

Explaining the Skill Premium: Technical Change or Capital-Skill Complements?*

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May 4, 2012

Abstract

Keywords: Multi-level CES production function, Factor-Augmenting Technical Progress, Capital-Skill Complementarity, Factor Substitution, Aggregation, Skill-premium.

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1 Introduction

Grilliches (1969) was the first to give an explanation to the widening skill premium. He showed that - for US manufacturing – capital and skilled labor were more complementary than capital and unskilled labor. This spawned a considerable literature examining the so-called capital-skill complementarity hypothesis, for example, Greenwood, Hercowitz, and Krussel (1997), Krussel, Ohanian, Rioss-Rull, and Violante (2000), and Duffy, Papageorgious, and Perez-Sebastian(2004). The hypothesis gained particular currency given the sharp decline in the constant-quality relative price of equipment and particularly in the relative price of information and communication technology equipment, Gordon (1990). This decline naturally led to an uptake in usage of such capital. Given complementarity between capital and skilled labor, the faster usage of such capital increased the relative demand for skilled labor and - despite the apparent increase in the supply of such labor – the widening skill or wage premium relative to unskilled labor increased in a dramatic and persistent manner (see Acemogly, 2009, for a textbook discussion). On the other hand, authors such as Katz and Murphy (1992), Acemgly (2002b), and Autor, Katz, and Kearney (2008) claimed that the skill premium can be attributed to technical change that was biased in favour of skilled workers. Given that skilled and unskilled workers are gross substitutes, an increase in skilled labor efficiency led to an increase in the relative wages (and factor income shares) of skilled workers. Both approaches rely on particular nesting and estimation values for elasticities of substitution between different categories of factors of production and their associated factor-biased technical progress parameters.

The aim of this paper is to examine these alternative hypothesis on a more general level. We examine the three-level and the two-level nested Constant Elasticity of Substitution production (CES) functions where labor is disaggregated into skilled and unskilled labor and the capital stock into structures and equipment capital. Using four-equation system approach and several nesting alternatives we retrieve estimates of the inter- and intra-class elasticities of substitution and factor augmenting-technical progress coefficients. The system is estimated for US data for the 1963-2006 period. In this multi-equation environment we study, whether the source of the observed skill-premium between the wage rates of skilled and unskilled labor is capital-skill complementarity or the skill biased technical change. Our estimation results strongly reject the capital skill-complementarity hypothesis. Instead, our results favour the specification, where skilled and unskilled labor are gross substitutes, whilst structures and equipment capital are cross substitutes both with each other and with skilled and unskilled labor. This result is favored both by two- and three-level CES function. Technical progress is the most strongly biased towards skilled labor explaining the skill-premium between the wage rates of skilled and unskilled labor.

The structure of the paper is ...

2 Multi-level Multi-Factor CES Production Functions

2.1 Four-Factor Three-Step CES

Let us write the normalized four-factor, three level-CES production function for production Y as follows,

$$Y_t = Y_0 \left[\alpha \left(e^{\gamma_1 \tilde{t}} \frac{V1_t}{V1_0} \right)^{\frac{\psi-1}{\psi}} + (1 - \alpha) Z_t^{\frac{\psi-1}{\psi}} \right]^{\frac{\psi}{\psi-1}} \quad (1)$$

$$Z_t = \left[(1 - \beta) \left(e^{\gamma_2 \tilde{t}} \frac{V2_t}{V2_0} \right)^{\frac{\sigma-1}{\sigma}} + \beta X_t^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \quad (2)$$

$$X_t = \left[(1 - \pi) \left(e^{\gamma_3 \tilde{t}} \frac{V3_t}{V3_0} \right)^{\frac{\eta-1}{\eta}} + \pi \left(e^{\gamma_4 \tilde{t}} \frac{V4_t}{V4_0} \right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \quad (3)$$

where $\tilde{t} = (t - t_0)$, ψ is the elasticity of substitution between the input $V1$ and the compound input Z , σ is the elasticity of substitution between the compound input $V2$ and the compound input X and η is the elasticity of substitution between inputs $V3$ and $V4$. Parameters γ_i measures the (constant-growth) rate of the factor i augmenting technical change. Subscripts zero indicate variable values at the point of normalization. It is straightforward to see that (1)-(3) imply that $Z_0 = X_0 = 1$.

Denoting factor prices by w_i ($i = 1, 2, 3, 4$) the normalization implies that the distribution parameters α , β and π in (2)-(3) are defined by factor incomes of the normalization point as follows,

$$\alpha = \frac{w1_0 \cdot V1_0}{w1_0 \cdot V1_0 + w2_0 \cdot V2_0 + w3_0 \cdot V3_0 + w4_0 \cdot V4_0} \quad (4)$$

$$\beta = \frac{w3_0 \cdot V3_0 + w4_0 \cdot V4_0}{w2_0 \cdot V2_0 + w3_0 \cdot V3_0 + w4_0 \cdot V4_0} \quad (5)$$

$$\pi = \frac{w4_0 \cdot V4_0}{w3_0 \cdot V3_0 + w4_0 \cdot V4_0} \quad (6)$$

After inserting (2) and (3) into (1), the three step-CES function for production can be written as:

$$\frac{Y}{Y_0} = \left\{ \alpha \left(e^{\gamma_1 \tilde{t}} \frac{V1_t}{V1_0} \right)^{\frac{\psi-1}{\psi}} + (1-\beta) \left(e^{\gamma_2 \tilde{t}} \frac{V2_t}{V2_0} \right)^{\frac{\sigma-1}{\sigma}} + \beta \left[(1-\pi) \left(e^{\gamma_3 \tilde{t}} \frac{V3_t}{V3_0} \right)^{\frac{\eta-1}{\eta}} + \pi \left(e^{\gamma_4 \tilde{t}} \frac{V4_t}{V4_0} \right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1} \frac{\sigma-1}{\sigma}} \right\}^{\frac{\sigma-1}{\sigma-1} \frac{\psi-1}{\psi}} \quad (7)$$

Assume that a firm faces an isoelastic demand curve, $Y_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\varepsilon} Y_t$. The profit maximizing under the specified CES technology implies the following four first order conditions.

$$\log w1_t = \log \left[\frac{\alpha}{(1+\mu)} \frac{Y_0}{V1_0} \right] + \left(\frac{\psi-1}{\psi} \right) \gamma_1 \tilde{t} + \frac{1}{\psi} \left[\log \left(\frac{Y_t}{Y_0} \right) - \log \left(\frac{V1_t}{V1_0} \right) \right] \quad (8)$$

$$\begin{aligned} \log w2_t = & \log \left[\frac{\alpha(1-\beta)}{(1+\mu)} \frac{Y_0}{V2_0} \right] + \left(\frac{\sigma-1}{\sigma} \right) \gamma_2 \tilde{t} + \frac{1}{\psi} \log \left(\frac{Y_t}{Y_0} \right) - \frac{1}{\sigma} \log \left(\frac{V2_t}{V2_0} \right) \\ & + \frac{(\psi-\sigma)}{\psi(\sigma-1)} \log \left\{ (1-\beta) \left(e^{\gamma_2 \tilde{t}} \frac{V2_t}{V2_0} \right)^{\frac{\sigma-1}{\sigma}} + \beta \left[(1-\pi) \left(e^{\gamma_3 \tilde{t}} \frac{V3_t}{V3_0} \right)^{\frac{\eta-1}{\eta}} + \pi \left(e^{\gamma_4 \tilde{t}} \frac{V4_t}{V4_0} \right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1} \frac{\sigma-1}{\sigma}} \right\} \end{aligned} \quad (9)$$

$$\begin{aligned} \log w3_t = & \log \left[\frac{(1-\alpha)\beta(1-\pi)}{(1+\mu)} \frac{Y_0}{V3_0} \right] + \left(\frac{\eta-1}{\eta} \right) \gamma_3 \tilde{t} + \frac{1}{\psi} \log \left(\frac{Y_t}{Y_0} \right) - \frac{1}{\eta} \log \left(\frac{V3_t}{V3_0} \right) \\ & + \frac{(\psi-\sigma)}{\psi(\sigma-1)} \log \left\{ (1-\beta) \left(e^{\gamma_2 \tilde{t}} \frac{V2_t}{V2_0} \right)^{\frac{\sigma-1}{\sigma}} + \beta \left[(1-\pi) \left(e^{\gamma_3 \tilde{t}} \frac{V3_t}{V3_0} \right)^{\frac{\eta-1}{\eta}} + \pi \left(e^{\gamma_4 \tilde{t}} \frac{V4_t}{V4_0} \right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1} \frac{\sigma-1}{\sigma}} \right\} \\ & + \frac{(\sigma-\eta)}{\sigma(\eta-1)} \log \left[(1-\pi) \left(e^{\gamma_3 \tilde{t}} \frac{V3_t}{V3_0} \right)^{\frac{\eta-1}{\eta}} + \pi \left(e^{\gamma_4 \tilde{t}} \frac{V4_t}{V4_0} \right)^{\frac{\eta-1}{\eta}} \right] \end{aligned} \quad (10)$$

$$\begin{aligned}
\log w_{4t} = & \log \left[\frac{(1-\alpha)\beta\pi}{(1+\mu)} \frac{Y_0}{V_{40}} \right] + \left(\frac{\eta-1}{\eta} \right) \gamma_4 \tilde{t} + \frac{1}{\psi} \log \left(\frac{Y_t}{Y_0} \right) - \frac{1}{\eta} \log \left(\frac{V_{4t}}{V_{40}} \right) \\
& + \frac{(\psi-\sigma)}{\psi(\sigma-1)} \log \left\{ \begin{array}{l} (1-\beta) \left(e^{\gamma_{24} \tilde{t} \frac{V_{2t}}{V_{20}}} \right)^{\frac{\sigma-1}{\sigma}} \\ + \beta \left[(1-\pi) \left(e^{\gamma_{34} \tilde{t} \frac{V_{3t}}{V_{30}}} \right)^{\frac{\eta-1}{\eta}} + \pi \left(e^{\gamma_{44} \tilde{t} \frac{V_{4t}}{V_{40}}} \right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1} \frac{\sigma-1}{\sigma}} \end{array} \right\} \\
& + \frac{(\sigma-\eta)}{\sigma(\eta-1)} \log \left[(1-\pi) \left(e^{\gamma_{34} \tilde{t} \frac{V_{3t}}{V_{30}}} \right)^{\frac{\eta-1}{\eta}} + \pi \left(e^{\gamma_{44} \tilde{t} \frac{V_{4t}}{V_{40}}} \right)^{\frac{\eta-1}{\eta}} \right] \quad (11)
\end{aligned}$$

where $\mu = \varepsilon/(\varepsilon - 1)$. Equations (7)-(11) define a 5-equation system with strong cross equation parameter constraints. This encompasses the 3-equation system estimated by Krusell et al. (2000). They, however, constrained the elasticity of substitution ψ between variable $V1$ (structures capital) and the compound factor Z (capturing unskilled labor $V2$, equipment capital $V3$ and skilled labor $V4$) to equal unity, i.e. the Cobb-Douglas function. To have a better comparability between our specification and Krusell et al. (2000) specification we in the following make a closer look to this special case of the nested CD-CES production function.

