Structural change in agriculture - an equilibrium approach

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Abstract

Empirical investigations suggest that the farm size distribution in Western European countries has changed over the last decades. This paper analyses the impact of limited sectoral production capacity on entry and exit of farms in the agricultural industry. We present a dynamic stochastic framework which accounts for firm specific uncertainty and captures the close interdependency between exit of firms and fluctuating entry costs. In an equilibrium firms base their entry/exit decision on its expectations on future output prices and entry costs. We illustrate that the output price is higher and inefficient firms tend to stay longer in the industry if capacity constraints are binding at the sector level.

Keywords: dynamic stochastic equilibrium, uncertainty, capacity constraints, firm entry and exit

1 Introduction

Agricultural sectors dynamically evolve over time. This structural change is the result of entry and exit of firms but also of growth and shrinkage activities in combination with changes in the production structure or the adoption of new technologies. In the last decades, basic food production has been replaced by complex (bio-)technological production systems. As a result the structure of primary production in Western countries has fundamentally altered: the number of farms has declined, whereas their average size has increased. In West Germany, for example, the average farm size was about 14 hectares in 1949 and it increased to 44.5 hectares in 2010 on average. This phenomenon of structural change in the agricultural sectors can be understood in a broader sense as the results of several adjustment processes of economic entities as a response to various driving forces
such as price or policy changes, but also technical progress (cf. among others Kimhi and Bollman, 1999; Pietola, Väre, and Lansink, 2003). Entities are single farms, value chains, local markets, or institutions depending on the respective perspective.

A peculiarity of agricultural sectors, which needs specific attention, is that some production factors are short in supply, i.e. their availability is limited. Examples of such factors are agricultural land or milk quota. This shortage of production factors causes a strong interdependence of farms’ decisions within a region (e.g. Chavas, 2001). That is, farms usually cannot grow, e.g., in terms of land endowment, unless other farms exit, since only the capacity of ceasing firms results in newly available resources (Balmann, Dautzenberg, Happe, and Kellermann, 2006). Since free capacities are a precondition for growth, exits are crucial for any industry development. The price for acquiring additional production capacity thus depends strongly on the exit and shrinking rate of other firms determining free capacity (e.g. Weiss, 1999; Zepeda, 1995; Richards and Jeffrey, 1997). Since production capacity is a valuable asset, firm’s liquidation value increases under capacity constraints. This is further strengthened if some firms believe to benefit from economies of size and have the option to increase profitability and competitiveness by expanding and increase the capacity demand.

The relationships are complex. This might be a reason why the literature about modelling structural change in agriculture analyses either farm growth or farm exit, but only little attention is given to the interrelation between both. Here, we opt for an aggregated view and we analyse the result of the interrelated individual decisions at the sectoral level. Against this background, our objective is to investigate how farms’ entry and exit decisions are affected by limited production capacities, and how the availability of scarce production factors is endogenously determined by entry and exit decisions. We aim to shed light on the question whether the inability to expand production capacity increases the likelihood that less profitable firms leave the market. Our analysis will improve the understanding of the interdependency between entry and exit of farms and the implications of this interrelation under capacity constraints for structural change in agriculture. Such knowledge is desirable from a policy perspective because it allows to assess the pace of structural change and the corresponding adjustments pressure for the involved farms. Moreover, we can contribute to a long-lasting puzzle in agricultural economics: What is the impact of capacity constraints like land or production quotas on the dynamics of structural change? It is frequently hypothesized that the introduction of a production quota slows down structural change and hinders efficient adjustment processes (Colman, 2000). But is this also true if quotas are tradable (Barichello, 1995)?

To pursue these objectives, we develop a Dynamic Stochastic General Equilibrium
Model (DSGE) framework that supports the analysis of structural change in agriculture and takes into account three important characteristics: First, entry and exit decisions of farms as well as prices and production output are determined endogenously. Second, decisions are made in a dynamic framework. By means of this approach it is possible to track changes in the composition of the sector. Third, the model is driven by a stochastic component. Based on this DSGE modelling framework we introduce constrained capacities at the sector level.

We employ a dynamic stochastic framework which has been proposed by authors like Jovanovic (1982) or Hopenhayn (1992). The base is a perfectly competitive, heterogeneous industry and the firms differ according to their productivity level, which is further assumed to be stochastic. The firm specific productivity follows a Markov process and is supposed to be the only source of uncertainty in the model. The entry/exit decision of a firm is based upon its future expected profits and is thus affected by the development of the output price as well as by the current state and evolution of its own productivity. In contrast to Hopenhayn (1992) we analyse competition in a finite time horizon which allows us to keep track of farms’ adjustment processes in each period. This makes the model applicable to any sector of the agricultural industry and enables us to predict the prospective evolution of market structure. We further enhance the Hopenhayn-model and let the entry costs, as well as the liquidation value of exiting firms correlate with the industry structure. The limited availability of additional capacity is accounted for by means of sharply increasing entry cost. If the capacity constraint is not binding, the entry cost reduce to a constant term. We find that the price in the output market tends to be higher if entry of new firms is constrained by a production quota / limited sectoral production capacity. Although having the option the sell the capacity in case of exit, less productive farms benefit from a higher output price and stay longer in the industry compared to a situation with free market access.

The remainder of this article is structured as follows. Starting with the discussion of structural change in the agricultural sector, we describe the phenomenon for Germany and discuss the existing literature about industry dynamics. This is followed by the modelling idea. In section 3 we present the model and prove the existence of a finite dynamic equilibrium. Moreover, we demonstrate a way to calculate the equilibrium. We make use of this theoretic approach in section 4 where we illustrate the effect of limited capacity supply on the exit and entry decisions of firms. Section 5 concludes and gives an outlook on further research directions.
2 Background discussion

2.1 Related literature

The literature dealing with industry dynamics - in the sense of development paths of firm numbers and sizes - is huge. The main questions behind these analyses are how do industries evolve over time, how the selection process in the market works, which firms are more likely to leave the market and what role plays the market structure. Based on these insights the response of the whole industry to changes in policy or environment / institutions can be analysed. Early research in industrial organisation has been based on the structure-conduct-performance-paradigm. This approach assumes a one-dimensional causality between the market structure, the behaviour of firms in the market and the efficiency of the firms. Briefly stated, a more concentrated market structure facilitates coordinated behaviour of firms. A major shortcoming is that the respective market structure is taken as given and differences in the market shares and in the firm size are observed (cf. Sutton, 1991). As stated by Dunne, Klimek, Roberts, and Xu (2009) the respective market structure is determined by entries and exits; these in turn are influenced by future expectations of prices that depend on the nature of competition within the market. Accordingly, dynamic and stochastic approaches are required to analyse industry dynamics.

