Asymmetric tax competition with public inputs and imperfect labour markets

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This paper examines efficiency in public input provision in two large jurisdictions with imperfect labour markets. It analyses how equilibrium capital tax rates and public input provision levels differ between asymmetric jurisdictions that can strategically influence the interest rate on the common capital market in an international tax competition setting. In contrast to the scenario assuming competitive labour markets, the non-cooperative equilibrium is inefficient also when governments have capital and head taxes at disposal. As a source of both the distortion in the capital allocation between jurisdictions and the inefficiency in public input provision, which can be determined in at least one of the jurisdictions, we identify the governments’ incentives to decrease unemployment, and a pecuniary externality [DePater, J., Myers, G., 1994. Strategic capital tax competition: a pecuniary externality and a corrective device. Journal of Urban Economics 36, 66-78.] in both jurisdictions. Efficiency in public input provision can be restored, however, if the set of fiscal instruments available for regional policy makers is extended by a labour tax.

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1. Introduction

Recent research stresses that the implications of fiscal competition depend to a significant degree on the institutional setting of the competing jurisdictions’ labour markets. This is because, assuming imperfect labour markets, the representative government needs not only to account for the effects that a variation of available policy parameters may have on the amount of capital invested in the jurisdiction, but also on the level of employment. A potential inefficiency in the non-cooperative equilibrium can then be attributed to the fact that the change in employment arising from a region’s choice of tax and/or expenditure levels creates an additional incentive for regional governments to deviate from public good provision levels according to the first-best provision rule, where the marginal utility of public good provision equals the marginal cost.

The early tax competition literature (for example, Wilson 1986; Zodrow and Mieszkowski 1986) abstracts from possible labour market distortions and assumes a large number of small and identical jurisdictions. As the underlying reason for a potential inefficiency in the provision of public consumption goods, fiscal externalities resulting from international capital mobility are identified (Wildasin 1989). While fiscal competition between symmetric jurisdictions is an appropriate approximation in some cases, it is more realistic to assume that the regions competing for mobile capital are asymmetric. Capital flows between competing jurisdictions, resulting from international trade, for instance, do not occur between symmetric jurisdictions. The fact that a large part of the literature on fiscal competition abstracts from the asymmetry assumption can be justified by the fact that the obtained results frequently hold for more realistic characterisations of the competing jurisdictions. However, as will be shown below, also under the assumption of tax competition with imperfect labour markets, the assumption of asymmetry is valuable for providing new insights into the challenges faced by decentralised jurisdictions in providing public goods.

It is well known that if the jurisdictions competing for mobile capital are large and asymmetric, for instance, with respect to the size of the jurisdictions, a further source for inefficiency in both public good provision and the capital allocation between jurisdictions can be identified. In a non-price-taking environment, where competing jurisdictions have market power on the common capital market, additional externalities arise. This stems from the fact that, as argued by Bucovetsky (1991) and DePater and Myers (1994), jurisdictions have an incentive to strategically influence the net-of-tax return to capital to manipulate interest payments from or to other jurisdictions. While a jurisdiction, which is a net importer of capital, will want to decrease interest rates, a capital-exporting jurisdiction has an incentive to increase interest payments from the capital-importing region by increasing the interest rate. The resulting change in the price of capital and the associated effect on the well being of the residents in the other region is referred to as ‘pecuniary externality’ by DePater and Myers (1994), who also present a corrective device for this externality in a tax competition setting with full employment and governmental provision of public consumption goods.

Also in the present analysis, where the economy can be characterised by a situation with less than full employment, inefficiency of the non-cooperative equilibrium can be traced back to (but it is not limited to) both fiscal and pecuniary externalities. Under this scenario,
however, it remains to examine the implications for efficiency in the decentralised equilibrium that can be attributed to imperfect labour markets, and to the possible interactions of this new feature in tax competition with the well-known externalities that have been derived for situations without labour market distortions.

Overall, there are three major aspects in which the present paper differs from the basic tax competition setting of the earlier literature: i) competing jurisdictions are asymmetric and large enough to manipulate the interest rate on the capital market, ii) jurisdictions provide a public intermediate good (public input), and, iii) jurisdictions can be characterised by involuntary unemployment. Each of the features i) - iii) has been examined independently in an international tax competition setting in literature. This is not the case, however, for a combination of these characterisations of the competing jurisdictions in an economy.

Bucovetsky (1991), Wilson (1991), and DePater and Myers (1994), for instance, analyse tax competition under assumption i), where each of the competing jurisdictions can be characterised by full employment, and where governments provide a public consumption good. In contrast, Zodrow and Mieszkowski (1986), Noiset (1995), Fuest (1995), and Matsumoto (1998) allow for the possibility that decentralised jurisdictions provide a productive public input used in the production process of firms instead of (or in addition to) a public consumption good, which is typically considered as an argument in the utility function of individuals. Therefore, the authors focus on the efficient provision of public goods, which do have an impact on production in the economy, such as roads, air and seaports, the telecommunication infrastructure or other productive infrastructure (assumption ii)). The authors do not, however, account for the features i) and iii). As a central finding of the analysis in Matsumoto (1998), public inputs tend to be underprovided in the case of sole capital-tax financing, and if the public input is of the ”creation of atmosphere” type (Meade 1952), i.e. the production function is linear homogeneous in primary inputs of production (in their case, capital and labour). In this respect, we follow a similar approach. In the present setting, regional governments provide a public input, however, with regards to the primary factors of production, land is assumed as additional, fixed production factor, while both capital and labour are endogenous. The endogeneity of capital arises from capital mobility, while labour is endogenous due to the assumption of imperfect labour markets. In addition, also the net-of-tax rate on capital is endogenous in our setting as jurisdictions have market power on the common capital market.

Some of the more recent examinations of fiscal competition allowing for imperfect labour markets (assumption iii)) include Ogawa et al. (2006a,b), Aronsson and Wehke (2008), Exbrayat et al. (2012), and Eichner and Upmann (2012). Exbrayat et al. (2012) argue that the consideration of unemployment is a realistic and necessary extension of the existing literature on fiscal competition. In fact, it is also one of the main reasons for jurisdictions to engage in tax competition. Policy makers, for instance, who frequently emphasise on the possible employment gains in association with the attraction of foreign direct investments seem to be well aware of this additional aspect in tax competition, and also of the interacting effects that are at work between the employment of capital and labour. One aim of the present analysis is also to take a closer look at the potential implications of these aspects in
an international tax competition setting.

