GOVERNMENT EXPENDITURES AND BUSINESS CYCLES
- POLICY REACTION AND SURPRISE SHOCKS

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Abstract
This paper analyzes the effects of discretionary fiscal policy on German business cycle fluctuations by means of an estimated DSGE model based on low frequency oscillations. The results highlight that fiscal policy has a strong impact on the amplitude of fluctuations while hardly any on the duration of business cycles. To the extent that fiscal policy shocks are an important source for triggering economy wide fluctuations, fiscal policy also strongly reacts to shocks originating elsewhere. Thus standard fiscal policy specifications in terms of simple AR(1) processes for government expenditures ignore the fact that there can indeed be a strong endogenous dependence of governmental variables on economic fluctuations.

JEL Classifications: C13, C32, E3, E62
Key Words: Fiscal Policy, Business Cycles, DSGE Models, Estimation
1 Introduction

This paper seeks to understand the impact of fiscal policy on business cycles once it is characterized as a responder to observed fluctuations versus a scenario where it acts as a source of fluctuations. To this end a dynamic stochastic general equilibrium (DSGE) model is formulated with an extended fiscal policy apparatus. There are two different types of policy mechanisms at work. On the one hand, the model specifies fiscal policy as reacting to changes in the economic stance. On the other hand, fiscal policy is extended with an exogenous stochastic term in order to elaborate on surprises in fiscal policy. The model is used to judge the importance of surprise fiscal policy shocks as well as to analyze the impact of its endogenous component. I do so by estimating the model using only low frequency oscillations of German time series data.

As far as the profoundness of the analysis of fiscal policy in DSGE models is concerned, up to now the literature surrounding this issue has received considerably less attention than the literature on monetary policy reaction functions. In DSGE models the specification of the fiscal authority typically involves some type of fiscal closure rule. Except for a few studies (Gal, Lopez-Salido and Valles (2007), Perez and Hiebert (2004)) this rule is still completely exogenous and the fiscal authority is not allowed to react to various shocks affecting the model economy. This is of course a rather limited description of the fiscal authorities reaction to shocks other their own ones. In addition to this, the government is usually described in a very limited way. Rather than setting a profound government sector, it is usually preferred to stick to lump sum taxes in a framework where there is no public debt.

The model that I construct has two key features. First it nests the now standard new Keynesian model, as for instance in Smets and Wouters (2007), and the Diamond-Mortensen-Pissarides model. This allows to disentangle the adjustment in the labour market into its extensive and intensive margins. Second, I model a tax and an unemployment payments structure to act as so called automatic stabilizers in order to separate fiscal policy fluctuations which are due to automatic stabilizers as opposed to those which are due to their discretionnary component.

The key findings are as follows. First, fiscal policy has a strong impact on the amplitude of fluctuations, however, there is no effect on the duration of business cycles. In particular, fiscal policy can smooth cyclical fluctuations but there is no possibility of fundamentally disturbing cyclicalities in their lengths.

Second, relative to the other shocks, surprise fiscal policy shocks are a major source of fluctuations. Fiscal policy shocks explain a large amount of the fluctuations in inflation and unemployment for short horizons (up to 28%), but for longer forecasting horizons this magnitude declines strongly. For output, fiscal policy shocks explain around 20-25% of the fluctuations rather independently of the specific horizon considered.

Third, as the estimates for the fiscal policy rules indicate, government expenditures significantly contribute to dampening cyclical oscillations. Considering a fiscal authority’s aim of dampening cyclical fluctuations indicates that especially for unemployment and output, countercyclical fiscal policy is a powerful tool, whereas for inflation its impact is limited due to the rather rigid fluctuations therein.

I pursue a particular limited information econometric strategy to estimate and evaluate the model. The basic idea behind this approach can be described as follows. DSGE models
and especially new Keynesian and Real Business Cycle models are theoretical models aiming at explaining business cycles. However, usually the data used for estimation are a composition of cycles of different periodicities; there are long periodic cycles describing growth patterns, seasonal cyclicalities, purely random components and among them, business cycle patterns. Since the model is only aimed at explaining low cyclicalities and especially business cycle frequencies, I hence make only use of this part of information in the data. Carrying out this estimation procedure hence requires a frequency domain approach. I proceed by using a procedure proposed by Smith (1993) and Gourieroux, Monfort and Renault (1993) called Indirect Inference. I minimize a measure of distance between the model and empirical spectral density functions as proposed by Diebold et al. (1998) using only those fluctuations which correspond to business cycle fluctuations.

The paper is structured as follows. Section 2 describes the model. Section 3 addresses the econometric methodology used to appropriately adjust the model to the data; section 4 discusses the impact of the various fiscal policy rules and analyzes the effects of different specifications on the fluctuations triggered by fiscal policy. Section 5 concludes.

2 The model

The present model has the key features that many authors have found useful for capturing the data; these include habit formation, investment adjustment costs, variable capital utilization, nominal price and real wage rigidities.

The key changes in this model are in the labour market. Rather than considering aggregate hours worked irrespective of its dependence on the intensive and extensive margin, I introduce variation occurring at both margins. I do so by extending the new Keynesian Model by the Mortensen-Pissarides model of search and matching.

The economy consists of two types of households, a continuum of firms, called the wholesale firms who produce differentiated intermediate goods, the retail firms, a fiscal authority and a central bank in charge of monetary policy. I use a representative family construct for each household type, similar as in Merz (1995), in order to introduce complete consumption insurance. The model features two important non-Ricardian elements: liquidity constraint agents and distortionary labour and capital taxation.

2.1 Households

I assume a continuum of infinitely lived households, indexed by $j \in [0, 1]$. Following Gal, Lopez-Salido and Valles (2004) and Gal, Lopez-Salido and Valles (2007), a fraction $1 - \varsigma$ of the total population has access to capital markets where they are able to trade a full set of contingent securities and buy and sell physical capital which they accumulate over time and rent out to firms. These type of consumers are called Ricardian consumers. The remaining fraction $\varsigma$ of households do not own any assets nor do they have access to capital markets and hence they just consume their current labour income. These types of households are referred to as non-Ricardian consumers.
2.1.1 Ricardian households

The starting point for the Ricardian consumers is based on the now conventional monetary DSGE model developed by Christiano, Eichenbaum and Evans (2005), Smets and Wouters (2007) and others. There is a representative household for the Ricardian consumers. The number of family members currently employed is \( N_t^o = (1 - \zeta) N_t \), where \( o \) refers to optimizing, i.e.: Ricardian consumers, and \( N_t \) is the amount of people currently employed. Employment is determined through a search and matching process described in section 2.2. Individuals gain utility from consumption and leisure. Each family member offers \( H_t^o \) hours of work. Individuals not currently working are searching for a job. Hours worked will be determined jointly within the bargaining over the real wage between firms and households. Accordingly, conditional on \( z_t \) are searching for a job. Hours worked will be determined jointly within the bargaining over the real wage between firms and households. Accordingly, conditional on \( N_t^o \) and \( H_t^{o,s} \), the household chooses consumption \( C_t^o \), government bonds \( B_t^{o,s} \), capital utilization \( z_t \), investment \( I_t^o \) and physical capital \( K_t^o \) to maximize the intertemporal utility function

\[
E_t \left[ \sum_{s=0}^{\infty} \beta^s \cdot \frac{1}{\xi_{t+s}} \cdot u \left( C_{t+s}^o - h \cdot C_{t+s-1}^o, H_{t+s}^{o,s} \right) \right] = 0 \tag{1}
\]

where \( h \) is the degree of habit persistence in consumption of Ricardian consumers and \( \xi_t^o \) is a preference shock with mean unity which obeys \( \log \xi_t^o = \xi^u + \xi^s_t \) where \( \xi^s_t \sim WN(0, \sigma^2_s) \) and the instantaneous utility function satisfies \( u(C_t^o - hC_{t-1}^o, H_t^{o,s}) = \frac{1}{1 - \sigma_c} (C_t^o - hC_{t-1}^o)^{1 - \sigma_c} - v(H_t^{o,s}) \) with \( \sigma_c \) being the coefficient of relative risk aversion and \( v(H_t^{o,s}) = \chi H_t^{o,s} \).

Let \( D_t^o \) be lump sum profits, \( B_t^o \) the quantity of nominally riskless one period government bonds carried over from period \( t - 1 \) and paying one unit of the numeraire in period \( t \), \( R_t \) the gross nominal rate of return on bonds purchased in period \( t - 1 \), \( r_t^K \) the rental rate on physical capital, \( \Gamma \) the amount of unemployment benefits an unemployed household member receives, \( W_t \) is the real market wage rate and \( \{\tau^K, \tau^N\} \) the tax rates on capital and labour income, respectively; then the households’ intertemporal budget constraint is

\[
C_t^o + I_t^o + R_t^{-1} B_t^o = \frac{P_{t+1} B_{t+1}^o}{P_t} + (1 - \tau^N) W_t N_t^o H_t^{o,s} + (1 - \tau^K) \nu^K z_t K_t^o - \psi(z_t) K_{t-1}^o + D_t^o + \Gamma U_t^o \tag{2}
\]

Households of the Ricardian consumers type own capital and choose the capital utilization rate \( z_t \), which transforms physical capital \( K_t^o \) into effective capital \( \tilde{K}_t^o \) according to: \( \tilde{K}_t^o = z_t K_{t-1}^o \). The cost of capital utilization per unit of physical capital is \( \psi(z_t) \) and I assume that in the steady state \( \psi(1) = 0 \) and \( z = 1 \).

The physical capital accumulation dynamics are given by

\[
K_t^o = (1 - \delta) K_{t-1}^o + \xi^I_t \left( 1 - f \left( \frac{I_t^o}{I_{t-1}^o} \right) \right) I_t^o \tag{3}
\]

where \( \xi^I_t \) is an investment specific technology shock with mean unity, \( \delta \) is the depreciation rate and the function \( f(\cdot) \) satisfies the following steady state conditions: \( f(1) = f'(1) = 0 \) and \( f''(1) = \frac{1}{\nu^K} > 0 \).

