for 1) and shows that with a relatively smaller value, even a big shock is not causing ZLB to bind (in a way, the ZLB case never binds). Gali et al. (2007) sets it to $\eta = 0.2$. If $\eta = 1$ then in no-investment case same results as in Eggertsson and Woodford are obtained, yet for investment case, results are very sensitive to value of η. Christiano (2004) argues if parameters in Woodford (2003) are used, in a model with investment, then probability of output collapse and negative inflation is reduced a lot. Yet an elasticity of 100 (very high compared with literature) is needed for the worst case scenario to happen.

Sensitivity analysis for the fiscal multipliers (model features that change the multiplier):

Table 4: Robustness Tests for Various Parameter Values

Targets	Initial value ($f = 0,3$)	at the ZLB	ω_w up to $0,95$	f $= 0,5$	$f = 0,7$
$\frac{\Delta \hat{Y}_{s+k}}{-\Delta \hat{\tau}_s^w}$	$1,2$ percent	2,1	3,35	4,05	4,70

f, share of the Keynesian households, is first set to 1/3 and then will be changed for sensitivity analysis. I only use range of f values consistent with a unique equilibrium. Campbell and Mankiw (1989) find a fraction of 1/2 captures the importance of the rule-of-thumb behavior in the industrial economies. Forni et al. (2009) find a fraction of non-Ricardian agents around 30 to 40 percent for the Euro area. While Gali et al. (2007), in an estimated DSGE model for the Euro area, finds (baseline 1/2) fraction of Keynesians over 1/4 is needed for a positive response of C to fiscal shocks in a monopolistically competitive labor market (for perfect competition case in labor market, very high-unrealistic- fractions of K are needed). Uhlig and Drautzburg (2011) take the fraction of the constrained households between $(0, 0.5)$.

Instead of considering pure myopic Keynesian agents, Uhlig and Drautzburg (2011) consider rates of time preferences ranging between 7 and 30 percent higher for the credit-constrained households, compared to unconstrained households. Which means a higher β for the unconstrained households again. They also find that with the rates of time preferences around or higher than 20 percent, the constrained agents get substantial positive welfare gains.

Gali et al. (2007) find the impact multipliers are changing by the degree of price stickiness γ, with a higher stickiness meaning higher multiplier and the multiplier changing in nonlinear way (increasing), and that $\gamma > 0.5$ are consistent (with micro evidence and) with positive multiplier resulting from stronger consumption response. Fiscal multipliers are also sensitive to the capital adjustment cost (ϵ_x), but not affected by elasticity of substitution for labor η. Rise in capital adjustment cost, increases consumption further and decreases the negative impact on investment and thus a higher output is observed.

Sensitivity to policy parameters: a higher ϕ_π means stronger response to increasing inflation, and thus a higher real interest rate, r. Higher real interest rate, decreases consumption (of the Ricardians) and thus the output, therefore it negatively affects the multiplier. Gali et al. (2007) further find that positive co-movement of C and output requires a high response of debt financing, ϕ_b in the fiscal rule, and a low response of tax - ϕ_t. This basically means, the more tax cuts are financed by debt finance in future, the more better off the Keynesians are and therefore the higher C and Y response we get.

10 Extensions & Future Work

One problem is that since we have some Keynesian agents in the model, implementation lags matter, as claimed in the Keynesian theory. Timing of tax cuts matters since some of the agents make their decisions per period. In models where we have only Ricardian agents, implementation lags would not matter that much because households optimize inter-temporally and take all future policies into account. So its the announcement of a policy, rather than timing of implementation that matters.[69]

The other issue is, as Krugman (1998) points out, if current income has very high effect on

[69] See also, the arguement by Christiano et al. (2009) and Eggertsson (2010), expectations of a future policy matter if agents expect a policy in all future states of the world.

spending, then economy could have multiple equilibria as well. However, there is no evidence for such multiple equilibria. If it is the case, this could lead a sufficient temporary fiscal stimulus to take the economy out of the trap (where conventional MP is effective again) and thus a temporary fiscal shock have permanent effects. But it should not mean that FP was not effective and should not be used, as he suggests both the fiscal policy and inflation expectation.

Extensions of this study could include income and consumption tax cuts. I may follow Hall and Woodward (2008), Feldstein (2002) and Correia et al. (2010) who use the following setup. Decrease current consumption taxes and increase future taxes. Increase in taxes goes on until the recession is over. Meanwhile, labor taxes go down not to have any distortionary affect through MRS between consumption and labor.

$$E_s\hat{\tau}^c_{s+1} - \hat{\tau}^c_s = r^n_s$$

s.t. in the IS curve, it will satisfy $\hat{Y}_s = E_s\hat{Y}_{s+1} = 0$ and $E_s\pi_{s+1} = 0$, and

$$\hat{\tau}^w_s = -\hat{\tau}^c_s \tag{63}$$

s.t. in the Phillips curve, it will satisfy $\hat{Y}_s = 0$ and $\pi_s = E_s\pi_{s+1} = 0$, and another condition for capital taxes,

$$\hat{\tau}^A_s = -\hat{\tau}^c_s$$

I use debt payment for current tax cuts and assume tax cuts are financed by current or future lump-sum taxes. An alternative approach, as in Correia et al. (2011), is to use other distortionary taxes to finance current tax cuts in future.

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