Abstract

This study summarizes an approach of Czech Ministry of Finance to macroeconomic modelling. We rely on a small open economy structural model that serves for analytical purposes. Namely, for prediction purposes as a support in preparations of regular quarterly Macroeconomic Forecast and also for preparing simulations, e.g. an analysis of macroeconomic impacts of various fiscal policy measures.

The model relies on a general equilibrium and the resulting consistency of all blocks in the model. The government regulates tax and transfer policy. The Central Bank, operating under the inflation targeting regime, determines a short term policy rate. Households supply labour to firms and spend their income for consumption. We distinguish between two types of consumers - Ricardians and so called Rule-of-Thumb consumers. Business sector in the model is represented by three different firms: (i) importers buying goods for consumption and/or further production; (ii) producers combining imported and domestic inputs to produce intermediate products for retailers; (iii) retailers selling final goods to domestic agents (households and/or government) and/or export abroad. Since the Czech economy is a small open and raw materials dependent economy, foreign trade (mainly with other EU countries) is of the importance. The model also allows for fiscal simulations through implicit tax rates on consumption, wages and benefits to households. The paper includes some sensitivity analysis and illustrative simulation results.

Keywords: New Keynesian macroeconomics, dynamic stochastic general equilibrium model, fiscal policy, solution of a DSGE model, impulse response functions.

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The views expressed in the paper do not necessarily reflect those of the Ministry of Finance of the Czech Republic. And of course, all mistakes are my own.
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1 Non-technical summary

The Economic Modelling Unit at the Ministry of Finance has developed macroeconomic model called "HUBERT", using a simple dynamic stochastic general equilibrium (DSGE) approach. The model provides support for macroeconomic forecasts carried out by the Ministry. Beside this, simulations of various macroeconomic scenarios are provided. The model also aims at filling the gap in analysis of fiscal policy measures, which is of crucial importance for the Ministry. This paper is an aggregate study of current Czech Ministry of Finance approach and stems from two original papers Stork et al. (2009, 2010).\(^1\)

The model is of New Keynesian type and describes the behaviour of four basic agents in the economy. First, we assume infinitely lived households maximizing their intertemporal utility function subject to budget constraint. There are two types of households in the economy. Liquidity unconstrained Ricardians with a free access to the financial market in order to smooth their consumption and non-Ricardians\(^2\) spending the whole income in each period of time. We incorporate habit formation according to Abel (1989) and Fuhrer (2000). Households also have a power in wage negotiation. Following Erceg, Henderson and Levin (1999) we incorporate a wage rigidity to the labour market, by assuming households to negotiate their wage only after receiving some random signal.

Second, we distinguish three types of firms: importers, intermediate-goods producers and final-good producers (retailers). Since the Czech economy is small and open one, importers are assumed to buy imports at given prices. They maximize their profit function with respect to a demand function. We assume monopolistic competition in case of producers, that maximize production function with respect to costs of inputs (imports and wages). The production function is of Cobb-Douglas type and following Hamermesh and Pfann (1996) adjustment costs for input factor are incorporated. This type of firm is also the only one which is able to handle the price on the market. Following Calvo (1983) and Gali and Gertler (1999) we consider price rigidity on the market. Finally, retailers are assumed to behave in perfect competitive environment. They aggregate intermediate goods from producers and sell them to consumers in domestic and foreign economy. In this competitive environment, the price is given by producers and retailers could optimize the quantity.

Third, the Central Bank, operating under the inflation targeting regime, determines a short term policy rate. This is set with respect to an extended Taylor rule (considering Taylor (1993) and Svensson (1998)) and interest rate smoothing (according to Srour (2001)). On the side of the government a simple expenditure fiscal rule focused on primary deficit is introduced.

\(^1\) Besides our model, another DSGE of the Czech economy has been developed by the Czech National Bank recently, see Beneš et al. (2005) for more information.

\(^2\) Also called "Rule-of-Thumb households".
Finally, because of the openness of the Czech economy, foreign trade is of crucial importance. The "rest-of-the-world" is approximated using a simplified version of Smets and Wouters (2002), capturing behaviour of three European agents: households, firms and government. An overview of the main blocks of the model can be found in Figure 1.

**Figure 1:** A bird’s eye view of the model

The paper is organized as follows. Chapter 2 introduces a structure of the model. Following Chapter 3 shows model solution, linearized version of the model and its parameters. Chapter 4 discusses main simulation results and illustrates an analysis of fiscal policy shocks. Chapter 5 concludes.

### 1.1 Introduction to model documentation

For a clear understanding of the model documentation we present a basic concept of data transformation. A variable denoted by a capital letter (e.g. $X_t$) is a variable in levels (thousands of employees, billions of CZK, etc.). A variable denoted by a lower case letter (e.g. $x_t$) is natural logarithm of the original variable $x_t \equiv \ln X_t$ or some ratio in percentage (unemployment rate, government debt as a percentage of GDP, etc.).
2 Structure of the model

2.1 Households

We assume that domestic economy consists of infinitely lived consumers, maximizing their intertemporal utility function subject to a lifetime budget constrain. We distinguish between Ricardian and non-Ricardian types of households, which is important for capturing proper dynamics in the model. The latter agents do not have an access to a capital market and they consume all resources in each period of time. Ricardians, on the other hand, smooth their consumption by accumulation and decumulation of assets.

The utility function is positively affected by consumption $C_t$ and negatively by labour supply $N_t$. Specific forms of objective functions in the RBC/DSGE literature seem to be somewhat arbitrary but they are usually chosen with respect to some theory and/or empirical findings. In the case of the households’ optimization problem we have to assume that an instantaneous utility function is concave and a budget constrain is a convex set in order to achieve a unique solution of the problem. However, there are many functions satisfying this condition. Therefore, we have to impose additional restrictions on households’ preferences. For example, we know from business cycle facts that consumption and output exhibit approximately the same constant growth rate over time which implies that consumption-output ratio is approximately constant over time. From this reason, we should specify the utility function of households from the admissible set of functions ensuring a balanced growth path of the model. King et al. (1988) proofed that the constant relative risk aversion function (CRRA) satisfies all necessary conditions. Moreover, this function is concave, continuous and easy to differentiate.

We also incorporate habit formation\footnote{Although we found some weak evidence in favour of the habit formation from empirical data sets, there is virtually no single test for distinguishing between different types of habits (external, internal and deep habits). Hence, the specification of habit formation is fully up to model builders. We use the external additive form of habits from two reasons: a) simplicity of the derived FOCs; b) the relative risk parameter can be a constant. This feature does not have to hold for other types of habits.} according to Abel (1989) and Fuhrer (2000). Habit formation of consumption is then defined as $H^c_t = \gamma_c C_{t-1}$, where $\gamma_c$ represents the habit persistence parameter, measuring the effect of past consumption ($0 \leq \gamma_c \leq 1$). The same applies for habit formation of labour supply, so $H^n_t = \gamma_n N_{t-1}$, where $\gamma_n$ is the habit persistence parameter, which is also ($0 \leq \gamma_n \leq 1$).
The optimization problem of Ricardian households concentrates in following utility function:

\[
\max_{\{C_t^R, A_t, A_t^*, N_t\}} E_t \sum_{t=0}^{\infty} \beta^t \left[ \frac{(C_t^R - H_t^c)^{1-\psi_c}}{1 - \psi_c} - \frac{(N_t^R - H_t^n)^{1+\psi_n}}{1 + \psi_n} \right], \tag{1}
\]

with respect to their budget constraint

\[
A_t + S_t A_t^* + (1 + \tau_t^c) P_t C_t^R = (1 + i_{t-1}) A_{t-1} + (1 + i_t^* + \zeta_{t-1}^*) S_t A_{t-1}^* + (1 - \tau_t) W_t N_t^R + (1 - \tau_f^f) \Pi_t, \tag{2}
\]

where:

- \(A_{it}, A_t^*\) bunch of net domestic and net foreign assets held by a household,
- \(C_t^R\) individual real consumption of Ricardian household,
- \(H_t^c\) habit level of consumption,
- \(H_t^n\) habit level of labour supply,
- \(i_t, i_t^*\) nominal rate of returns on domestic and foreign assets,
- \(N_t^R\) individual labour supply of Ricardian household,
- \(P_t\) consumer price index,
- \(S_t\) nominal exchange rate,
- \(W_t\) nominal wage,
- \(\beta\) discount factor,
- \(\psi_c\) inverse of substitution of consumption,
- \(\psi_n\) inverse of substitution of labour supply,
- \(\Pi_{it}\) aggregated profit of firms,
- \(\zeta_t^*\) risk premium,
- \(\tau_t^b\) rate of benefits to households,
- \(\tau_t^w\) personal income tax rate,
- \(\tau_t^f\) implicit tax rate on consumption (value added tax and excise tax),
- \(\tau_f^f\) corporate income tax rate.