Based on this set-up, the first order conditions for the Ricardian consumers’ problem can
be written as

$$(C_t^g)$$

$$\lambda^g_t = \frac{1}{(C_t^o - hC_{t-1}^o)\sigma_c} - h\beta E_t \left[ \frac{\xi^u_{t+1}(C_{t+1}^o - hC_t^o)\sigma_c}{\xi^u_{t+1}(C_{t+1}^o - hC_t^o)\sigma_c} \right]$$

$$(B_t^g)$$

$$1 = \beta E_t \left[ \Lambda_{t,t+1} \cdot \frac{P_t}{P_{t+1}} \right] \cdot R_t$$

$$(z_t)$$

$$(1 - \tau^K) \cdot r^K_t = \psi'(z_t)$$

$$(I_t^g)$$

$$1 = \xi^I_t q^K_t \left( 1 - f\left( \frac{I^o_t}{I^o_{t-1}} \right) - f'\left( \frac{I^o_t}{I^o_{t-1}} \right) \right) + \beta E_t \left[ \Lambda_{t,t+1} q^K_{t+1} \cdot f'\left( \frac{I^o_{t+1}}{I^o_t} \right) \cdot \frac{(I^o_{t+1})^2 \xi^I_{t+1}}{(I^o_t)^2} \right]$$

$$(K_t^g)$$

$$q^K_t = \beta E_t \left[ \Lambda_{t,t+1} \left( (1 - \tau^K) r^K_{t+1} z_{t+1} - \psi(z_{t+1}) + (1 - \delta) q^K_{t+1} \right) \right]$$

where

$$\Lambda_{t,t+1} := \frac{\xi^o_{t} \lambda^g_{t+1}}{\xi^o_{t+1} \lambda^o_{t}}$$

is the stochastic discount factor. $q^K_t$ is the real shadow value of physical capital in place, i.e.: Tobin’s Q. Except for the treatment of the labour supply condition, the household’s optimization problem is conventional. Among the first order conditions, I have not listed an intratemporal optimality condition linking the real wage to the consumer’s marginal rate of substitution between consumption and labour. The reason is that wages and hours worked are determined within a bargaining process outlined in section 2.4.

2.1.2 Non-ricardian households

The Non-Ricardian consumers do neither borrow nor save due to a lack of access to capital markets. Hence they are not able to smooth their consumption path as a reaction to fluctuations in labour income or substitute intertemporally in response to interest rate changes. The number of family members currently employed is $N^r_t = c N_t$, where an $r$ refers to Non-Ricardian (i.e.: rule of thumb) consumers. Each family member employed offers $H^r_{t,s}$ hours of work. Each period these households solve a static optimization problem; let their instantaneous utility function be given by $u(C^r_t, H^r_{t,s}) = \frac{1}{1 - \sigma_c}(C^r_t)^{1 - \sigma_c} - v(H^r_{t,s})$ with $v(H^r_{t,s}) = \chi \left( \frac{(H^r_{t,s})^{1+\sigma_c}}{1+\sigma_c} \right)$, which is maximized given the following static budget constraint

$$C^r_t = (1 - \tau^N) W_t N^r_t H^r_{t,s} + \Gamma U^r_t$$

Consumption is determined simply by after tax labour income. As it was the case for the Ricardian consumers, hours worked $H^r_{t,s}$ is determined within the households' and firms' bargaining
over hours worked and the real wage. It is assumed that the tax rate imposed on the labour income of the two types of consumers is the same.

2.1.3 Aggregation

Aggregate consumption \((C_t)\) is specified by a weighted average of the relevant magnitudes for each household type: \(C_t = \varsigma C_t^r + (1 - \varsigma)C_t^o\), the amount of employed and unemployed family members of both household types is given by \(N_t = (1 - \varsigma)N_t^o + \varsigma N_t^r\) and \(U_t = (1 - \varsigma)U_t^o + \varsigma U_t^r\). Similarly, aggregate investment and the aggregate capital stock (both for the effective and physical capital stock) are defined by: \(I_t = (1 - \varsigma)I_t^o\) and \(K_t = (1 - \varsigma)K_t^o\).

For the further analysis I assume that, first of all, firms hire workers independently of their household type and that, once at work, the productivity of workers is independent of their household type, so that

\[H_t^s = H_t^{rs} = H_t^{os}\] (11)

This simplification allows consumption of the Ricardian households to be expressed in terms of aggregate consumption

\[C_t^o = \frac{1}{1 - \varsigma} (C_t - \varsigma^2(1 - \tau^N)W_t N_t H_t^s - \varsigma^2 \Gamma U_t)\] (12)

2.2 Matching of vacancies and unemployment

There is a continuum of intermediate goods producing firms, called the wholesale firms, indexed by \(i \in [0, 1]\). Firms post vacancies \(V_t(i)\) in order to attract unemployed workers and employs \(n_t(i)\) workers. The fixed labour force consists of \(n_t = \int_0^1 n_t(i)di\) workers and aggregate vacancies are given by \(V_t = \int_0^1 V_t(i)di\). Each worker who has a job supplies \(H_t\) units of hours worked, which are determined jointly with the real wage. Effort of each worker is constant. All unemployed workers at time \(t\) search for jobs. The timing assumption is such that in case an unemployed worker matches with a vacancy, he starts working there immediately within that period. This implies that the pool of unemployed people \(U_t\) searching for a job at time \(t\) is given by the difference between the total labour force, which is normalized to unity and the amount of employed people at the end of period \(t - 1\), that is

\[U_t = 1 - n_{t-1}\] (13)

The function matching unemployed workers \(U_t\) and firms with a vacancy \(V_t\) is

\[m_t = \xi^m U_t^{\zeta_m} V_t^{1 - \zeta_m}\] (14)

where \(\xi^m\) is the exogenous part in the matching process. \(\zeta_m\) is the elasticity to unemployment and \(m_t\) is the total amount of matches in period \(t\). Let \(q_t\) denote the probability of a vacancy being filled in period \(t\) by defining

\[q_t = \frac{m_t}{V_t} = \xi^m \cdot \left(\frac{V_t}{U_t}\right)^{-\zeta_m} = \xi^m \vartheta_t^{-\zeta_m}\] (15)
where \( \vartheta_t := \frac{V_t}{U_t} \) measures the degree of labour market tightness. Similarly, the probability \( p_t \) a searching worker finds a job is given by

\[
p_t = \frac{m_t}{U_t} = \xi m^{1-\zeta_m} = \vartheta_t \cdot q_t
\]  

(16)

Firms and workers take \( q_t \) and \( p_t \) as given. The definitions of \( q_t \) and \( p_t \) in (15) and (16) show that there is an intricate connection between the process linking workers to jobs and the one linking jobs to workers. The total flow of hires for an individual firm \((i)\) in period \( t + 1 \) is \( q_t V_t(i) \).

Finally, each period firms separate from a fraction \( s \) of their current workforce \( n_{t-1}(i) \). Once a worker loses his job, he is not allowed to search until the next period. This restriction implies that fluctuations are triggered by cyclical fluctuations in hiring rather than due to fluctuations in separations. Gertler and Trigghi (2006) use a similar timing assumption based on empirical evidence in support for this phenomenon outlined by Hall (2005) and Shimer (2005, 2007).

### 2.3 Wholesale firms

Production takes place at the wholesale firms. These firms are competitive. They hire workers and negotiate wage contracts with them. Wholesale firms and workers bargain over the real wage and the amount of hours worked based on an efficient Nash bargaining solution. The production for a typical wholesale goods firm depends on the amount of workers hired. Each output good \( Y_t(i) \) is produced by a firm \( i \) at time \( t \) using capital and labour as inputs. The firm considers the workers as identical and independent of the corresponding household type and the output of job \( j \) at firm \( i \) at time \( t \) is

\[
e^{\xi^A} \bar{k}_t(i)^\alpha H_t(j, i)^{1-\alpha}
\]

(17)

where \( \xi^A \) is an aggregate AR(1) random disturbance term with mean zero. Firms rent a unit of physical capital \( \bar{k}_t(i) = z_t k_{t-1}(i) \) from the households at the capital rental rate \( r^K_t \). \( H_t(j, i) \) denotes the amount of hours worked of worker \( j \) at firm \( i \). Assuming identical workers implies that \( H_t(j_1, i) = H_t(j_2, i) = H_t(i) \). Total output at firm \( i \) is given by a production function including the aggregate disturbance \( \xi^A \), the amount of employment relationships \( n_t(i) \) and average per worker capital \( \bar{k}_t(i) \), so that

\[
Y_t(i) = e^{\xi^A} \bar{k}_t(i)^\alpha \int_0^{n_t(i)} g \left( H_t(i)^{1-\alpha}, n \right) dn
\]

(18)

Following Pissarides (2000), the identical worker assumption implies that \( g(H, n) = \bar{g}(H) \bar{f}(n) \); since the mass of workers is uniformly distributed over \([0, 1] \), \( \bar{f}(n) = 1 \ \forall n \in [0, 1] \) and for \( \bar{g}(H) \) I assume: \( \bar{g}(H) = H \), which implies that the production function simplifies to

\[
Y_t(i) = e^{\xi^A} \bar{k}_t(i)^\alpha n_t(i) H_t(i)^{1-\alpha}
\]

(19)

Wholesale firms solve a profit maximization problem based on the following expected stream of revenues net of expenses\(^2\) for an individual firm \( i \) at time \( t \)

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\(^2\)The total real wage bill for firm \( i \) is the product of the mass of employment relationships at time \( t \) times the number of hours worked by each employee times the hourly real wage rate: \( n_t(i) H_t(i) W_t \). Similar reasoning applies
\[
E_t \left[ \sum_{s=0}^{\infty} \beta^s \Lambda_{t+1} \left( n_{t+s}(i) \cdot \left( \mu_t e^{k_{t+1}} H_t(i) \right)^{1-\alpha} - r_t^K \tilde{k}_{t+s}(i) - W_t H_t(i) + (1-s) \beta E_t [\Lambda_{t+1} \kappa_{t+1}(i)] \right) \right]
\]

where \( \Lambda_{t,t+1} \) is the households’ stochastic discount factor defined in equation (9), since households are the firms’ owners, profits are evaluated in terms of value attached to them. The employment flow equation for firm \( i \) is given by

\[
n_t(i) = (1-s) n_{t-1}(i) + q_{t-1} V_{t-1}(i)
\]

This implies that the adjustment cost for labour is \( \frac{1}{q_t} (n_{t+1}(i) - (1-s) n_{t-1}(i)) \). Therefore search theory is an alternative way of introducing adjustment costs which are linear in \( n_t(i) \) and the vacancy cost.