2.1.1 Special Case: Four-Factor-Nested CD-CES production function (Krusell et al.)

Under the special case of $\psi = 1$ we end up with the following nested CD-CES production function corresponding to the specification used by Krusell et al. (2000),

$$Y_t = Y_0 e^{\gamma_H \tilde{t}} \left(\frac{V_{1t}}{V_{10}} \right)^\alpha \left\{ \begin{array}{l} (1-\beta) \left(e^{\gamma_{24} \tilde{t} \frac{V_{2t}}{V_{20}}} \right)^{\frac{\sigma-1}{\sigma}} \\ + \beta \left[(1-\pi) \left(e^{\gamma_{34} \tilde{t} \frac{V_{3t}}{V_{30}}} \right)^{\frac{\eta-1}{\eta}} + \pi \left(\frac{V_{4t}}{V_{40}} \right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1} \frac{\sigma-1}{\sigma}} \end{array} \right\}^{\frac{\sigma(1-\alpha)}{\sigma-1}} \quad (12)$$

where distribution parameters α , β and π are defined by (4)-(6) and

$$\begin{aligned}
\gamma_H &= \alpha\gamma_1 + (1-\alpha)\gamma_4 \\
\gamma_{24} &= \gamma_2 - \gamma_4 \\
\gamma_{34} &= \gamma_3 - \gamma_4 \quad (13)
\end{aligned}$$

This nested CD-CES production function does not allow the identification of all four factor augmenting components of technical change. However, as (13)

shows three components of the augmenting technical change can be expressed as deviations from that of the reference factor. In (12) the reference factor is chosen to be $V4$, but it could have been any of the three factors $V2 - V4$. The implied first order maximization conditions with respect to inputs corresponding (8)-(11) equations are

$$\log w1_t = \log \left[\frac{\alpha}{(1 + \mu)} \frac{Y_t}{V1_t} \right] \quad (14)$$

$$\begin{aligned} \log w2_t = & \log \left[\frac{\alpha(1 - \beta)}{(1 + \mu)} \frac{Y_0}{V2_0} \right] + \left(\frac{\sigma - 1}{\sigma} \right) \gamma_{24} \tilde{t} + \log \left(\frac{Y_t}{Y_0} \right) - \frac{1}{\sigma} \log \left(\frac{V2_t}{V2_0} \right) \\ & - \log \left\{ \begin{aligned} & (1 - \beta) \left(e^{\gamma_{24} \tilde{t}} \frac{V2_t}{V2_0} \right)^{\frac{\sigma-1}{\sigma}} \\ & + \beta \left[(1 - \pi) \left(e^{\gamma_{34} \tilde{t}} \frac{V3_t}{V3_0} \right)^{\frac{\eta-1}{\eta}} + \pi \left(\frac{V4_t}{V4_0} \right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1} \frac{\sigma-1}{\sigma}} \end{aligned} \right\} \quad (15) \end{aligned}$$

$$\begin{aligned} \log w3_t = & \log \left[\frac{(1 - \alpha)\beta(1 - \pi)}{(1 + \mu)} \frac{Y_0}{V3_0} \right] + \left(\frac{\eta - 1}{\eta} \right) \gamma_{34} \tilde{t} + \log \left(\frac{Y_t}{Y_0} \right) - \frac{1}{\eta} \log \left(\frac{V3_t}{V3_0} \right) \\ & - \log \left\{ \begin{aligned} & (1 - \beta) \left(e^{\gamma_{24} \tilde{t}} \frac{V2_t}{V2_0} \right)^{\frac{\sigma-1}{\sigma}} \\ & + \beta \left[(1 - \pi) \left(e^{\gamma_{34} \tilde{t}} \frac{V3_t}{V3_0} \right)^{\frac{\eta-1}{\eta}} + \pi \left(\frac{V4_t}{V4_0} \right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1} \frac{\sigma-1}{\sigma}} \end{aligned} \right\} \\ & + \frac{(\sigma - \eta)}{\sigma(\eta - 1)} \log \left[(1 - \pi) \left(e^{\gamma_{34} \tilde{t}} \frac{V3_t}{V3_0} \right)^{\frac{\eta-1}{\eta}} + \pi \left(\frac{V4_t}{V4_0} \right)^{\frac{\eta-1}{\eta}} \right] \quad (16) \end{aligned}$$

$$\begin{aligned} \log w4_t = & \log \left[\frac{(1 - \alpha)\beta\pi}{(1 + \mu)} \frac{Y_0}{V4_0} \right] + \log \left(\frac{Y_t}{Y_0} \right) - \frac{1}{\eta} \log \left(\frac{V4_t}{V4_0} \right) \\ & - \log \left\{ \begin{aligned} & (1 - \beta) \left(e^{\gamma_{24} \tilde{t}} \frac{V2_t}{V2_0} \right)^{\frac{\sigma-1}{\sigma}} \\ & + \beta \left[(1 - \pi) \left(e^{\gamma_{34} \tilde{t}} \frac{V3_t}{V3_0} \right)^{\frac{\eta-1}{\eta}} + \pi \left(\frac{V4_t}{V4_0} \right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1} \frac{\sigma-1}{\sigma}} \end{aligned} \right\} \\ & + \frac{(\sigma - \eta)}{\sigma(\eta - 1)} \log \left[(1 - \pi) \left(e^{\gamma_{34} \tilde{t}} \frac{V3_t}{V3_0} \right)^{\frac{\eta-1}{\eta}} + \pi \left(\frac{V4_t}{V4_0} \right)^{\frac{\eta-1}{\eta}} \right] \quad (17) \end{aligned}$$

Assume as Krussel et al. (2000) that $V1$ is structures capital, $V2$ unskilled labor, $V3$ equipment capital and $V4$ is skilled labor. Under this interpretation the two first equations (factor share equations) of the three equation system estimated by Krussel et al. (2000) are direct transformations of the first-order conditions (16)-(17). Their third equation (the rate of return equality condition), in turn, may be linked to the conditions (14)-(15). However, as they do not show its explicit derivation the possible correspondence remains ambiguous. As regards the underlying production function (12) Krussel et al. (2000) left it outside their estimated 3-equation system.

2.1.2 Skill Premium

Assume that the four inputs $V1 - V4$ represent the variant combinations of the following four inputs: structures capital (KB), equipment capital (KQ), unskilled labor (NU) and skilled labor (NS). Assume the following two inter-variable correspondences,

$$CES \{V1, \psi, [V2, \sigma, (V3, \eta, V4)]\} : \begin{cases} 1. CES \{KB, \psi, [NS, \sigma, (KQ, \eta, NU)]\} \\ 2. CES \{KB, \psi, [NU, \sigma (KQ, \eta, NS)]\} \end{cases}$$

Equations (9) and (11) implies the following relative price of inputs $V2$ and $V4$, i.e. in the case 1 the relative price of skilled to unskilled labor or in the case 2 its inverse.

$$\log \left(\frac{w_{2t}}{w_{4t}} \right) = \left(\frac{\sigma - 1}{\sigma} \gamma_2 - \frac{\eta - 1}{\eta} \gamma_4 \right) t - \frac{(\sigma - \eta)}{\sigma \eta} \log X_t - \frac{1}{\sigma} \log V_{2t} + \frac{1}{\eta} \log V_{4t} + const \quad (18)$$

Differentiate (18) with respect to time (at the point of normalization) assume

the correspondence $[V2, \sigma, (V3, \eta, V4)] : [NS, \sigma, (KQ, \eta, NU)]$ to end up with the following skill-premium relation,

$$\begin{aligned} \Delta \log \left(\frac{w_{NS}}{w_{NU}} \right) &= \frac{1}{\sigma} (g_{NU} - g_{NS}) + \frac{\sigma - 1}{\sigma} (\gamma_{NS} - \gamma_{NU}) \\ &+ \underbrace{\frac{(\eta - \sigma)(1 - \pi)}{\sigma \eta} (g_{KQ} + \gamma_{KQ} - g_{NU} - \gamma_{NU})}_{\text{capital-skill complementarity}} \end{aligned} \quad (19)$$

where the growth rate of variable x is denoted by g_x . In terms of Krussel et al. (2000) terminology, the first right hand term - *the relative quantity effect* - states

that the faster growth of skilled than unskilled labor demand (as in the actual US data) affects negatively the skill premium. Also the second component - *the relative efficiency effect* - affects negatively (positively) the skill premium, if technical change is more skilled than unskilled labor augmenting and skilled labor is gross complement, $\sigma < 1$, (gross substitute, $\sigma > 1$) to equipment capital and unskilled labor. The third component - *the capital-skill complementarity effect* - shows that growth in the stock of equipment capital (both in physical and efficiency units) increases the relative wage rate (marginal product) of skilled to unskilled labor, if the elasticity of substitution between capital equipment and unskilled labor is higher than that between capital equipment and skilled labor, i.e. $\eta > \sigma$. Hence, under Hicks neutral technical change, coupled with the empirical fact $g_{KQ} > g_{NU}$, the capital-skill complementarity effect could explain the widening skill premium. However, under non-Hicks neutrality with technical change being more skilled than unskilled labor augmenting, the capital skill complementarity effect may be partly or completely overridden when skilled and unskilled labor are gross complements.

Next assume the correspondence $[V2, \sigma, (V3, \eta, V4)] : [NU, \sigma, (KQ, \eta, NS)]$ to end up with,

$$\begin{aligned} \Delta \log \left(\frac{w_{NS}}{w_{NU}} \right) &= \frac{1}{\sigma} (g_{NU} - g_{NS}) + \frac{\sigma - 1}{\sigma} (\gamma_{NS} - \gamma_{NU}) \\ &+ \underbrace{\frac{(\sigma - \eta)(1 - \pi)}{\sigma \eta} (g_{KQ} + \gamma_{KQ} - g_{NS} - \gamma_{NS})}_{\text{capital-skill complementarity}} \end{aligned} \quad (20)$$

Again the right-hand-side can be decomposed to the same three components. As earlier *the relative quantity* and *the relative efficiency effect* depend crucially on the size of the substitution elasticity between skilled and unskilled labor σ . The capital skill complementarity, in turn, requires that $\eta < \sigma$, i.e. the elasticity of substitution between capital equipment and skilled labor is lower than that between capital equipment and unskilled labor. Also as above the appropriately factor augmenting technical change may be either the alternative cause or the supplementary cause of the observed widening of the skill premium.

As discussed earlier one empirically feasible three-step CES specification is also: $CES \{V1, \psi, [V2, \sigma, (V3, \eta, V4)]\} = CES \{KB, \psi, [KQ, \sigma (NU, \eta, NS)]\}$. In this case (10) and (11) results in the following relation for the skill-premium:

$$\Delta \log \left(\frac{w_{NS}}{w_{NU}} \right) = \frac{1}{\eta} (g_{NU} - g_{NS}) + \frac{\eta - 1}{\eta} (\gamma_{NS} - \gamma_{NU}) \quad (21)$$

Under this specification the skill-premium is affected only by *the relative quantity effect* (the first component) and *the relative efficiency effect* (the second com-

ponent). From the factor substitution point of view it is only the size of substitution elasticity η between skilled and unskilled labor that matters. Hence, under this specification to compensate the effect of the observed faster growth of skilled than unskilled labor input, these labor inputs must be gross substitutes (complements), if $\gamma_{NS} > \gamma_{NU}$ ($\gamma_{NS} < \gamma_{NU}$) to be able to explain the widening skill-premium in the actual US data.

2.2 Four-Factor-Two Step CES

An alternative to the four-factor-three step CES production function is the four-factor-two-step CES function. It contains the same number of parameters as the three level function (7). In terms of the possible range of cross-factor substitution possibilities, however, the two step CES is somewhat more restrictive than the three level case. However as neither of them contains another as a special case, there is no a priori reason to favor either of them. It is, however, apparent that with some appropriate combinations of the estimated parameter values the two- and three-step-CES systems may quite closely approximate each other.

The four-factor two-step-CES production function is :

$$\frac{Y_t}{Y_0} = \left[\alpha X1_t^{\frac{\sigma-1}{\sigma}} + (1 - \alpha) X2_t^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \quad (22)$$

where σ is the elasticity of substitution between compound inputs $X1$ and $X2$ defined by the CES functions,

$$X1_t = \left[(1 - \beta) \left(e^{\gamma_1 \tilde{t}} \frac{V1_t}{V1_0} \right)^{\frac{\eta-1}{\eta}} + \beta \left(e^{\gamma_2 \tilde{t}} \frac{V2_t}{V2_0} \right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \quad (23)$$

$$X2_t = \left[(1 - \pi) \left(e^{\gamma_3 \tilde{t}} \frac{V3_t}{V3_0} \right)^{\frac{\theta-1}{\theta}} + \pi \left(e^{\gamma_4 \tilde{t}} \frac{V4_t}{V4_0} \right)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \quad (24)$$

where η and θ are the respective elasticity of substitutions between inputs $V1$ and $V2$, and between inputs $V3$ and $V4$. Denoting factor prices by w_i ($i = 1, 2, 3$) normalization implies that the distribution parameters α , β and π in (22)-(24) are defined by the normalized factor incomes as follows,

$$\alpha = \frac{w1_0 \cdot V1_0 + w2_0 \cdot V2_0}{w1_0 \cdot V1_0 + w2_0 \cdot V2_0 + w3_0 \cdot V3_0 + w4_0 \cdot V4_0} \quad (25)$$

$$\beta = \frac{w2_0 \cdot V2_0}{w1_0 \cdot V1_0 + w2_0 \cdot V3_0} \quad (26)$$

$$\pi = \frac{w_{4_0} \cdot V_{4_0}}{w_{3_0} \cdot V_{3_0} + w_{4_0} \cdot V_{4_0}} \quad (27)$$