In this subsection we focus on microeconomic approaches analysing the dynamics of firm size distribution; different approaches exist but they commonly acknowledge that industry dynamics are mainly characterized by the simultaneous entry and exit of firms as well as growth and shrinkage. While entry plays a minor role in agricultural markets, there is a intense debate about firms’ incentives to leave the market. Exit decisions are characterized by their (partial) irreversibility, uncertainty about the profitability and flexibility with regard to the optimal timing. Thus, is seems natural to analyse industry dynamics by means of the real options approach, which captures the aforementioned aspects. Its particular relevance for the analysis of structural change comes from the fact that the latter can be regarded as an outcome of individual decisions on growth, shrinkage, entry and exits which are basically (dis)investment decisions. Dixit (1989) introduces a very general framework for analysing investment and disinvestment decisions under uncertainty. The basic idea is the optimal investment (disinvestment) trigger exceeds (falls below) its counterpart from traditional investment theory. This, in turn, may offer an explanation for economic inertia. In order to apply this modelling framework to analyse structural change under production quotas or capacity constraints, two major shortcomings have to be overcome. First, the optimal (dis)investment strategy refers to a single firm for which
the price process is given exogenously. Interdependencies from joint entries and exits and their feedback on the price process are not explicitly taken into account in this basic model. Second, it is frequently observed that firms differ in their initial cost structure, their efficiency or strategic position. Such heterogeneity is in the single firm framework not accounted for; however is expected to have severe implications on industry dynamics.

Finding an endogenous price process in a dynamic uncertain market environment is a challenging task. In principle, one could proceed as follows: Start with an assumption on the price process expected by each firm and let firms react to this expectation according to their investment strategies. Next, aggregate firms’ optimal quantities to get the market supply. Calculate the market clearing price at each time point, compare this price with the assumed price processes and correct the initial beliefs if necessary until convergence is reached. The literature offers solutions how such tedious calculations could be bypassed, at least under some simplifying assumptions. Leahy (1993) shows that the thresholds are also valid in a competitive environment with free market entry and homogeneous firms which face aggregate uncertainty but no idiosyncratic risk and it is assumed that the sectoral inverse demand function consists of a deterministic part and an aggregated demand shock that follows a geometric Brownian motion. In that case the endogenous price process is characterized by a regulated geometric Brownian motion with reflecting barriers. If the price reaches the ceiling (the floor) firms will enter (leave) the market and the resulting increase (decrease) in quantity will prevent a further increase (decline) of the price. Though firms cannot realize option values due to the conform behaviour of competitors, waiting is still as attractive as before because the profit (loss) potential is now reduced so that a higher (lower) trigger than the Marshallian one is required for an immediate entry (exit). Caballero and Pindyck (1996) generalize this model by allowing for additional firm-specific risk. The insight, that the ignorance of entries and exits of competitors, i.e., myopic behaviour does not result in incorrect decisions rules for an individual firm, is an important one, since it eases the derivation of dynamic competitive equilibria considerably. It is worth emphasizing that sectoral real options models so far are based on the assumption of free market entry, i.e., competing firms can enter the market after paying fixed entry costs which do not depend on the state of the industry. Also, the liquidation value of exiting firms does not depend on the scarcity of binding production factors. Summarizing, the real options approach primarily focusses on the optimal timing of (dis)investments and the relation between entry/exit and (dis)investment. An exception is the analysis of Novy-Marx (2007) in which (heterogeneous) firm’s decisions in a competitive uncertainty environment using an equilibrium model. He explicitly accounts for competitive effects and shows that in a competitive industry firms could deviate more
from the standard neoclassical approach than the real options approach suggests (without considering competition effects). Neglecting adjustment costs real option premia could be shown to reduce to monopoly rents.

Focussing more on the competition theory and equilibrium models for instance, Jovanovic (1982), Ericson and Pakes (1992), Hopenhayn (1992) or Hanazono and Yang (2009) analyse industry dynamics at an aggregated level. In earlier studies like in Jovanovic (1982), the impact of idiosyncratic uncertainty at the cost level of the firms on the industry dynamics is analysed. The model is based on the theory of noisy selection using a Bayesian learning process: past profits’ informational content enables learning about future expectations. Allowing for simultaneous entry and exit he shows that inefficient firms decline and leave the market whereas the more efficient firms grow. Ericson and Pakes (1992) take into account that the firms’ production is affected by investments with uncertain outcomes. Based on a stochastic growth model with endogenous exit depending on the (uncertain) expected profitability they model the selection process that generates the industry structures. Considering exit and entry with sunk cost into an industry, Hopenhayn (1992) investigates based on the concept of a stationary equilibrium and idiosyncratic uncertainty high turnover rates within industries. The dynamic stochastic model for a competitive industry allows for endogenous exit and possible subsequent entry with sunk cost induced by exogenous firm specific productivity shocks. Exit as a precondition of entry takes place as soon as a firm’s productivity shocks falls below a reservation value. This enables the reallocation of resources between the firms. In the stationary equilibrium entry and exit occur. Together with the productivity shocks and the respective production decisions they determine the firm size and profit distribution within that industry. Further, his findings reveal that the size distribution is stochastically increasing with age, meaning that larger firms have a higher survival probability. Melitz (2003) extends the framework of Hopenhayn to consider monopolistic competition and he analyses intra-industry effects of international trade. He shows that the least productive firms are forced to leave the market while the most productive produce for the export market. From a more global perspective this will lead to an international re-allocation towards the more productive firms. The core model assumption that the patterns of entry and exit are systematically related to productivity differences among firms is confirmed by Fariñas and Ruano (2005). The authors further show that sunk cost are one source of persistent heterogeneity in productivity, that is, in markets with high and sunk entry cost, a lower productivity becomes more likely.