Eichner and Upmann (2012) derive some general results for tax competition with involuntary unemployment for an unspecified labour market model. They argue that the results derived in the non-cooperative equilibrium by Zodrow and Mieszkowski (1986) remain valid for some efficient bargaining solution on the labour market. Ogawa et al. (2006a), in contrast, emphasise that the provision level of the public consumption good may also be in excess of the first-best level. This result is obtained in the tax game where regions have no other instrument than a source-based capital tax at disposal, and when capital and labour are substitutes (the marginal productivity of capital decreases with an increase of the production factor labour). With respect to the labour market distortion they focus on the fixed-wage model, where an exogenous wage rate above the competitive wage causes unemployment. However, both Ogawa et al. (2006a) and Eichner and Upmann (2012) abstract from the features \( i \) and \( ii \) in tax competition. In the present approach, we follow Ogawa et al. (2006a) in considering the fixed-wage model as a source of unemployment. This is also because the focus of this paper is on the interplay between the features of unemployment, asymmetric tax competition, and public input provision, rather than on the implications of various labour market models.\(^1\)

There is only a small number of papers that analyse the implications of a combination of two of the features \( i \)-\( iii \) in international tax competition. Arnold and Fuest (1999) examine the potential implications of \( i \) and \( ii \), but abstract from unemployment. For the tax game with governmental provision of public consumption goods and public inputs, the authors find inefficiency in the provision of public inputs when regional governments rely on the head tax for public good financing. The deviation from the first-best provision level arises from a pecuniary externality, where public input provision serves as an instrument to manipulate the interest rate on the common capital market. If a jurisdiction imports capital it can reduce its interest payments through a downward deviation from the first-best provision level of the public input. In contrast, the public input is provided efficiently when regions have both head and capital taxes at disposal. The capital allocation, however, remains inefficient in this case as jurisdictions use the capital tax rate to favourably manipulate the terms of trade in capital. Aronsson and Wehke (2008) consider the features \( ii \) and \( iii \) in fiscal competition. Assuming that the wage rate is determined in a bargaining process between unions and firms, the authors find that for a coordination of tax rates, welfare increases even in the presence of labour market imperfections. The authors’ focus is on the welfare consequences from tax coordination with a public intermediate good and a public consumption good, however, not explicitly on issues regarding the equilibrium level of policy instruments and public goods. Including the same features in an international tax competition setting, and using the fixed-wage model as a source of unemployment, Ogawa et al. (2006b) identify

\(^1\)The fixed-wage model is, for instance, also considered in an international tax competition setting by Huang (1992), Leite-Monteiro et al. (2003) and Ogawa et al. (2006b). It can be appropriate for the analysis of unemployment that is caused by a minimum wage.
employment externalities from public input provision that depend also on the level of the fixed wage. As a result of their analysis, regions choose to tax or subsidise capital in the tax game with head and capital taxes. In addition, the authors discuss the provision level of the public input and the composition of public spending.

The aim of this paper is to examine efficiency in public input provision in two large (and asymmetric) jurisdictions when labour markets are imperfect. We combine the features $i)$, $ii)$, and $iii)$ in an international tax competition framework and extend the set of fiscal instruments by a labour tax. The approach solves for the equilibrium tax rate of capital (and of labour), and for the public input provision levels in both jurisdictions when unemployment is introduced in asymmetric tax competition. We find that the results of tax competition deviate from those obtained for undistorted labour markets. The fundamental source of the inefficiency in both the interjurisdictional capital allocation and governmental provision of public inputs can be traced back to an externality arising from the jurisdictions’ incentive to increase employment and a modified pecuniary externality resulting in the non-price-taking environment.

The remainder of the paper is structured as follows: Section 2 introduces a model of asymmetric tax competition with imperfect labour markets. Section 3 analyses the decentralised equilibrium for alternative tax games, where governments have a head tax (Section 3.1), both head and source-based capital taxes (Section 3.2), and a labour tax in addition to these tax instruments (Section 3.3) at disposal. Section 4 summarises the results and concludes the paper.
2. The model

Let us assume a model economy with two large, economically independent regions, in which \( N_i \) denotes the number of inhabitants in jurisdiction \( i (i = 1, 2) \), and \( N \) is the total number of individuals in the economy \( (N_1 + N_2 = N) \). Inhabitants in both jurisdictions are assumed to be entirely immobile. Each individual provides one unit of labour inelastically so that the maximum amount of labour available in each region is identical to the number of inhabitants \( N_i \). However, as regional labour markets in both jurisdictions are imperfect, \( L_i < N_i \) holds for the factual amount of labour \( (L_i) \) available in the private sector. Following Ogawa et al. (2006a), we consider a fixed wage \( w_i \) above the competitive wage rate as a source of the labour market distortion in both regions.

The total amount of capital \( (K) \) in the economy is fixed and mobile between both jurisdictions \( (K_1 + K_2 = K) \). Individuals are the owners of the private production factors capital, and of the fixed factor land \( (Z_i) \) in their region, and all individuals in a region are assumed to receive identical income from the ownership of these production factors. In addition to three private factors of production, firms employ a public input in the production process, where the quantity of the public factor is denoted with \( B_i \). The production technology in a representative jurisdiction reads
\[
G_i = F_i(K_i, L_i, Z_i, B_i).
\]

One unit of the final good \( G_i \) can be transformed into one unit of a private consumption good or into one unit of the public factor. The price of \( G_i \) and the number of firms in each jurisdiction are normalised to unity. Marginal products of all factors are positive and diminishing. In addition, cross derivatives between capital and labour as well as between the public input and both capital and labour are positive \((F_{iKL}, F_{IKB}, F_{iLB}) > 0\).\(^2\) The production function is linear homogeneous in primary inputs of production, capital, labour and land. For this specification of the production technology, we derive from the Euler-Theorem:
\[
F_i(K_i, L_i, Z_i, B_i) = F_i(K_i, L_i, Z_i, B_i).
\]

In addition, we define
\[
A_i = F_{iK^2}F_{iL^2} - (F_{iKL})^2 > 0,
\]
where the sign in (3) follows from decreasing returns to scale in capital and labour. With firm profits \( Q_i = F_i(\cdot) - (r + T_i)K_i - w_iL_i - z_iZ_i \), the first-order conditions for a maximum in firm profits can be derived as:
\[
F_{iK}(\cdot) = r + T_i,
\]
and
\[
F_{iL}(\cdot) = w_i.
\]