After inserting (23) and (24) into (22) the two-step-CES function for production Y is written as:

$$\frac{Y}{Y_0} = \left\{ \begin{array}{l} \alpha \left[(1 - \beta) \left(e^{\gamma_1 \tilde{t} \frac{V_{1_t}}{V_{1_0}}} \right)^{\frac{\eta-1}{\eta}} + \beta \left(e^{\gamma_2 \tilde{t} \frac{V_{2_t}}{V_{2_0}}} \right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1} \frac{\sigma-1}{\sigma}} + \\ (1 - \alpha) \left[(1 - \pi) \left(e^{\gamma_3 \tilde{t} \frac{V_{3_t}}{V_{3_0}}} \right)^{\frac{\theta-1}{\theta}} + \pi \left(e^{\gamma_4 \tilde{t} \frac{V_{4_t}}{V_{4_0}}} \right)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1} \frac{\sigma-1}{\sigma}} \end{array} \right\}^{\frac{\sigma}{\sigma-1}} \quad (28)$$

Isoelastic demand, and profit maximization, implies the following four first order conditions:

$$\begin{aligned} \log w_{1_t} = & \log \left[\frac{\alpha(1-\beta)}{(1+\mu)} \frac{Y_0}{V_{1_0}} \right] + \frac{(\eta-1)\gamma_1 \tilde{t}}{\eta} + \frac{1}{\sigma} \log \left(\frac{Y_t}{Y_0} \right) - \frac{1}{\eta} \log \left(\frac{V_{1_t}}{V_{1_0}} \right) \\ & + \frac{\sigma-\eta}{\sigma(\eta-1)} \log \left[(1-\beta) \left(e^{\gamma_1 \tilde{t} \frac{V_{1_t}}{V_{1_0}}} \right)^{\frac{\eta-1}{\eta}} + \beta \left(e^{\gamma_2 \tilde{t} \frac{V_{2_t}}{V_{2_0}}} \right)^{\frac{\eta-1}{\eta}} \right] \end{aligned} \quad (29)$$

$$\begin{aligned} \log w_{2_t} = & \log \left[\frac{\alpha\beta}{(1+\mu)} \frac{Y_0}{V_{2_0}} \right] + \frac{(\eta-1)\gamma_2 \tilde{t}}{\eta} + \frac{1}{\sigma} \log \left(\frac{Y_t}{Y_0} \right) - \frac{1}{\eta} \log \left(\frac{V_{2_t}}{V_{2_0}} \right) \\ & + \frac{\sigma-\eta}{\sigma(\eta-1)} \log \left[(1-\beta) \left(e^{\gamma_1 \tilde{t} \frac{V_{1_t}}{V_{1_0}}} \right)^{\frac{\eta-1}{\eta}} + \beta \left(e^{\gamma_2 \tilde{t} \frac{V_{2_t}}{V_{2_0}}} \right)^{\frac{\eta-1}{\eta}} \right] \end{aligned} \quad (30)$$

$$\begin{aligned} \log w_{3_t} = & \log \left[\frac{(1-\alpha)(1-\pi)}{(1+\mu)} \frac{Y_0}{V_{3_0}} \right] + \frac{(\theta-1)\gamma_3 \tilde{t}}{\theta} + \frac{1}{\sigma} \log \left(\frac{Y_t}{Y_0} \right) - \frac{1}{\theta} \log \left(\frac{V_{3_t}}{V_{3_0}} \right) \\ & + \frac{\sigma-\theta}{\sigma(\theta-1)} \log \left[(1-\pi) \left(e^{\gamma_3 \tilde{t} \frac{V_{3_t}}{V_{3_0}}} \right)^{\frac{\theta-1}{\theta}} + \pi \left(e^{\gamma_4 \tilde{t} \frac{V_{4_t}}{V_{4_0}}} \right)^{\frac{\theta-1}{\theta}} \right] \end{aligned} \quad (31)$$

$$\begin{aligned} \log w_{4_t} = & \log \left[\frac{(1-\alpha)\pi}{(1+\mu)} \frac{Y_0}{V_{4_0}} \right] + \frac{(\theta-1)\gamma_4 \tilde{t}}{\theta} + \frac{1}{\sigma} \log \left(\frac{Y_t}{Y_0} \right) - \frac{1}{\theta} \log \left(\frac{V_{4_t}}{V_{4_0}} \right) \\ & + \frac{\sigma-\theta}{\sigma(\theta-1)} \log \left[(1-\pi) \left(e^{\gamma_3 \tilde{t} \frac{V_{3_t}}{V_{3_0}}} \right)^{\frac{\theta-1}{\theta}} + \pi \left(e^{\gamma_4 \tilde{t} \frac{V_{4_t}}{V_{4_0}}} \right)^{\frac{\theta-1}{\theta}} \right] \end{aligned} \quad (32)$$

2.2.1 Skill premium

Within this setting there we get different skill-premium relations for each three alternative ways to combine inputs in the two-step CES:

$$CES[(V1, \eta, V2), \sigma, (V3, \theta, V4)] : \begin{cases} 1. CES[(KB, \eta, NS), \sigma, (KQ, \theta, NU)] \\ 2. CES[(KB, \eta, NU), \sigma, (KQ, \theta, NS)] \\ 3. CES[(KB, \eta, KQ), \sigma, (NU, \theta, NS)] \end{cases}$$

Corresponding to the first case we end up with the following skill-premium relation:

$$\begin{aligned} \Delta \log \left(\frac{w_{NS}}{w_{NU}} \right) &= \left(\frac{1-\pi}{\theta} + \frac{\pi}{\sigma} \right) g_{NU} - \left(\frac{1-\beta}{\eta} + \frac{\beta}{\sigma} \right) g_{NS} \\ &+ \left(1 - \frac{1-\beta}{\eta} - \frac{\beta}{\sigma} \right) \gamma_{NS} - \left(1 - \frac{1-\pi}{\theta} - \frac{\pi}{\sigma} \right) \gamma_{NU} \\ &+ (1-\pi) \left(\frac{1}{\sigma} - \frac{1}{\theta} \right) (g_{KQ} + \gamma_{KQ}) + (1-\beta) \left(\frac{1}{\eta} - \frac{1}{\sigma} \right) (g_{KB} + \gamma_{KB}) \end{aligned} \quad (33)$$

A key implication of capital-skill complementarity is that growth in the stock of equipment increases the marginal product and the wage rate of skilled labor but decreases the marginal product and the wage rate of unskilled labor. We find that this is the case, if $\theta > \sigma$, i.e. unskilled labor is a closer substitute to equipment capital than to the composed input of skilled labor and structures capital. We also find that the growth in the stock of structures capital has similar effect, if $\eta < \sigma$ i.e. skilled labor and structures capital are weaker substitutes to each other than to two other inputs. Hence, under the ordering $\theta > \sigma > \eta$ the growth of both capital input components impact positively the skill-premium. This is the main qualitative difference compared to the three step cases in Section 3, where the skill premium was not affected by the growth of the structures capital stock. We see, however, that this ordering tends to strengthen also the negative quantity effect related to the faster growth of skilled than unskilled labor compared to e.g. to the opposite ordering. As regards the effects of non-Hicks-neutral technical change on the skill premium, both the equipment and structures capital augmenting technical change affects positively on it under the same conditions as growth in physical amount of these inputs. The skilled labor augmenting technical change increases and the unskilled labor augmenting change decreases unambiguously the skill premium only if all three substitution elasticities are larger than one.

Under the second CES specification the skill premium relation is,

$$\begin{aligned}
\Delta \log \left(\frac{w_{NS}}{w_{NU}} \right) &= \left(\frac{1-\beta}{\eta} + \frac{\beta}{\sigma} \right) g_{NU} - \left(\frac{1-\pi}{\theta} + \frac{\pi}{\sigma} \right) g_{NS} \\
&+ \left(1 - \frac{1-\pi}{\theta} - \frac{\pi}{\sigma} \right) \gamma_{NS} - \left(1 - \frac{1-\beta}{\eta} - \frac{\beta}{\sigma} \right) \gamma_{NU} \\
&+ (1-\pi) \left(\frac{1}{\theta} - \frac{1}{\sigma} \right) (g_{KQ} + \gamma_{KQ}) + (1-\beta) \left(\frac{1}{\sigma} - \frac{1}{\eta} \right) (g_{KB} + \gamma_{KB})
\end{aligned} \tag{34}$$

We find that this is the case, if $\theta < \sigma$, i.e. skilled labor is a weaker substitute to equipment capital than to the composed input of unskilled labor and structures capital. We also find that the growth in the stock of structures capital has similar effect on the skill premium, if $\eta > \sigma$ i.e. unskilled labor and structures capital are closer substitutes to each other than to two other inputs. Hence, under the ordering $\eta > \sigma > \theta$ the growth of both capital input components impact positively the skill premium. However, as in the first case, this ordering strengthens also the negative quantity effect related to the faster growth of skilled than unskilled labor. As regards the effects of non-Hicks-neutral technical change on the skill premium, also likewise in the case 1, both equipment and structures capital augmenting technical change affect positively on it under the same conditions as growth in physical amount of these input. The skilled labor augmenting technical change increases and the unskilled labor augmenting technical change decreases unambiguously the skill premium only if all three substitution elasticities are larger than one.

The third CES specification implies the following relation for the skill premium,

$$\Delta \log \left(\frac{w_{NS}}{w_{NU}} \right) = \frac{1}{\theta} (g_U - g_{NS}) + \left(\frac{\theta-1}{\theta} \right) (\gamma_{NS} - \gamma_{NU}) \tag{35}$$

This is essentially the same relation as (21) with capital labor complementarity playing no role in the development of skill premium. It is only the size of substitution elasticity θ between skilled and unskilled labor that matters. Hence, to compensate the effect of the observed faster growth of skilled than unskilled labor input, these labor inputs must be gross substitutes (complements), if $\gamma_{NS} > \gamma_{NU}$ ($\gamma_{NS} < \gamma_{NU}$) to be able to explain the widening skill-premium in the actual US data.

3 The Elasticities of Substitution

The Allen (or Allen-Uzawa) partial elasticity of substitution has been the most common substitution statistic reported in empirical studies of production. It is

the share of the j th input in total cost (s_j) weighted cross-price elasticity that measures the percentage change in demand for input i induced by a one percent change in the price of input j (with output and other input prices constant),

$$AES_{ij} = \frac{1}{s_j} \frac{\partial \ln V_i}{\partial \ln w_j} = \frac{1}{s_i} \frac{\partial \ln V_j}{\partial \ln w_i} \quad (36)$$

It is a one-input one-price elasticity and, as compellingly argued by C. Blackorby and R. Russele [AER 1989 p. 882-888, "Will the Real elasticity of Substitution Please Stand Up? (A Comparison of the Allen/Uzawa and Morishima Elasticities)"], it is only in two input case the correct measure the ease of factor substitution, i.e. the curvature of the production isoquant. In the multi-factor (more than two inputs) environment Morishima elasticity (MES), they argue, is the correct elasticity concept,

$$\begin{aligned} MES_{ij} &= \frac{\partial \ln V_j}{\partial \ln w_i} - \frac{\partial \ln V_i}{\partial \ln w_i} \\ &= \frac{1}{s_i} (AES_{ij} - AES_{ii}) \end{aligned} \quad (37)$$

where $AES_{ii} = \frac{1}{s_i} \frac{\partial \ln V_i}{\partial \ln w_i} = -\frac{1}{s_i} \sum_{j \neq i} s_j AES_{ij}$, [C. Perroni and T. Rutherford (1995), "Regular flexibility of nested CES function", European Economic Review, 335-343]. We see that MES (unlike AES) accounts for besides the cross-price also the own price elasticity responses of factor demands V_j and V_i to a one percent change in input price w_i and, therefore, measures the percentage change of the factor share V_i/V_j . It is worth noting that, unlike the Allen elasticity, the Morishima elasticity is asymmetric so that $MES_{ij} \neq MES_{ji}$. Blackorby and Russel (1989) explain that this dependency of curvature on the direction in which price ratios are changed obviates symmetry as a natural property of an n -dimensional elasticity of substitution.

In this paper we report both the Allen and Morishima elasticities, because we, anyway, calculate the Morishime elasticites in terms of the Allen ealsticities as in (37). In evaluating the (time varying) Allen elaticities in the point of normalization in the case of the stree-step function (7) the Allen elasticites are [see Y. Sheinin (1980), "The Demand for Factor Inputs Under a Three Level CES Four Factor Production Function, dissertation, University of Pennsylvania.],

$$\begin{aligned}
AES_{1j} &= \psi \\
AES_{23} &= AES_{24} = \psi + \frac{1}{1-\alpha} (\sigma - \psi) \\
AES_{34} &= \psi + \frac{1}{1-\alpha} (\sigma - \psi) + \frac{1}{(1-\alpha)\beta} (\eta - \sigma) \\
AES_{ij} &= AES_{ji} \\
AES_{ii} &= -\frac{1}{s_i} \sum_{j \neq i} s_j AES_{ij}
\end{aligned} \tag{38}$$

where $s_1 = \alpha$, $s_2 = (1 - \alpha) (1 - \beta)$, $s_3 = \beta (1 - \alpha) (1 - \pi)$ and $s_4 = \beta (1 - \alpha) \pi$.