Another strand of literature with the aim of understanding market structures considers the role of competition issues in industry dynamics since it is empirically shown that com-
petition may shape the firm’s distribution of the surviving firms (Syverson, 2004). Using
game theoretic approaches the relationship between market structure and competition is
of major interest but to handle the dynamics it is of importance to consider entry and
exit under uncertain future expectations. Murto (2004) for instance, explores exit in a
duopoly model (perfect Nash equilibrium framework) with uncertain revenues where the
firms negatively affect each other’s profitability. He finds that there exists only a unique
equilibrium if the uncertainty is sufficiently low or the asymmetry between the two firms is
sufficiently high. This allows one firm to commit successfully to stay longer in the market
in case the other firm leaves. As a consequence, one firm is forced to leave the market first.
Under high uncertainty and if the firms are nearly about the same size, the reverse order
may happen but no unique equilibrium will result. Such studies are often motivated to
explain asymmetric industry structures. It is argued that they can arise as the outcome
of a game in which firms differ in their economic fundamentals, e.g., cost structures, or
their strategic positions at the outset of the game, e.g., first versus last mover. But this
begs the question: How do such differences in initial conditions arise in the first place?
The seminal work by Ericson and Pakes (1995) considers a dynamic stochastic game and
tracks the development of an oligopolistic structure over time with heterogeneous firms.
Applying this framework Besanko and Doraszelski (2004) use a dynamic model of capacity
allocation with the aim is to show how asymmetric industry structures can arise endoge-
nously as the outcome of a capacity accumulation game played by ex ante identical firms.
Based on firms’ learning about their relative uncertain cost positions, Hanazono and Yang
(2009) explain that during shakeouts firms that entered just before the shakeout are more
likely to exit than earlier entrants. They consider a dynamic game with an infinite time
horizon where the firms decide in each period whether to enter or not. Their equilibrium
findings confirm the empirical observations: the firms leaving the market first are those
that entered the market later.1

However, a direct application of these models to the agricultural sector with its capacity
constraints is not possible since farms can only grow if free capacities are available and so
the exit-stage is crucial for any further industry development. Game theoretic models are
capable to model growth and shrinkage of firms in a given market with endogenous supply
or constrained capacities, but they are difficult to handle, in particular if there are more
than two firms within the market. For instance, Esö, Nocke, and White (2010) consider a
framework with ex-ante identical firms that compete for scarce resources in an upstream
market and subsequently for sales in the downstream market. Firms are assumed to have

1We do not intend to provide a detailed overview about game theoretic approaches used for the analysis
of industry dynamics; details can be found for instance in Doraszelski and Pakes (2007).
symmetric production technologies and cost structures. The game involves two stages, in
the first stage the capacities are allocated among the firms. The allocation is presumed to
be efficient such that the each unit of capacity ends up with the firm that values it most.
Given the respective capacity allocation resulting from the first stage, these firms com-
pete in a second stage a la Cournot in the downstream market. The major finding is that
an asymmetric industry structure becomes more likely the larger the pool of resources.
This modelling approach, however, does not consider uncertain future expectations and
a dynamic adjustment path.

2.2 Structural change in agriculture

Exemplary for western Europe agricultural sectors we describe dynamics of the German
agricultural sector; since the East and West German sectors still considerably differ in
their dynamics and their structure, we provide the findings for the West German sec-
tor. In 1984, the EU introduced the milk quota system with intervention prices limiting
farms’ milk production. In the first years, the production right was not transferable; this
restriction has been relaxed over time, from family transfer, regional but rental transfer
to official sales within auctions for East and West Germany separately.²

Quotas as such are as land short, immobile and overall limited production factors.
Both limit farms’ expansion possibilities and individual farm adaptations have stronger
inter-linkages because farm growth is possible only if some farmers quit production and
the free capacity is available for other active farms that expect to benefit from economies
of scale. From the mainly empirically based literature it is reasoned that tradable produc-
tion rights allow for a better allocation of production quotas (e.g. Burrell, 1989; Naylor,
1990; Guyomard and Mahé, 1994). Bailey (2002) shows for EU dairy sector that after a
quota removal, structural change in the dairy sector might be accelerated; this effect is
expected to be stronger the tighter the transfer rules of the milk quota in the quota pe-
riod are. Nevertheless, even in EU Member States where the quota trade scheme is rather
well organized, e.g. the United Kingdom (UK), the milk quota scheme can be shown
to impose inefficient production structures (Colman, 2000; Colman, Burton, Rigby, and

²Since 1984, the EU’s Common Agricultural Policy (CAP) dairy policy has been characterized by
a milk quota system with intervention prices. Within the 2003 CAP reform, the decoupling of direct
payments from the production levels and the further reduction of intervention prices induced higher price
volatility and lowered the certainty level of expectations. More recently, the 2008 health check of the
CAP, the further stages of the 2003 milk market reform and falling milk prices have induced further
pressure on farms. Moreover, the end of the milk quota scheme in 2014/15 has been confirmed so far.
Moreover, as Oskam and Speijers (1992) show, the capital costs of farms that bought or leased the quota increase considerably. Furthermore, Richards (1995) and Richards and Jeffrey (1997) show that the milk quota scheme reduces the investment rate of dairy farms in Canada, hindering farm growth and necessary adaptations of technical progress. Thus, it is undisputed that even tradable quotas have an impact on the dairy production industry dynamics.

The dairy sector faces both kinds capacity constraints; the rather strong consolidation process for the West German dairy sector is visualized in the following figures: the number of dairy farms declined from 1,216,700 in 1960 to 90,200 in 2010 (Statistisches Bundesamt), while the average farm size increased, viz. from an average of 5 cows per farm in 1960 to 43 cows per farm in 2009 (Agrarmarkt Informationsgesellschaft 2010). Furthermore, considerable increases in farm productivity, in particular in the average milk yield per cow have been observed: it increased from 3.6 in 1964 to 6.9 tons in 2009. Moreover, the dairy farm size structure altered over time; while the share of the small farms (less than 10 cows per farm) sharply declined over time, the medium (10-49 cows) and large (more than 50 cows) increased in numbers and shares of total number of dairy farms. The share of the large and very large increased in particular in the more recent years (starting in the mid-nineties). This is illustrated in Figure 1.