Following Benigno (2001) or Schmitt-Grohe and Uribe (2003), we apply a hypothesis that a foreign interest rate realized on the international financial market differs with a borrower/lender position of a country. The interest rate is then a sum of “risk free” interest rate \(i_t^*\) and a “risk” premium \((\zeta_t^*)\). The later is a function of net foreign assets.\(^4\) So, if the domestic economy is in the position of a net borrower then the domestic households are charged by the extra premium and therefore \(\zeta_t^* > 0\), and vice versa.

\(^4\)The risk premium is a function of \(NFA/GDP\). From a balance of payments we know that \(\Delta NFA_t \approx CA_t + FA_t\), where \(CA_t\) stands for the current account and \(FA_t\) the financial account. Since many transactions from the financial account are sometimes sterilized, we decided to ignore this component of net financial assets in the model. Although it is a rough approximation, it can be shown that the main relations are still preserved.
Non-Ricardian households maximize slightly different utility function lacking habit formation elements, i.e.

$$\max_{\{C_{t}^{NR}, N_{t}\}} \mathbb{E}_{t} \sum_{t=0}^{\infty} \beta^{t} \left[ \frac{(C_{t}^{NR})^{1-\psi_{c}}}{1 - \psi_{c}} - \frac{(N_{t}^{NR})^{1+\psi_{n}}}{1 + \psi_{n}} \right],$$

subject to a simple budget constraint

$$(1 + \tau_{t}^{c}) P_{t} C_{t}^{NR} = (1 - \tau_{t}^{w} + \tau_{t}^{b}) W_{t} N_{t}^{NR},$$

where:

- $C_{t}^{NR}$ individual real consumption of non-Ricardian household,
- $N_{t}^{NR}$ individual labour supply of non-Ricardian household.

Their initial consumption demands and labour supplies are then aggregated according to the formulas

$$C_{t} = C_{t}^{R} + C_{t}^{NR}, \quad N_{t} = N_{t}^{R} + N_{t}^{NR}.$$  \hspace{1cm} (5)

The distinction between two types of households requires defining their respective shares. According to a partial analysis and Forni et al. (2007), we assume the ratio of non-Ricardian households on consumption and labour supply would be identical. However, setting the value is not straightforward. In this sense, the literature provides a variety of proposals from 25% (Coenen and Straub (2005)) to 34–37% (Forni et al. (2007)) for Euro-area. Higher shares of non-Ricardians are used for the US as in Gali et al. (2007), often reaching level of 50%.

We stem from the EU-SILC database\(^5\) providing quite detailed characteristics of households in our analysis of households’ share. Despite using quality data, setting the share of non-Ricardians requires expert judgements taking on board country specificities (e.g. larger share of retirees in eastern Europe should be considered as non-Ricardians comparing to ”old” EU member countries). We divided population into several categories. In the baseline scenario we assume non-Ricardians those, who are non-working retirees and long-term unemployed. Beside these, using an expert opinion, certain parts of other groups are considered in this share: 20% of employees, 10% of self-employed, 50% of working retirees, 70% of those unemployed for less than a year, and half of others. Thus, our baseline assumption of the non-Ricardians share is equal to 37% which is illustrated by columns in Figure 2. We use this assumption in the analysis in the rest of the paper.

However, we also employ other opinion and set the non-Ricardian share on 50%, by three changes in previous assumptions: (i) half of employees are rather non-Ricardians, since they are well below an average wage\(^6\); (ii) those self-employed and (iii) working

\(^5\)European Union - Statistics on Income and Living Conditions.
\(^6\)They have approximately 80% and less of average wage.
retirees are all Ricardians. These alternative thresholds between the two groups are displayed by lines in the Figure 2.\footnote{We provide a sensitivity analysis of this alternative scenario in Appendix B.}

![Figure 2: Share of Ricardians and non-Ricardians](image)

Having specified the model, we can derive the optimal behaviour of households. An optimal decision constitutes from two parts: intertemporal and intratemporal optimization.

**Intertemporal optimization**

We solve an optimization problem where households seek an optimal allocation with respect to their limited resources. This relationship is not difficult to derive from the first order conditions (FOCs) of the maximization problem.

The Euler equation of consumption expresses the determinants of relative consumption depending on the risk aversion factor $\psi_c$ and the habit formation parameter $\gamma_c$. In case of Ricardians’ the equation is such as

$$1 = \beta E_t \left[ \left( \frac{C^R_{t+1} - H^c_{t+1}}{C^R_t - H^c_t} \right)^{-\psi_c} \left( \frac{1 + \tau^t_t}{1 + \pi_{t+1}} \right) \left( \frac{1 + \tau^c_{t+1}}{1 + \tau^c_{t+1}} \right) \right]$$

and for non-Ricardians

$$C^{NR}_t = \left[ \frac{(1 - \tau^w_t + \tau^b_t)W_t}{(1 + \tau^c_t)P^c_t} \right]^{\frac{1 + \psi_n}{\psi_n + \psi_c}}.$$  

\footnote{We provide a sensitivity analysis of this alternative scenario in Appendix B.}
In the case of labour supply, households offer labour with respect to the real wage and a consumption decision. Again distinguished for Ricardians

\[(N_t^R - H_t^n)^{\psi_n} = \left(\frac{(1 - \tau_t^w + \tau_t^b)W_t}{(1 + \tau_t^c)P_t}\right) (C_t^R - H_t^R)^{-\psi_c}\]  

and non-Ricardians

\[N_t^{NR} = \left(\frac{(1 - \tau_t^w + \tau_t^b)W_t}{(1 + \tau_t^c)P_t}\right)^{1-\psi_c} \left(\frac{\psi_n + \psi_c}{\psi_n}\right).\]  

We can also derive determinants of the exchange rate in the form of an uncovered interest rate parity\(^8\)

\[\frac{E_t(S_{t+1})}{S_t} = \frac{1 + i_t}{1 + i^*_t + \zeta_t^*}.\]  

**Intratemporal optimization**

Households do not optimize consumption from intertemporal perspective only but they also consider which goods and services to consume, whether those domestically produced or imported ones. They try to minimize consumption expenses, so the important determinants are relative prices of goods in the market. An optimization problem may be formalized as follows

\[\min_{C_t^d, C_t^m} P_t C_t = P_t^d C_t^d + P_t^m C_t^m,\]  

subject to a consumption bundle

\[C_t = \left[(1 - \mu_{cm})^{1-\theta_c} (C_t^d)^{\theta_c} + (\mu_{cm})^{1-\theta_c} (C_t^m)^{\theta_c}\right]^{\frac{1}{\theta_c}},\]  

where:

- \(C_t\) aggregate consumption,
- \(C_t^d\) consumption of domestically produced goods,
- \(C_t^m\) consumption of imported goods,
- \(P_t\) consumption price index,
- \(P_t^d\) price of domestically produced goods for consumption,
- \(P_t^m\) price of imported goods for consumption,
- \(\theta_c\) parameter of substitution between domestic and imported goods,
- \(\mu_{cm}\) weight of the imported consumption in the bundle.

\(^8\)The way how to derive the uncovered interest rate parity in DSGE models is from the optimization problem of households. This is also the reason why the risk premium occurs in households’ budget constraint. Otherwise we would not be able to consistently derive exchange rate including risk premium.
Both consumption bundles are assumed to be Dixit-Stiglitz consumption aggregates of individual consumption goods

\[ C^d_t = \int_0^1 (C^d_{jt})^{\theta_d} \, dj \left[ \frac{1}{\theta_d} \right], \quad C^m_t = \int_0^1 (C^m_{jt})^{\theta_m} \, dj \left[ \frac{1}{\theta_m} \right], \]  

where \( \theta_d \) and \( \theta_m \) are parameters of substitution among goods in baskets of domestically produced and imported goods. Households decide for domestic and foreign goods respectively, in compliance with the weight of respective goods in their consumption and with their substitutability. A parameter of substitution \( \theta_c \) is defined using the elasticity of substitution in consumption \( (\sigma_c) \) as \( \theta_c = 1 - 1/\sigma_c \). We may derive another key macroeconomic relationship, such as aggregate consumer price index \( P_t \), which depends on prices of domestic and foreign goods weighted by their shares in consumption

\[ P_t = \left[ (1 - \mu_{cm}) \left( P^d_t \right)^{1-\sigma_c} + \mu_{cm} \left( P^m_t \right)^{1-\sigma_c} \right] \left[ \frac{1}{1-\sigma_c} \right], \]  

and also demand for the two consumption categories

\[ C^d_t = (1 - \mu_{cm}) C_t \left( \frac{P^d_t}{P_t} \right)^{-\sigma_c}, \quad C^m_t = \mu_{cm} C_t \left( \frac{P^m_t}{P_t} \right)^{-\sigma_c}. \]  

The latter is determined by the household’s decision about the level of current consumption, weights of domestic and imported goods in consumption, their substitutability and relative prices.