At each point in time, the firm maximizes its value choosing its capital stock, the work force and the amount of vacancies posted. The first order conditions for the firm’s optimization problem are

\[
(\tilde{k}_t(i)) \quad \quad \alpha \mu_t e^{k_t} \left( \frac{H_t(i)}{k_t(i)} \right)^{1-\alpha} = r_t^K
\]

\[
n_t(i) \quad \quad \kappa_t(i) = \mu_t e^{k_t} \tilde{k}_t(i)^{1-\alpha} - r_t^K \tilde{k}_t(i) - W_t H_t(i) + (1-s) \beta E_t [\Lambda_{t+1} \kappa_{t+1}(i)]
\]

\[
V_t(i) \quad \quad \frac{c_v}{q_t} = \frac{c_v}{q_t} [\Lambda_{t,t+1} \kappa_{t+1}(i)]
\]

The Lagrange multiplier \( \kappa_t \) is the shadow value of employment. Equation (22) states that the real rental rate on capital equals the marginal product of capital.

Combining equations (23) and (24) yields the following job creation condition

\[
\frac{c_v}{q_t} = \beta E_t [\Lambda_{t,t+1} \kappa_{t+1}(i)]
\]

The job creation condition equates the expected return of creating a new job, expressed by the right hand side, to the cost of a vacancy creation, which is given by the expected duration it takes to fill a vacancy \( 1/q_t \) times the associated costs of a vacancy, given here by the vacancy cost to the capital costs.
2.4 Wage bargaining

The matching friction gives rise to a bilateral monopoly context. When workers and firms match, there are rents to be split: if one party - either the worker or the firm - turns down a wage offer, finding another potential partner is costly. I proceed by following Pissarides (2000) and Mortensen and Pissarides (1994) and specifically Christoffel and Linzert (2005) by applying the efficient Nash bargaining solution. In this framework, the bargained real wage ($w_t$) and hours worked ($H_t$) are determined as the outcome of a Nash bargaining between workers and firms - bargaining over the nominal wage would lead the the same wage schedule to be derived in what follows.

2.4.1 Bellman equations

I assume that each firm-worker pairing is equally productive so that the wage rate is the same everywhere. Firms consider workers independent of their corresponding household type. However, a worker’s valuation of his employment and unemployment status depends on his household type in terms of the marginal rate of substitution between consumption and leisure. Consider worker $i$ of either household type. This worker’s value function in the employment and unemployment states satisfy

$$Y_{t}^{E,h}(i) = (1 - \tau^N)w_tH_t - \frac{v(H_t)}{\lambda_t^h} + \beta E_t \left[ \Lambda_{t,t+1} \left( (1 - s)Y_{t+1}^{E,h}(i) + sY_{t+1}^{U,h}(i) \right) \right] \forall h \in \{o, r\} \quad (26)$$

$$Y_{t}^{U,h}(i) = \Gamma + \beta E_t \left[ \Lambda_{t,t+1} \left( p_{t+1}Y_{t+1}^{E,h}(i) + (1 - p_{t+1})Y_{t+1}^{U,h}(i) \right) \right] \forall h \in \{o, r\} \quad (27)$$

where $v(H_t)$ and $\lambda_t^o$ are defined as in section 2.1 and $\lambda_t^r = (C_t^r)^{-\sigma_c}$.

Note that in fact only the value function for employment depends on the consumer type due to a different valuation of marginal consumption and $\Gamma$ is the amount of unemployment benefits given to an unemployed worker by the fiscal authority. The value of unemployment hence depends on the current flow value $\Gamma$ and the likelihood of being employed versus unemployed in the next period.

The value function of firm $i$ for job $j$ is (compare equation (28))

$$J_t(i,j) = \mu_t y_t(i,j) - r_t k_t(i) - w_t H_t(i) + \beta E_t [\Lambda_{t,t+1}(1 - s)J_{t+1}(i,j)] \quad (28)$$

Finally the free entry condition implies that (compare equation (24))

$$\frac{c_v}{q_t} = \beta E_t [\Lambda_{t,t+1}J_{t+1}(i,j)] \quad (29)$$

The stochastic discount factor $\Lambda_{t,t+1}$ is as defined in equation (9) and the output in firm $i$ of job $j$ - $y_t(i,j)$ - is as defined in equation (17). Bargaining takes place over the real wage $w_t$ and
hours worked $H_t$ taking the aggregate price level $P_t$ as given.

**2.4.2 Wage setting**

Due to the presence of Ricardian and non-Ricardian consumers, the Nash product is extended to take both kind of consumers into account. In particular, I assume that both groups bargain jointly with the firms where each group’s weight is given by its corresponding share in the population.

The equilibrium real wage and hours worked are derived from the maximization of the following Nash product

$$\max_{w,H} \left( (1-\varsigma) \left[ Y^{E,o} - Y^{U,o} \right] + \varsigma \left[ Y^{E,r} - Y^{U,r} \right] \right)^{\eta} \cdot J^{1-\eta} \quad (30)$$

where $\eta \in (0,1)$ measures the bargaining power of firms and households.

The first order condition with respect to the the real wage implies the following expression for the target wage $w_t$

$$w_t H_t = \frac{1 - \eta}{1 - \tau^N} \left( \Gamma + \frac{mrs t H_t}{1 + \phi} \right) + \eta \left( \mu_t \delta_t(i,j) - r^k_t \tilde{k}_t(i) + c_v \varphi_t \right) \quad (31)$$

where the marginal rate of substitution is given by

$$mrs t = \chi H_t^{\beta} \left( \frac{(1-\varsigma)}{\lambda^o} + \frac{\varsigma}{\lambda^r} \right) \quad (32)$$

The result is intuitive. The real wage workers get is increasing in the unemployment benefit $\Gamma$, the relative bargaining strength of the worker $1 - \eta$, the tightness in the labour market and the tax rate on households’ labour income. Workers get a weighted average of the unemployment benefits and the surplus, which consists of the output of job $j$ minus the costs on capital needed for job $j$ to be productive.

Finally, the first order condition with respect to hours worked implies the following two optimality conditions

$$w_t = (1 - \alpha) \mu_t \delta_t^{\alpha} \left( \frac{\tilde{k}_t(i)}{H_t(i)} \right)^{\alpha} \quad \text{and} \quad (1 - \tau^N) w_t = mrs_t \quad (33)$$

These two equations close the labour market of the model economy.

**2.4.3 Real wage rigidity**

Due to some un-modeled friction, I assume that not all labour supply reaches the market; specifically, the steady state supply of labour is fixed at some exogenously imposed level below the level of the frictionless economy. Locally around the steady state, households are therefore willing to supply labour at the going real market wage $W_t$, assumed to be governed by

$$W_t = (W_{t-1})^\nu \cdot (w_t)^{1-\nu} \cdot e_t^W \quad (34)$$
where \(c^W_t \sim WN(0, \sigma^2_W)\). This is a way of introducing real wage rigidity as recently postulated by e.g. Hall (2005) and Shimer (2005). The particular specification here follows Blanchard and Gal (2007).

### 2.5 Retailers

Retailers are monopolistic competitors and set prices for their goods in a staggered fashion. They buy intermediate goods from the wholesale firms, transform them and sell them as either investment/capital or as a consumption good.

Let \(Y_t(r)\) be the output sold by retailer \(r\) and let \(p_t(r)\) be its corresponding sale price in the goods market. Final goods denoted with \(Y\) are a Dixit-Stiglitz type aggregator of the individual goods \(Y_t(r)\) according to: \(Y_t = \left( \int_0^1 Y_t(r)^{1-\frac{\xi_t^R}{2}} dr \right)^{\frac{\xi_t^R}{2-1}}\) where \(\xi_t^R\) is the elasticity of substitution between two individual goods and evolves according to \(\log(\xi_t^R) = \xi_t^R + \epsilon_t\) where \(\epsilon_t \sim WN(0, \sigma_{\epsilon_t}^2)\).

As in Calvo (1983) firms are not allowed to change prices unless they receive a random price-change-signal. The likeliness that firm \(r\) can re-optimize its price in any period is constant and equal to \(1 - \theta\). Let \(Y_{t+k\mid t}(r)\) be the output of retail firm \(r\) at time \(t + k\) that could adjust its price \(p_{t+k\mid t}(r)\) optimally in period \(t\) the last time. Cost minimization on the final goods sector can be written as: \(Y_{t+k\mid t}(r) = \left( \frac{p_{t+k\mid t}(r)}{P^{\omega}_t} \right)^{-\frac{\xi_t^R}{\xi_t}} Y_{t+k}\) where \(p_{t+k\mid t}(r)\) is the price of intermediate good \(r\). Firms that are allowed to adjust their prices in period \(t\), set the price to maximize expected discounted profits. In the periods between price re-optimization firms adjust their prices according to the following indexation rule:

\[
\begin{align*}
    p_{t+k\mid t} &= \begin{cases} 
    p_{t+k-1\mid t}(\Pi_{t+k-1})^{\omega_p} \Pi_t^{1-\omega_p} & \text{for } k = 1, 2, 3, \ldots \\
    p_t & \text{for } k = 0
    \end{cases} \\
\end{align*}
\]

(35)

where \(p_{t+k\mid t}\) denotes the price effective in period \(t + k\) of a firm that last re-optimized its price in period \(t\). \(\Pi_t := \frac{P_t}{P_{t-1}}\) is the aggregate gross inflation rate and \(\omega_p \in (0, 1)\) is an exogenous parameter that measures the degree of inflation indexation. The term \(\Pi_t^{1-\omega_p}\) is an adjustment for trend inflation. The aggregate price level \(P_t\) can be expressed in the following way:

\[
P_t = \left[ (1 - \theta)(p_t^*)^{1-\xi_t^R} + \theta \left( \Pi_{t-1}^{1-\omega_p} \Pi_{t-1}^{\omega_p} \right)^{1-\xi_t^R} \right]^{\frac{1}{1-\xi_t^R}}
\]

(36)

The probability that a firm cannot re-optimize its price for \(k\) periods is given by \(\theta^k\). Profit maximization by retailer \(r\) who is allowed to re-optimize his price at time \(t\) chooses a target price \(p_t^*\) to maximize the following stream of future profits

\[
\max_{p_t^*} \frac{E_t}{\beta} \sum_{k=0}^\infty (\beta)^k \Lambda_{t,t+k} \left( \frac{p_t^*}{P_{t+k}} \cdot Y_{t+k\mid t}(r) \cdot \Pi_t^{(1-\omega_p)} \prod_{j=1}^k \Pi_{t+j}^{\omega_p} - C_{t+k}(Y_{t+k\mid t}(r)) \right)
\]

(37)

where \(C_{t+k}(Y_{t+k\mid t}(r))\) is the associated cost function expressed in real terms and \(\beta^k \Lambda_{t,t+k}\) is the associated stochastic discount factor. The first order condition of the retailers’ optimization
problem is

\[
E_t \left[ \sum_{k=0}^{\infty} (\theta \beta)^k \lambda_{t+k} t_{t+k+k} Y_{t+k+k} (r) \left( \frac{P_t^*}{P_{t+k}} (\xi_{t+k}^R - 1) \prod_{j=1}^{k} \Pi_{t+j}^{\nu} - \xi_{t+k}^R \mu_{t+k+k} t_{t+k+k} (r) \right) \right] = 0 \tag{38}
\]

where \( C'_{t+k+k} (Y_{t+k+k} (r)) = \mu_{t+k+k} t_{t+k+k} (r) \) is the marginal cost function given by equation \( 26 \).