In the two step case (28) the Allen elasticities are, Sato (1972),

$$\begin{aligned}
AES_{12} &= \sigma + \frac{1}{1-\alpha} (\eta - \sigma) \\
AES_{13} &= AES_{14} = AES_{23} = AES_{24} = \sigma \\
AES_{34} &= \sigma + \frac{1}{\alpha} (\theta - \sigma) \\
AES_{ij} &= AES_{ji} \\
AES_{ii} &= -\frac{1}{s_i} \sum_{j \neq i} s_j AES_{ij}
\end{aligned} \tag{39}$$

where $s_1 = \alpha (1 - \beta)$, $s_2 = \alpha \beta$, $s_3 = (1 - \alpha) (1 - \pi)$ and $s_4 = (1 - \alpha) \pi$.

4 Data

Annual data were obtained from various sources for the US economy for the 1963-2006 period. The annual frequency is determined by the availability of skilled/unskilled hours and wages. Data for output, capital, total employment, and labor compensation are for the US private non-residential sector. Most of the data come from NIPA series available from the Bureau of Economic Analysis. The output series are thus calculated as total output minus net indirect tax revenues, public-sector, and residential output. After these adjustments, the output concept used is compatible with that of the capital stock series used which is the quantity index of net stock of non-residential private capital from NIPA tables. We also pay special attention to the construction of the hours and wage series by skill level, and the user cost of capital.

Data by skill levels were obtained from Autor et al. (2008).¹ Skilled workers are defined as those with (some) college education and above. Unskilled workers are defined as those with education levels up to (and including) high school. Autor et al. (2008) provide relative supply and relative wages (the skill premium) for both categories. Relative supply is defined in terms of hours worked.² Because the coverage of these data coming from the Current Population Survey is different from our coverage for the non-residential private sector, we combined these data with Bureau of Labor Statistics (BLS) data. While preserving relative wages and relative labor supply, we correct both so as to be compatible with the evolution of total private employment and labor compensation. Hence, we proceed as follows. We define unskilled workers' wages (WU) as,

$$WU = \frac{W}{NU/N + (NS/N) * \tilde{W}}$$

where W are wages of all workers, NU number of unskilled workers, N is total private sector workers, NS is number of skilled workers and, finally, \tilde{W} is the skilled/unskilled wage ratio. Then WS , skilled wages, is simply defined as $W \times \tilde{W}$. We now need to define how some of these variables are obtained. We define W as labor income ($NINC$) over total private sector employment. A problem in calculating labor-income is that it is unclear how the income of proprietors (self-employed) should be categorized in the labor-capital dichotomy. Some of the income earned by self-employed workers clearly represents labor income, while some represents a return on investment or economic profit. Following Klump et al. (2007), we use compensation per employee as a shadow price of labor of self-employed workers:

$$NINC = \left(1 + \frac{\text{self-employed}}{\text{total private employment}} \right) \cdot \text{Comp}$$

where $Comp$ = private sector compensation of employees.

We then define $W = \frac{NINC}{\text{total private sector employment}}$. Finally, we define NS as total private sector employment times relative skilled/unskilled hours worked, and $NU = N - NS$. These transformations preserve relative quantities but correct the levels in order to comply with our previous definitions and the self-employment transformation. This assumes, of course, that relative wages and relative labor supply in the private sector evolve in a similar fashion to those in the (wider) definition provided by Autor et al. (2008).

¹We thank David Autor for providing the files for annual data by skill levels.

²See Autor et al. (2008) for further detail on data construction. We chose to use relative supply in terms of hours rather than the 'efficiency units' measure which is also provided by the authors.

Our capital stock concept is private non-housing capital disaggregated into structures and equipment capital. As NIPA presents these data as the end-of-year levels, in our estimation we use the two year averages of these end of year stocks. The user cost of aggregate capital K was obtained using a residual method.³ In order to do so, we first need to make an assumption about the share of income belonging to a pure mark-up. The mark-up share can be estimated directly within the normalized system. However, because of the relatively short sample and demands imposed by the system with three factors, we imposed an average mark-up of 10%, $\mu = 0.1$. This is consistent with estimates of the system using two factors (capital and aggregate labor). Under this assumption, the real user cost of capital, r , is defined as:

$$r = \frac{Y/(1 + \mu) - NINC}{K}$$

Similarly, in calculating the user cost measures also for the two disaggregates of the total capital stock, i.e. non-residential structures and equipment capital, we first decomposed the total capital income $Y/(1 + \mu) - NINC$ into components associated to structures and equipment capital and then proportioned them to the stocks. These capital income shares were based on capital income estimates obtained by multiplying - for scaling purposes - current dollar capital stocks by the relevant real user cost term of each type of capital.

To calculate the real user cost term, the real interest component was defined as the difference of the sample averages of the ten year government bond rate and inflation in terms of the net investment deflators. As inflation of structures investment was higher than equipment investment, depreciation rates in turn were calculated on the basis of current dollar values of depreciations and current dollar value capital stocks and were markedly higher for structures than for equipment capital.

Figure 1 plots some relevant ratios related to capital and labor inputs. The top panel of 1 shows that the equipment capital (KQ) to output (Y) ratio displays a positive trend and the structures capital (KB) to output ratio has a negative trend over the sample and, hence, as the middle panel shows, the size of equipment capital relative to the structures capital rises reflecting the downward trend in their relative user prices (UCQ and UCB). As these opposite trends largely compensate each other their relative factor income shares remain relatively stable only marginally favoring equipment capital.

As regards skilled and unskilled labor inputs, corresponding trend developments look quite different. The bottom panel of Figure 1 shows that both relative input (NS/NU) and wage (WS/WU) developments favors skilled labor, i.e. both

³Direct measures such as those used in León-Ledesma et al. (2010) did not change the results substantially.

of them have an upward trend implying an even steeper trend in the skilled labor income to unskilled labor income ratio. This provides indication against a unit substitution elasticity between these two labor inputs, since in the Cobb-Douglas case factor shares are constant.

– Insert Figure 1 Here –

5 Estimation

We estimated the systems based on both the three-step and the two-step CES production function specifications. In the following we examine first the estimation results based on the three-step CES function and thereafter the results based on the two-step specification.

5.1 Estimation Results: (The Four-Factor) Three-Step CES

Our underlying hypothesis was that the structures capital KB is simultaneously either gross complement ($\psi < 1$) or gross substitute ($\psi > 1$) to three other inputs.

$$CES \{V1, \psi, [V2, \sigma, (V3, \eta, V4)]\} : \begin{cases} 1. CES \{KB, \psi, [NS, \sigma (KQ, \eta, NU)]\} \\ 2. CES \{KB, \psi, [NU, \sigma, (KQ, \eta, NS)]\} \\ 3. CES \{KB, \psi, [KQ, \sigma (NU, \eta, NS)]\} \end{cases}$$

The first specification alternative implies *capital-skill complementarity*, if $\sigma (< 1) < \eta$. With our notation, in the second alternative the size order of these parameters should be reversed, i.e. $\sigma (> 1) > \eta$, for *capital-skill complementarity* to hold. The third alternative does not allow *capital-skill complementarity*, at least, in a conventional sense of the term. For a better comparison to Krussel et al. (2000), we also estimated the systems implied by these three production function alternatives under the unit elasticity constraint $\psi = 1$, i.e. the systems under the the nested CD-CES production function constraint (12)-(17). As in the cases, where at least one of three elasticity estimates tends towards infinity, our numerical estimation algorithm was unable to converge, substitution elasticity parameters were estimated under the upper bound constraint of 10^4 .

The results are shown in **Tables 1 to 3**. These Tables report the Augmented Dickey-Fuller (ADF) t-test statistics as measures for residual stationarity of the equations of the system and the Log Determinant as the statistical measure of the overall fit of the system. In all Tables the first column (A) shows results with Hicks Neutrality imposed on the components of technical progress. In the

three subsequent columns (B)-(D) neutrality constraints related to four technical progress components are gradually relaxed. The last column (E) presents the estimation results under the nested CD-CES production function constraint, i.e. $\psi = 1$. Tables are supplemented by residual and fit graphs of the the best estimated system in each table.

Estimation results based on the production function specification

$$CES \{KB, \psi, [NS, \sigma (KQ, \eta, NU)]\}$$

are presented in **Table 1**. In this case *capital-skill complementarity* would require that $\eta (> 1) > \sigma$. However, the estimation results of Table 1 do not support *the capital-skill complementarity hypothesis*. In addition, especially, under Hicks neutrality constraint estimation results are also statistically poor; the fit is bad and residuals are non-stationary. The overall fit improves somewhat, when the two components of capital augmenting technical change are estimated freely but both labor augmenting components are constrained to be equal (see column B). However, residuals remain still strongly non-stationary. In column (C), where the components of labor augmenting technical progress are estimated freely but both capital augmenting components are constrained to be equal, the overall fit is improved further and now also stationarity properties of the estimated residuals look quite good as also **Graph 1** shows. In terms of economic interpretation the results of column (C) are the most reasonable of Table 1, although the overall fit of the system presented in column (D) is even somewhat better. However, in column (D) the estimated growth rate of structures capital augmenting technical change is unfeasibly high (84% per annum). This is related to the close to unity estimate of the substitution elasticity parameter ψ . As discussed e.g. in Klump et al. (2007) it is typical that numerical estimation algorithms tend to find a maximum with some combination of close to unity substitution elasticity and, in absolute terms, unrealistically high augmented technical progress component(s). This is confirmed by column (E), where the three-step CES is estimated under the explicit nested CD-CES constraint, i.e. $\psi = 1$. Although the statistical properties of this equation look quite satisfactory its overall fit is worse than those of the systems presented in columns (D) and (C). Hence, under this production function specification column (C) suggests that the structures capital is gross complement ($\psi = 0.87$) to other inputs. Compatibly with the capital-skill complementarity hypothesis equipment capital is gross substitute to unskilled labor ($\eta = 1.27$) but violates the hypothesis by being even closer substitute to skilled labor ($\sigma = 2.60$). The upper Panel A of Graph1 shows that general trend developments of factor prices (incl. the skill-premium) and output are quite satisfactorily explained by this equation system. Column (C) indicates strongly capital and skilled labor augmenting technical change, whilst the unskilled labor augmenting component is markedly negative.

The lower pane B of Graph 1 presents the growth contributions of total factor productivity (TFP) decomposed into underlying factor augmenting components. The estimated growth contribution of TFP has upward trend reflecting mainly developments in contributions of its largest positive (skilled labor augmenting) and negative (unskilled labor augmenting) components. Hence, under this specification the widening skill-premium, in terms of equation (19), is explained by the relative efficiency effect $\frac{\sigma-1}{\sigma}(\gamma_{NS} - \gamma_{NU})$ that dominates the negative effects of two other right-hand-side terms of (19).