In order to illustrate how the dairy production industry evolves over time we compare annual distributions of the dairy farm size in West Germany using farm individual data from the agricultural census; unfortunately these data are only available for every 4th year for the period 1999-2010. Farm size is defined by the number of dairy cows and part-time farms have been excluded. The histogram is shown in the left part of Figure 2. On the y-axis we opted for percent and used the natural logarithm of the average farm size. It can easily be seen that the percentage of the number of farms with a lower farm size declined
within the period and an over right-shift of the distribution is visible. We further estimate a kernel density function using a Gaussian kernel for the natural logarithm of the farm size. As illustrated in the right part of Figure 2, the shift between the years 1999 and 2010 towards the right is confirmed and the distribution becomes less concentrated. This begs the question whether it has to do with the relaxation policy of the quota transfer rule in the more recent years (starting in 2005)? How would this distribution look like without quota limitations? That is, would the distribution look different, would there be a stronger shift or a lower one? Since data are not available for a longer period, it is not possible to compare distributions before and after the milk quota introduction. This does not allow us to answer this question empirically.

![Figure 2: Dairy farm size distribution 1999 and 2010 for West Germany](image)


3 Modelling structural change under capacity constraints

3.1 From stylized facts to the model

Our aim here is to model capacity constraints at the sectoral level to provide a base for measuring the difference between industry development with and without capacity constraints. This gives insights about the impact of constrained capacities on structural development. The basic set-up of the model draws closely upon the seminal papers of
Jovanovic (1982) and Hopenhayn (1992). These approaches explicitly allow for endogenous entry and exit of the firms, which is crucial to analyse structural change in agriculture under capacity constraints. We consider a perfectly competitive industry with a continuum of firms.\(^3\)

Firms are supposed to produce a homogeneous good and to act as price takers in the output market. The respective output price is assumed to be determined by market clearance and the firms choose their optimal output level for the given prices. It is further assumed that all firms have the same production technology but they differ with respect to their productivity level. The firm specific productivity follows a Markov process and is the only source of uncertainty faced by the firms. By means of this we account for firm specific productivity differences, e.g., through farm size, capital stock, feed management, livestock management or natural conditions. Further output price risks which are likely present in the agricultural sector could not be accounted for to keep the model tractable. This implies that firms have perfect foresight of output prices. Based on their optimal profit maximizing choice of output level firms generate profits for each period. We further consider fixed costs.

New entrants to the industry are assumed to face entry cost and also uncertainty induced by the productivity shock drawn from a distribution function which is common for all firms. Thus, the expected discounted future profit of the entering firms expressed in terms of the value function must exceed the entry cost. Since the new firms must costly acquire production capacity, this holds also for established firms which want to grow. That is, expanding production is accompanied by investing in additional capacity and the growing firms are thus part of the mass of entering firms. We assume a perfectly competitive market environment and do not account for any strategic effects. At the end of each period the active farms have the option to leave the market. The exit decision is based upon the value function: if discounted future expected profits expressed in terms of the value function do not exceed a critical threshold productivity (minimum productivity to survive) the firm will leave the industry. Through the firm’s value function this decision is directly affected by the the output prices as well as by the current state and evolution of the firm specific productivity. The state of the industry will be described by the measure over all productivity levels of the active firms. This measure is crucial for interpreting the model results and reflects the firm (size) distribution.

The timing of the model is as follows:

- at the end of each period active firms decide whether to stay or to leave the market through comparing their expected profits with the critical productivity value

\(^3\)Farms and firms are used interchangeably.
• at the same time, entrants decide whether to enter or not through comparing the expected profits with the entrance cost
• at the beginning of the next period firms that stay pay the fixed costs, thereafter get the realization of their productivity (firm specific) according to the Markov process and then start producing
• at the same time, entrants pay the entry costs, thereafter get their productivity realization drawn from a common distribution function and then start producing

If production factors like land or milk quota are only available to a limited extent this leads to strongly interlinked production decisions and forms a constraint to whole industry, i.e., limited supply of production capacity in the whole sector. The result is that firms should take into account in their optimal factor allocation decision that potential entry and exit of other firms may have an impact on the output prices. We use the total mass of the industry including the respective productivity shocks as a proxy for the total sector capacity. This could be either land or quota. We further assume that all capacity is under usage in the starting period. In order to account for a binding capacity constraint, the entry cost are defined for each period and are assumed to to be monotonically increasing. The entry costs are further assumed to decline if the capacity is not fully under usage and will reduce to a constant term if the capacity constraint is not binding. The entry costs directly depend on the size of the industry, that is, the larger the industry in terms of number of firms, the higher are the entry costs. The latter generate an exit premium for active firms leaving the market. The value function of the active firm accounts for this terminating premium. It depends on the price for capacity which is driven by the demand for the capacity generated by all entering and growing firms. The price is further influenced by the capacity supply that is determined by the exiting firms. This in turn leads to adjusted entry and exit rules. The value of the active firm must be lower or equal to the exit premium to induce the firm to leave the market. Note that by means of the entry costs and the liquidation value of exiting firms there is a direct relation between the decisions and the industry structure which is reflected by the distribution of the productivity shocks.

The original model from Hopenhayn refers to a steady state model in which firms grow or decline, enter and exit but the overall distribution of the firms remains stable. The dynamics are created through the productivity shock assumed to follow a Markov process. The most important property is that farm activities like entry and exit are endogenously determined. This basic model set-up being expanded by the consideration of the capacity constraints through the increasing entry cost allows us to derive the following comparative statics: the firm size distribution varies with and without capacity constraints, under dif-
different initial distributions and different developments of the productivity shock (Random Walk versus AR(1)-process).