### 2.2 Firms

There are assumed to be three types of firms, indexed by \( j \in [0,1] \), operating in the domestic economy: intermediate-goods producers (hereafter only producers), final-goods producers (retailers), and import firms (importers). In order to improve the performance of a supply side of the model, it seems to be necessary to incorporate some form of nominal and real rigidities into the model. We use a simple Cobb-Douglas production function, extended for adjustment costs for input factors. Only for the sake of simplicity, we assume quadratic (symmetric) adjustment costs. An example of adjustment cost function is as follows

\[ \Upsilon_t^x = \frac{\nu}{2} \left( \frac{\Delta X_t}{\bar{X}_{t-1}} - \bar{x} \right)^2, \]  

9We derive parameters \( \theta_d \) and \( \theta_m \) analogically.

10 Another advantage of using this approach is in an explanation of some issues known from the RBC literature. A textbook example is rigidity of employment during the economic recession despite rigidity in nominal wages. An explanation can be found in real rigidity: especially in lower efficiency of new (additional) unit of workers (input factor) which can be captured, at least to some extent, by above mentioned costs of adjustment functions.
where $X_t \in \{L_t, M_t\}$ denotes employment and imports, and $\dot{x} \in \{\dot{L}, \dot{M}\}$ means their steady state growth rates, respectively, $\nu$ is a constant term. An interested reader is referred to Hamermesh and Pfann (1996), for a formal explanation and excellent discussion about adjustment cost functions.

**Retailers**

Retail firms are assumed to behave in a perfect competitive environment.\(^{11}\) They buy intermediate goods from producers, aggregate and sell them to households or government in the domestic or foreign economy. The competitive environment causes that prices of goods are set by producers and retailers can optimize only according to quantities $Q_{jt}$. Maximization a profit function is thereby equivalent to minimization a cost function due to a zero profit margin condition. Each retail firm tries to minimize the following cost function

$$
\min_{Q_{jt}} P_t^d Q_t = \int_0^1 P_{jt}^d Q_{jt} dj,
$$

where:

- $P_{jt}^d$ individual price of a good $j$,
- $P_t^d$ aggregate price index,
- $Q_{jt}$ individual intermediate good $j$,
- $Q_t$ aggregated output.

subject to a bundle generated by the Dixit-Stiglitz aggregate

$$
Q_t = \left[ \int_0^1 (Q_{jt})^{\theta_q} dj \right]^{\frac{1}{\theta_q}},
$$

where $\theta_q$ is a parameter of substitution among goods in the whole basket. From the FOC follows a demand function for an individual good $Q_{jt}$ given by

$$
Q_{jt} = Q_t \left( \frac{P_{jt}^d}{P_t^d} \right)^{-\sigma_q}
$$

and the aggregate price index

$$
P_t^d = \left[ \int_0^1 \left( P_{jt}^d \right)^{1-\sigma_q} dj \right]^{\frac{1}{1-\sigma_q}},
$$

where $\sigma_q$ is a price elasticity.

\(^{11}\)A perfect competition assumption is used for the convenience since it simplifies the derivation of an aggregate price index, see below.
Producers

We assume that there is a continuum of monopolistically competitive intermediate-goods firms producing a differentiated output $Q_{jt}$ using a simple production function with costs of adjustment. Due to monopolistic competition, producers are given some power in price setting behaviour. Following Calvo (1983), we assume that firms are allowed to reset their output prices only after receiving a random signal. The probability of receiving such the signal at time $t$ is $(1 - \xi_p) \in [0, 1]$. Moreover, in line with Gali and Gertler (1999) we assume that a fraction of firms $\mu_p$, allowed to reoptimize their price at time $t$, uses some simple rule for price updating: $P_{jt}^b = (1 + \pi_t - 1) \tilde{P}_{t-1}$, where $\tilde{P}_{t-1}$ is a lagged optimal price of their competitors. Loosely speaking, a part of firms that can reoptimize output prices is backward-looking and the rest of them is forward-looking. Those firms that cannot reoptimize are assumed to follow a simple price setting rule: $P_{jt} = P_{jt-1}$.

Each producer, indexed by $j \in [0, 1]$, tries to maximize the following profit function

$$\max_{\{P_{jt}^d, L_{jt}, M_{jt}\}} \Pi_{jt}^d = E_t \sum_{t=0}^{\infty} (\beta \xi_p)^t (P_{jt}^d - MC_{jt}^d) Q_{jt},$$

subject to a very simple production function in the form

$$Q_{jt} = Z_t L_{jt}^\alpha M_{jt}^{1-\alpha} - \gamma_l^t L_{jt} - \gamma_m^t M_{jt},$$

where:
- $P_{jt}^d$ individual price of a good $j$,
- $MC_{jt}^d$ domestic marginal costs,
- $Q_{jt}$ individual output of a good $j$,
- $L_{jt}$ employment,
- $M_{jt}$ imported goods for consumption,
- $Z_t$ technological progress,
- $\alpha$ production function parameter,
- $\gamma_l^t, \gamma_m^t$ adjustment cost functions.

From the standard FOCs result the optimal demand for labour and imports in the form as follows

$$W_t = \frac{\alpha Q_{jt}^l}{P_{jt}^d} - \gamma_l^l - \nu \left( \frac{\Delta L_{jt}}{L_{jt-1}} - \dot{l} \right) \frac{L_{jt}}{L_{jt-1}} + \nu \beta \xi_p E_t \left[ (1 + \pi_{t+1}) \left( \frac{\Delta L_{jt+1}}{L_{jt}} - \dot{l} \right) \left( \frac{L_{jt+1}}{L_{jt}} \right)^2 \right],$$

$$S_t \frac{P_{jt}^m}{P_{jt}^d} = \frac{(1 - \alpha) Q_{jt}^m}{M_{jt}} - \gamma_m^m - \nu \left( \frac{\Delta M_{jt}}{M_{jt-1}} - \dot{m} \right) \frac{M_{jt}}{M_{jt-1}} +$$

$$+ \nu \beta \xi_p E_t \left[ (1 + \pi_{t+1}) \left( \frac{\Delta M_{jt+1}}{M_{jt}} - \dot{m} \right) \left( \frac{M_{jt+1}}{M_{jt}} \right)^2 \right],$$

14
where $P^*_t$ represents the foreign consumer price index and $S_t P^*_t$ represents marginal costs of imported intermediate goods.\textsuperscript{12}

The FOC for producers allowed to reoptimize their prices is as follows

$$P_{jt}^f = \left(\frac{\sigma_q}{\sigma_q - 1}\right) \left[ \frac{E_t \sum_{i=0}^{\infty} (\beta \xi_p)^i MC_{t+1}^d Q_{jt+i}}{E_t \sum_{i=0}^{\infty} (\beta \xi_p)^i Q_{jt+i}} \right], \quad (25)$$

where $P_{jt}^f$ is the optimal price of a forward-looking producer, $\theta_q$ is the parameter of substitution among goods in a basket, $MC_t^d$ is the marginal costs specification, and $Q_{jt}$ is the output quantity. The aggregate price index is a function of newly set prices and updated prices from the previous period

$$P_t^d = \left[ \int_0^1 \left( P_{jt}^d \right)^{1-\sigma_q} \, dj \right]^{\frac{1}{1-\sigma_q}} = \left[ (1 - \xi_p) (\tilde{P}_t^d)^{1-\sigma_q} + \xi_p (P_t^d)^{1-\sigma_q} \right]^{\frac{1}{1-\sigma_q}}, \quad (26)$$

where $\sigma_q$ is the elasticity of substitution among goods, and $\tilde{P}_t^d$ is a function of forward-looking price setters using $P_{jt}^f$ as the optimal price and backward-looking price setters using an optimal price $P_{jt}^b$.