Estimation results based on the production function specification

$$CES \{KB, \psi, [NU, \sigma(KQ, \eta, NS)]\}$$

are presented in **Table 2**. This alternative (under the CD-CES production technology constraint) was supported by the estimation results of Krussel et al. (2000). They found *capital-skill complementarity* with the elasticity of substitution $\eta = 0.67$ between skilled labor and equipment capital and $\sigma = 1.67$ between unskilled labor and equipment capital. In fact, estimation results presented in columns (A), (B) and (E) of Table 2 are, at least, qualitatively supportive to the results of Krussel et al. (2000). The estimates of σ are above and the estimates of η below unity. Column (A), which is estimated under the assumption of Hicks-neutral technical progress and column (B), which is estimated under the assumption of common capital augmenting change for both capital augmenting components and no labor augmenting technical progress, proposes unit elasticity between the structures capital and the composite of other three inputs, i.e. they suggest the nested CD-CES specification. Column (E) present the best estimation results under this constraint. These results are closest to the Krussel et al. (2000) results. Substitution between skilled labor and equipment capital $\eta = 0.89$ and between unskilled labor and equipment capital $\sigma = 1.84$. Hence, there is *the capital-skill complementarity effect* via which the faster growth of (physical) equipment capital than that of skilled labor input is transmitted to skill-premium, see (20). In addition, unlike in Krussel et al. (2000) the estimated ordering of factor augmenting components of technical change, $\gamma_{KQ} > \gamma_{NS} > \gamma_{NU}$, essentially strengthens *the capital-skill complementarity effect* on the skill premium, as we discussed in the context of equation (20). Column (E) results, however, are not the best results of Table 2 and markedly worse than estimation results of Table 1. In terms of the overall fit and the stationarity properties of the equations of the estimation results of the columns (C) and (D) of Table 2 are better, although still somewhat worse than best results in Table 1. In these columns, on one hand, structures capital and skilled labor augmenting technical changes were constrained to equal and, on the other hand, equipment capital and unskilled labor augmenting technical change were constrained to equal. In column (C) the latter component is constrained

to equal zero whilst in column (D) it is estimated freely resulting in the slightly improved overall fit.⁴ Noteworthy, these results do not any longer support *the capital-skill complementarity hypothesis*. Now, instead of being below unity, the substitution elasticity between equipment capital and skilled labor is estimated to exceed markedly unity ($\eta = 2.5$) and, and against *the skill-complementarity hypothesis* it is higher than the substitution elasticity between equipment capital and unskilled labor ($\sigma = 2.4$). However, as the **Panel A of Graph 2** shows, the estimated system reported in **Column (D)** is able relatively well to track the general trend developments of factor prices (incl. the skill-premium) and output. **Panel B of Graph 2** present the growth contributions of TFP and its components of augmenting technical change. As earlier the growth contribution of TFP is growing in time reflecting even stronger growth contribution of skilled labor augmenting component than the results in Column (D) of Table 1. Accordingly, also in this setting the widening skill premium is explained by *the relative efficiency effect* reflecting the estimated markedly faster skilled than unskilled labor augmenting technical change coupled with high substitution elasticity between unskilled and skilled labor.

Estimation results based on the third three-step-CES production function specification

$$CES \{KB, \psi, [KQ, \sigma (NU, \eta, NS)]\}$$

are presented in **Table 3**. From our three alternative ways to combine inputs this way proved to be the most compatible with the actual US data and, hence, our general conclusion is that *the capital capital-skill complementarity hypothesis* is not favored by the data. However, in common with results presented in Tables 1 and 2 the ability of estimated systems to track the data was improved markedly the more freely the components of augmenting technical progress were estimated. Constraints and freeing them affected strongly also the estimated sizes of substitution elasticities. In **Column (D)** which presents statistically and from the point of view economic interpretation best results all technical progress components were estimated freely. According these results, in terms of estimated parameter sizes, technical progress augments most structures capital and skilled labor. However, as **Panel B of Graph 3** shows in growth contribution terms skilled labor augmenting component is again the most dominating. Unskilled labor augmenting component is quite small and not statistically significant, whilst the estimated equipment capital augmenting technical change is negative. Although one can think that technical change may be largely embodied in equipment capital, the estimation results of column (D) do not support it being equipment capital saving. This is reflected by the fast growth of equipment capital coupled with its

⁴Trials to estimation the specification without any constraints on augmenting augmenting technical change did not converge.

decreasing prices. Accordingly to the estimation results of column (D) suggest that structures capital is gross substitute ($\psi = 0.80$) to the composite of other inputs and equipment capital gross substitute ($\sigma = 0.71$) to the composite of labor inputs. Skilled and unskilled labor, in turn, are quite close substitutes for each other ($\eta = 3.6$). **Panel A of Graph 3** shows that this specification is not so much superior to track the observed development of the skill premium compared to those presented in Tables 1 and 2 than in its ability to explain the price developments of the two capital stock components. In those respects the tracking ability of the estimated system is improved remarkably.

5.2 Estimation results: (The Four-Factor) Two-Step CES

We estimated the alternatives,

$$CES [(V1, \eta, V2), \sigma, (V3, \theta, V4)] : \begin{cases} 1. CES [(KB, \eta, NS), \sigma, (KQ, \theta, NU)] \\ 2. CES [(KB, \eta, NU), \sigma, (KQ, \theta, NS)] \\ 3. CES [(KB, \eta, KQ), \sigma, (NU, \theta, NS)] \end{cases}$$

Estimation results based on the production function specification 1 above are presented in **Table 4**. In this case *capital-skill complementarity*, in a sense that the growth of equipment capital impacts positively on the skill-premium, would require that $\theta > \sigma$. In addition, the growth of structures capital has similar effect on the skill premium if $\sigma > \eta$. However, the estimation results of Table 4 do not support *the capital-skill complementarity hypothesis*. Rather the opposite is supported by this specification. However, as with the three-step CES cases we find that the ability of the specified system to track properly the data depends crucially on imposed constraints on technical change. Especially, under the Hicks neutrality constraint estimation results are statistically poor, i.e. the fit is bad and residuals are non-stationary. The overall fit and the stationarity properties of residuals improve gradually, when the neutrality constraints of technical progress are loosened and the best results are presented in column (D) with free augmenting technical change. According these estimates technical progress augments most strongly skilled labor and equipment capital. Also structures capital augmenting component is estimated to be positive, although statistically insignificant, whilst the unskilled labor augmenting component is negative. All substitution elasticity estimates are above unity with the elasticity of substitution between skilled labor and structures capital being highest ($\eta = 4.6$) and between unskilled labor and equipment capital lowest ($\theta = 1.4$). As the **Panel A of Graph 4** shows this estimated system tracks the overall trends in factor price and production relatively

well although as the ADF test statistics indicate some stationarity problems in the residuals. All in all the ability of this system to explain the observed data is quite comparable with that presented in column (D) of Table 1. Also the growth contributions of TFP and its components of augmenting technical change (Panel B of Graph 4) resemble quite closely those presented in Graph 1 with the contribution of the skilled labor augmenting being overwhelmingly dominant. Accordingly, also in this setting the widening skill premium is explained by the estimated markedly faster skilled than unskilled labor augmenting technical change coupled with high substitution elasticity between unskilled and skilled labor.

Estimation results based on the production function specification 2 above are presented in **Table 5**. In this case *capital-skill complementarity*, in a sense that the growth of equipment capital impacts positively on the skill-premium, would require that $\theta < \sigma$. In addition, the growth of structures capital has similar effect on the skill premium if $\sigma < \eta$. The results under this CES-specification are broadly in line with this hypothesis. The estimated elasticity of substitution between skilled labor and equipment capital is the lowest and the highest (practically infinite) between unskilled labor and structures capital, except in column (B) that, however, is markedly dominated by the estimated systems of columns (C) and (D). All in all, however, the results in Table 5 are somewhat worse than in Table 4. Also the estimated very strongly equipment capital augmenting technical change - 17 per cent per annum in column (C) and 10 per cent in (column D) - raises the question of empirical feasibility of these results. As **Panel B of Graph 5** shows, **Column D** results imply that the growth contribution of equipment capital augmenting technical change would have exceeded the growth contribution of TFP through the whole estimation period.

Estimation results based on the production function specification 2 above are presented in **Table 6**. From our three alternative ways to combine inputs this way proved to be the most compatible with the actual US data being even marginally better than the results in column (D) of Table 3. All in all the results in columns (D) in Table 3 and Table 6 are quite comparable with each other. Both results indicate elasticity of substitution between structures and equipment capital around 0.8 and between skilled and unskilled labor practically identically 3.6. Also the estimated substitution elasticities between capital inputs and labor inputs are in line with each other. Whilst the three step-CES substitution elasticity estimate is slightly above 0.7, the two step-CES estimate is 0.5. Finally, also the estimates of the factor augmenting technical change are close to each other. According to these results technical progress augments most structures capital and skilled labor. Unskilled labor augmenting component is quite small and now statistically significant, whilst the estimated equipment capital augmenting technical change is negative

that is compatible with the fast growth of equipment capital coupled with its decreasing prices. All in all, the results of Table 6 corroborates our earlier conclusion in the three-step CES case that, firstly, the main explaining factor behind the widening skill premium has been the skill-biased technical change combined with high substitution elasticity between skilled and unskilled labor and, secondly, that *the capital capital-skill complementarity hypothesis* is not favored by the data.

6 Conclusions

Our estimation results reject the capital skill-complementarity hypothesis. Instead, our results favour the production function specification, where skilled and unskilled labor are gross substitutes, whilst structures and equipment capital are cross substitutes both with each other and with skilled and unskilled labor. Technical progress is the most strongly biased towards skilled labor explaining the skill-premium between the wage rates of skilled and unskilled labor.

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Figure 1:

Some key ratios related to capital and labor inputs

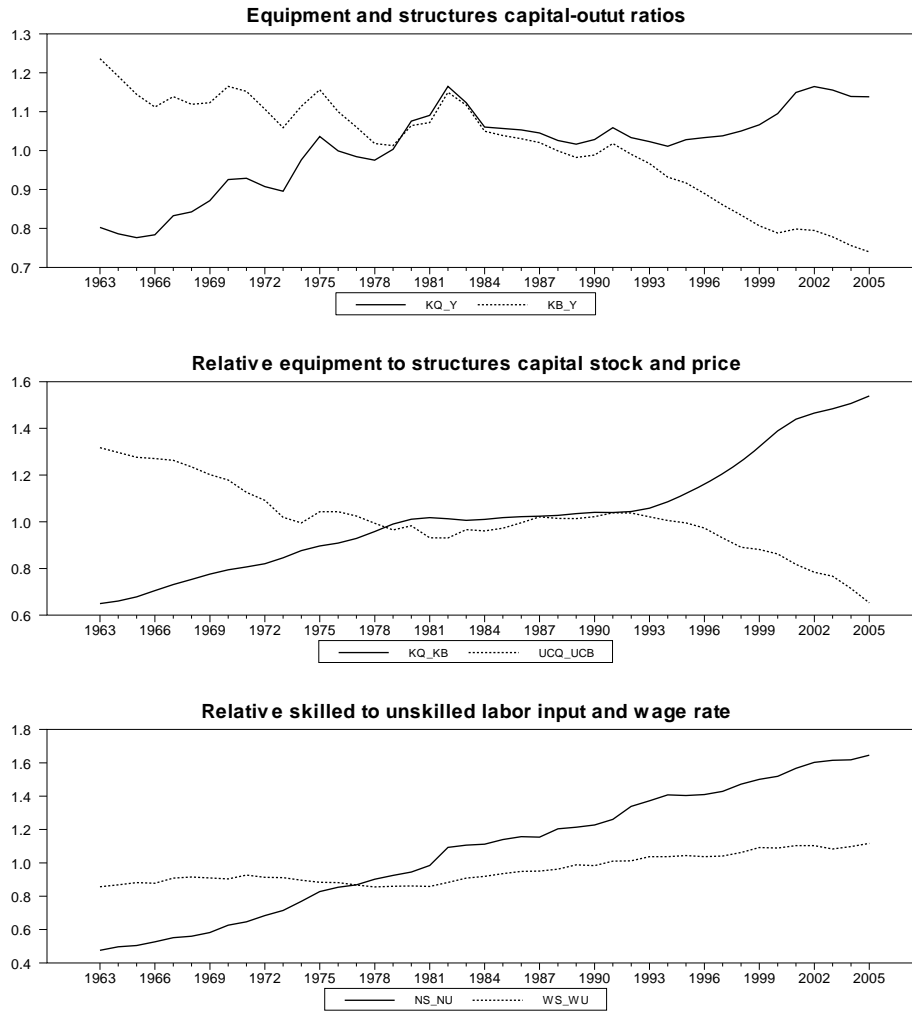


Table 1.
4-Factor, 3-Step CES{KB, ψ , [NS, σ , (KQ, η , NU)]}

	A	B	C	D	E
ψ	0.7476 (0.0088)	0.7259 (0.0043)	0.8684 (0.0053)	0.9944 (0.0047)	1 (-)
σ	u.b.	9.9107 (2.5132)	2.6003 (0.2464)	2.2143 (0.3562)	2.8234 (0.4154)
η		0.6874 (0.0352)	1.2674 (0.0595)	1.1968 (0.0775)	1.3929 (0.1006)
γ_{KB}	0.0072 (0.0003)	0.0379 (0.0039)	0.0403 (0.0071)	0.8361 (0.5979)	-
γ_{KQ}		-0.0459 (0.0030)		0.0112 (0.0210)	
γ_{NS}		0.0155 (0.0009)	0.0266 (0.0015)	-0.0162 (0.0315)	
γ_{NU}			-0.0178 (0.0044)	-0.0737 (0.0452)	
$\gamma_H = \alpha\gamma_{KB} + (1-\alpha)\gamma_{NS}$	-	-	-	-	0.0252 (0.0019)
$\gamma_{KQ} - \gamma_{NS}$					0.0126 (0.0085)
$\gamma_{NU} - \gamma_{NS}$					-0.0380 (0.0073)
<hr/>					
$\gamma_{KB} = \gamma_{KQ} = \gamma_{NS} = \gamma_{NU}$	[0.????]	-	-	-	-
$\gamma_{KB} = \gamma_{KQ}$	-		[0.????]		
$\gamma_{NS} = \gamma_{NU}$		[0.????]	-		
<hr/>					
ADF(FOC _{KB})	-3.1661	-0.7764	-3.5513	-3.3081	-3.3165
ADF(FOC _{KQ})	-1.8079	-1.2830	-3.5228	-3.1955	-3.5102
ADF(FOC _{NU})	-2.3342	-1.7246	-3.1324	-2.9529	-2.9300
ADF(FOC _{NS})	-1.2352	-1.2334	-2.9717	-3.2557	-3.5631
ADF(CES)	-0.9824	-1.1263	-4.1399	-3.4763	-3.7162
Log. Det.	-33.2729	-37.5914	-39.4977	-39.6080	-38.9067

Note: Robust Standard errors in parenthesis; probability values in squared brackets; “-” denotes not applicable; “u.b.” refers to numerical upper bound, here and hereafter arbitrarily set to 10^4 .