### 3.2 The formal model

The inverse demand function $D(Q) > 0$ should be continuously differentiable and strictly monotonic decreasing. We assume that $\lim_{Q \to +\infty} D(Q) = 0$. The time horizon $T < \infty$ is finite and competition takes place in discrete time ($t = 0, ..., T$). We model the firm’s individual productivity as a stochastic parameter $\varphi_t \in \mathbb{R}$ which follows the AR(1)-process

$$\varphi_{t+1} = \rho \varphi_t + \varepsilon_{t+1}, \quad \rho \in (0, 1]$$ and $\varepsilon_{t+1} \overset{iid}{\sim} N(\nu, \sigma^2_{\varepsilon})$. (1)

The stochastic process defined in (1) describes the evolution of a firm’s productivity and is the same for all incumbents. Nevertheless, the realization of the error term $\varepsilon_{t+1}$ is independent across firms and over time. It is obvious that this process has the Markov property and is time homogeneous. Under the hypothesis $\varphi_t = \varphi$ we have $\varphi_{t+1} \sim N(\rho \varphi + \nu, \sigma^2_{\varepsilon})$. If we denote the density of this normal distribution by

$$f(z, \varphi) := \frac{1}{\sqrt{2\pi\sigma^2_{\varepsilon}}} \exp\left(-\frac{(z - (\rho \varphi + \nu))^2}{2\sigma^2_{\varepsilon}}\right)$$ (2)

the conditional cumulative distribution function $F(\varphi' | \varphi) = \text{Prob}(\varphi_{t+1} \leq \varphi' | \varphi_t = \varphi)$ is given by

$$F(\varphi' | \varphi) = \int_{-\infty}^{\varphi'} f(z, \varphi) \, dz.$$ (3)

The function $F(\varphi' | \varphi)$ constitutes a probability kernel and is continuous with respect to both arguments. Moreover, it is strictly decreasing in $\varphi$ if we keep $\varphi'$ fixed. All active firms can be explicitly distinguished by their current productivity level $\varphi_t$. The distribution of these values across all firms thus expresses the state of the industry in period $t$. This should be displayed by a measure $\mu_t : \mathcal{B}(\mathbb{R}) \to \mathbb{R}_+$ defined on the Borel sets of the real numbers. Hence, any changes of the industry structure, caused by the stochastic productivity process as well as entry/exit of firms, translate into changes of $\mu_t$.

We make the assumption that firms with a higher productivity level are able to produce any amount of output $q$ at lower costs. This property is represented by a twice continuously

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4 If $\varphi_1 < \varphi_2$, the distribution $F(\cdot | \varphi_2)$ stochastically dominates $F(\cdot | \varphi_1)$.

5 $\mu_t$ does not need to be a probability measure. The total mass $\mu_t(\mathbb{R})$ may be smaller/bigger than one, indicating the size of industry.
differentiable cost function $c(q, \varphi)$ which is monotonic decreasing in $\varphi$ and has the limits

$$\lim_{\varphi \to +\infty} c(q, \varphi) = 0 \quad \text{and} \quad \lim_{\varphi \to -\infty} c(q, \varphi) = \infty, \quad \forall q \geq 0.$$  \hspace{1cm} (4)

Furthermore, the function $c : \mathbb{R}_+^0 \times \mathbb{R} \to \mathbb{R}_+^0$ should satisfy

$$c(0, \varphi) = 0, \quad \frac{\partial c}{\partial q} > 0 \quad \text{with} \quad \frac{\partial c}{\partial q}(0, \varphi) = 0, \quad \frac{\partial^2 c}{\partial q^2} > 0, \quad \lim_{\bar{q} \to +\infty} \frac{\partial c}{\partial q}(\bar{q}, \varphi) = \infty \hspace{1cm} (5)$$

In each period $t$ of the planning horizon all active firms have to choose their own optimal production output. They take the output price $p_t \geq 0$, as well as their current productivity level $\varphi_t$, as given and maximize:

$$\max_{q_t \geq 0} \quad p_t q_t - c(q_t, \varphi_t) \hspace{1cm} (6)$$

The imposed restrictions on the cost function guarantee that for all valid combinations of $p_t$ and $\varphi_t$ a unique solution $q_t^* = q^*(p_t, \varphi_t)$ to (6) exists. The firm specific optimal output is thus a function of the output price and the productivity.

**Proposition 3.1.** (i) The function $q^*(p, \varphi)$ is continuous and (strictly) monotonic increasing in $p$ and $\varphi$. (ii) For all $\varphi \in \mathbb{R}$, we have $q^*(p, \varphi) > 0$ if $p > 0$ and $q^*(0, \varphi) = 0$. (iii) The firm specific output will tend to infinity, if the price does.

**Proof.** The first order condition for a maximum in (6) is

$$p_t \leq \frac{\partial c}{\partial q}(q_t, \varphi_t), \quad \text{with equality if } q_t > 0. \hspace{1cm} (7)$$

The statements (i)-(iii) follow immediately from (7) and the properties of the cost function. \hfill \Box

The aggregate industry output $Q_t = Q(p_t, \mu_t)$ depends on the structure of the industry and is given by

$$Q(p_t, \mu_t) = \int_{\mathbb{R}} q^*(p_t, \varphi) \, d\mu_t(\varphi). \hspace{1cm} (8)$$

In case the integral on the right hand side exists for all prices, we infer from Proposition 3.1 that $Q(p, \mu)$ is continuous and increasing with respect to $p$.

Production incurs a fixed cost $c_f > 0$ which is the same for all incumbents and has to be paid at the beginning of each period before the new productivity level is revealed. Hence, it is sunk by the time firms choose their production output. A firm’s profit per
period is
\[ \pi(p_t, \varphi_t) := p_t q_t^* - c(q_t^*, \varphi_t) - c_f \] (9)

with \( q_t^* = q^*(p_t, \varphi_t) \) being the optimal firm specific output level.

**Proposition 3.2.** (i) \( \pi \) is continuous in \( p \) and \( \varphi \). (ii) \( \pi \) is strictly increasing in \( p \) and if \( p > 0 \) it is strictly increasing in \( \varphi \). (iii) \( \pi(p, \varphi) \to \infty \), if either \( p \to +\infty \) or \( \varphi \to +\infty \). (iv) \( \pi(p, \varphi) \to -c_f \), if either \( p \to 0 \) or \( \varphi \to -\infty \).