\(^2\)In addition, it follows from Young’s theorem that \( F_{iKL} = F_{iLK}, F_{iKB} = F_{iBK}, F_{iLB} = F_{iBL} \).
which state that the marginal products of capital and of labour are equal to the marginal cost of both factors. Per unit costs of capital consist of the interest rate \( r \) and - depending on the assumption on available policy instruments - a source-based capital tax rate \( T_i \). \( w^i \) and \( z^i \) are the per-unit costs of labour and of land. Assuming zero firm profits in the competitive equilibrium, we derive for the income that can be attributed to land ownership:

\[
z^i \bar{Z}^i = F^i(\cdot) - (r + T^i)K^i - \bar{w}^iL^i. \tag{6}
\]

As we intend to analyse tax competition between two large jurisdictions, the factor demand functions are derived from:

\[
F^i_K(K^i, L^i, \bar{Z}^i, B^i) - T^i = F^j_K(\bar{K} - K^i, L^j, \bar{Z}^j, B^j) - T^j, \tag{7}
\]

\[
F^i_L(K^i, L^i, \bar{Z}^i, B^i) = \bar{w}^i, \tag{8}
\]

\[
F^j_L(\bar{K} - K^i, L^j, \bar{Z}^j, B^j) = \bar{w}^j. \tag{9}
\]

The equilibrium of the common capital market of jurisdictions \( i \) and \( j \) is characterised by (7), and equilibria in the local imperfect labour markets by (8) and (9), where \( i, j = 1, 2 \) and \( i \neq j \), which also holds in all following referrals to both regions. From equations (7)-(9) we are able to derive the factor demand functions

\[
\frac{\partial K^i}{\partial T^i} = \frac{F^i_{LL} F^j_{LL} F^j_{LL}}{F^i_{LL} A^i + F^j_{LL} A^i} < 0, \tag{10}
\]

\[
\frac{\partial L^i}{\partial T^i} = \frac{-F^i_{KL} F^j_{LL}}{F^i_{LL} A^i + F^j_{LL} A^i} < 0, \tag{11}
\]

\[
\frac{\partial K^i}{\partial B^i} = \frac{(F^i_{KL} F^j_{LB} - F^i_{KB} F^j_{LL}) F^j_{LL}}{F^i_{LL} A^i + F^j_{LL} A^i} > 0, \tag{12}
\]

\[
\frac{\partial L^i}{\partial B^i} = \frac{(F^i_{KB} F^j_{LK} - F^i_{KK} F^j_{LB}) F^j_{LL} - F^i_{LB} A^j}{F^i_{LL} A^i + F^j_{LL} A^i} > 0. \tag{13}
\]

According to (10) - (13), the private sector in a representative jurisdiction increases employment of capital and labour when public input provision is increased and capital tax rates are decreased in this region. Note that in the basic model of fiscal competition, which abstracts from distortions in the labour market, the effects of a change in tax and expenditure levels are

\[\text{Observe also that a profit tax would generate zero income in this setting.} \]

\[\text{See Appendix 5.1 for a detailed derivation.} \]
limited to capital, while under the present setting regional governments are also able to improve local labour market conditions through the appropriate variation in both instruments.\footnote{5} As the sign of the effects of a change in tax and expenditure levels are identical for both production factors, the comparative statics results point to an intensified tax competition compared to the tax game with full employment in both jurisdictions. This argument will be further elaborated throughout the analysis. The intuition behind the signs obtained from a change in tax and expenditure levels is as follows: First, a decline in the capital endowment occurs from capital tax financing of the public input as an increase in the source-based capital tax raises the cost of capital for each firm in that jurisdiction (according to the right-hand side of (4)). The result of a decreasing employment from an increase in this tax instrument comes from the complementary relationship between labour and capital, which decreases the marginal productivity of labour once the capital endowment in the region falls. As a result, also the demand for workers decreases. The rise in public input provision levels works in the opposite direction. The capital demand in the private sector increases as a consequence of a higher marginal productivity of capital, resulting from the fact that also private and public capital are complements. Moreover, as labour and the public factor are complements, labour demand is higher compared to a situation with lower public input provision levels.\footnote{6} Observe, that a change in fiscal policy parameters in one region also affects the situation in the other region. The externalities in the neighbouring region can be traced back to the associated change in factor endowments: $\partial K^j / \partial T^i > 0, \partial K^j / \partial B^i < 0, \partial L^j / \partial T^i > 0, \partial L^j / \partial B^i < 0$. Therefore, in addition to the fiscal externality, an externality arises in the neighbouring region from the effects on the labour market, which is captured by the latter two effects.\footnote{7}

Contrary to the case of symmetric tax competition with a large number of small jurisdictions, tax and expenditure decisions of both jurisdictions also affect the interest rate $r$ on the common capital market as both governments have market power in the capital market.\footnote{8}

\[
\frac{\partial r}{\partial T^i} = \frac{-F^i_{LL} A^i}{F^i_{LL} A^i + F^j_{LL} A^j} < 0, \tag{14}
\]

\footnotetext[5]{Both variables are also strategic choice variables in tax competition between both jurisdictions in section 3.}
\footnotetext[6]{Note that there are also second-round effects as the resulting increase in the labour demand augments the marginal productivity of capital, raising the capital demand. Second, also a higher demand for capital according to (12) increases the demand for labour further.}
\footnotetext[7]{Both production factors are affected in the opposite manner in the neighbouring region compared to the region that alters fiscal policy parameters. This follows from capital mobility. The resulting increase in capital in region $j$, resulting from a higher capital tax in region $i$, increases the marginal productivity of labour, and firms in the neighbouring country increase their labour demand. A higher expenditure level in jurisdiction $i$, on the other hand, results in a lower capital endowment in region $j$, which decreases labour demand through the channel of a lower marginal productivity of this production factor. The respective comparative statics results are derived in detail in the Appendix 5.1.}
\footnotetext[8]{An identical sign for the reaction of the interest rate to a variation in both fiscal tools is detected in the tax game with two large regions and for the institutional setting of competitive labour markets (see Arnold and Fuest 1999). Appendix 5.2 contains a derivation of (14) and (15).}
\[
\frac{\partial r}{\partial B^i} = \frac{A^i (F_{KB}^i F_{LL}^i - F_{KL}^i F_{LB}^i)}{F_{LL}^i A^j + F_{LL}^i A^i} > 0. \tag{15}
\]
As a result, increasing the capital tax rate not only decreases the capital endowment in the respective region, but also the interest rate. On the other hand, increasing the level of the public input provided in one of the jurisdictions increases both the capital invested in that region and the interest rate on the capital market. Tax and expenditure policies in one jurisdiction therefore also affect the second jurisdiction through the common interest rate, which is part of the per unit costs of capital in both jurisdictions.\(^9\)