Table 2.
4-Factor, 3-Step CES{KB, ψ , [NU, σ , (KQ, η , NS)]}

	A	B	C	D	E
ψ	1.0410 (0.0190)	1.0059 (0.0087)	0.8379 (0.0063)	0.8246 (0.0058)	1 (-)
σ	u.b.	3.2871 (0.1208)	2.8065 (0.0785)	2.3779 (0.1608)	1.8432 (0.2851)
η	0.4027 (0.0322)	0.8906 (0.0108)	3.7934 (0.7114)	2.5051 (0.5046)	0.8938 (0.0776)
γ_{KB}	0.0115 (0.0003)	0.0622 (0.0022)	0.0255 (0.0006)	0.0288 (0.0018)	-
γ_{KQ}			-	-0.0035 (0.0018)	
γ_{NS}		-	0.0255 (0.0006)	0.0288 (0.0018)	
γ_{NU}			-	-0.0035	
$\gamma_H = \alpha\gamma_{KB} + (1 - \alpha)\gamma_{NS}$	-	-		-	0.0077 (0.0121)
$\gamma_{KQ} - \gamma_{NS}$					0.0828 (0.0679)
$\gamma_{NU} - \gamma_{NS}$					-0.0186 (0.0084)
<hr/>					
$\gamma_{KB} = \gamma_{KQ} = \gamma_{NS} = \gamma_{NU}$	[0.????]	-	-		
$\gamma_{KB} = \gamma_{KQ}$	-	[0.????]			-
$\gamma_{NS} = \gamma_{NU}$				-	
<hr/>					
ADF(FOC _{KB})	-3.2927	-3.2641	-3.9098	-3.8398	-3.3165
ADF(FOC _{KQ})	-2.1088	-3.0452	-3.2176	-2.9574	-3.0502
ADF(FOC _{NU})	-1.7105	-2.8055	-2.7872	-2.8622	-2.9921
ADF(FOC _{NS})	-1.6141	-2.5364	-3.7423	-3.3185	-2.7303
ADF(CES)	-2.2272	-2.3188	-3.0423	-3.6018	-3.2688
Log. Det.	-34.1982	-37.1617	-38.4490	-38.5993	-37.7373

Note: See notes to Table 1.

Table 3.
4-Factor, 3-Step CES{KB, ψ , [KQ, σ , (NU, η , NS)]}

	A	B	C	D	E
ψ	4.2630 (0.8319)	0.5471 (0.0012)	1.1490 (0.0123)	0.7973 (0.0043)	1 (-)
σ	1.1546 (0.0462)	0.7384 (0.0058)	0.8270 (0.0508)	0.7113 (0.0403)	0.9286 (0.0065)
η	u.b.		3.7960 (0.7577)	3.5917 (0.7278)	5.326 (1.6867)
γ_{KB}	0.0088 (0.0004) 4.2630 (0.8319)	0.0278 (0.0011)	-0.0200 (0.0040)	0.0279 (0.0057)	-
γ_{KQ}				-0.0146 (0.0024)	
γ_{NS}		-0.0056 (0.0007)	0.0279 (0.0020)	0.0241 (0.0018)	
γ_{NU}			0.0084 (0.0022)	0.0036 (0.0026)	
$\gamma_H = \alpha\gamma_{KB} + (1 - \alpha)\gamma_{NS}$					0.0342 (0.0078)
$\gamma_{KQ} - \gamma_{NS}$	-	-	-	-	-0.1242 (0.0504)
$\gamma_{NU} - \gamma_{NS}$					-0.0144 (0.0034)
$\gamma_{KB} = \gamma_{KQ} = \gamma_{NS} = \gamma_{NU}$	[0.????]	-	-		
$\gamma_{KB} = \gamma_{KQ}$			[0.????]		-
$\gamma_{NS} = \gamma_{NU}$		[0.????]	-		
ADF(FOC _{KB})	-2.9151	-0.2770	-3.3577	-3.7102	-3.3165
ADF(FOC _{KQ})	-2.5326	-1.0573	-3.3096	-3.5620	-3.3897
ADF(FOC _{NU})	-1.2513	0.9363	-2.7479	-2.9017	-2.6980
ADF(FOC _{NS})	-0.9166	0.2620	-2.8632	-2.7869	-2.8460
ADF(CES)	-1.2333	0.8635	-2.9904	-2.9832	-2.2723
Log. Det.	-33.7849	-35.0542	-39.8952	-40.0125	-39.0726

Note: See notes to Table 1.

Table 4.
4-Factor, 2-Step CES[(KB, η ,NS), σ , (KQ, θ ,NU)]

	A	B	C	D
σ	u.b.	u.b.	1.5213 (0.0184)	2.3373 (0.0485)
η			0.9700 (0.0184)	4.5545 (5.5121)
θ		0.7682 (0.0748)	1.8961 (0.4750)	1.3609 (0.0800)
γ_{KB}	0.0063 (0.0002)	-0.0274 (0.0005)	0.4662 (0.0357)	0.0112 (0.0094)
γ_{KQ}			0.0022 (0.0132)	0.0267 (0.0079)
γ_{NS}		0.0100 (0.0003)	-0.0193 (0.0048)	0.0296 (0.0015)
γ_{NU}			-0.0122 (0.0029)	
<hr/>				
$\gamma_{KB} = \gamma_{KQ} = \gamma_{NS} = \gamma_{NU}$	[0.????]	-	-	-
$\gamma_{KB} = \gamma_{KQ}$	-	[0.????]	[0.????]	
$\gamma_{NS} = \gamma_{NU}$		[0.????]	-	
<hr/>				
ADF(FOC _{KB})	-2.8595	0.6015	-2.8977	-2.8311
ADF(FOC _{KQ})	-2.4445	-0.5633	2.9946	-2.5009
ADF(FOC _{NU})	-2.1190	-2.9054	-2.8206	-2.5982
ADF(FOC _{NS})	-0.5297	-0.7431	-3.1892	-3.0182
ADF(CES)	-0.7843	-0.3094	-2.9371	-4.0740
Log. Det.	-33.0194	-36.0090	-37.6810	-39.035

Note: See notes to Table 1.

Table 5.
4-Factor, 2-Step CES[(KB, η ,NU), σ , (KQ, θ ,NS)]

	A	B	C	D
σ	u.b.	3.7313 (0.1887)	1.63651 (0.0284)	1.65783 (0.0312)
η		1.2541 (0.0267)	u.b.	
θ	0.4056 (0.0297)	1.0617 (0.0480)	0.9468 (0.0070)	0.8743 (0.0513)
γ_{KB}	0.0115 (0.0003)	0.0839 (0.0052)	-0.0131 (0.0014)	-0.0156 (0.0172)
γ_{KQ}			0.1726 (0.0018)	0.1031 (0.0254)
γ_{NS}		-0.0060 (0.0016)	-0.0144 (0.0016)	0.0088 (0.0086)
γ_{NU}				-0.0135 (0.0017)
$\gamma_{KB} = \gamma_{KQ} = \gamma_{NS} = \gamma_{NU}$	[0.????]	-	-	
$\gamma_{KB} = \gamma_{KQ}$	-		[0.????]	
$\gamma_{NS} = \gamma_{NU}$		[0.????]	-	
ADF(FOC _{KB})	-2.7845	-2.6634	-2.8977	-2.7780
ADF(FOC _{KQ})	-1.5700	-2.7843	2.9946	-2.7384
ADF(FOC _{NU})	-1.7350	-3.0383	-2.8206	-2.5368
ADF(FOC _{NS})	-2.2423	-2.8329	-3.1892	-2.7345
ADF(CES)	-2.2203	-2.8455	-2.9371	-3.4562
Log. Det.	-34.1941	-36.4174	-38.5186	-38.7229

Note: See notes to Table 1.

Table 6.
4-Factor, 2-Step CES[(KB, η ,KQ), σ , (NU, θ ,NS)]

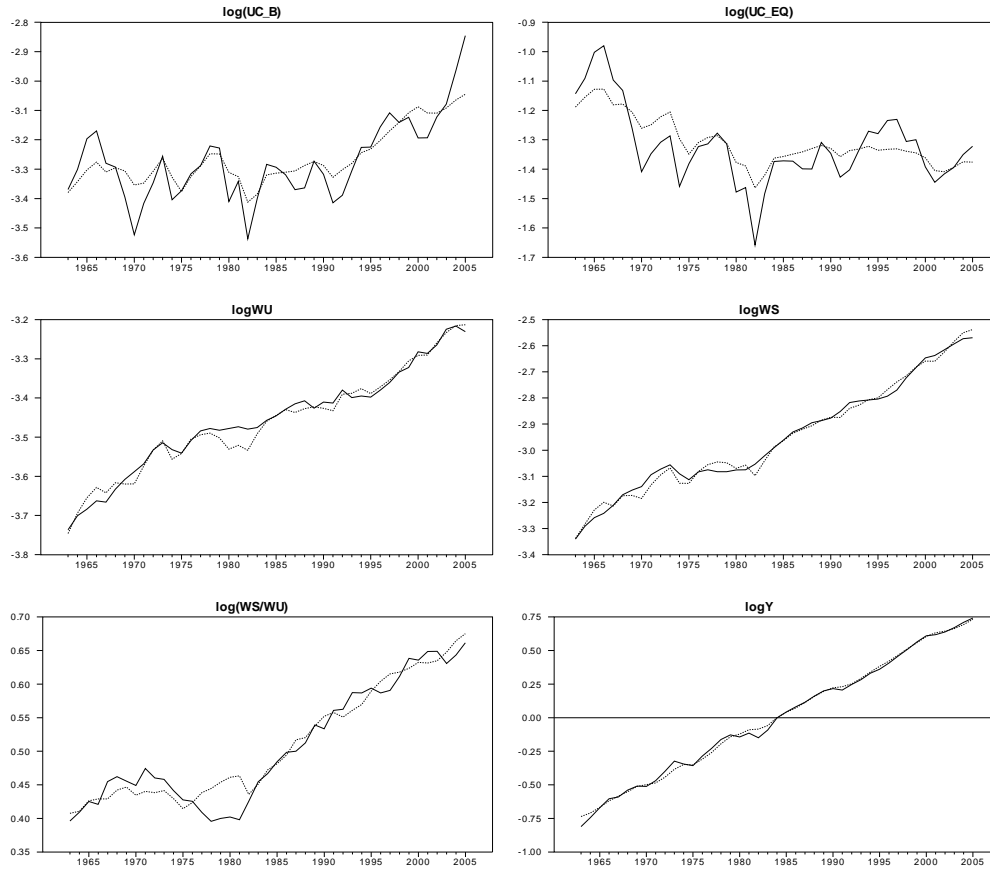
	A	B	C	D
σ	0.4624 (0.0014)	0.5823 (0.0027)	0.51125 (0.0014)	0.5512 (0.0016)
η	1.6405 (0.0462)	1.0617 (0.0480)	1.6132 (0.1163)	0.8222 (0.0403)
θ	u.b.		3.7236 (0.6142)	3.6148 (0.7278)
γ_{KB}	0.0015 (0.0003)	-0.0262 (0.0006)	-0.0033 (0.0007)	0.0326 (0.0139)
γ_{KQ}				-0.0180 (0.0054)
γ_{NS}		0.0223 (0.0006)	0.0238 (0.0013)	0.0240 (0.0016)
γ_{NU}				0.0045 (0.0017)
$\gamma_{KB} = \gamma_{KQ} = \gamma_{NS} = \gamma_{NU}$	[0.????]	-	-	
$\gamma_{KB} = \gamma_{KQ}$	-		[0.????]	
$\gamma_{NS} = \gamma_{NU}$			[0.????]	-
ADF(FOC _{KB})	-0..6409	-1.6996	-1.9556	-3.0954
ADF(FOC _{KQ})	-1.9326	-1.1764	-3.7173	-3.6978
ADF(FOC _{NU})	0.0022	-1.7270	-1.7270	-2.9620
ADF(FOC _{NS})	0.60410	-2.2404	-2.9416	-2.9845
ADF(CES)	0.3358	-1.9442	-2.98812	-2.9161
Log. Det.	-34.7830	-35.8258	-39.9410	-40.1712

Note: See notes to Table 1.