### 2.6 Fiscal and monetary policy

Monetary policy is modeled by a simple Taylor type rule

\[
R_t = \left( \frac{R_{t-1}}{R} \right)^{\rho_r} \left( \left( \frac{\pi_t}{\pi} \right)^{a_{\pi}} \left( \frac{Y_t}{Y} \right)^{a_Y} \right)^{1-\rho_r} \cdot e_r^M \tag{39}
\]

where \( e_r^M \sim WN(0, \sigma_r^2) \) and \( R, \pi, Y \) are the corresponding steady state variables of the nominal interest rate (or nominal bond yield), the steady state inflation rate and steady state output level. The parameter \( \rho_r \) captures the degree of interest rate smoothing.

The fiscal authority’s budget restriction satisfies

\[
T_t + R_t^{-1} B_t^* = \frac{B_{t-1}^*}{\Pi_t} + G_t + U_t \Gamma \tag{40}
\]

where real government bonds satisfy \( B_t^* = B_t / P_t \) and \( \Pi_t = P_t / P_{t-1} \). \( \Gamma \) is the amount of unemployment benefits given to an unemployed worker as outlined in section 2.4.1. Hence the total amount of expenditures due to unemployment payments is \( U_t \Gamma \). The total amount of taxes collected satisfies

\[
T_t = \tau N W_t N_t H_t + \tau K_t K_t z_t K_{t-1} \tag{41}
\]

The government budget constraint implies a stable difference equation in nominal debt as long as \( R_t < 1 \) and in real debt as long as \( R_t / \Pi_t < 1 \). Hence the fiscal policy sector has to be modified such as to control for the public debt in case the previous stability conditions are not satisfied.

### 2.7 The fiscal policy rule

In the present model the fiscal authority controls government consumption by making use of the following fiscal policy rule (expressed in log-deviations from the corresponding steady state)

\[
\hat{G}_t = \phi_G \hat{G}_{t-1} + (1 - \phi_G) \left( \phi_G B_t^* + \phi_Y \hat{Y}_t + \phi_U \hat{U}_t \right) + \epsilon_G^F \tag{42}
\]

where \( \epsilon_G^F \sim WN(0, \sigma_G^2) \)

where it is assumed that \( \phi_G^2 \in [0, 1], \phi_B^2 < 0, \phi_Y^2 < 0, \phi_U^2 > 0 \). The policy rule extends the
standard model set up in terms of a higher degree of flexibility for the fiscal authority.

The rule for $G_t$ is defined such that increases in debt trigger declines in government expenditures in order to control for the deviation of public debt from its long run level and hence to guarantee stability. Based on the log-linearized equilibrium equations outlined in appendix C the condition for the government budget constraint to be a stable difference equation in $B_t^*$ requires the following to be satisfied: $b_y(R^{-1} - \Pi^{-1}) < \gamma_G(1 - \phi^G_G)\phi^G_B$ where $b_y = B^*/Y$ and $\gamma_G = G/Y$.

The current specification of the fiscal sector implies that there are two different types of fiscal policy mechanisms at work. On the one hand, the tax specification and the unemployment specification are two types of the so called automatic stabilizers. These automatic stabilizers function without the government taking any specific actions. In the current model set up, recessions triggering lower income and higher unemployment automatically imply lower tax revenues and higher expenditures due to higher unemployment payments which finally both lead to expansionary fiscal policy effects.

However, the model also has room for discretionary fiscal policy of two types. The fiscal policy rule implies that the government actively intervenes to changes in the economic stance both by relating its actions to current economic conditions and to an exogenous part. The reaction of the government to changes in unemployment and output determines that part of the rules which dependent on the current economic stance, while $\epsilon^G$ is a purely random intervention, i.e. surprises in the fiscal authority’s actions.

Throughout the analysis, fiscal policy is analyzed taking monetary policy as given.

2.8 Equilibrium and solution of the model

Assuming representative firms next to representative households in equilibrium the aggregate capital supply ($K_t$) in period $t$ is equal to total capital demand so that: $K_t = n_t k_{t-1}$. Moreover, labour supply ($N_t$) in period $t$ is equal to labour demand ($n_t$): $N_t = n_t$. The aggregate resource constraint satisfies

$$C_t + I_t + G_t + c_v V_t = Y_t - \psi(z_t \xi^t)K_{t-1}$$

(43)

and, by using equation (19) and the labour market equilibrium conditions, total output supplied can be rewritten in the following way

$$Y_t = e^{\xi^t} (z_t K_{t-1})^\alpha (N_t H_t)^{1-\alpha}$$

(44)

In order to solve the model, I first log-linearize the model around the non-stochastic steady state, provided in appendix B, where it is assumed that all variables except for the aggregate price level $P_t$ are stationary. The model has a well defined steady state. Appendix C contains the complete log-linear model.
3 Econometric methodology

Recently, various different methodologies have been developed to estimate the underlying structural parameters of DSGE models given its reduced form representation. Here I proceed by using a procedure proposed by Smith (1993) and Gourieroux, Monfort and Renault (1993) called Indirect Inference. I minimize a measure of distance between the model and empirical spectral density functions as proposed by Diebold et al. (1998). Working with spectral densities enables a decomposition of variation across frequencies which is often useful and the multivariate focus facilitates the examination of cross-variable correlations and lead-lag relationships at those frequencies of interest. Specifically I only use those frequencies in the data which correspond to low cyclicalities including business cycle periodicities. In particular, the estimation methodology carried out here does not use all the information being contained in the data. This is intentional for the following reason. DSGE models and especially New Keynesian and Real Business Cycle models are theoretical models aiming at explaining business cycles. However, usually the data used for estimation are a composition of cycles of different periodicities; there are long periodic cycles describing growth patterns, seasonal cyclicities, purely random components and among them, business cycle patterns. Since the model is only aimed at explaining low cyclicalities and especially business cycle frequencies, I hence make only use of this part of information in the data.

More formally, the model is solved using the algorithm of Uhlig (1999) which yields the following state space representation:

\[
\begin{bmatrix}
I_{(x)} & -Q(\theta) \\
0 & I_{(z)}
\end{bmatrix}
\begin{bmatrix}
x_t \\
z_t
\end{bmatrix}
= \begin{bmatrix}
P(\theta) & 0 \\
0 & N(\theta)
\end{bmatrix}
\begin{bmatrix}
x_{t-1} \\
z_{t-1}
\end{bmatrix}
+ \begin{bmatrix}
0 \\
\epsilon_t
\end{bmatrix}
\]  

(45)

where \( \epsilon_t \sim WN(0, \Sigma(\theta)) \)

\[
y_t = \begin{bmatrix}
R(\theta) & S(\theta)
\end{bmatrix}
\begin{bmatrix}
x_t \\
z_t
\end{bmatrix}
\]  

(46)

where \( x_t \) is a vector of endogenous state variables, \( y_t \) is a vector of control variables, \( z_t \) is a vector of exogenous state variables and \( \theta \) is a vector containing the structural parameters and variances of the exogenous shock processes. \( \Sigma(\theta) \) is a diagonal matrix with the variances of the exogenous shock processes on its main diagonal and zeros everywhere else since it is assumed that the structural shocks are all orthogonal to each other.

Let \( H(\omega) \) \( \forall \omega \in [0, \pi] \) denote the \( n \times n \) matrix of spectral densities of observed time series (\( n \) denotes the number of variables). Denote by \( H(\theta, \omega) \) the synthetic counterpart of \( H(\omega) \) of artificial spectral densities generated by the DSGE model. Specifically, for the solution of the DSGE model given by equations (45)-(46), the spectral densities of the variables are given by

\footnote{The alternative to this approach is to filter the data before using them. However, the problems with filtering are outlined well in Canova (1998).}
Figure 1: Plot of the data (all in logs except the inflation, nominal interest and employment rate which are in levels) and their spectral densities (in logs) over all frequencies ($\omega \in [0, \pi]$).

\[
g_z(\omega, \theta) = \frac{1}{2\pi} (I - N(\theta)^{-1} \Sigma(\theta) (I - N'\theta e^{i\omega})^{-1}
\]

\[
g_x(\omega, \theta) = (I - P(\theta)^{-1}) Q(\theta) \cdot g_z(\omega, \theta) \cdot Q(\theta) (I - P'(\theta) e^{i\omega})^{-1}
\]

\[
g_y(\omega, \theta) = R(\theta) \cdot g_z(\omega, \theta) \cdot R'\theta + S(\theta) \cdot g_z(\omega, \theta) \cdot S'(\theta) \omega \in [0, \pi]
\]

where $\omega \in \mathbb{R}$, however, it is sufficient to consider $\omega \in [0, \pi]$. Then, the Indirect Inference estimator of $\theta$, call it $\hat{\theta}$, is given by a vector which solves

\[
\hat{\theta} = \arg\min_\theta \int_{\omega_L}^{\omega_U} f(\hat{H}(\omega), H(\theta, \omega)) \odot V(\omega) d\omega
\]

where $\odot$ is the Hadamard-product. In practice the integral is replaced with a sum over frequencies $\omega \in [\omega_L, \omega_U] \subset [0, \pi]$. For what follows, I specify a quadratic loss function with uniform weighting over all frequencies.