**Proof.** The statements can be proven quite easily with the help of Proposition 3.1. \( \Box \)

At the end of each period firms have the option to leave or enter the market. New firms entering the market or expanding firms must acquire production capacity and have to pay entry costs \( k_t > 0 \). This is supposed to be the only investment possibility for established firms. We do not distinguish between new and expanding firms explicitly and will refer to both groups as entering firms. Each new firm is assigned with a productivity level which is drawn from the common distribution function \( G \). Through the constrained capacity at the sector level, entry and exit are directly related since the entry costs depend on the number of firms, which in turn is determined by the total entry and exit of the firms. Every incumbent possesses production capacity which can be sold in case of exit. If a firm decides to leave the industry it will release its capacity and get a compensation payment \( r_t \). This liquidation value will depend on the demand for capacity generated by entering firms. In order to capture this interdependency between entry costs and liquidation value we model both as a function of the total industry mass \( \mu_t(\mathbb{R}) \). The industry mass depends on the number of firms leaving or entering the market and describes growth/shrinkage of the industry. It can be interpreted as a proxy for the availability of the production capacity at the sector level. We introduce a continuous and nondecreasing function \( k \) and define \( k_t := k(\mu_t(\mathbb{R})) \). The exit premium should be smaller but proportional to the entry costs at time \( t \). Therefore, we define \( r_t := \delta k_t \) with a fixed factor \( \delta \in [0, 1) \). A scenario without limited capacity supply can be modelled by setting entry costs and compensation payment constant.

A firm bases its entry/exit decision on the expected discounted future profits. The discount rate for all firms is supposed to be \( 0 < \beta < 1 \). If the output prices for all periods are known and denoted by the vector \( p = (p_0, \ldots, p_T) \), the value of an incumbent with productivity \( \varphi \) at time \( t \) can be defined recursively by

\[
 v_t(\varphi, p) = \pi(\varphi, p_t) + \beta \max \left\{ r_{t+1}, \int_{\mathbb{R}} v_{t+1}(\varphi', p) dF(\varphi' | \varphi) \right\}, \quad \forall t = 0, \ldots, T - 1. \tag{10}
\]
It is composed of the current profits plus the optional liquidation or continuation value. Since we assume a finite planning horizon, this definition holds true for all periods but the last one. The value at the end of competition is just equal to the profits generated in the final period \( v_T(\varphi, p) = \pi(\varphi, p_T) \). A firm stays in the industry as long as its continuation value offsets the exit premium \( r_{t+1} \). The continuation value indicates the expected future profits conditioned on the firm’s current productivity level. The exit-point \( x_t \) describes the critical threshold for being indifferent between staying in or leaving the market.

\[
x_t := \inf \left\{ \varphi \in \mathbb{R} : \int_{\mathbb{R}} v_{t+1}(\varphi', p) dF(\varphi'|\varphi) \geq r_{t+1} \right\}
\] (11)

The assumptions made on the stochastic process and Proposition 3.2 imply that all firms with a productivity above the exit-point \( \varphi_t \geq x_t \) stay in the industry while all firms with a lower productivity \( \varphi_t < x_t \) take the exit premium and quit. If the infimum in (11) does not exist, we are in a situation where no exit occurs in period \( t \) and we formally set \( x_t = -\infty \).

The expected profits of a firm willing to enter the industry at the end of period \( t \) are given by

\[
v_{t+1}^e(p) = \int_{\mathbb{R}} v_{t+1}(\varphi, p) dG(\varphi).
\] (12)

We denote the mass of firms which decide to enter at time \( t \) and start production in the following period by \( M_t \). An increasing number of active firms will lead to a higher aggregate industry output and result in a lower market price. New firms will be entering the industry as long as their expected future profits cover the entry costs, i.e. in an equilibrium we have \( v_{t+1}^e \leq k_{t+1} \). This condition must hold with equality if \( M_t > 0 \).

Due to the large number of firms in the industry (recall that firms are assumed to constitute a continuum), we do not have to deal with aggregate uncertainty. The frequency distribution of productivity levels in upcoming periods is completely specified by the stochastic productivity process and the entry/exit behaviour of firms.\(^6\) For a given exit-point \( x_t \) and entry-mass \( M_t \) the industry structure in period \( t+1 \) is

\[
\mu_{t+1}((-\infty, \varphi]) = \int_{\varphi \geq x_t} F(\varphi'|\varphi) d\mu_t(\varphi) + M_tG(\varphi').
\] (13)

If both \( \mu_t \) and \( G \) have Lebesgue densities \( m_t(z) \) and \( g(z) \), the state of the sector \( \mu_{t+1} \) can

\(^6\)A deterministic development the of industry structure is justified by the law of large numbers. Evidence can be found in Judd (1985) or Feldman and Gilles (1985)
also be characterized by its density

\[ m_{t+1}(z) = \int_{\varphi \geq x_t} f(z, \varphi) m_t(\varphi) d\varphi + M_t g(z). \]  
(14)

### 3.3 Equilibrium analysis

As a direct consequence of (13) both industry output and market price follow deterministic sequences. Firms are atomistic and cannot affect the price by the choice of their output quantity. However, they have perfect information about the strategic decisions of others and are thus able to foresee the development of output prices. In a dynamic equilibrium they adjust their output as well as their entry/exit decisions to the anticipated prices. These output prices, on the other hand, must be reinforced by the strategic behaviour of firms. Keeping this in mind we define a dynamic stochastic equilibrium as follows:

**Definition 3.3.** Given a starting distribution \( \mu_0 \) a dynamic equilibrium consists of a finite sequence of measures \( \{\mu^*_t\} \) and vectors \( p^*, Q^*, x^*, M^* \) containing the market prices, aggregate industry output, exit-points and entry-masses for each period such that for all times \( t = 1, \ldots, T \) the following conditions are satisfied:

(i) the output market is cleared

\[ p^*_t = D(Q^*_t) \]
\[ Q^*_t = Q^*(p^*_t, \mu^*_t) \]

(ii) the exit-rule (11) holds with \( x^*_t \)

(iii) no more firms have an incentive to enter the industry, i.e. \( v^*_t(p^*) \leq k_t \)

(iv) \( \mu^*_{t+1} \) is determined recursively by (13)