3. The non-cooperative equilibrium

As convenient in a large part of the tax competition literature, we assume that regional policy makers choose the available tax instruments strategically to maximise the utility of individuals in their region. The budget constraint of a representative jurisdiction reads

\[
B^i = T^i K^i + H^i \bar{N}^i, \tag{16}
\]
where \(H^i\) is the head tax rate on the individuals in that region. Without savings, the consumption of the employed \((c^{i,e})\) and unemployed \((c^{i,u})\) individuals can be derived as:

\[
c^{i,e} = \bar{w}^i + r \frac{\bar{K}}{\bar{N}} + z^i \frac{\bar{Z}^i}{\bar{N}^i} - H^i, \tag{17}
\]
and

\[
c^{i,u} = r \frac{\bar{K}}{\bar{N}} + z^i \frac{\bar{Z}^i}{\bar{N}^i} - H^i. \tag{18}
\]
As the utility of a representative individual depends on private consumption only (there is no public consumption good) regional governments maximise:

\[
\max: \frac{L^i}{\bar{N}^i} c^{i,e} + \frac{\bar{N}^i - L^i}{\bar{N}^i} c^{i,u}, \tag{19}
\]
where \(L^i/\bar{N}^i\) and \((\bar{N}^i - L^i)/\bar{N}^i\) are the shares of individuals that are employed and unemployment in region \(i\).

3.1. Expenditure competition

Let us assume in a first scenario that both jurisdictions can only use head taxes on both employed and unemployed individuals, and that regional governments play Nash in public

\(^9\)Section 3 analysis in detail how the tax and expenditure policies affect the interest rate.
expenditure levels $B^i$. The relevance of fiscal competition with public expenditure levels has recently been validated empirically by Hauptmeier et al. (2012). Using (16), (17), (18) and (6) in (19), the maximisation problem of a representative government reads:

$$\max \frac{1}{N^i}[F^i(K^i, L^i, \bar{Z}^i, B^i) - rK^i] + \frac{\bar{K}}{N} - \frac{B^i}{N^i},$$

(20)

where the representative region accounts for the reactions of the endogenous variables to a change in public input provision: $K^i = K^i(B^i), L^i = L^i(B^i)$ and $r = r(B^i).^{10}$ Maximising with respect to $B^i$, and using the profit maximisation conditions (4) and (5), the first-order condition for a maximum can be expressed as:

$$F^i_B = 1 - \bar{w}^i \frac{\partial L^i}{\partial B^i} + \frac{\partial r}{\partial B^i} M^i,$$

(21)

where

$$M^i \equiv K^i - \bar{N}^i \frac{\bar{K}}{N}$$

(22)

are the capital imports (exports) in (from) region $i$ if $M^i$ is positive (negative).^{13} As a first result, efficiency in public input provision depends on the assumption of (a)symmetry in both jurisdictions. For identical regions ($M^i = 0$), we have $F^i_B = 1 - \bar{w}^i (\partial L^i / \partial B^i) < 1$, and we derive Proposition 1 for public input provision in the non-cooperative equilibrium:

**Proposition 1.** If two large and symmetric regions compete over public input provision levels with a head tax as the sole financing instrument, public inputs will be provided in excess of the first-best provision level, where the marginal product of the public input equals marginal costs ($F^i_B = 1$).

The intuition behind Proposition 1 is that regional decision makers have an incentive to augment regional employment by increasing the level of public input provision in excess of the first-best level where $F^i_B = 1$ holds. Stated differently, at the efficient provision level of the public input, the representative jurisdiction perceives that it can further increase the utility of individuals by augmenting employment through a rise in the provision level of the

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10They do not, however, account for the effects of a policy change on the other region. This means the changes in capital and labour in region $j$ associated with policy changes in region $i$ ($K^j = K^j(B^i)$ and $L^j = L^j(B^i)$) remain unaccounted for in region $i$.

11In the maximisation problem (20), the term in brackets reflects the income from land and labour. With a modified representation of the maximisation problem it becomes obvious that governments maximise (per-capita) production in the jurisdiction less payments for the capital imports and less the spending for the public input: $\frac{1}{N^i}[F^i(K^i, L^i, \bar{Z}^i, B^i) - r(K^i - \bar{N}^i \frac{\bar{K}}{N}) - B^i]$.

12Appendix 5.3 contains a detailed derivation (21).

13For a presentation of the optimal provision rule as a function of the capital flows, see also DePater and Myers (1994) who analyse fiscal competition with public consumption goods and full employment.
public input. We are able to distinguish between a direct and an indirect effect arising from an increasing provision level of the public input. To see this, observe that (13) can also be presented as 
\[ \frac{\partial L}{\partial B_i} = \frac{\partial L}{\partial B_i} \bigg|_{L} + \frac{\partial L}{\partial K_i} \cdot \frac{\partial K_i}{\partial B_i}, \]
where the index \( L \) denotes the effects that can be derived in the labour market equilibrium.\(^\text{14}\) 
First, as a direct effect from a higher public input provision level, the increased marginal productivity of labour augments employment. Second, according to the indirect effect, employment rises from an increase in the marginal productivity of labour resulting from a higher capital endowment in a jurisdiction, where the latter is a consequence from the rise in the public input provision level.