**Importers**

The Czech economy, as a small open economy, is in the position of a price taker in foreign trade markets. Importers purchase foreign goods at given foreign prices (marginal costs) and turn it into differentiated import goods used for consumption. We assume that each importing firm, indexed by $j \in [0, 1]$, tries to maximize a profit function given by

$$\max_{P_{jt}^m} \Pi_{jt}^m = E_t \sum_{t=0}^{\infty} (\beta \xi_p)^t (P_{jt}^m - MC_t^m) C_{jt}^m, \quad (27)$$

subject to a very simple demand function in the form

$$C_{jt}^m = C_t^m \left( \frac{P_{jt}^m}{P_t^m} \right)^{-\sigma_m}, \quad (28)$$

where:

- $C_{jt}$: individual imported good $j$,
- $C_t$: imported goods,
- $MC_t^m$: marginal costs of imported goods,
- $P_{jt}^m$: individual price of imported good $j$,
- $P_t^m$: aggregate price of imported goods,
- $\sigma_m$: elasticity of substitution among imported goods.

\textsuperscript{12}Only for simplicity we assume that firms have a direct link to foreign producers.
The FOC for importers, which is an analogue to the FOC of domestic producers, results in the following aggregate price index of imported goods

\[ P_m^t = \left[ \int_0^1 (P_m^t)^{1-\sigma_m} \, dj \right]^{\frac{1}{1-\sigma_m}} = \left[ (1 - \xi_p)(\tilde{P}_m^t)^{1-\sigma_m} + \xi_p(P_m^{t-1})^{1-\sigma_m} \right]^{\frac{1}{1-\sigma_m}}, \quad (29) \]

where the notation is identical to the previous section.

2.3 Labour market

Each household, indexed by \( i \in [0, 1] \), is assumed to supply a differentiated type of labour to intermediate producers. Imperfect substitution of labour provides some monopoly power to households in wage negotiation. Following Erceg, Henderson and Levin (1999), we assume that households can negotiate wage only after receiving some random signal. The probability of receiving such a signal at time \( t \) is \((1 - \xi_w) \in [0, 1]\). Since wages are set in the form of staggered contracts, each households reoptimize its wage rate by maximizing utility function in equation (1) with respect to \( W_t \) and subject to a standard labour demand function. The resultant FOC gives

\[ \tilde{W}_t = \left( \frac{-\sigma_l}{1 + \tau_c} \right) \left[ \frac{E_t \sum_{j=0}^\infty (\beta \xi_w)^j U_{nt+j} N_{nt+j}} {E_t \sum_{j=0}^\infty (\beta \xi_w)^j (1 - \tau_w + \tau_b) U_{ct+j} N_{ct+j} / P_{t+j}} \right], \quad (30) \]

where:
- \( \tilde{W}_t \) newly negotiated wage,
- \( U_{nt} \) marginal disutility of labour,
- \( U_{ct} \) marginal utility of consumption,
- \( \theta_l \) parameter of substitution.

Those households who cannot re-set their wages follow a simple wage rule: \( W_t = W_{t-1} \), it means there is no indexation of wages. The aggregate wage is a function of newly negotiated wage and updated wage from the previous period

\[ W_t = \left[ \int_0^1 (W_{it})^{1-\sigma_l} \, di \right]^{\frac{1}{1-\sigma_l}} = \left[ (1 - \xi_w)(\tilde{W}_t)^{1-\sigma_l} + \xi_w(W_{t-1})^{1-\sigma_l} \right]^{\frac{1}{1-\sigma_l}}, \quad (31) \]

where \( \sigma_l \) is the elasticity of labour substitution.

2.4 Monetary policy

Conducting of monetary policy has undergone through two different strategies recently in the Czech Republic. The first one, adopted and carried out during a transformation period, was based on targeting specific monetary aggregates. However, this strategy was
not very successful especially due to a fixed exchange rate attracting foreign speculative capital. After exchange regime fluctuations in 1997, the Czech National Bank (CNB) adopted a new strategy for monetary policy based on inflation targeting. Roughly speaking, the main point of this approach is to set the main instrument of the central bank according to key macroeconomic variables such as inflation, output gap and possibly other relevant variables, in order to achieve and maintain a price stability.

We approximate bank’s behavior by the extended Taylor rule, developed by Taylor (1993) and discussed by Svensson (1998).\(^\text{13}\) The rule takes the following form

\[ i_t = (1 - \phi_i)[\bar{i} + \lambda_i \hat{\pi}_t + \lambda_y \hat{y}_t] + \phi_i i_{t-1}, \tag{32} \]

where
- \( i_t \) is short-term nominal interest rate,
- \( \bar{i} \) steady state value of short-term interest rate,
- \( \hat{\pi}_t \) deviation of inflation rate from its steady state (target) value,
- \( \hat{y}_t \) output gap,
- \( \lambda_y \) output gap weight,
- \( \lambda_p \) inflation weight,
- \( \phi_i \) interest rate smoothing parameter.

According to Srour (2001), there are many reasons for interest rate smoothing. First, the behaving of the central bank is important for investors and smoothing of interest rates can reduce volatility of a term premium and therefore volatility of long-term interest rates and other financial market instruments. Second, the central bank has usually limited information about the shocks hitting the economy. Third, many shocks are serially correlated.

According to Levin et al. (1998), simple monetary policy rules with a high degree of interest rate smoothing (\( \phi_i \rightarrow 1 \)) are also surprisingly robust against model uncertainty and misspecification. Unfortunately, this is probably a characteristic feature for large closed economies only. Coté et al. (2002) find that the most robust rule is the original Taylor rule (\( \phi_i \rightarrow 0 \)) for small open economy. Much worse, Natvik (2006) showed that extending a DSGE model for a fiscal block can lead to a serious determinacy problem. From this point of view, a cautionary note should be made for straightforward application of Taylor rules.

### 2.5 Fiscal policy

Although we can find a large body of the literature analyzing different issues of monetary policy using DSGE models, fiscal policy applications are rather rare and leading often to controversial results. For instance, one of the most daunting issue of fiscal policy is...

\(^{13}\)We do not consider an inflation forecast-based rules since we have witnessed rather accommodative approach of monetary policy recently.
related to contradictionary effects of government expenditures on key macroeconomic variables such as employment and/or output coming from the empirical (VAR) and structural (DSGE) models. VAR models usually predict a positive effect of government expenditures on both output and employment which is in sharp contrast to the main findings from DSGE models, see Fatás and Mihov (2001), Gali et al. (2007), or Coenen and Straub (2005) for details. However, we strongly believe that fiscal policy can be a very efficient and powerful tool for economic policy especially due to many different instruments that can be used. So, the main purpose of this section is to present a simple fiscal policy rule closing the model. Although this task may seem quite simple, it is by no means easy to introduce even apparently unsophisticated rule into the model. An ad-hoc fiscal and monetary rules may lead to unintended implications, see Ascari and Rankin (2007) for details.

**Government budget and deficit**

Government revenues $GR_t$ are defined as follows

$$GR_t = PIT_t + CIT_t + VAT_t + EXCISE_t = \tau^w_t W_t L_t + \tau^f_t \Pi_t + \tau^c_t P_t C_t,$$

(33)

where:

- $GR_t$: total government revenues,
- $PIT_t$: revenues from personal income tax,
- $CIT_t$: revenues from corporate income tax,
- $VAT_t$: revenues from value added tax,
- $EXCISE_t$: revenues from excise tax.

Income from taxes is the essential revenue of the state budget. Personal income tax revenues are dependent on wages and employment ($W_t L_t$) and tax rate imposed – here represented by implicit personal income tax rate ($\tau^w_t$).  

CIT revenues are determined by corporate income tax rate ($\tau^f_t$) and profits ($\Pi_t$). VAT and excise taxes are modelled together and are represented by one implicit tax rate on consumption ($\tau^c_t$), which is imposed on nominal consumption ($P_t C_t$).