### 3.1 Estimation

I use data of Germany from the OECD data base on ten variables for the estimation, these are: (1) real GDP, (2) real personal consumption of non-durables, (3) real private investment, (4) hours worked per employee, (5) inflation, measured by the percentage change of the GDP deflator, (6)
the employment rate, (7) the nominal interest rate on 3-month government bonds, (8) the real wage rate which is defined here as the compensation per hour worked in the non-farm industry, (9) real government consumption and (10) the number of vacancies posted. The GDP-Deflator is used in all cases where nominal variables need to be converted to real ones. The sample is: 1990q1:2006q4.

The empirical spectral densities are estimated based on a parametric estimation. In order to compute them, I estimate a stationary Bayesian Vector Autoregression (BVAR) using a non-informative prior density, four lags, a time trend and a constant term (see Uhlig (1994) and Canova (2007) for further details). The reason for sticking to a Bayesian VAR is due to the fact that this procedure facilitates the computation of standard errors of the estimated spectra as well as confidence intervals. All variables enter the BVAR in logarithmic terms except the nominal interest rate, inflation and the employment rate which are used in levels. A plot of the data used in the BVAR is given in figure 1. Given these estimates, I transform the BVAR to get an estimator for the spectrum of the variables (see for instance Hamilton (1994), chapter 10). Specifically, let the VMA of the corresponding VAR be given by

\[ y_t = \mu(t) + \Psi(L)e_t, \quad e_t \sim N(0, \Sigma) \quad \forall \ t = 1, \ldots, T \]  

(51)

An estimator for \( H(\omega) \) is then given by

\[ \hat{H}(\omega) = \frac{1}{2\pi} \left( \Psi(e^{-i\omega}) \right) \Sigma \left( \Psi(e^{i\omega}) \right)' \quad \forall \ \omega \in [0, \pi] \]  

(52)

I proceed by numerically drawing 10,000 Monte Carlo replications of the posterior density of the BVAR coefficients and taking the median across all draws. Figure 1 shows the spectra of all variables next to the data used in the BVAR. The graphs indicate that there are indeed lots of fluctuations which DSGE models are not able to account for. Especially hours worked, government expenditures and vacancies have many cyclicalities outside the typical business cycle interval. To take into account the failure of the DSGE model to explain fluctuations outside the business cycle interval, I henceforth use only fluctuations which correspond to 2-year up to 15-year periodicities; this implies that the frequencies considered are: \( \omega \in [2\pi/60, 2\pi/8] \). In order to evaluate \( \hat{H}(\omega) \) and \( H(\theta, \omega) \) I use a grid of size \( N \) for the frequencies: \( \{\omega_i\}_{i=1}^N \in [2\pi/60, 2\pi/8] \).

Note that \( H(\omega) \) as well as \( g_x(\omega, \theta), g_y(\omega, \theta) \) and \( g_z(\omega, \theta) \) are matrices with real numbers on the main diagonal and complex numbers on the off-diagonal elements. Due to the fact that within the estimation algorithm the same element in \( H(\omega) \) and \( H(\theta, \omega) \) need not necessarily have the same form of number (either real or complex) a problem which can emerge henceforth is that the estimates of the structural parameters are complex numbers too, which does not make any sense at all. Diebold et al. (1998) proceed by using only the spectra as well as the coherence and the phase shift functions in their matching exercise. In what follows I proceed by making use only of the absolute value of each element in the spectral matrices.

A further important point concerns the level of the spectrum which is matched. Since each variable has a different level in its spectrum, which expresses the differences in the variances, the standard question of how to adjust the model to the empirical data also applies here. However, in the current procedure, this problem is solved rather easily. Here I proceed by matching normalized spectra; that is, as can be seen in figure 2 each spectral density function is normalized to unity at
Table 1: Calibrated parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$ discount factor</td>
<td>0.98</td>
</tr>
<tr>
<td>$\delta$ depreciation rate</td>
<td>0.025</td>
</tr>
<tr>
<td>$p$ probability a searching worker finds a job</td>
<td>0.5</td>
</tr>
<tr>
<td>$\alpha$ capital share in output</td>
<td>0.33</td>
</tr>
<tr>
<td>$\zeta_m$ elasticity to unemployment in matching function</td>
<td>0.5</td>
</tr>
<tr>
<td>$\tau_K$ capital tax rate</td>
<td>0.35</td>
</tr>
<tr>
<td>$\tau_N$ labour tax rate</td>
<td>0.40</td>
</tr>
<tr>
<td>$\eta$ bargaining power of workers</td>
<td>0.5</td>
</tr>
<tr>
<td>$\xi^R$ elasticity of substitution between two final goods</td>
<td>6</td>
</tr>
<tr>
<td>$\sigma_c$ coefficient of relative risk aversion</td>
<td>1</td>
</tr>
<tr>
<td>$b_N$ steady state fiscal deficit</td>
<td>0.50</td>
</tr>
<tr>
<td>$c_N$ consumption share in output</td>
<td>0.55</td>
</tr>
<tr>
<td>$\gamma_r$ unemployment payments share in output</td>
<td>0.01</td>
</tr>
<tr>
<td>$\gamma_G$ government expenditures share in output</td>
<td>0.20</td>
</tr>
</tbody>
</table>

the lowest frequency. I do so for the following two reasons:

First, to the extent that different variances are expressed by different levels of the spectral density function, the estimation procedure gives higher weight to those variables within the matching process whose spectral density function are at a higher level, independently of using an optimal weighting matrix in the distance function \(50\) or not. This implies that the fit of the spectral density functions of those variables with rather small variance is indeed poor. Matching the spectral density functions based on normalized spectra gives equal weight to all variables.

Second, the empirical spectral density functions are based on the level of the variables while those of the theoretical ones only on the deviation of the variables from their corresponding steady state. This does not affect the shape of the density functions, however, it matters for the level since the area under the spectrum is the unconditional variance and (log) linear transformations, as it is done within log-linearization, transforms the variables by a constant term which crucially influences the level of the variance and hence the level of the spectrum. Since the transformation of a time series with a constant term effects its variability across all frequencies by the same amount, the spectral shape is henceforth unaffected. To this extent a normalization of the empirical and theoretical spectral density functions wipes out this discrepancy.

There are a couple of parameters which are not estimated but calibrated instead which are given in table \[1\]. These values are based partly on long run statistics as in Trabandt and Uhlig (2006) as well as on estimates obtained in Pytlarczyk (2005) and Christoffel et al. (2007).

3.2 Estimation results

The estimation results are presented in table \[2\]. In order to elaborate on the research questions stated in the introduction, I estimate three different versions of the previously specified DSGE model. The distinctions among them only refer to different fiscal policy specifications. Model 1 in table \[2\] refers to a fiscal policy structure where fiscal policy is governed as outlined in section \[2.7\]. The second model defines fiscal policy as being completely exogenous to fluctuations other than its own ones. Specifically, I estimate $\sigma^2_G$, $\phi^G_0$ and $\phi^B_0$ only. The third model is as model 1 but omits $\sigma^2_G$ implying that no surprises are allowed in fiscal policy.

In order elaborate on the way the current estimation procedure works and how well the
Table 2: Estimated parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho^A ) productivity shock</td>
<td>0.741</td>
<td>0.697</td>
<td>0.710</td>
</tr>
<tr>
<td>( \sigma ) productivity shock</td>
<td>0.0942</td>
<td>0.1078</td>
<td>0.1088</td>
</tr>
<tr>
<td>( \sigma ) discount factor shock</td>
<td>0.0932</td>
<td>0.1129</td>
<td>0.2236</td>
</tr>
<tr>
<td>( \sigma ) government expenditures</td>
<td>0.2236</td>
<td>0.1960</td>
<td>-</td>
</tr>
<tr>
<td>( \sigma ) monetary policy shock</td>
<td>0.1941</td>
<td>0.2129</td>
<td>0.2352</td>
</tr>
<tr>
<td>( \sigma ) investment shock</td>
<td>0.0928</td>
<td>0.1366</td>
<td>0.1475</td>
</tr>
<tr>
<td>( \sigma ) markup shock</td>
<td>0.1032</td>
<td>0.1125</td>
<td>0.1310</td>
</tr>
<tr>
<td>( \sigma ) wage shock</td>
<td>0.0600</td>
<td>0.0805</td>
<td>0.1132</td>
</tr>
<tr>
<td>job separation rate (s)</td>
<td>0.036</td>
<td>0.054</td>
<td>0.046</td>
</tr>
<tr>
<td>real wage rigidity (( \nu ))</td>
<td>0.741</td>
<td>0.691</td>
<td>0.719</td>
</tr>
<tr>
<td>vacancy posting cost (( c_{v} ))</td>
<td>8.490</td>
<td>9.045</td>
<td>7.991</td>
</tr>
<tr>
<td>non-ricardian share (( \varsigma ))</td>
<td>0.201</td>
<td>0.212</td>
<td>0.286</td>
</tr>
<tr>
<td>habit persistence (h)</td>
<td>0.734</td>
<td>0.737</td>
<td>0.780</td>
</tr>
<tr>
<td>labour supply (( \phi ))</td>
<td>4.120</td>
<td>4.197</td>
<td>4.310</td>
</tr>
<tr>
<td>capital utilization cost (( \psi_2/\psi_1 ))</td>
<td>0.872</td>
<td>0.931</td>
<td>0.902</td>
</tr>
<tr>
<td>investment adjustment cost (( \kappa_I ))</td>
<td>3.249</td>
<td>2.925</td>
<td>2.750</td>
</tr>
<tr>
<td>Calvo prices (( \theta_p ))</td>
<td>0.902</td>
<td>0.886</td>
<td>0.872</td>
</tr>
<tr>
<td>price indexation (( \omega_p ))</td>
<td>0.981</td>
<td>0.969</td>
<td>0.986</td>
</tr>
<tr>
<td>government expenditures smoothing (( \phi^G_P ))</td>
<td>0.567</td>
<td>0.620</td>
<td>0.598</td>
</tr>
<tr>
<td>government debt (( \phi^G_G ))</td>
<td>-0.579</td>
<td>-1.293</td>
<td>-1.317</td>
</tr>
<tr>
<td>output (( \phi^G_Y ))</td>
<td>-2.047</td>
<td>-2.773</td>
<td>-2.973</td>
</tr>
<tr>
<td>unemployment (( \phi^G_U ))</td>
<td>2.762</td>
<td>-2.973</td>
<td>0.31</td>
</tr>
<tr>
<td>interest rate smoothing (( \rho_R ))</td>
<td>0.759</td>
<td>0.775</td>
<td>0.751</td>
</tr>
<tr>
<td>inflation in monetary policy rule (( a_\pi ))</td>
<td>1.567</td>
<td>1.530</td>
<td>1.498</td>
</tr>
<tr>
<td>output in monetary policy rule (( a_Y ))</td>
<td>0.039</td>
<td>-0.080</td>
<td>-0.058</td>
</tr>
</tbody>
</table>

| SSR | 196.15 | 1665.89 | 437.07 |
| BK satisfied | yes | no | yes |
| prob(BK satisfied) | 0.69 | 0.40 | 0.38 |

Notes: SSR denotes the sum of squared residuals, BK satisfied = yes if the Blanchard-Kahn conditions are satisfied for the point estimates and prob(BK satisfied) gives the probability of these conditions being satisfied once having simulated the model 5,000 times based on the assumption that the structural parameters follow a normal distribution.
As far as the robustness of the results is concerned, the estimates of the variances of the exogenous shock processes are highly sensitive to the chosen frequency band, which is, however, rather intuitive. Due to the fact that the area under the spectrum of a time series equals its variance, using only a part of the spectrum, as is done in the current estimation exercise, while ignoring the rest crucially affects the estimates of the variances of the exogenous shock processes. Moreover, this induces a discrepancy between commonly estimated values and the ones here. Apart from this, however, the current estimation procedure is invariant to the stochastic singularity problem common in the estimation of DSGE models.