The question arises in which situations a dynamic equilibrium exists and how it can be detected. We assume that the structure of the industry at the beginning of competition \( \mu_0 \) is given. Both the distribution \( \mu_t \) and the aggregate industry output \( Q_t \) in period \( t \) can be regarded as functions of previous entry/exit of firms and the output price respectively. Therefore, the challenge is basically to find values for \( p^*, x^*, M^* \) such that the four equilibrium conditions are fulfilled. Due to (i), the equilibrium output price in period \( t \) is implicitly determined by

\[ p_t = D(Q^*(p_t, \mu_t)). \]

(15)
The properties of the demand function $D$ and the aggregate industry output $Q$ make sure that for any given industry structure $\mu_t$ a unique solution $p_t^* > 0$ to (15) exists. According to the industry dynamics (13) the structure at time $t$ depends on the whole history of exit/entry decisions made by the firms up to this point. This means, the distribution $\mu_t$ is explicitly determined by the starting distribution $\mu_0$ and all previous exit points $x_0, ..., x_{t-1}$ and entry masses $M_0, ..., M_{t-1}$. Obviously, the equilibrium output price can thus be expressed as a function of these variables as well, i.e. $p_t^* = p_t^*(x_0, ..., x_{t-1}, M_0, ..., M_{t-1})$. It can be shown by means of the Implicit Function Theorem that $p_t^*$ is a continuously differentiable function and the partial derivatives satisfy

$$\frac{\partial p_t^*}{\partial x_j} \geq 0, \quad \frac{\partial p_t^*}{\partial M_j} \leq 0 \quad \forall j = 0, ..., t - 1.$$  

The equilibrium values for all $x_t$ and $M_t$ are determined by the exit and entry conditions. In each period $t = 0, ..., T - 1$ the following pair of equations has to be satisfied

$$\int_{\mathbb{R}} v_{t+1}(\varphi, p^*) dF(\varphi | x_t) = r_{t+1} \quad (17)$$

$$\int_{\mathbb{R}} v_{t+1}(\varphi, p^*) dG(\varphi) \leq k_{t+1}, \quad \text{with equality if } M_t > 0. \quad (18)$$

Since both $r_{t+1}$ and $k_{t+1}$ are defined as a function of $\mu_{t+1}(\mathbb{R})$, this adds up to a system of $2T$ equations with $\mathbf{x} = (x_0, ..., x_{T-1})$ and $\mathbf{M} = (M_0, ..., M_{T-1})$ being the only unknown variables. Consequently, a dynamic equilibrium exists if this system of equations has a solution $(\mathbf{x}^*, \mathbf{M}^*)$. It is possible, but rather difficult to show that a solution will always exist in the assumed framework. We do not present the proof here, as it is rather technical and requires the application of general fixed point theorems. In any case, an explicit solution cannot be found analytically and has to be computed with numerical methods.

To some extent, the equilibrium outcome will be affected by the assumed length of the planning horizon. Due to the finite time framework all exit and entry conditions, represented by (17) and (18), are essentially discounted sums of expected future profits. Thus, the value of a firm at time $t$ will depend on the number of time periods which are still to come. At the beginning firms take the industry development over the whole time span into consideration, while they base their entry/exit decision on just a few upcoming periods at the end of competition. An extension of the time horizon by one period may thus have a strong impact on the value of a firm in the final periods. As firms discount future profits by the factor $\beta < 1$, however, the impact on a firm’s value in the first periods is less harsh and will possibly diminish in the long run. For this reason, we expect results to stabilize if the time horizon tends to infinity. But, the numeric effort to calculate an
equilibrium in this case will be enormous.

4 Illustration

4.1 Assumptions

Below we aim to illustrate how a limited sectoral production capacity, which is present particularly in the agricultural industry, affects the dynamic equilibrium outcome. The functions and values we utilize are in line with the general framework presented in section 3 and exemplify the competition in an arbitrary industry. We make the following explicit assumptions:

- planning horizon $T = 5$
- demand function $D(Q) = Q^{-2}$
- starting distribution $\mu_0 \sim N(0, 1)$
- productivity distribution for new firms $G \sim N(0, 1)$
- cost function $c(q, \varphi) = q^2 \exp\left(-\frac{\varphi^2}{5}\right)$
- fixed costs $c_f = 0.3$
- discount factor $\beta = 0.8$

The optimal firm specific output in this situation is given by

$$q^*(p_t, \varphi_t) = 0.5 \ p_t \ \exp\left(\frac{\varphi_t}{5}\right)$$

(19)

and the profits per period are

$$\pi(p_t, \varphi_t) = 0.25 \ p_t^2 \ \exp\left(\frac{\varphi_t}{5}\right) - 0.3.$$  

(20)

The market clearing output price is determined by

$$p_t^* = \left[ 0.5 \int_{-\infty}^{\infty} \exp\left(\frac{z}{5}\right) m_t(z) \ dz \right]^{-\frac{2}{3}}$$

(21)

with $m_t(z)$ being the density function of the frequency distribution $\mu_t$ as defined in (14).

The results may be influenced by the question whether the stochastic productivity process is stationary or non stationary. We account for this and consider on the one hand the AR(1)-process

$$\varphi_{t+1} = 0.7 \ \varphi_t + \epsilon_{t+1}, \quad \epsilon_{t+1} \sim N(0, 1)$$

(22)
and on the other hand the Random Walk

$$\varphi_{t+1} = \varphi_t + \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim N(0, 1)$$ (23)

To quantify the effect of constrained sectoral production capacity on the industry development we analyze two different scenarios. In a base scenario we assume that no constraints are present and new firms have free access to the industry by paying constant entry costs $$\bar{k} = 0.3$$. The second scenario is supposed to incorporate constraints and we define variable entry costs depending on the total industry mass $$k_t = \bar{k} \varepsilon^{10(\mu(R) - 1)}$$. This definition guarantees that $$k_t > \bar{k}$$ as soon as $$\mu_t(R) > 1$$. The compensation value for exiting firms is supposed to be $$\bar{r} = 0.5 \bar{k}$$ or $$r_t = 0.5 k_t$$ respectively.