Government expenditures $GE_t$ are represented by two groups

$$GE_t = G_t + G^s_t = G_t + \tau^b_t W_t N_t,$$

(34)

---

14 For further explanation of implicit tax rate concept follows.
15 Currently we do not work with endogenous corporate income taxes since the sector of firms (namely capital and investments) is not developed enough to ensure meaningful analysis of corporate income tax measures.
16 We are aware of simplification in using this notation since VAT is calculated in current prices, while excise taxes have the assessment base in constant prices.
where:

- $GE_t$ total government expenditures,
- $G_t$ government consumption,
- $G^s_t$ paid out social benefits,

where social expenditures are determined by implicit rate ($\tau^b_t$) and wage development. By subtracting revenues from expenditures we may easily derive a primary government deficit

$$D_t = GE_t - GR_t,$$

that is cumulated into debt

$$B_t = D_t + (1 + i_{t-1})B_{t-1},$$

where:

- $B_t$ government debt,
- $D_t$ primary government deficit.

**Fiscal policy rule**

In the previous section we defined main budgetary items that are of the most importance when analyzing public finances. Two important questions must be answered when deriving a fiscal rule. First, what will be a reference variable activating the fiscal policy rule - debt, deficit, or both. Second issue concerns an instrument that should be adjusted by the rule. Unfortunately, there is no clear evidence in the economic literature which instrument should play the main role. In general, most analysis rely on tax rules where fiscal policy rectifies the debt dynamics by changes in tax rates. Unluckily various difficulties are related with introducing tax rules into the model (an optimal taxation problem, omitted interactions with monetary policy and internal consistency of the model).

In our view, the best way how to deal with these difficulties is to introduce an expenditure fiscal rule. First, the expenditure based rule is much easier. We do not need to arbitrarily decide which tax rate should be adjusted. Second, changes in tax rates require a change in legislation which can be very inflexible. Contrary, government expenditures may be adjusted quite promptly. Third, changes in taxation has an impact on relative prices. Our fiscal policy rule is based on an assumption of a balanced primary government budget (zero primary deficit) in equilibrium.\[^7\] Formally, the rule is given by the following equation

$$\frac{G_t}{P_t} = (1 - \phi_g) \frac{G_{t-1}}{P_{t-1}} + \phi_g \frac{G_{t-1}}{P_{t-1}},$$

\[^7\]We use primary deficit in order to avoid the coincident restriction of public finances implied by the monetary restriction through interest rate payments, and vice versa.
where $\bar{G}_t$ stands for equilibrium government consumption. The rule is derived from equation (35) under the following condition: $D_t = GE_t - GR_t = 0$. The parameter $\phi_g$ reflects a speed of convergence of public finances.$^{18}$

**Implicit tax rates**

In order to be able to simulate fiscal policy measures, we endogenized implicit tax rate on consumption (value added tax and excise tax) $\tau_c^c$, rate of benefits to households $\tau_b^b$ and personal income tax rate $\tau_w^w$. Because of insufficient specification of investment and capital in the model, we keep corporate income tax rate $\tau_f^f$ as a constant and do not allow for simulations at this stage.

All variable tax rates are decomposed for modelling purposes as

$$\tau^x_t = \bar{\tau}^x + \hat{\tau}^x_t,$$  \hspace{1cm} (38)

where:

- $\tau^x_t$: respective tax rate,
- $\hat{\tau}^x_t$: deviation from the steady state value,
- $\bar{\tau}^x$: steady state value of tax rate.

The description of tax behaviour in the model is described in the fashion of Forni et al. (2007). We simply assume that deviation of each tax rate from its steady state follows AR(1) process, i.e.

$$\hat{\tau}^x_t = (1 - \lambda^x)\hat{\tau}^x_{t-1} + \varepsilon_t,$$  \hspace{1cm} (39)

where $\lambda^x$ is estimated tax parameter.$^{19}$ Steady state values of implicit taxes $\bar{\tau}^x$ are derived based on data from National Accounts. For this purpose we define tax rates as an implicit share of tax yield ($T^x_t$) on respective tax base.

**Tax rate on consumption $\bar{\tau}^c$**

This rate contains value added tax ($VAT_t$) and excise tax ($EXCISE_t$) altogether and is defined as

$$\bar{\tau}^c = \frac{T^c_t}{P_tC_t} = \frac{VAT_t + EXCISE_t}{P_tC_t},$$  \hspace{1cm} (40)

where:

- $T^c_t$: budgetary income from taxes on consumption.

---

$^{18}$We provide a sensitivity analysis on this parameter in Appendix C.

$^{19}$After a tax shock, we assume a gradual (exponential) convergence to steady state, which is mathematically approximated by AR process.

20
**Tax rate on wages** $\bar{\tau}_w$

This implicit tax rate consists of personal income tax and social contributions. We defined it as

$$\bar{\tau}_w = \frac{T^w_t}{W_tL_t}, \quad (41)$$

where:

$T^w_t$ budgetary income from taxes on wages.

**Rate of benefits to households** $\bar{\tau}_b$

The implicit rate is defined as a share of social benefits ($G^s_t$) on the base, i.e.

$$\bar{\tau}_b = \frac{G^s_t}{W_tN_t}, \quad (42)$$

It is obvious from the definitions, that we do not cover the whole government revenues and expenditures. We exclude several items that are of minor importance and that are hard to consistently implement into the model. Revenue side is thus covered by 86% and expenditures by 75%. Specific items included in implicit tax rates can be found in Appendix A.

### 2.6 World

The Czech economy can be characterized as a small open and raw materials dependent economy. For this reason a transmission of world economy shocks (oil price, etc.) into the Czech key macroeconomic variables is of the crucial importance for economic policy. Therefore, we decided to incorporate a small model of the EU economy into our model.

There are two alternatives how to deal with this issue. The first solution is to build up a small structural model of the EU economy. The second one is to follow results from simple VAR models, see Lindé et al. (2004) for an example. VAR models are easy to estimate and operate with but they can give us confusing results without any sign/parameter restrictions. On the other hand, a small calibrated structural model can have a problematic fit with actual data and it is by no means easy to calibrate. All in all, we decided to rely on a small structural model due to Cho and Moreno (2003), which is a simplified version of Smets and Wouters (2002) model. The world model captures the behaviour of three European agents: households, firms, and government: households are assumed to maximize a simple utility subject to budget constraint (analogically to equations (1) and (2); monopolistically competitive producers maximize a profit and set output prices following the Calvo price setting mechanism (analogically to equations (25) and (26); the government is assumed to determine a short-term interest rate following the Taylor rule (analogically to equation (32))).
3 Solution of the model

3.1 Steady-states

To determine steady state values of variables means to determine long-run dynamic properties of the core model. We suppose that all variables on the equilibrium path are either constant (stationary variable) or exponentially growing with the constant growth rate (nonstationary variables).

Determination of steady state values can be illustrated using the following examples. The steady state inflation rate, denoted as $\bar{\pi}$, is not derived explicitly from the static version of the NK Phillips curve but from the inflation target of the Czech National Bank. According to CNB’s strategy, the inflation rate target is set to be 3% from January 2006 (and 2% from January 2010). However, this approach can simply lead to inconsistency of the model provided that a given inflation target is incorrectly set.

The steady state growth rate of domestic output, denoted as $\dot{q}$, is derived explicitly from a given production function in Section 2.2. Clearly, from the log-linear production function follows that $\dot{q} = \dot{\bar{z}}/\alpha + \dot{n}$, where $\dot{\bar{z}}$ is the steady state growth rate of (exogenous) technology progress, and $\dot{n}$ denotes the steady state growth rate of population. Both quantities are calibrated according to Czech business cycle facts.

3.2 Model

Using the log-linearization we are able to rewrite the model into following system of equations that can be solved easily. We sort the equation to show how the main macroeconomic variables are determined. The equations are simplified using reduced-form parameters denoted by $\omega_{ij}$, where the index $i$ denotes a particular equation and $j$ a given variable. We also use a simplified notation for shocks.