The estimates of the structural parameters other than those of the fiscal policy rules, are similar in size with commonly estimated values; specifically, the estimates of the parameters capturing the degree of price rigidity are similar in size as those obtained by Pytlarczyk (2005) and Christoffel, Kuester and Linzert (2007) using Bayesian estimation techniques. Moreover, the estimates of the interest rate smoothing parameter and the inflation reagibility parameter in the monetary policy rule closely reflect those of Pytlarczyk (2005) and Christoffel, Kuester and Linzert (2007). The estimates of the exogenous job separation rate highlight that there is a rather high exogenous degree of inertia in the labour market, especially along the extensive margin. This high

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5I compute confidence bands for the spectral density functions by numerically drawing 10,000 Monte Carlo replications of the posterior density, ordering them and extracting the 68% confidence band (from the 16th to the 84th quantile) as suggested by Canova (2007).
degree of rigidity in the labour market transmits into a rather sticky real wage which can be seen well by the high value of $\nu$.

Prior to commenting on the estimation results specific to the fiscal policy rules I analyze them in detail regarding their impact on the variability of key macroeconomic variables.

4 Analysis

Using the previously estimated structural parameters, specifically those of model 1, this section focuses on an analysis of the impact of the discretionary fiscal policy rule specified in section 2.7 where the impact of each variable and coefficient is explored in isolation of the remaining parameters. The analysis focuses, among others, on second moments, impulse response functions, forecast error variance decompositions and spectral densities. It is important to realize that these statistics are, given the variances of the stochastic processes, not stochastic and hence their values can be directly obtained from the structural parameters without simulating the model. Hence there is no need for specifying a density for the stochastic terms in the processes for the exogenous state variables.

4.1 Variability

Fluctuations are at the core for governmental intervention to achieve less variability and smaller amplitudes of fluctuations. Figure 3 plots the standard deviations of inflation, unemployment and output as functions of the policy parameters of the fiscal policy rule for government expenditures.

\[ vec(\Gamma_z(\theta)) = [I - N(\theta) \otimes N(\theta)]^{-1} vec(\Sigma(\theta)) \]  
\[ vec(\Gamma_x(\theta)) = [I - P(\theta) \otimes P(\theta)]^{-1} (Q(\theta) \otimes Q(\theta)) vec(\Gamma_z(\theta)) \]  
\[ \Gamma_y(\theta) = R(\theta)\Gamma_x(\theta)R^*(\theta) + S(\theta)\Gamma_z(\theta)S^*(\theta) \]
While letting one specific parameter vary to investigate its impact on the standard deviations, the remaining coefficients are fixed at their estimated values, such that the mechanism at work in creating changes in the standard deviations can be assigned uniquely to the one analyzed. The results highlight interesting patterns.

The effect of the coefficient specifying the sensitivity of government expenditures to changes in debt ($\phi_{GB}$) clearly shows that reactions to changes in debt imply a continuous decline in the standard deviation of inflation, unemployment and output for larger values of $\phi_{GB}$. The intuition behind this is the following. A strong reaction of the government to deviations of public debt from its steady state value imply also large impacts in size on output, inflation and employment through the aggregate resource constraint. Hence the high variability of government expenditures due to debt fluctuations amounts into higher instability of those variables due to the nature of government expenditures as interventions on the demand side.

The effect of $\phi_{GU}$ also shows rather homogeneous effects on the standard deviations of the three variables under consideration. In all cases a declining pattern of the variability can be observed with increasing values of $\phi_{GU}$. This is in line with countercyclical fiscal policy. It is apparent that only procyclical fiscal policy significantly increases instability, whereas a fiscal policy rule reacting positively to unemployment deviations or not at all does not imply large differences in the variability of inflation, unemployment and output.

The effect of stabilization in terms of reacting to output fluctuations shows a rather clear pattern for all three variables. It is interesting to note that here, fiscal stabilization policy in terms of a reaction to output indeed is a powerful tool for not only dampening fluctuations in output, but in unemployment and inflation too.

From a qualitative point of view, changes in the structural variance of surprise fiscal policy shocks as well as its degree of persistency described by the parameter $\phi_{GG}$, do not imply any specific changes in the pattern of the standard deviations of unemployment, inflation and output outlined previously. What changes, however, is the size of the overall variance of unemployment, output and inflation in absolute terms. The calibrated model replicates empirical variances for these three variables appropriately. It is interesting to note that changes in the various policy parameters of the fiscal policy reaction function cause only small changes in the value of the standard deviation of inflation in absolute terms, whereas, the range of variation is much larger for the other two variables. This implies that fiscal policy as such has a much stronger impact on output and unemployment than on inflation; more on this in section 4.4.

4.2 Cyclical interdependences

Since the various previously outlined fiscal policy specifications imply rather different patterns for the variability of the model economy, it is likely that also the cyclical pattern of fluctuations is influenced. Due to the fact that the area under the spectrum is the variance, the current section extends the previous one in elaborating on the changes in the variability of inflation, unemployment and output. Specifically the question is whether these changes are induced by changes in the amplitude or frequency of fluctuations. The important thing to note is that a change in the spectrum does not necessarily imply a change in the duration of cyclicalities since it can equally well affect the amplitude of fluctuations. To this extent a movement of the spectrum form one
Figure 4: Spectrum of $\pi$, $U$, and $Y$; $\omega \in [0, \pi]$ is the frequency, however, only those frequencies corresponding to the (quarterly) business cycles are plotted ($\omega \in \left[\frac{2\pi}{\hat{\omega}^1}, \frac{2\pi}{\hat{\omega}^2}\right]$ which corresponds to cycles of length 8 quarters to 60 quarters).

frequency band onwards to another amounts into a change in the duration of cycles while a general, not necessarily, proportional upward or downward movement would be in favour of a change in the amplitude of fluctuations.

Figure 4 shows the spectrum of inflation, unemployment and output for frequencies corresponding to business cycles of two to fifteen years length ($\omega \in \left[\frac{2\pi}{60^1}, \frac{2\pi}{8^2}\right]$, for quarterly data). The figures, similarly as before, show the impact of one parameter of the fiscal policy rule for government expenditures in isolation of the remaining ones. The parameter space over which the coefficients vary is the same as before. The graphs show some interesting patterns.

For the three variables at focus, the cyclicalities are affected rather homogeneously by various fiscal policy specifications. The graphs of the spectral density functions highlight that for any chosen parameter of the fiscal policy rule for government expenditures, the impact on the spectral densities is such that the new policy induces proportional changes in the spectra for a rather narrow frequency band only, while leaves the form of the spectral density functions outside this band completely unchanged. Across the parameter space, the spectrum does not change its shape, it only indicates a higher or lower value for specific calibrations for a rather narrow band across the frequency space. This implies that different fiscal policies primarily affect the amplitude of fluctuations rather than the duration of cyclicalities.

Hence in terms of the variability of the variables induced by the government only the amplitude of fluctuations is affected, however, hardly the duration of cycles. To this extent the effect of the reagibility of government expenditures to inflation, unemployment and output affects the overall fluctuations in a similar way as surprise fiscal policy shocks which also only lead to proportional upward or downward movements of the spectral density functions.
4.3 The adjustment paths

In general, the impulse response functions due to a shock in government expenditures (shown in figure 5 with an approximate 68% confidence interval for the response functions\footnotemark[7] are in line with general wisdom - inflation and output increase while unemployment declines, which is supported by various other empirical studies (see for instance Mountford and Uhlig (2008), Coenen and Straub (2005) and Blanchard and Perotti (2002) among others).

Both output and unemployment display a rather strong effect initially which, however, dies out very quickly. The initial reaction of these two variables to this surprise shock is rather heterogeneous. Unemployment drops by around 0.2% while output increases by around 0.2% points. Eventhough the shock as such is rather persistent (\( \phi_G = 0.55 \)) its long-lasting effects on these two variables is negligible. Inflation displays a hump-shaped adjustment pattern with both positive and negative deviations. To the extent that these deviations are of rather equal length, their size is negligibly small.

Within the DSGE model, the source of the fiscal policy shock is uniquely defined. From an empirical point of view, however, this is everything else but clear. To this extent, there are three realistic sources: (a) changes in the relative weights defined by the fiscal authority when reacting to fluctuations in output, unemployment, etc., (b) imperfect information on the part of the fiscal authority about the current or future economic stance and (c) changes in the fiscal policy stance unrelated to the current or future economic stance as well as unrelated to the prevailing policy reaction function.

The first source of fiscal policy shocks refers to the decision making process within the fiscal authority. Different fiscal policy positions on how to set government expenditures are likely described by different preferences concerning the relative weights determining the reagibility of those with respect to fluctuations in output, unemployment, etc. As a result, the decision making process itself can be random. In this case the random component in the fiscal policy reaction

\footnotetext[7]{The confidence intervals are obtained by assuming an (approximate) normal distribution for the estimated parameters, simulating the model and extracting the 16th and 84th percentile.}

---

Figure 5: Impulse response functions of inflation (\( \pi \)), unemployment (\( U \)) and output (\( Y \)) to a government expenditures shock.
function corresponds to random fluctuations in the preferences of the fiscal authority.