We are now turning to the case of non-identical (asymmetric) regions, where (21) is obtained in the non-cooperative equilibrium, and, as a consequence, the provision level of the public input depends also on the fact whether a jurisdiction is a net importer (\( M^i > 0 \)) or exporter (\( M^i < 0 \)) of capital. For a capital exporter it is straightforward to show that the public input is overprovided relative to the first-best scenario. This is because, in addition to the inefficiency that is caused from imperfect local labour markets (see above), the pecuniary externality works towards an overprovision of the public factor. With an increase of the public input provision level, the capital-exporting jurisdiction (e.g., \( i \)) can increase interest payments from the capital-importing jurisdiction \( j \) (\( \partial r / \partial B_i > 0 \)), and this creates an incentive for the capital-exporting jurisdiction to raise public input provision in excess of the optimal level. According to the same logic, the capital-importing region \( j \) has an incentive to decrease the level of public input provision in order to decrease its interest payments. If the incentive to appropriately manipulate the interest rate exceeds (is equal to, is below) the incentive to increase the level of the public input arising from the inefficiency in the labour market, the capital-importing jurisdiction provides the public input below (according to, in excess of) the first-best efficient level.\(^\text{15}\)

**Proposition 2.** Assume public input financing with head taxes in two large and asymmetric regions that play Nash in public expenditure levels \( B^i \). 
\textbf{a.} The capital-exporting region will overprovide the public input. 
\textbf{b.} In the capital-importing jurisdiction the public input can be either under- or overprovided, or be provided according to the first-best provision rule.

The results clearly deviate from the outcome of fiscal competition with full employment, where public input provision is not distorted from a jurisdiction’s incentive to improve local labour market conditions. With \( \partial L^i / \partial B^i = 0 \), we arrive at \( F_B^i = 1 + (\partial r / \partial B^i) M^i \), in which case public input provision in asymmetric jurisdictions is distorted from a pecuniary

\(^{14}\)Applying the implicit function theorem, we derive from (5): \( \frac{\partial L^i}{\partial B^i} \bigg|_{L} = -\frac{F^i_{LB}}{F^i_{LL}} > 0 \) and \( \frac{\partial L^i}{\partial K^i} \bigg|_{L} = -\frac{F^i_{KL}}{F^i_{LL}} > 0 \) (Appendix 5.4 contains a detailed derivation; see also Ogawa et al. (2006b) for the tax game with a large number of small countries.).

\(^{15}\)Taken alone, the former effect results in an underprovision in the capital-importing region. In detail, the former effect will outweigh the latter, if the savings in interest payments from a reduction in \( B^i \) exceed the increase in the sum of wage payments for the newly hired workers (see (21)).
externality, while governments will provide the optimal level of a public input for symmetric jurisdictions (Arnold and Fuest 1999).

3.2. Competition in tax and expenditure levels

Considering now the tax game where both jurisdictions have the capital tax and the head tax at disposal for the provision of the public factor. If the capital tax and the public input are the strategic choice parameters, the jurisdictions solve the maximisation problem:

$$\max \frac{1}{N_i} [F_i(K_i^i, L_i, \bar{Z}_i, B_i) - (r + T_i^i) K_i^i] + r \frac{K}{N} - \frac{1}{N_i} (B_i - T_i^i K_i^i),$$

(23)

with $K_i^i = K_i^i(T_i^i, B_i)$, $L_i^i = L_i^i(T_i^i, B_i)$ and $r = r(T_i^i, B_i)$. In this case, the first-order conditions for a maximum read:

$$- \bar{w}^i \frac{\partial L_i^i}{\partial T_i^i} + T_i^i \frac{\partial K_i^i}{\partial T_i^i} - \frac{\partial r}{\partial T_i^i} M^i = 0$$

(24)

and

$$F_B^i = 1 - \bar{w}^i \frac{\partial L_i^i}{\partial B_i^i} - T_i^i \frac{\partial K_i^i}{\partial B_i^i} + \frac{\partial r}{\partial B_i^i} M^i.$$  

(25)

From (24) we derive for the optimal capital tax rate: \(^16\)

$$T_i^i = -\bar{w}^i \left( \frac{\partial L_i^i}{\partial K_i^i} \right)_{L} - \left[ F_{KK}^i + F_{KL}^i \frac{\partial L_i^i}{\partial K_i^j} \right]_{L} M^i.$$  

(26)

Let us first assume again that both jurisdictions are identical. Then each jurisdiction will choose $T_i^i = -\bar{w}^i \left( \frac{\partial L_i^i}{\partial K_i^i} \right)_{L}$, which is negative as capital and labour are complements in production ($F_{KL}^i > 0$). \(^17\)

**Proposition 3.** Assume public input financing with capital and head taxes, and that two large and identical jurisdictions play Nash in capital taxes and the public input. Both jurisdictions will subsidise capital in the non-cooperative equilibrium and rely on the head tax for public input financing.

A subsidisation of capital in the equilibrium with labour market distortions is also derived for fiscal competition between a large number of small jurisdictions. For identical assumptions on the labour market model, this result is well-known for governmental provision of public

\(^16\) See Appendix 5.5 for the derivation.

\(^17\) As the assumption on the symmetry of jurisdictions includes that the fixed wage is determined at an identical level in both jurisdictions ($\bar{w} = \bar{w}^i$), this implies that $T_i^i = T_j^i$, and therefore efficiency in the capital allocation.
consumption goods (see Ogawa et al. 2006a and Eichner and Upmann 2012), and can be shown to hold also for publicly provided public inputs (see Ogawa et al. 2006b and Pauser 2013).

Observe that in two large and asymmetric jurisdictions, non-identical capital tax rates will result in a distortion in the capital allocation between jurisdictions. The equilibrium capital tax rate, which is well-known from tax competition assuming competitive labour markets \( T^i = -F^j_{KK}M^i \) is, next to the term that accounts for the distortion in the labour markets \( -w^i \partial L^i/\partial K^i |_L \), modified through the additional term \(-F^j_{KL}(\partial L^j/\partial K^j)M^i \). The effect of a change in the capital tax rate on the interest rate, and therefore also on the well being of the residents in the neighbouring region (pecuniary externality), is not as easily determined as for non-distorted labour markets.\(^{18}\) In the latter case, the effect that the marginal productivity of capital in the capital-importing region (e.g., \( j \)) rises once the amount of capital invested in that region falls, is accounted for by the term \( F^j_{KK} < 0 \), where the decreased capital endowment in the capital-importing region is due to the fact that the capital-exporting country (\( i \)) subsidises capital. This effect contributes to a rise in the interest rate. However, a second effect is obtained when labour markets are distorted as a decline in the amount of capital invested in region \( j \) decreases employment in that region \( (\partial L^j/\partial K^j > 0) \). As a consequence, as capital and labour are complements in production, this effect decreases the marginal productivity of capital in jurisdiction \( j \), and thus counteracts the first effect which has been found to increase the marginal productivity of capital. We find that for our assumptions on the production technology, the former effect dominates the latter \( (|F^j_{KK}| > F^j_{KL}(\partial L^j/\partial K^j)|_L) \),\(^{19}\) so that the marginal productivity of capital rises in region \( j \) when capital is subsidised in the capital-exporting region \( i \). This implies that - as for the institutional setting of competitive labour markets - the common interest rate rises should the capital tax rate be decreased in the capital-exporting region.