Output

Following equations illustrate how the model derives the determinants of the output – consumption, government expenditures and net exports. We assume in this version investments to be a part of the consumption expenditures. Consumption itself stems from domestic producers or from imports, while factors of their mutual share are their relative prices. Firms output ($\hat{q}_t$) is produced using the technology, labour and imported sources.
\( \hat{y}_t = \omega_y \hat{c}_t^d + (1 - \omega_y) \hat{y}_t + \omega_{yx} (\hat{e}_t - \hat{m}_t), \)  
(43a)

\( \hat{c}_t = \omega_c \hat{c}_{t-1} + (1 - \omega_c) \hat{c}_{t+1} - 
- \omega_{ctw} (\hat{\tau}^b_{t-1} - \hat{\tau}^w_{t-1}) + (\omega_{ctw} + \omega_{ctul}) (\hat{\tau}^b_t - \hat{\tau}^w_t) - \omega_{ctul} (\hat{\tau}^b_{t+1} - \hat{\tau}^w_{t+1}) - 
- \omega_{ctul} (\hat{w}_{t-1} - \hat{p}_{t-1}) + (\omega_{ctul} + \omega_{ctc}) (\hat{w}_t - \hat{p}_t) - \omega_{ctul} (\hat{w}_{t+1} - \hat{p}_{t+1}) + 
+ \omega_{ctc} \hat{\tau}^c_{t-1} - \omega_{ctc} \hat{\tau}^c_{t+1} - \omega_{ct} (\hat{e}_t - \hat{m}_t), \)  
(43b)

\( \hat{g}_t = \omega_g \hat{d}_{t-1} - \hat{p}_t + \omega_{gg} \hat{g}_{t-1} + u^g_t, \)  
(43c)

\( \hat{e}_t = (1 - 2\omega_e) (\hat{g}_t^* - (\hat{p}_t^* - \hat{p}_t^* - \hat{s}_{t-1})) + \omega_{ee} \hat{e}_{t-1} + \omega_{ee} E_t (\hat{e}_{t+1}), \)  
(43d)

\( \hat{m}_t = (1 - 2\omega_{mm}) (\hat{q}_t - (\hat{p}_t^* + \hat{s}_t - \hat{p}_t^d)) + \omega_{mm} \hat{m}_{t-1} + \omega_{mm} E_t (\hat{m}_{t+1}). \)  
(43e)

\[ \begin{align*}
\hat{c}_t^d &= \hat{c}_t - \sigma_c (\hat{p}_t^d - \hat{p}_t), \\
\hat{c}_t^m &= \hat{c}_t - \sigma_c (\hat{p}_t^m - \hat{p}_t).
\end{align*} \]
(44a)

\[ \begin{align*}
\hat{q}_t &= \hat{z}_t + \alpha \hat{q}_t + (1 - \alpha) \hat{m}_t, \\
\hat{z}_t &= \omega_{zz} \hat{z}_{t-1} + u^z_t.
\end{align*} \]
(45a)

**Prices**

The model works with different price indexes defined with following relations. Weighted prices of domestic and imported production are source of consumer price index \( \hat{\pi}_t \). These components – price indexes of domestic, imported and exported production – having both forward and backward looking component are derived using a concept of marginal costs.

\[ \begin{align*}
\hat{\pi}_t &= (1 - \omega_{cm}) \hat{\pi}_t^d + \omega_{cm} \hat{\pi}_t^m, \\
\hat{\pi}_t^d &= \omega_{pc} \hat{m}_t^d + \omega_{pp} \hat{\pi}_{t-1}^d + (1 - \omega_{pp}) E_t (\hat{\pi}_{t+1}^d), \\
\hat{m}_t^d &= \alpha (\hat{w}_t - \hat{p}_t) + (1 - \alpha) (\hat{m}_t^m - \hat{p}_t) - \hat{z}_t + u^p_t, \\
\hat{\pi}_t^m &= \omega_{pc} \hat{m}_t^m + \omega_{pp} \hat{\pi}_{t-1}^m + (1 - \omega_{pp}) E_t (\hat{\pi}_{t+1}^m), \\
\hat{m}_t^m &= \hat{p}_t^* + \hat{s}_t, \\
\hat{\pi}_t^e &= \omega_{pc} \hat{m}_t^e + \omega_{pp} \hat{\pi}_{t-1}^e + (1 - \omega_{pp}) E_t (\hat{\pi}_{t+1}^e), \\
\hat{m}_t^e &= \hat{m}_t^d + \hat{s}_t.
\end{align*} \]
(46a)

**Labour market**

The extent of a labour supply \( \hat{n}_t \) depends on households’ decision while labour demand is ruled by firms. Wage on the labour market is a result of bargaining between employers
and employees when only a segment of wage contracts is reset in each period of time.

\[
\hat{w}_t = (1 - 2\omega_{ww})(\hat{y}_t - \hat{l}_t + \hat{p}_t) + \omega_{ww}\hat{w}_{t-1} + \omega_{ww}E_t(\hat{w}_{t+1}), \quad (47a)
\]

\[
\hat{n}_t = \omega_{nw}(\hat{c}_t - \hat{w}_t) - \omega_{nw}(\hat{c}_{t-1} - \hat{w}_{t-1}) + \omega_{nw}(\hat{w}_t - \hat{p}_t) - \omega_{nw}(\hat{w}_{t-1} - \hat{p}_{t-1}) - \omega_{ntc}\hat{c}_t - \omega_{ntc}\hat{c}_{t-1} + \omega_{nn}\hat{n}_{t-1}, \quad (47b)
\]

\[
\hat{l}_t = (1 - 2\omega_{lt})(\hat{q}_t - (\hat{w}_t - \hat{p}_t)) + \omega_{lt}\hat{l}_{t-1} + \omega_{lt}E_t(\hat{l}_{t+1}). \quad (47c)
\]

**Central bank and financial market**

Currently the most important financial market variable is short term interest rate set by the central bank using an extended Taylor rule. Modified uncovered interest rate parity determines an exchange rate.

\[
\hat{\iota}_t = \omega_{\iota\pi}\hat{\pi}_t + \omega_{\iota\gamma}\hat{\gamma}_t + \omega_{\iota\hat{l}_{t-1}} + u_{\iota t}, \quad (48a)
\]

\[
\hat{s}_t = \omega_{ss}[E_t(\hat{s}_{t+1}) + \hat{s}_{t-1}] - \omega_s\iota \hat{\iota} - \omega_{ss}(\Delta \hat{\iota}_t - \Delta \hat{\iota}_t^*) + u_{\iota t}. \quad (48b)
\]

**Government**

Besides very simple structure of government revenues and expenditures, important variables for fiscal policy are deficits and debts.

\[
\hat{g}_{rt} = \omega_{gr\hat{w}}(\hat{w}_t + \hat{\iota}_t) + \omega_{gr\hat{p}}(\hat{p}_t + \hat{\iota}_t) + \omega_{gc}\hat{\iota}_t + \omega_{gwc}\hat{w}_t + \omega_{gc}\hat{c}_t, \quad (49a)
\]

\[
\hat{g}_{ct} = \omega_{gb\hat{g}}(\hat{g}_t + \hat{\iota}_t) + \omega_{gb}\hat{w}_t + \hat{n}_t + \omega_{gbt}\hat{\iota}_t, \quad (49b)
\]

\[
\hat{b}_t = \omega_{bbd}\hat{d}_t + \omega_{bb}\hat{b}_{t-1}. \quad (49c)
\]

**Trade**

Current account balance is used to describe an external balance. It is also an important factor of exchange rate development.

\[
\hat{\iota}_t = \omega_{\iota\pi}[\hat{\pi}_t - \omega_{\iota\pi}\hat{\pi}_t - (1 - \omega_{\iota\pi})\hat{\pi}_t^m] + \omega_{\iota\gamma}[\hat{\gamma}_t - \hat{\gamma}_t^m]. \quad (50)
\]

**World**

The development of other economies, approximated by the EU in the model, has not negligible impact on domestic agents. Thus we derived a small model with results of an output, price level and interest rates.

\[
\hat{y}_t^* = \omega_{yy}\hat{y}_{t-1} + (1 - \omega_{yy})E_t(\hat{y}_{t+1}) - \omega_{yy}E_t(\hat{\gamma}_t + \hat{\gamma}_{t-1}^*) + u_{y t}^y, \quad (51a)
\]

\[
\hat{\pi}_t^* = \omega_{pp}\hat{\pi}_{t-1} + (1 - \omega_{pp})E_t(\hat{\pi}_{t-1} + \hat{\pi}_{t+1}^*) + u_{p t}^p, \quad (51b)
\]

\[
\hat{\iota}_t^* = \omega_{\iota\gamma}\hat{\gamma}_t + \omega_{\iota\gamma}\hat{\gamma}_t^* + \omega_{\iota\iota}\hat{\iota}_t + u_{\iota t}^\iota. \quad (51c)
\]
3.3 Parameters

Parameters in our model are rather calibrated using other DSGE studies (especially those mentioned in Introduction) and also using a limited information estimation method. Following table summarizes deep parameters of the model with values currently used in the model.