The second source of fiscal policy shocks refers to measurement errors caused by lags in the collection of the data of those variables which are essential within the fiscal authority’s decision making process. The fiscal authority can observe the actual economic stance and reverse policy actions due to measurement errors only after the final data has become available. Hence, with a fiscal reaction function based on revised data due to previous misperceptions of the economic stance, all previous policy actions show up in the model as deviations from the rule, which can then be interpreted as unexpected fiscal policy shocks.

The third source refers to genuine surprise shocks which are indeed unrelated to the current fiscal policy reaction function as well as to the current or future economic stance.

In order to quantify the importance of these surprise shocks, the next section presents some evidence using forecast error variance decompositions.

### 4.4 Surprise fiscal policy shocks

Surprise fiscal policy shocks are an important source of fluctuations. This can be best seen in table 2 from the estimates of model 1. In model 1 all fiscal policy parameters are estimated including those of the variances ($\sigma^2_G$). The estimates for the fiscal policy rule for government expenditures clearly indicate countercyclical fiscal policy with especially a rather strong reaction to output deviations. The estimate for the surprise in fiscal policy ($\sigma^2_G$) has a rather large value compared to the other shocks. Now, the crucial thing to notice is that when $\sigma^2_G$ is omitted in the estimation (consider model 3), specifically its value is set to zero, higher variability is introduced by a strong procyclical fiscal policy of government expenditures in terms of a procyclical behaviour with respect to unemployment. This implies that when ignoring surprise fiscal policy shocks, an important source of fluctuation is discarded such that a different fiscal policy stance has to account for this lack of variability.

Moreover, despite in-significant effects of unemployment and output on government expenditures (consider the estimates of model 1 in table 2), ignoring those variables leads to a change in the variance of surprise fiscal policy shocks which henceforth captures the omitted effects of those variables. Specifically, the variance of surprise fiscal policy shocks is much larger in model 1 where countercyclical policy is allowed in terms of a reaction to output and unemployment, while in model 2 this damping factor is not prevalent and hence also the standard deviation of surprise fiscal shocks is much smaller. As it turns out, the drop in the standard deviation is rather severe: from 0.22 down to 0.09. Moreover, for the point estimates, model 1 satisfies the Blanchard-Kahn conditions (Blanchard and Kahn (1980)) while model 2 does not. As Lubik and Schorfheide (2004) argue one can expect a richer pattern of the autocovariance functions which cannot be reproduced with parameters from determinate models. Hence model 1 offers a richer environment as compared to model 2 such as to adjust the model to the data given that determinacy is satisfied.

When focusing on the estimates of model 1, it is apparent that fiscal policy is both an important force in contributing to fluctuations, due to its strong dependence on output and unemployment fluctuations, as well as an important source of fluctuations. To the extent that government expenditures cause rather different persistences across various variables, a surprise shock triggers important fluctuations in many of them. This is confirmed by the forecast error variance
decomposition outlined in table 3. As can be seen from the table, fiscal policy shocks explain a large amount of the fluctuations in inflation and unemployment for short horizons, but then this magnitude declines strongly. For output, fiscal policy shocks explain around 20-25% of the fluctuations rather independently of the specific horizon considered. Relative to the other shocks, fiscal policy shocks are a major source of fluctuations.

5 Conclusion

This study analyzed the influence of discretionary fiscal policy based on an estimated DSGE model for Germany. Discretionary fiscal policy was designed to have government expenditures react to output and unemployment with an additional stochastic error term. In particular, the focus was to

\[ \sigma^A \] for the model’s solution given by equation (55) and (56), the forecast error variance decomposition for the variables in the endogenous state vector $x_t$ is given by

\[ \gamma_{m,n}(h) = \sum_{j=0}^{h-1} \left( \frac{1}{1_m(z)} A_j(\theta) \Sigma(\theta)^{1/2} 1_{n(z)} \right)^2 \quad (56) \]

where the $A_j(\theta)$ matrices satisfy $A(L) = (I - P(\theta)L)^{-1} Q(\theta) (I - N(\theta)L)^{-1}$ so that $A_0(\theta) = Q(\theta)$, $A_1(\theta) = Q(\theta) N(\theta) + P(\theta) Q(\theta)$ and $A_j(\theta) = A_{j-1}(\theta) N(\theta) + P(\theta) Q(\theta) \forall j > 1$. $1_m(z)$ is a column vector of zeros of the same size as $x_t$ with a one in the $m^{th}$ row and $1_{m,z}$ is a column vector of zeros of the same size as $x_t$ with a one in the $m^{th}$ row.

For the control variables $y_t$, the forecast error variance decomposition is given by

\[ \gamma_{m,n}(h) = \sum_{j=0}^{h-1} \left( \frac{1}{1_m(y)} B_j(\theta) \Sigma(\theta)^{1/2} 1_{n(z)} \right)^2 \quad (57) \]

where the $B_j(\theta)$ matrices satisfy $B(L) = R(\theta) A(L) + S(\theta) (I - N(\theta)L)^{-1}$ so that $B_0(\theta) = R(\theta) A_0(\theta) + S(\theta) N(\theta) \forall j \geq 0$ and the $A_j(\theta)$ matrices are as defined above. $1_m(z)$ is a column vector of zeros of the same dimension as $x_t$ with a one in the $m^{th}$ row.

$\gamma_{m,n}(h)$ then denotes the fractions of the forecast error variance until horizon $h$ of variable $m$ which is accounted for by the $m^{th}$ shock.
investigate, for output, unemployment and inflation, in how far fluctuations therein can be affected by fiscal policy. The important point here, among others, was to figure out the effect of fiscal policy once it is characterized as a force in contributing to the changing characteristics of cycles versus a scenario where it acts as a cause of fluctuations.

The following conclusions can be drawn from the analysis. Discretionary fiscal policy has a strong impact on the amplitude of fluctuations while basically no on the duration of business cycles. The results highlight that, similar as for monetary policy, fiscal policy can smooth cyclical fluctuations but there is no possibility of fundamentally disturbing cyclicalities in their lengths.

Apart from the fact that countercyclical discretionary fiscal policy can even be destabilizing (Friedman (1953)) the results obtained here clearly indicate that countercyclical fiscal policy indeed contributes strongly to diminish the amplitude of fluctuations.

Fiscal policy shocks originating in government expenditures are an important source of economic fluctuations as especially the forecast error variance decomposition displayed. To the extent that this technique indicates the importance of surprises in fiscal policy for explaining economic fluctuations, the applied econometric methodology strengthens this result since once the variance of the policy rule is omitted from the model, implying that there are no surprises in fiscal policy to be allowed, the discretionary fiscal policy stance changes significantly from a countercyclical to a strongly procyclical one so as to account for the fluctuations triggered by surprise shocks. Relative to the other shocks, fiscal policy shocks are a major source of fluctuations.

But besides the fact that fiscal policy is an important source for causing fluctuations, it is also an important responder such that, as the estimates for the fiscal policy rules indicate, government expenditures significantly contribute to dampening cyclical oscillations. To the extent that government expenditures play the most important role of governmental intervention to the economy, the prevalence of countercyclical fiscal policy can clearly be stated.

Considering a fiscal authority’s aim of dampening cyclical fluctuations indicates that for unemployment and output, countercyclical fiscal policy is a powerful tool, whereas for inflation its impact is limited due to the rather rigid fluctuations therein.

Moreover, the results highlight that once a researcher’s aim is to describe the fluctuations observed empirically by means of a DSGE model, a standard set up of the fiscal authority in terms of simple AR(1) processes ignores that there can indeed be a strong endogenous dependence of governmental variables on economic fluctuations.

References


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Appendix

A Data and variables

The Bayesian VAR analysis applied to compute the empirical spectral density functions uses quarterly German data on the following variables: (1) real GDP ($Y_t$), (2) real personal consumption of non-durables ($C_t$), (3) real private investment ($I_t$), (4) hours worked per employee ($h_t$), (5) inflation, measured by the percentage change of the GDP deflator ($P_t$), (6) the unemployment rate ($U_t$) from which the employment rate can be obtained ($N_t = 1 - U_t$), (7) the nominal interest rate on long term government bonds ($i_t$), (8) the real wage rate which is defined here as the compensation per hour worked in the non-farm industry ($W_t$), (9) real government consumption ($G_t$) and (10) the number of vacancies posted ($V_t$). All data series are taken from the OECD database and put into the BVAR without further transformations. The series range from 1990Q1 until 2007Q1 and the data are not seasonally adjusted. All variables entering the BVAR are expressed in logarithmic terms except for the employment rate, the nominal interest rate and the inflation rate.

The data vector in the BVAR is defined as follows

$$\mathbf{y}_t = \begin{bmatrix} \log(Y_t), \log(C_t), \log(I_t), \log(h_t),... \\ ...\pi_t, N_t, i_t, \log(W_t), \log(G_t), \log(V_t) \end{bmatrix}'$$

(58)

Table 4: Data: definitions and sources

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_t$</td>
<td>Vacancies; OECD, MEI: DEU Unfilled job vacancies</td>
</tr>
<tr>
<td>$I_t$</td>
<td>Investment; OECD, MEI: DEU Gross fixed capital formation, constant prices</td>
</tr>
<tr>
<td>$C_t$</td>
<td>Consumption; OECD, MEI: DEU Private final consumption expenditure, constant prices</td>
</tr>
<tr>
<td>$h_t$</td>
<td>Hours worked; OECD, EO: DEU Hours worked per employee - total economy</td>
</tr>
<tr>
<td>$U_t$</td>
<td>Unemployment rate; OECD, MEI: DEU Unemployment rate: survey-based all persons</td>
</tr>
<tr>
<td>$Y_t$</td>
<td>Gross Domestic Product; OECD, MEI: DEU Gross domestic product, constant prices</td>
</tr>
<tr>
<td>$P_t$</td>
<td>Price level; OECD, EO: DEU Gross domestic product, deflator, market prices</td>
</tr>
<tr>
<td>$W_t$</td>
<td>Real wage rate; OECD, MEI: DEU Hourly wage rate: manufacturing</td>
</tr>
<tr>
<td>$G_t$</td>
<td>Real government consumption; OECD, MEI: DEU Government final consumption expenditure, constant prices</td>
</tr>
<tr>
<td>$i_t$</td>
<td>Nominal interest rate; OECD, EO: DEU Long term interest rate on government bonds.</td>
</tr>
</tbody>
</table>

Notes: MEI stands for Main Economic Indicators (URL: http://oecd-stats.ingenta.com/OECD/TableViewer/DimView.aspx?TableName=6jmeicd.ivt&IF=eng and EO for Economic Outlook (URL: http://oecd-stats.ingenta.com/OECD/TableViewer/DimView.aspx?TableName=4deo.ivt&IF=eng). All data are freely accessible.
B  Steady state

The derivations here are part of the solution of the model as outlined in section 2.8.