According to (26), the sign of the capital tax rate depends on the magnitude of two counteracting effects in the capital-importing region \( (M^i > 0) \): While the fact that labour markets are imperfect works towards a subsidisation of capital, the effect of the tax rate on the common interest rate (which triggers pecuniary externalities) creates incentives to tax capital.

**Proposition 4.** Assume tax competition between two large and asymmetric regions with capital taxes and public input provision, where capital and head taxes are disposable for financing. \( a. \) The capital-exporting region subsidises capital. \( b. \) The capital-importing region may choose negative, positive or no taxation of capital in the non-cooperative equilibrium.

\(^{18}\)This effect is captured by the second term in (26). Note therefore that (14) can also be presented as \( \partial r/\partial T^i = -F^j_{KK} + F^j_{KL}(\partial L^j/\partial K^j)|_L \) \((\partial K^i/\partial T^i) \), and (26) as \( T^i = -w^i \partial L^i/\partial T^i + \partial r/\partial T^i M^i \) (see (A-10) and (A-15) in the Appendix.).

\(^{19}\)In detail, this follows from decreasing returns to scale in capital and labour (equation (3)).
Again, results differ for the institutional setting of competitive labour markets, where the derivation of the capital tax rate in the Nash equilibrium is more straightforward. With $\partial L^i/\partial K^i|_L = \partial L^j/\partial K^j|_L = 0$ in (26), one derives $T^i = -F^i_{K,K} M^i$. In that case, identical jurisdictions will not tax capital, while for asymmetric jurisdictions the optimal tax rate is negative for the capital-exporting jurisdiction ($M^i < 0$) and positive for the capital-importing jurisdiction ($M^i > 0$).

Finally, when both choose the capital tax rate according to (26), we derive from (25)\(^{21}\)

$$F_B^i = 1 - \bar{w}^i \frac{\partial L^i}{\partial B^i} \bigg|_L < 1 \quad (27)$$

for the optimal provision rule.

**Proposition 5.** Assume tax competition between two large regions with a source-based capital tax and public inputs, where also head taxes are available for revenue generation. Both regions will provide public inputs in excess of the first-best provision level, irrespective of the asymmetry assumption.

The intuition behind Proposition 5 is that decentralised regions have an incentive to augment employment in their region (i.e. to decrease unemployment) by raising the public input provision level in excess of the first-best provision level. In addition, as the capital tax is available to strategically influence the net-of-tax price of capital, jurisdictions abstract from using public input provision for this purpose, and public input provision levels are independent of the fact whether a region imports or exports capital.\(^{22}\)

In contrast, for undistorted labour markets, public input provision levels are efficient for fiscal competition between two large regions in the tax game with capital and head taxes (see Arnold and Fuest 1999).

### 3.3. A labour tax as an additional fiscal policy parameter

In this section, we consider a labour tax ($V^i$) as a potential further source of revenue generation for regional governments. We then examine the implications for the non-cooperative

\(^{20}\)See DePater and Myers (1994) for the provision of public consumption goods and Arnold and Fuest (1999) for the provision of public inputs.

\(^{21}\)See Appendix 5.5 for a derivation.

\(^{22}\)Ogawa et al. (2006b) derive a provision rule identical to (27) for the tax game with head and capital taxes, when a large number of identical regions is assumed. Equilibrium capital tax rates, however, differ for the two specifications as they depend on the assumption of asymmetry between jurisdictions. Ogawa et al. (2006b) find, in addition, that public input provision according to (27) is second-best efficient, where the second-best efficiency conditions are derived under the constraint of the fixed-wage.
equilibrium in asymmetric tax competition with unemployment. The view that labour taxes provide a realistic extension in an international tax competition setting is, for instance, shared by Eichner and Upmann (2012). With this additional fiscal instrument at hand, the balanced-budget constraint of a representative government reads $B^i = T^i K^i + H^i \bar{N}^i + V^i L^i$, while the profit maximisation condition of firms, that determines the optimal employment of labour, changes to $F^i_{L}(\cdot) = \bar{w}^i + V^i$ under this setting. Note that, such as for the capital tax, a positive (negative) value of $V^i$ is a tax (a subsidy) on the production factor. For the reactions of the endogenous variables $K^i$ and $L^i$ to a change in the policy instruments $T^i$, $B^i$ and $V^i$, we derive in addition to (10)-(13):

$$\frac{\partial K^i}{\partial V^i} = \frac{-F^i_{KK}F^i_{LL}}{F^i_{LL}A^j + F^i_{LL}A^i} < 0, \quad (28)$$

$$\frac{\partial L^i}{\partial V^i} = \frac{A^j + F^i_{KK}F^i_{LL}}{F^i_{LL}A^j + F^i_{LL}A^i} < 0. \quad (29)$$

As for an increase in the capital tax rate, both the capital endowment and the employment decrease in a jurisdiction if the representative government opts for a higher labour tax. The intuition behind the detected reactions of the endogenous production factors, labour and capital, can be traced back to the fact that the labour tax - such as the source-based capital tax - is distortionary. A higher labour tax decreases employment as firms face higher labour costs. Moreover, reduced labour demand declines the marginal product of capital, as a result of which also the demand for capital declines in that region. In addition, it is straightforward to show that the interest rate is affected by a jurisdiction’s variation in the labour tax by:

$$\frac{\partial r}{\partial V^i} = \frac{F^i_{KL}A^j}{F^i_{LL}A^j + F^i_{LL}A^i} < 0. \quad (30)$$

Again, the sign of the effect on the net-of-tax return to capital is identical to that of an increase in the capital tax, and of opposite direction to that of an increase in public input provision.

In analogy to the previous section, we aim to solve the maximisation problem of a representative jurisdiction. Assuming that the labour tax serves as additional policy parameter in both jurisdictions, we derive

$$(\bar{w}^i + V^i) \frac{\partial L^i}{\partial T^i} + T^i \frac{\partial K^i}{\partial T^i} - \frac{\partial r}{\partial T^i} M^i = 0, \quad (31)$$

23 Also in this tax game, the head tax is endogenous, and it is chosen so that the public budget constraint is fulfilled.