<table>
<thead>
<tr>
<th>parameter</th>
<th>description</th>
<th>block</th>
<th>value</th>
<th>method</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>discount factor</td>
<td>H</td>
<td>0.99</td>
<td>calibrated</td>
</tr>
<tr>
<td>$\psi_c$</td>
<td>consumption utility (risk aversion)</td>
<td>H</td>
<td>2.00</td>
<td>calibrated</td>
</tr>
<tr>
<td>$\psi_n$</td>
<td>labour utility (risk aversion)</td>
<td>H</td>
<td>5.00</td>
<td>calibrated</td>
</tr>
<tr>
<td>$\gamma_c$</td>
<td>consumption habit</td>
<td>H</td>
<td>0.70</td>
<td>calibrated</td>
</tr>
<tr>
<td>$\gamma_n$</td>
<td>labour habit</td>
<td>H</td>
<td>0.80</td>
<td>calibrated</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>production function parameter</td>
<td>F</td>
<td>0.50</td>
<td>calibrated</td>
</tr>
<tr>
<td>$\lambda_{\pi}$</td>
<td>sensitivity on inflation</td>
<td>M</td>
<td>1.50</td>
<td>calibrated</td>
</tr>
<tr>
<td>$\lambda_y$</td>
<td>sensitivity on output gap</td>
<td>M</td>
<td>0.50</td>
<td>calibrated</td>
</tr>
<tr>
<td>$\mu_p$</td>
<td>weight of backward-looking firms</td>
<td>F</td>
<td>0.50</td>
<td>calibrated</td>
</tr>
<tr>
<td>$\mu_{yc}$</td>
<td>weight of consumption on GDP</td>
<td>H</td>
<td>0.80</td>
<td>calibrated</td>
</tr>
<tr>
<td>$\mu_{yx}$</td>
<td>weight of export on GDP</td>
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<td>0.85</td>
<td>calibrated</td>
</tr>
<tr>
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<td>weight of PIT on gov. revenues</td>
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</tr>
<tr>
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<td>calibrated</td>
</tr>
<tr>
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<td>weight of social benefits on gov. exp.</td>
<td>G</td>
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<td>calibrated</td>
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<td>share of non-Ricardians in consumption</td>
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<td>calibrated</td>
</tr>
<tr>
<td>$\mu_{hn}$</td>
<td>share of non-Ricardians in labour supply</td>
<td>H</td>
<td>0.37</td>
<td>calibrated</td>
</tr>
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<td>$\phi_g$</td>
<td>autoregressive parameter</td>
<td>G</td>
<td>0.70</td>
<td>calibrated</td>
</tr>
<tr>
<td>$\phi_i$</td>
<td>autoregressive parameter</td>
<td>M</td>
<td>0.80</td>
<td>calibrated</td>
</tr>
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<td>0.40</td>
<td>calibrated</td>
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<td>F</td>
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<td>calibrated</td>
</tr>
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<td>$\phi_e$</td>
<td>autoregressive parameter</td>
<td>F</td>
<td>0.45</td>
<td>calibrated</td>
</tr>
<tr>
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<td>probability of price reoptimization</td>
<td>F</td>
<td>0.50</td>
<td>calibrated</td>
</tr>
<tr>
<td>$1-\xi_w$</td>
<td>probability of wage reoptimization</td>
<td>LM</td>
<td>0.50</td>
<td>calibrated</td>
</tr>
</tbody>
</table>

Note: In column “block”, H stands for households, F for firms, M for monetary policy, G for fiscal policy (government), LM for labour market.
3.4 Solution

It is important to note that the model consists of both lagged variables (e.g. $\hat{c}_{t-1}$) and expected future endogenous variables (e.g. $E_t\hat{c}_{t+1}$). That means, the model consists of backward and forward-looking variables and the unique solution to this type of models does not have to be easy to find. But in general, we can rewrite the model into the matrix form as follows

\[
\begin{align*}
G\hat{y}_t &= FE_t(\hat{y}_{t+1}) + H\hat{y}_{t-1} + L\hat{z}_t, \\
\hat{z}_t &= N\hat{z}_{t-1} + u_t,
\end{align*}
\]

where $\hat{y}_t$ is a vector of endogenous variables, $\hat{z}_t$ is a vector of exogenous variables or shocks, $G, F, H, L, N$ are matrices of structural (unknown) parameters which are functions of deep parameters, and $u_t$ is a vector of disturbances in the model. After having the model in the matrix form, we can focus on a closed-form solution given by

\[
\begin{align*}
\hat{y}_t &= P\hat{y}_{t-1} + Q\hat{z}_t, \\
\hat{z}_t &= N\hat{z}_{t-1} + u_t,
\end{align*}
\]

where $P, Q$ are reduced form parameters of the model but still nonlinear and complicated functions of deep parameters. Finally, we can rewrite the whole model in the compact form as follows

\[
\begin{align*}
\hat{x}_t &= \Phi\hat{x}_{t-1} + \Theta\varepsilon_t, 
\end{align*}
\]

which is in fact a VAR(1) model, where $\hat{x}_t = (\hat{y}_t', \hat{z}_t')'$ and $\varepsilon_t = (0', u_t')'$ is a vector of errors. A macroeconomic model written in this form can be used simply both for predictions and policy analysis simulations. An interested reader is referred to Hamilton (1994), Juselius (2006), or Lutkepohl (2007) for more information about VAR models.
4 Simulation results

Performance of the model is presented in illustrative simulation results, graphically interpreted as impulse response functions (IRF). In following graphs we present impact of fiscal policy measures on main macroeconomic variables. All simulations represent a positive unit shock into respective variables.

When interpreting the IRF’s, we have to bear in mind that we shock tax rates (and government consumption respectively) that are deflected from their steady state values. A specific position within business cycle is not considered. In reality, effects of the tax rate change in periods of economic boom could (and they do) differ from effects in times of economic slowdown.

4.1 Government consumption

An unit shock into the government consumption has a positive impact on real GDP. The higher government consumption elevate a demand for production, which afterwards results in higher firms’ labour demand, decrease of unemployment and increase in real wages. The lower consumption is a result of Ricardian households behaviour, which defer their consumption to the future due to higher real interest rate.

![Graphs showing impulse response functions for various economic indicators after government consumption shock.](image)

Figure 3: Government consumption shock
The higher demand in line with appreciation of real exchange rate worsen a current account. On the other hand, thanks to openness of the Czech economy, strong exchange rate causes a reduction of firms marginal costs. This helps to limit inflation tensions from higher domestic demand.

Higher government spending results in higher primary government deficit that is consequently cumulated in higher debt.

4.2 Tax rate on consumption

Higher tax rate on consumption reduces consumption demand. This means lower demand for imported goods and consequently also lower demand for imported goods as inputs of production. The difference between export and import increases and therefore net export is growing. GDP is also falling, dragged down by the lower consumption. Net export has a positive influence on GDP, but the increase is only the effect of lower demand for imported consumption goods and therefore it does not offset the negative effect of consumption on GDP.

![Figure 4: Tax rate on consumption shock](image_url)

Lower demand for production forces firms to reduce labour demand and pushes down wages (with respect to their negotiation position). Lower wage and higher taxation of consumption demotivate households from work. On the other hand, cut-down in
consumption has a stronger influence and the final labour supply is therefore rising. Lower labour demand and higher labour supply leads to unemployment rate increase. This is in line with wage reduction.

As the lower demand for import increases net export, also a current account run surpluses. Firms, which cash their profits in foreign currency create an additional demand for czech currency and push the exchange rate to appreciate.

Wages and exchange rate are the main factors of firms’ marginal costs (import prices are given). Reduction in wages and exchange rate appreciation lower marginal costs, consequently. This limits inflation pressures and together with lower GDP push down interest rates. This effect, however marginal, limits the exchange rate appreciation.

Higher government revenues from increase of tax rate on consumption itself leads to a positive primary government deficit and decrease of debt.

4.3 Tax rate on wages

The primary effect of higher tax rate on wages is reflected in households’ budget through lower disposable income and thus lower consumption demand. The reaction is therefore analoga to the increase in the tax rate on consumption.

Figure 5: Personal income tax rate shock
The increase of the tax rate therefore reduces consumption, GDP and increases net export. Lower demand for production mutes wages as well as labour demand. Together with increasing labour supply unemployment rate grows. Wages, net inflation and interest rates are falling and exchange rate appreciates. Higher tax rate brings additional revenues, lower primary government deficit and consequently lower debt.