- First obtain
  \[ N = \frac{p}{s + p} \]
  \[ U = 1 - N \]
  \[ MC = \mu = \frac{\xi^R - 1}{\xi^R} \]
  \[ r^K = \frac{1 - \beta + \beta \delta}{\beta(1 - \tau^K)} \]

- Then get
  \[ k = H \left( \frac{\alpha \mu}{\gamma^K} \right)^{\frac{1}{1-\alpha}} \]
  \[ K = k \cdot N \]
  \[ Y = K^\alpha (HN)^{1-\alpha} \]
  \[ I = \delta K \]

- Note that
  \[ z = q^K = 1 \]
  \[ C^r = C^\alpha = C \]

- Finally
  \[ C = c_y \cdot Y \]
  \[ V = Up \frac{1}{\gamma^K} \]
  \[ \vartheta = \frac{V}{U} \]

  \[ w = W = (1 - \alpha) \mu \left( \frac{k}{H} \right)^\alpha \]
  \[ mrs = (1 - \tau^K) \mu (1 - \alpha) \left( \frac{k}{H} \right)^\alpha \]
  \[ \chi = \frac{mrs}{(1 - \varsigma) \frac{H^\alpha}{\chi^r} + \varsigma \frac{H^r}{\chi^r}} \]

C  Log-linearized equilibrium

The following equations describe the equilibrium of the model in log-linearized form as referred to in section 2.8.
• Physical capital accumulation dynamics

\[ \dot{K}_t = (1 - \delta)K_{t-1} + \delta \left( \dot{I}_t + \epsilon_t^I \right) \]  

(59)

• Consumption-saving

\[- \left( \dot{R}_t - E_t[\hat{\pi}_{t+1}] \right) = \epsilon_t^{\hat{\nu}} - \hat{\lambda}_t^{\sigma} + E_t \left[ \lambda_{t+1}^{\sigma} - \epsilon_t^{\hat{\nu}} \right] \]  

(60)

• Capital utilization

\[ \hat{r}_t^K = \frac{\psi_2}{\psi_1} z_t \]  

(61)

where \( \psi_1 := \psi'(1) = 1/\beta - 1 + \delta \) and \( \psi_2 := \psi''(1) \).

• Investment

\[ \dot{I}_t = \frac{\kappa I}{1 - \beta} (\hat{\nu}_t^K + \epsilon_t^I) + \frac{1}{1 - \beta} \dot{I}_{t-1} + \frac{\beta}{1 - \beta} E_t \left[ \hat{I}_{t+1} \right] \]  

(62)

• Tobin’s Q

\[ \hat{q}_t^K = E_t \left[ - \left( \dot{R}_t - \hat{\pi}_{t+1} \right) + (1 - \beta(1 - \delta))\hat{r}_t^K + \right. \]  

\[ \beta(1 - \delta)\hat{d}_{t+1} \]  

(63)

• Marginal consumption of Ricardian consumers

\[ (1 - h\beta) \cdot \hat{\lambda}_t^{\sigma} = \frac{\sigma_c h}{1 - h} \hat{C}_{t-1}^{\sigma} + \frac{\sigma_c h \beta}{1 - h} E_t \left[ \hat{C}_{t+1}^{\sigma} \right] + \]  

\[ h \beta \left( E_t \left[ \hat{\nu}_t^{\hat{\nu}_t} \right] - \hat{\epsilon}_t^{\hat{\nu}_t} \right) - \frac{\sigma_c (1 + h^2 \beta)}{1 - h} \hat{C}_t^{\sigma} \]  

(64)

• Marginal consumption of Non-Ricardian consumers

\[ \hat{\lambda}_t^r = -\sigma_c \hat{C}_t^r \]  

(65)

• Consumption of Non-Ricardian consumers

\[ \hat{C}_t^r = c_1 \left( \hat{W}_t + \hat{N}_t + \hat{H}_t \right) + c_2 \cdot \hat{U}_t \]  

(66)

where \( c_1 = \frac{\sigma(1 - \sigma)(1 - \alpha)\delta}{\sigma_\sigma} \) and \( c_2 = \frac{3\Gamma U}{C^r} \).

• Aggregate Consumption

\[ \hat{C}_t = \zeta \hat{C}_t^r + (1 - \zeta)\hat{C}_t^{\sigma} \]  

(67)

• Marginal rate of substitution

\[ (b_3 + \zeta)\mu \hat{r}_t s_t = \phi(b_3 + \zeta)\hat{H}_t - \phi b_3 \hat{\lambda}_t^{\sigma} - \phi \hat{\lambda}_t^r \]  

(68)

where \( b_3 = \frac{(1 - h)(1 - \zeta)}{1 - h \beta} \).
• Capital renting

\[ \hat{r}_t^k = \hat{\mu}_t + \hat{\xi}_t^A + (1 - \alpha) \left( \hat{H}_t - \hat{k}_{t-1} - \hat{z}_t \right) \]  

(69)

• Unemployment

\[ \hat{U}_t = -\frac{p}{s} \cdot \hat{N}_{t-1} \]  

(70)

• Matching

\[ \hat{m}_t = \zeta_m \cdot \hat{U}_t + (1 - \zeta_m) \cdot \hat{V}_t \]  

(71)

• Transition probabilities

\[ \hat{q}_t = \hat{m}_t - \hat{V}_t \]  

(72)

\[ \hat{p}_t = \hat{m}_t - \hat{U}_t \]  

(73)

• Market tightness

\[ \hat{d}_t = \hat{V}_t - \hat{U}_t \]  

(74)

• Employment flow

\[ \hat{N}_t = (1 - s) \cdot \hat{N}_{t-1} + s \cdot \hat{m}_{t-1} \]  

(75)

• Aggregate wage

\[ \hat{w}_t = \frac{1 - \eta}{1 + \phi} \left[ \hat{m}rs_t - \phi \hat{H}_t \right] + \frac{\eta}{1 - \alpha} \left[ \hat{\mu}_t + \hat{\xi}_t^A - \alpha \cdot \hat{r}_t^K + \frac{c_v \gamma \nu p}{\mu s} \cdot \hat{d}_t \right] \]  

(76)

where \( \gamma \nu = V/Y \)

• Hours worked

\[ \hat{m}rs_t = \hat{\xi}_t^A + \hat{\mu}_t + \alpha \left( \hat{k}_{t-1} + \hat{z}_t - \hat{H}_t \right) \]  

(77)

• Real market wage

\[ \hat{W}_t = \nu \cdot \hat{W}_{t-1} + (1 - \nu) \cdot \hat{w}_t + \epsilon^w_t \]  

(78)

• Phillips curve

\[ \hat{\pi}_t = \frac{\omega_p}{1 + \beta \omega_p} \hat{\pi}_{t-1} + \frac{(1 - \theta)(1 - \theta \beta)}{\theta(1 + \beta \omega_p)} \left( \hat{\mu}_t + \frac{\epsilon^n_t}{\xi^R - 1} \right) + \frac{\beta}{1 + \beta \omega_p} E_t \left[ \hat{\pi}_{t+1} \right] \]  

(79)

• Employment

\[ b_1 \cdot \hat{k}_t = \hat{\mu}_t + \hat{\xi}_t^A - (1 - \alpha)\hat{W}_t - \alpha \hat{r}_t^K + \left( 1 - s \right) b_1 \beta E_t \left[ \hat{k}_{t+1} - \hat{R}_t + \hat{\pi}_{t+1} \right] \]  

(80)

where \( b_1 = \frac{c_v \gamma \nu p}{\mu s} \cdot \frac{N}{\nu p} \)

• Vacancies

\[ - \hat{q}_t = E_t \left[ \hat{\pi}_{t+1} - \hat{R}_t + \hat{\pi}_{t+1} \right] \]  

(81)
\[ \hat{R}_t = \rho_r \cdot \hat{R}_{t-1} + (1 - \rho_r) \left( a_\pi \hat{\pi}_t + a_Y \hat{Y}_t \right) + \epsilon_t^M \]  

(82)

\[ \hat{T}_t + b_y R^{-1} \left[ \hat{B}_t - \hat{R}_t \right] = b_y \Pi^{-1} \left[ \hat{B}_{t-1} - \hat{\pi}_t \right] + \gamma_G \cdot \hat{G}_t + \gamma_\Gamma \cdot \hat{U}_t \]  

(83)

where \( \gamma_G = G/Y, \ R^{-1} = \frac{\beta}{\Pi}, \ \gamma_\Gamma = \frac{\Gamma U}{\tau} \) and \( b_y = B^*/Y \)

\[ \hat{T}_t = \tau^N (1 - \alpha) \mu \left[ \hat{W}_t + \hat{N}_t + \hat{H}_t \right] + \tau^K \alpha \mu \left[ \hat{B}_t + \hat{\pi}_t + \hat{K}_{t-1} \right] \]  

(84)

\[ \hat{G}_t = \phi_G^G \hat{G}_{t-1} + (1 - \phi_G^G) \left( \phi_B^G \hat{B}_t^* + \phi_Y^G \hat{Y}_t + \phi_U^G \hat{U}_t \right) + \epsilon_t^G \]  

(85)

\[ \hat{K}_t = \hat{N}_t + \hat{k}_{t-1} \]  

(86)

\[ \hat{Y}_t = \xi_A^t + (1 - \alpha) \cdot \left( \hat{H}_t + \hat{N}_t \right) + \alpha \cdot \left( \hat{K}_{t-1} + \hat{\pi}_t \right) \]  

(87)

\[ \hat{Y}_t = c_g \hat{C}_t + i_y \hat{I}_t + \gamma_G \hat{G}_t + \psi_1 \frac{K}{Y} \hat{\pi}_t + c_v \gamma_v \hat{V}_t \]  

(88)

where \( i_y = I/Y \)