Graph 1. The system of column (C) in Table 1.

Panel A

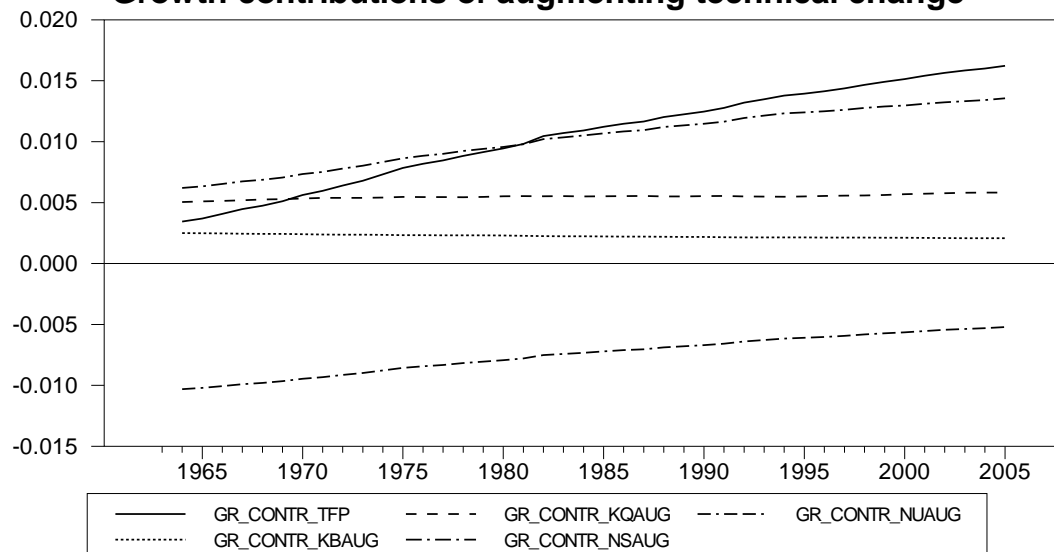
Actual vs fitted



Note: solid line represents data, and dotted line indicates fit of estimated systems.

Panel B

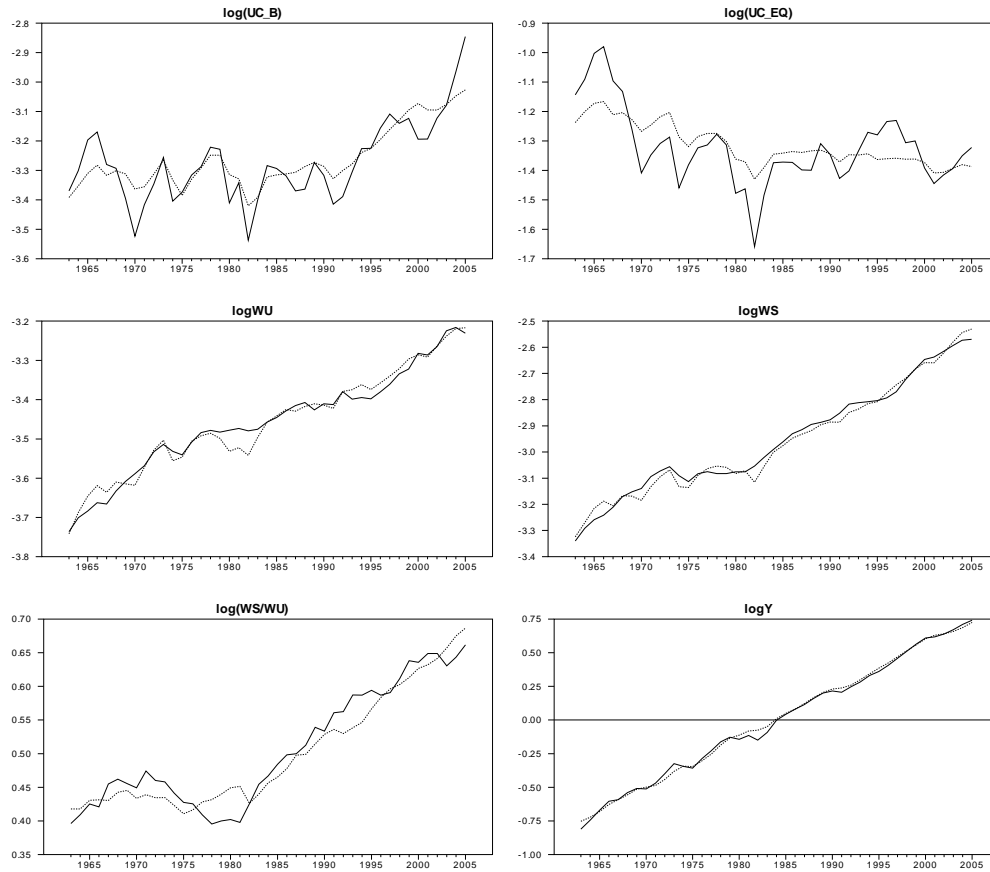
Growth contributions of augmenting technical change



Graph 2. The system of column (D) in Table 2.

Panel A

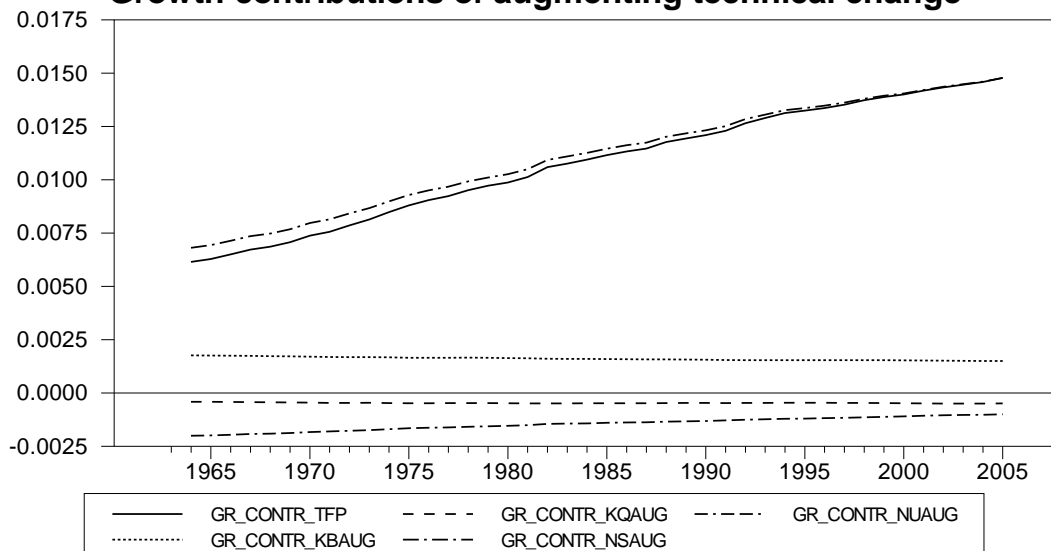
Actual vs fitted



Note: See note to Graph 1.

Panel B

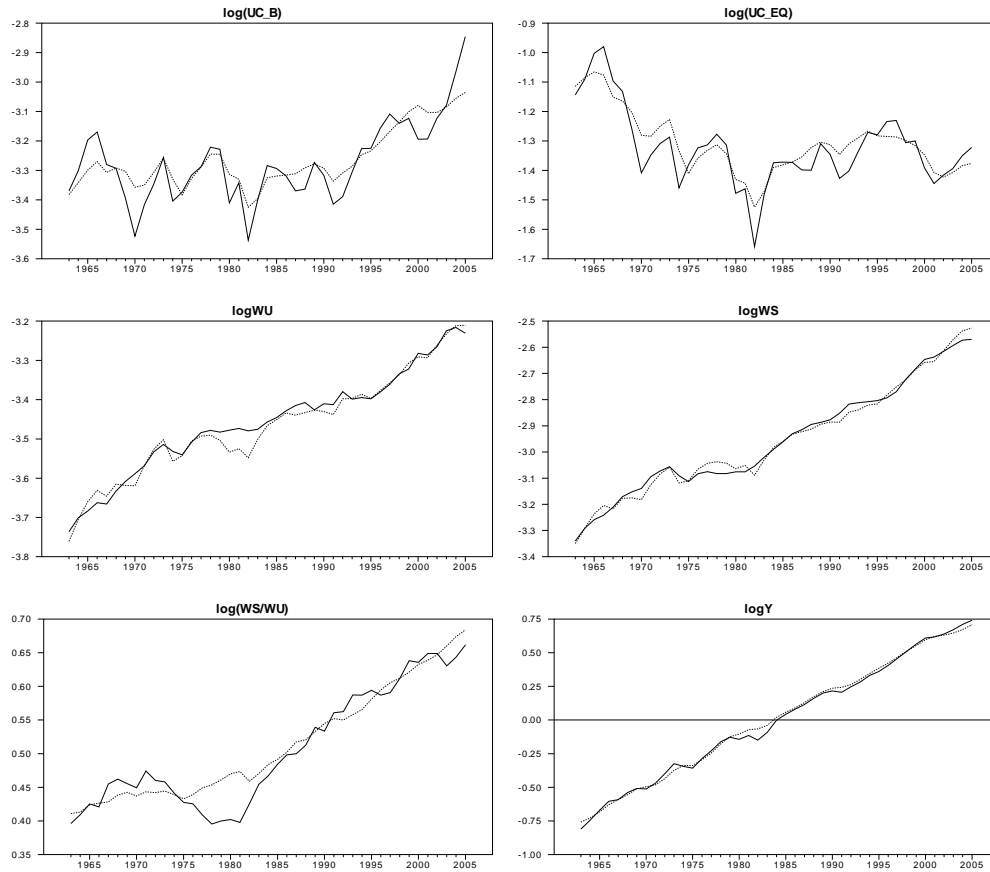
Growth contributions of augmenting technical change



Graph 3. Residuals and fits of column (D) in Table 3.

Panel A

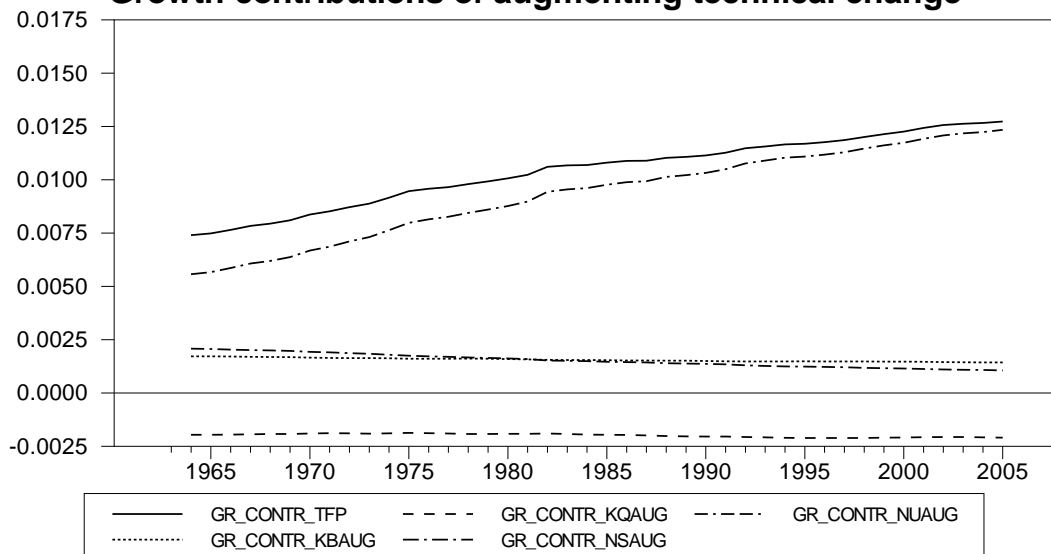
Actual vs fitted



Note: See note to Graph 1.