4.4 Rate of benefits to households

A positive shock into households’ benefits has a primary impact on consumption, which starts to grow. The higher demand means also a higher demand for consumption of imported goods, imports are rising and net export is declining. The later pushes GDP growth down, but this is more than offset by higher consumption and as a result GDP is increasing.

Both, higher GDP and demand for production enable firms to increase wages and labour demand. This motivates households to offer their labour supply in larger extent. On the other side, higher level of consumption has a stronger effect and pushes working incentives down and labour supply is falling in the end. Thus the higher GDP growth, higher labour demand and lower labour supply cause the unemployment decrease and raises the wage growth.

Figure 6: Rate of benefits shock
The higher demand for consumption of imported goods increases imports and induces net export and current balance decrease. The fall of net export causes the exchange rate depreciation.

With regard to wage increase and exchange rate depreciation, marginal costs are increasing. As a result the inflation pressures intensify. In addition, interest rates are pushed to increase too. Higher interest rates, on the contrary, has a negative impact on exchange rate, but this is rather marginal effect.

As benefits grow, government expenditures are higher and higher is also a primary deficit, which translates into the debt.
5 Conclusion

In this paper, we introduce an approach of Ministry of Finance to modelling of the Czech economy. To do this, we use a DSGE model with common features, analyzing behaviour of four basic agents in the economy. Households supply labour to firms and spend their income on consumption. Two type of households are distinguished. Besides so called Ricardian households, smoothing their consumption via capital market, non-Ricardians (Rule-of-Thumb households), spending all income on consumption in each period of time, has been included. A share between them is usually a subject to an expert judgment. We stem from data of the European Union, namely Statistics on Income and Living Conditions (EU-SILC), from which we derive the assumption. To show a sensitivity of our model on this parameter we include an analysis showing not very substantial vulnerability.

Business sector in the model is represented by three different firms: (i) importers buying goods for consumption and/or further production; (ii) producers combining imported and domestic inputs to produce intermediate products for retailers; (iii) retailers selling final goods to domestic agents (households and/or government) and/or export abroad. The Central Bank, operating under the inflation targeting regime, determines a short term policy rate using extended Taylor rule.

The government regulates tax and transfer policy. We simply assume taxes to follow AR(1) process. To derive taxes as implicit shares we use National Accounts in the structure also mentioned in the paper. Variables of implicit taxes are always calculated as a share of income from the tax and its respective base. Trying to keep the model medium scaled and easily manageable, we include two tax groups on revenue side - tax on consumption and tax on wages - and two types of expenditure items - paid out benefits and government consumption. Through these implicit rates, we are able to translate certain policy measures into the model. Our fiscal rule is expenditure oriented and stems from the condition of balanced primary government balance.

In the analytical part of the paper we try to illustrate results using impulse response analysis. We present how unit shocks into government consumption, taxes on consumption, taxes on wages and paid out benefits affect real economy. A story behind those results is attached. Besides the sensitivity for the share of non-Ricardian households, we show how results are dependent on the estimated parameter of the fiscal rule reflecting a speed of convergence of public finances. Also in this case there are not significant distinctions between scenarios.

The paper does not include an analysis of corporate income taxes, since the block of firms is not established in a proper manner to get reasonable results. This issue remains

\[^{20}\]The latter of course differs from the others since it is not defined as an implicit tax rate.
among plans for future work. Besides a proper definition of investments and capital in the model (see Woodford (2004), Woodford (2005) and Altig et al. (2005)), we would like to introduce some additional labour market imperfections by incorporating a matching function that can capture a limited matching between vacancies and unemployment in the economy, see Moyen and Sahuc (2004), Trigari (2004) and Stevens (2007). And finally some important parameters that are rather difficult to calibrated will be estimated via to different estimation techniques: Bayesian MLE and SMM, see Ruge-Murcia (2007) for details.
References


Appendix A

Data used for construction of implicit tax rates

Data from National Accounts that are used to define incomes of the state budget from respective taxes and social expenditures are defined as follows.

<table>
<thead>
<tr>
<th>ESA95 code</th>
<th>Item</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Taxes on consumption $T^c_t$</strong></td>
<td></td>
</tr>
<tr>
<td>D211</td>
<td>Value-added type taxes (VAT)</td>
</tr>
<tr>
<td>D2122C</td>
<td>Excise taxes</td>
</tr>
<tr>
<td>D214A</td>
<td>Excise taxes</td>
</tr>
<tr>
<td><strong>Taxes on wages $T^{wp}_t$</strong></td>
<td></td>
</tr>
<tr>
<td>D51A</td>
<td>Taxes on individual or household income</td>
</tr>
<tr>
<td>D51C1</td>
<td>Taxes on holding gains</td>
</tr>
<tr>
<td><strong>Benefits $G^i_t$</strong></td>
<td></td>
</tr>
<tr>
<td>D62</td>
<td>Social benefits other than social transfers in kind</td>
</tr>
</tbody>
</table>
Appendix B

Sensitivity: the share of non-Ricardians

The share of non-Ricarian households (NR) in the population has been set with a non-negligible degree of arbitrariness (see Chapter 2.1). This has an implication on the composition of households – namely their consumption and labour supply. For the time being, we suppose that the share of non-Ricardian consumption on total consumption bundle would be equal to the non-Ricardian share on labour supply. According to the analysis we set the ratio equal to 37% in the baseline. For the fact that this number contains non-negligible portion of arbitrariness and that the value has been rather set at a lower level, we provide a sensitivity test on different value of 50%. Following analysis compares the two different settings\(^{21}\) with the same positive unit shocks as in the Section 4.

**Government consumption**

As apparent from the Figure (7) the increase in government consumption has nearly the same impact on GDP growth in both cases. As described before, higher demand for production consequently increases labour demand and wages and limits the unemployment. Higher inflation, higher interest rates and appreciation of the Czech currency are other implications.

The difference between scenarios occurs in the case of consumption. The lower the share of NR the greater decrease in consumption. Larger share of Ricardians postpones a part of their consumption into the future as a result of real interest rate increase.

**Tax rate on consumption**

There are not large differences also in the case of taxes on consumption. Diverse reactions occur when the higher share of NR is chosen especially in the first several quarters after shock. It then leads to a higher attenuation of consumption causing higher decrease of GDP, wages and a higher rate of unemployment.

**Tax rate on wages**

Although reactions on personal income tax rate increase are quite similar to those on consumptions’ tax increase (both shocks have primary impact on households and then on production), they differ in intensity and persistence.

The tax increase lowers households’ disposable income and thus consumption too. The higher the NR share, the greater drop in consumption, since NR have no chance to smooth their consumption. The same applies in case of imported goods leading to lower demand for imports, which raises net exports and improves a current account

\(^{21}\)It is worth noting, that effects of higher and lower shares of non-Ricardian households are symmetric.
balance. The impact of more diminishing consumption is reflected in a lower GDP growth comparing to the baseline, higher unemployment, lower real wages, interest rates and inflation. Slower debt amortization is an effect on the fiscal policy side.

**Rate of benefits to households**
The rate of benefits is related to households' income in the same way as the tax on wages. The effects showed in graphs (9) and (10) are analogically opposite.

Thus the higher NR share on total consumption has a stronger positive effect on disposable income, which supports the consumption demand and higher GDP growth.
Figure 7: Government consumption shock

Figure 8: Tax rate on consumption shock
Figure 9: Tax rate on wages shock

Figure 10: Rate of benefits shock
Appendix C

Sensitivity: fiscal rule parameter

The fiscal rule that closes the model is driven by estimated parameter of $\phi_g$, which is set to 0.7 in the baseline. Let us assume alternative values of 0.6 and 0.8 respectively. Results of these alternatives are illustrated in Figures (11) to (14).

We do not see large distinctions between scenarios, which shows the stability of the model. The differences are hardly visible on Figures illustrating shocks into taxes on wages, consumption and rate of benefits. The minor exception is the government consumption shock since it directly affects the reference variable from the fiscal rule. The slower pace of consolidation (due to higher $\phi_g$) maintains slightly higher demand for production with all consequences: lower unemployment, higher wages, higher increase of interest rates following by somewhat lower inflation pressures.

A budgetary effect of maintaining relatively higher government consumption shows somewhat slower consolidation of budget balance with consequently higher debt cumulation.
Figure 11: Government consumption shock

Figure 12: Tax rate on consumption shock
Figure 13: Tax rate on wages shock

Figure 14: Rate of benefits shock