24 Observe from (11) and (28) that the reaction of a region’s capital endowment to a change in the labour tax is identical to the change in employment resulting from capital taxation ($\partial K^i/\partial V^i = \partial L^j/\partial T^i$).
\begin{equation}
(\tilde{w}^i + V^i) \frac{\partial L^i}{\partial V^i} + T^i \frac{\partial K^i}{\partial V^i} - \frac{\partial r}{\partial V^i} M^i = 0, \tag{32}
\end{equation}

\begin{equation}
F^i_B = 1 - (\tilde{w}^i + V^i) \frac{\partial L^i}{\partial B^i} - T^i \frac{\partial K^i}{\partial B^i} + \frac{\partial r}{\partial B^i} M^i \tag{33}
\end{equation}

as the first-order conditions for a maximum. Accounting for the comparative statics results, one obtains for the (interior) solution of the decentralised equilibrium:

\begin{equation}
T^i = - \left[ F^j_{KK} + F^j_{KL} \frac{\partial L^j}{\partial K^j} \right] M^i, \tag{34}
\end{equation}

\begin{equation}
V^i = -\tilde{w}^i, \tag{35}
\end{equation}

and

\begin{equation}
F^i_B = 1. \tag{36}
\end{equation}

The non-cooperative equilibrium can be described as follows:

**Proposition 6.** Assume tax competition with two large jurisdictions, where \( T^i, B^i \) and \( V^i \) (and not \( H^i \)), are the strategic instruments. **a.** Regional governments of both regions will subsidise labour at a rate equal to the exogenously determined wage rate. **b.** For two asymmetric jurisdictions, the capital-exporting region subsidises capital and the capital-importing region chooses a positive tax rate on capital. Symmetric jurisdictions will abstract from capital taxation. **c.** Both regions provide public inputs according to the first-best provision rule.

A comparison with the efficiency conditions (26) and (27) of the tax game with capital and head taxes makes obvious how the availability of the labour tax alters the outcome in the non-cooperative equilibrium. Optimality condition (35) leads to Proposition 6.a. As a result of the subsidisation of labour, labour supply of the previously unemployed individuals is absorbed. As a direct consequence, potential distortionary effects on the equilibrium capital tax rate and on public good provision that can be traced back to the wage rigidity, and which are derived for the tax game with head and capital taxes, are no longer present. According to efficiency condition (34), which determines the equilibrium tax rate, the pecuniary externality remains as the sole cause for a potential distortion in the capital allocation, contrary to the situation in the tax game with capital and head taxes, which states that the equilibrium tax on capital is also affected from unemployment (see (26)). As a result, the capital-exporting jurisdiction subsidises capital while the capital-importing region taxes capital according to (34).\(^{26}\) Probably the most interesting result is that efficiency in public input provision in the non-cooperative equilibrium can be restored through the introduction

\(^{25}\)See the Appendix 5.6.

\(^{26}\)The sign of the capital tax was ambiguous for the capital-importing region in the tax game with head and capital taxes (Section 3.2).
of labour taxes (Proposition 6.c).\textsuperscript{27} This follows directly from inserting (34) and (35) in (33). As the underlying reason we find that, first, from optimality condition (35), imperfect labour markets can be excluded as a potential source for inefficiency in public input provision in the non-cooperative equilibrium. The distortion in the labour market has in contrast been identified as a potential reason for inefficiency in our examined tax games with an absence of the labour tax (sections 3.1 and 3.2). Second, as evident in section 3.2, if the capital tax is available as an instrument to manipulate the net-of-tax interest rate appropriately in each region, this eliminates a jurisdiction’s incentive to use public input provision as a means to manipulate the net-of-tax interest rate (section 3.1). As a result, when all of the introduced policy instruments are available, each region provides the public input according to the modified Samuelson-condition for the efficient provision of public inputs.

4. Concluding remarks

The present paper examines efficiency in public input provision in two large and asymmetric regions with regional labour market distortions. Inefficiency in the decentralised economy may arise from both imperfect labour markets and asymmetry in tax competition. In addition to fiscal and pecuniary externalities, the distortion in the local labour markets adds a third cause for inefficiency in fiscal competition. As a consequence, also under the realistic scenario of unemployment in asymmetric tax competition with productive government spending, incentives for regional governments to use source-based capital taxes remain limited. When labour markets are distorted, jurisdictions use the available policy instruments not only to augment capital endowment and to manipulate the net-of-tax price of capital appropriately but also to increase the share of employed workers in their jurisdiction. In this sense, allowing for the additional feature of imperfect labour markets may even intensify interjurisdictional competition for mobile production factors.

As a general implication for the optimal fiscal policy in a decentralised economy, adding a potential source of inefficiency in fiscal competition requires that policy makers have an appropriate policy tool at disposal to correct for the associated externality. With an additional instrument for the taxation/subsidisation of labour, the respective government’s use of the remaining policy instruments is not distorted from unemployment as governments choose a subsidy on labour that eliminates the inefficiency caused by the wage rigidity. This argument also relates to the key result in public economics that governments should have at least as many instruments as economic targets (Tinbergen). Another implication is that the classic result of underprovision of public goods does not necessarily survive when one incorporates some real world aspects in interjurisdictional tax competition. As emphasised in the introduction, there is no reason to abstract from economic factors such as regional labour market distortions, asymmetry between jurisdictions, and the possibility that government

\textsuperscript{27}Note that for the tax game with symmetric tax competition and an identical set of tax instruments, Eichner and Upmann (2012) obtain the result of an efficient governmental provision of public consumption goods.
spending is productive in jurisdictions that engage in tax competition.

Of course, our results are derived from a highly-stylized model of interjurisdictional competition. To bring theoretical work more in line with economic reality, focusing on the composition of governmental spending between productive and consumptive public goods within the presented framework could be the next step in research. It is also well known from the literature in public economics that most public goods are subject to at least some degree of congestion, and the introduction of congestion externalities as a further source of inefficiency would bring an interesting additional element into the analysis. Third, it is also worth considering some alternative labour market model in asymmetric tax competition with productive government spending.