Panel B

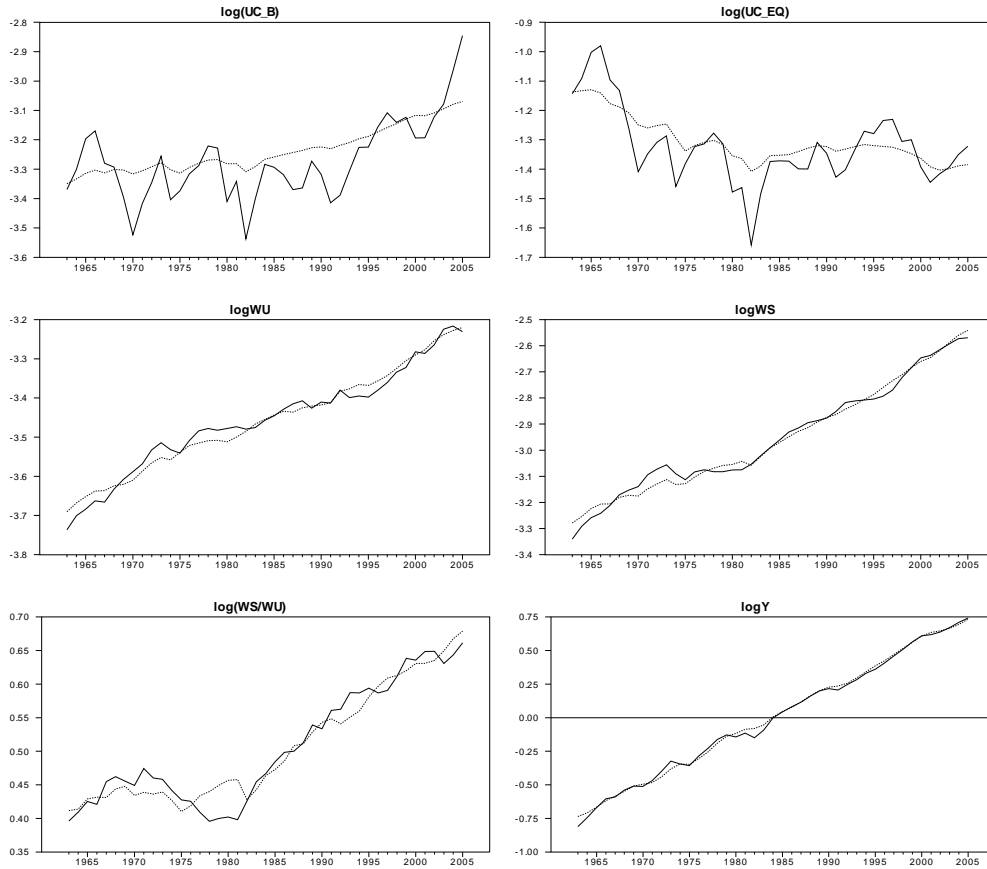
Growth contributions of augmenting technical change



Graph 4. The system of column (D) in Table 4.

Panel A

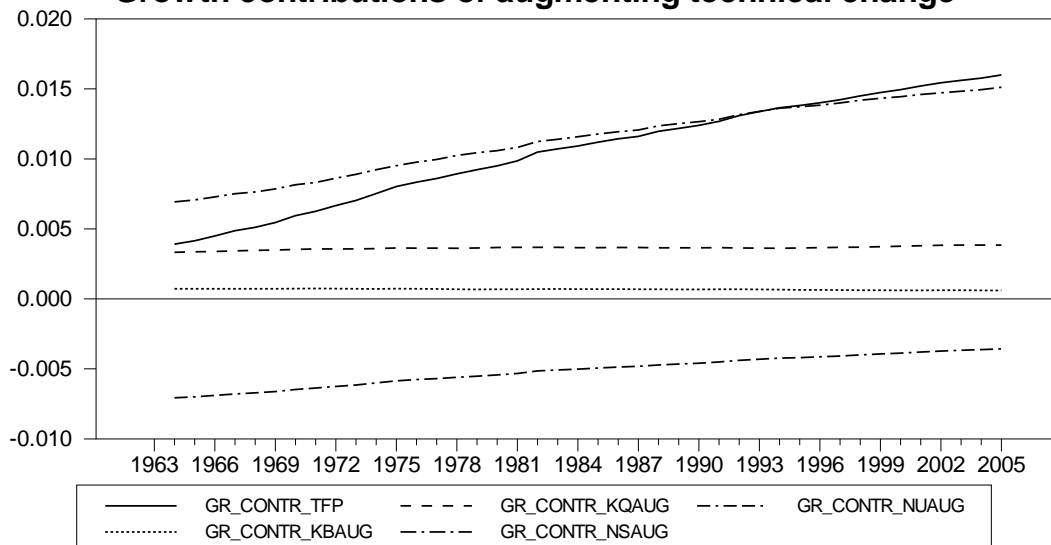
Actual vs fitted



Note: See note to Graph 1.

Panel B

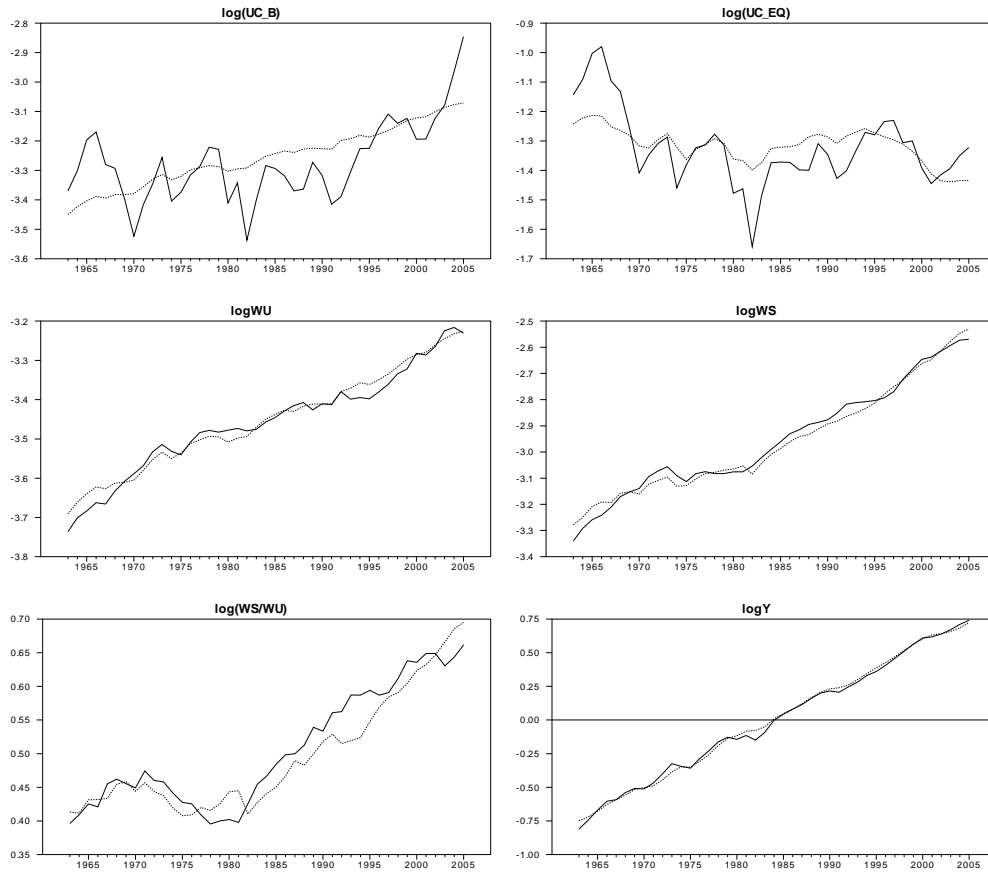
Growth contributions of augmenting technical change



Graph 5. The system of column (D) in Table 5.

Panel A

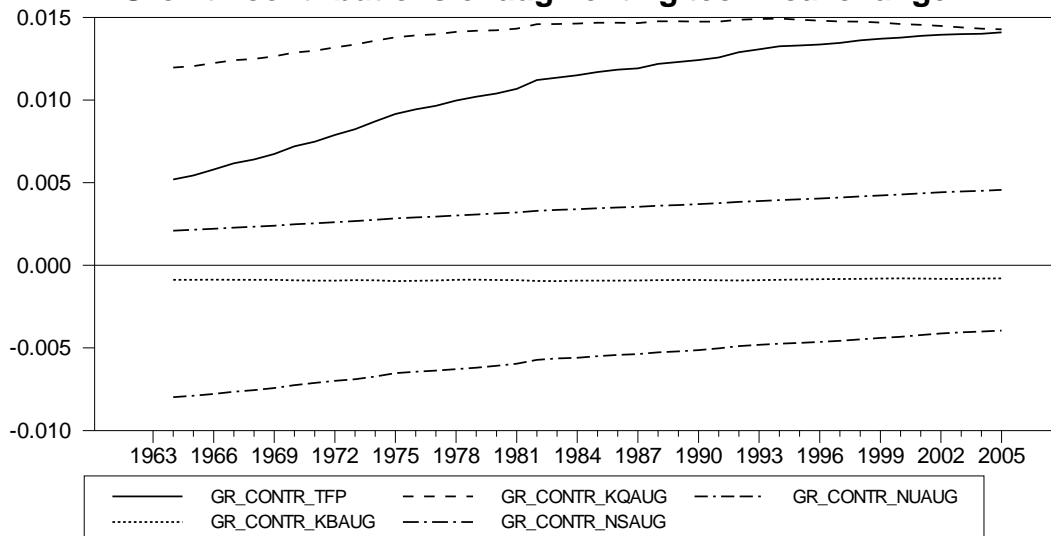
Actual vs fitted



Note: See note to Graph 1.

Panel B

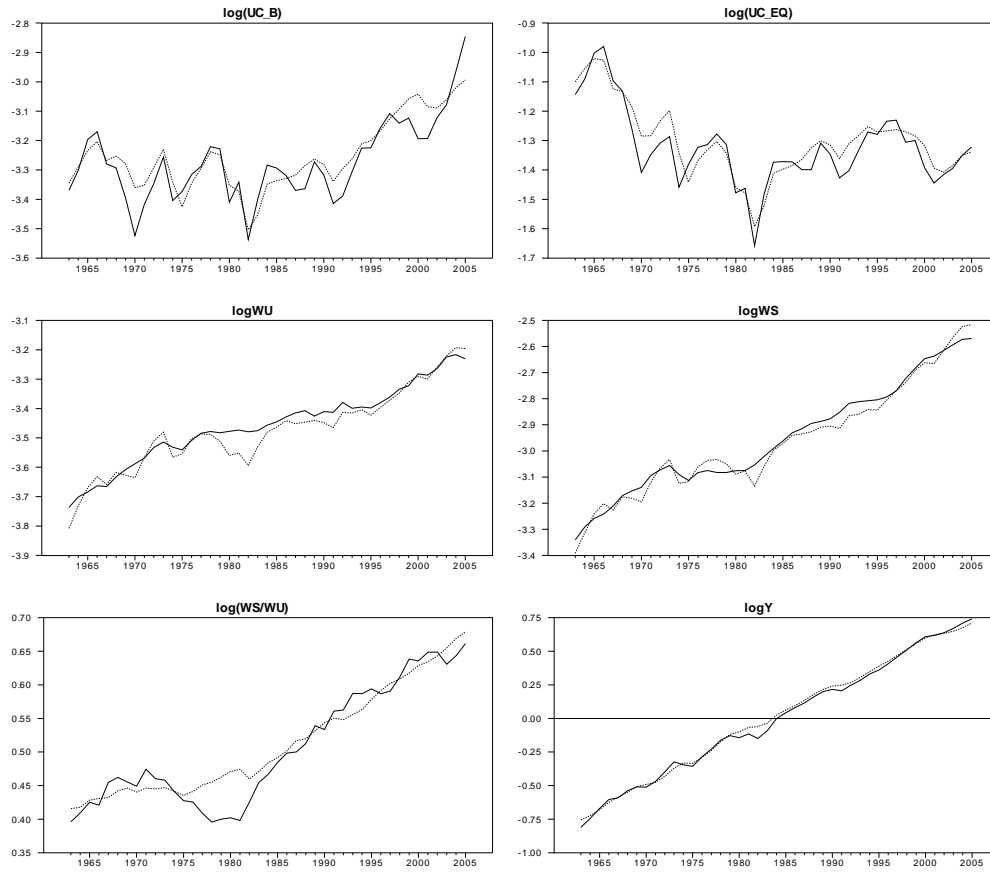
Growth contributions of augmenting technical change



Graph 6. Residuals and fits of column (D) in Table 6.

Panel A

Actual vs fitted



Note: See note to Graph 1.

Panel B

Growth contributions of augmenting technical change