4.2 Findings

At first we present the equilibrium outcome in case the firm specific productivity level follows the AR(1)-process (22). Table 1 contains the equilibrium values for $$x^*$$ and $$M^*$$ which were found numerically by solving the corresponding system of exit/entry equations as described in (17) and (18). The industry structures $$\mu_t$$ resulting from these exit points and entry masses are depicted by its density functions in Figure 3. In both scenarios the majority of firms enters the industry right at the beginning of competition. Obviously, the entry mass is much higher if firms have unrestricted access to the industry. This causes an increase of aggregate industry output and leads to lower output prices in the subsequent periods. In contrast to this, the prices stay on a higher level if entry of new firms is constrained. In this case incumbents are protected against too much entry by the variable costs $$k_t$$ which would rise extremely if more firms were willing to enter. The values for the total industry mass $$\mu_t(R)$$ indicate that entry prices paid by new firms in an equilibrium

\[
\begin{array}{cccccccccc}
\text{Period} & t & x_t^* & M_t^* & p_t^* & \mu_t^*(R) & x_t^* & M_t^* & p_t^* & \mu_t^*(R) \\
0 & 0 & -1.65 & 0.56 & 1.57 & 1.00 & -3.24 & 0.11 & 1.57 & 1.00 \\
1 & 1 & -1.66 & 0.07 & 1.18 & 1.51 & -2.89 & 0.00 & 1.46 & 1.11 \\
2 & 2 & -1.76 & 0.00 & 1.17 & 1.49 & -2.69 & 0.00 & 1.46 & 1.09 \\
3 & 3 & -1.69 & 0.00 & 1.21 & 1.40 & -2.38 & 0.01 & 1.47 & 1.08 \\
4 & 4 & -0.85 & 0.00 & 1.25 & 1.31 & -2.02 & 0.00 & 1.48 & 1.05 \\
5 & 5 & - & - & 1.41 & 1.05 & - & - & 1.52 & 0.99 \\
\end{array}
\]

Table 1: Equilibrium parameter values under an AR(1)-process
are higher than $\bar{k}$. Due to the linkage between entry costs and liquidation value the exit
premium is also higher than in the unconstrained scenario. This seems to make the exit
option more attractive to inefficient firms, but it is rather the case that especially less
productive firms benefit from higher output prices and stay longer in the industry. This is
reflected by the critical productivity threshold $x_t^*$ which is smaller for every single period
in the constrained scenario.

A similar impact of constraints can be observed for the Random Walk (23). The results
in Table 2 show that analogous to the situation for an AR(1)-process the exit points are
smaller and the output prices higher if entry is constrained. However, compared to the
stationary process the values for $x_t^*$ as well as for $M_t^*$ are on a higher level. Particularly
the increased number of firms entering the industry in the final periods of competition is
striking. This suggests a higher exchange rate of old firms by new ones if the stochastic
productivity process is non stationary. Another interesting finding is the progressive in-
crease of exit points occurring in the constrained scenarios. This means that the critical

![Firm's productivity distribution WITHOUT capacity constraints](image1)

![Firm's productivity distribution WITH capacity constraints](image2)

Figure 3: Effect of capacity constraints for an AR(1)-process

<table>
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<tr>
<th>Period t</th>
<th>$x_t^*$</th>
<th>$M_t^*$</th>
<th>$p_t^*$</th>
<th>$\mu_t^*(\mathbb{R})$</th>
<th>$x_t^*$</th>
<th>$M_t^*$</th>
<th>$p_t^*$</th>
<th>$\mu_t^*(\mathbb{R})$</th>
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</thead>
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<td>0.76</td>
<td>1.57</td>
<td>1.00</td>
<td>-1.66</td>
<td>0.14</td>
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<td>1.00</td>
</tr>
<tr>
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<td>-1.48</td>
<td>0.10</td>
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<tr>
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<td>0.00</td>
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<td>0.09</td>
<td>1.41</td>
<td>1.07</td>
</tr>
<tr>
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<td>0.00</td>
<td>1.19</td>
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<td>0.08</td>
<td>1.38</td>
<td>1.04</td>
</tr>
<tr>
<td>4</td>
<td>-0.20</td>
<td>0.00</td>
<td>1.24</td>
<td>1.10</td>
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<td>1.36</td>
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<td>0.85</td>
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<td>-</td>
<td>1.36</td>
<td>0.99</td>
</tr>
</tbody>
</table>

Table 2: Equilibrium parameter values under a Random Walk
productivity threshold for being active and staying in the industry rises in the course of time. As a consequence of this the density functions in Figure 3 and Figure 4, which represent the distribution of productivity across firms in every single period, shift to the right. This phenomenon is not visible at first sight for the stationary case but it is pretty evident if the stochastic productivity process follows a Random Walk. It shows that firms surviving the selection process tend to become more productive. To some extent these results are consistent with the shift of farm size distributions in Figure 2.

5 Conclusions

This article has examined how the limited sectoral production capacity in the agricultural industry affects farms’ entry and exit decisions. We have presented a method to incorporate this feature into a dynamic stochastic framework and to model entry and exit of firms endogenously. The focus is on firm specific uncertainty thus neglecting random events like demand shocks which would concern the whole industry. Due to the huge number of heterogeneous firms in the industry we do not have to deal with uncertainty on the aggregate level and are able to determine the industry structure and output price deterministically. However, a detailed analysis of adjustment processes and industry dynamics within an infinite context is hardly possible. Authors like Hopenhayn (1992) or Melitz (2003) thus concentrate on stationary equilibria and their properties. We refer to a finite time horizon but have to bear the consequences that the equilibrium outcome will depend on the number of considered time periods. In particular the last periods of competition may be
biased. We have argued that the deficit for the first periods can be reduced if the planning horizon is extended.

The effect of capacity constraints on the industry development has been demonstrated with the help of an example. Although the functions and parameter values are arbitrarily chosen and have not been calibrated to data the impact of limited capacity supply becomes clear. Incumbents are protected against entry of new firms and benefit from higher output prices if capacity constraints are binding. Nevertheless, the critical productivity threshold for staying active increases in the course of competition and the productivity distribution across all firms shifts to the right.

One advantage of our method is that it allows to keep track of industry dynamics and changes in the productivity distribution very precisely. To some extent this may also be useful for a quantitative analysis. If the required functions and distributions are fitted to any sector of the agricultural industry the development of this sector for a couple of time periods can be simulated. This could also provide an answer to the question how the current farm size distribution in a region affects prospective adjustment processes.

References