5. Appendix

5.1. Derivation of the reaction of capital and labour to a change in policy instruments

This Appendix derives the reactions of $K^i$, $L^i$, and $L^j$ to a change in all policy instruments assumed in the paper, $B_i$ and $T_i$ and $V_i$, where $V_i$ is the labour tax, which is introduced in section 3.3. The reactions can be obtained from the system:

$$
\begin{pmatrix}
F_{iK} + F_{iK}^j & F_{iL}^j & -F_{iL}^j \\
F_{iK}^j & F_{iL} & 0 \\
-F_{iL}^j & 0 & F_{iL}^j
\end{pmatrix}
\begin{pmatrix}
dK^i \\
dL^i \\
dL^j
\end{pmatrix} =
\begin{pmatrix}
-F_{iK}B_i & 1 & 0 \\
-F_{iL}B_i & 0 & 1 \\
0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
 dB^i \\
 dT^i \\
 dV^i
\end{pmatrix}. \tag{A-1}
$$

For the determinant of (A-1), we derive with (3):

$$
F_{iK}^j F_{iL}^j - (F_{iK}^j)^2 F_{iL}^j + F_{iL}^j F_{iK}^j F_{iL}^j - F_{iL}^j (F_{iK}^j)^2 = F_{iL}^j A^j + F_{iL}^j A^i > 0. \tag{A-2}
$$

By applying the Cramer rule to the system, one gets (10) - (13), as well as (28) and (29) for the change of the endogenous variables in jurisdiction $i$ to a change in policy instruments in that jurisdiction. In addition, we derive for the change in employment in jurisdiction $j$ from a change in policy instruments in $i$:28

$$
\frac{\partial L^j}{\partial T^i} = \frac{F_{iL}^j F_{iL}^j}{F_{iL}^j A^j + F_{iL}^j A^i} > 0 \tag{A-3}
$$

$$
\frac{\partial L^j}{\partial B^i} = -\frac{F_{iK}^j (F_{iL}^j F_{iK}^j - F_{iL}^j F_{iL}^j)}{F_{iL}^j A^j + F_{iL}^j A^i} < 0 \tag{A-4}
$$

28Because of $dK^i = -dK^j$ we can, in analogy, derive the reactions of $K^j$ to the change of the endogenous variables in jurisdiction $i$.  

18
\[
\frac{\partial L^j}{\partial V^i} = -\frac{F^j_{KL} F^i_{KL}}{F^j_{LL} A^j + F^i_{LL} A^i} > 0.
\] (A-5)

5.2. The effects of fiscal policy on the interest rate

This Appendix derives the reaction of the net-of-tax interest rate to a change in policy parameters. From and (4) and (10) - (13), we derive:

\[
r = F^i_K [K^i(T^i, T^j, B^i, B^j, \bar{Z}^i, \bar{Z}^j, \bar{w}^i, \bar{w}^j, \bar{K}), L^j(T^j, T^j, B^i, B^j, \bar{Z}^i, \bar{Z}^j, \bar{w}^i, \bar{w}^j, \bar{K}), B^i, \bar{Z}^i] - T^i.
\] (A-6)

Derivation with respect to \(B^i\) yields:

\[
\frac{\partial r}{\partial B^i} = F^i_{KK} \frac{\partial K^i}{\partial B^i} + F^i_{KL} \frac{\partial L^i}{\partial B^i} + F^i_{KB}.
\] (A-7)

Substitution with the comparative statics results derived in (12) and (13) yields equation (15). In analogy, we derive (14).

Note that with (12), equation (15) can, in addition, be presented as:

\[
\frac{\partial r}{\partial B^i} = -A^j \frac{\partial K^i}{\partial B^i}.
\] (A-8)

Using also (3) and (A-13), we obtain

\[
\frac{\partial r}{\partial B^i} = - F^i_{KK} + F^i_{KL} \left( \frac{\partial L^j/\partial K^j}{L^j} \right) \left( \frac{\partial K^i/\partial B^i}{L^j} \right).
\] (A-9)

In analogy, we derive:

\[
\frac{\partial r}{\partial T^i} = - F^i_{KK} + F^i_{KL} \left( \frac{\partial L^j/\partial K^j}{L^j} \right) \left( \frac{\partial K^i/\partial T^i}{L^j} \right).
\] (A-10)

5.3. Derivation of the first-order condition (21)

Derivation of (20) with respect to \(B^i\) and setting the result equal to zero yields:

\[
\max: \frac{1}{N^i} \left[ F^i_K \frac{\partial K^i}{\partial B^i} + F^i_L \frac{\partial L^i}{\partial B^i} + F^i_B - r \frac{\partial K^i}{\partial B^i} - \frac{\partial r}{\partial B^i} K^i \right] + \frac{\partial r}{\partial B^i} \bar{K} - \frac{1}{N^i} = 0.
\] (A-11)

Using the profit maximisation conditions (4) and (5), and accounting for the fact that \(T^i = 0\), the first-order condition for a maximum can be presented as (21).

5.4. Comparative statics in the labour market equilibrium

Using (5):

\[
W^i = F^i_L (K^i, L^i, B^i, \bar{Z}^i) - \bar{w}^i = 0.
\] (A-12)
Applying the implicit function theorem, we derive for the change in $L^i$ in the labour market equilibrium that results from a change in $K^i$ and $B^i$:

$$\frac{\partial L^i}{\partial K^i} \bigg|_L = -\frac{\partial W^i/\partial K^i}{\partial W^i/\partial L^i} = -\frac{F_{KL}^i}{F_{LL}^i} > 0$$

(A-13)

and

$$\frac{\partial L^i}{\partial B^i} \bigg|_L = -\frac{\partial W^i/\partial B^i}{\partial W^i/\partial L^i} = -\frac{F_{LB}^i}{F_{LL}^i} > 0.$$  

(A-14)

### 5.5. Derivation of the first-order conditions (26) and (27)

Solving (24) for $T^i$ yields:

$$T^i = -w^i \frac{\partial L^i}{\partial T^i} - \frac{\partial r/\partial T^i}{\partial K^i/\partial T^i} \cdot M^i.$$  

(A-15)

We are also able to derive:

$$\frac{\partial L^i}{\partial T^i} = \frac{\partial L^i}{\partial K^i} \bigg|_L.$$  

(A-16)

Using that and (A-10), we get (26). Substituting (26) for $T^i$ in (25) yields:

$$F_B^i = 1 - w^i \frac{\partial L^i}{\partial B^i} + \tilde{w}^i \frac{\partial L^i}{\partial K^i} \bigg|_L \frac{\partial K^i}{\partial B^i} + \left( F_{KK}^i + F_{KL}^i \frac{\partial L^j}{\partial K^j} \bigg|_L \frac{\partial K^i}{\partial B^j} \right) F_{BB}^i + \frac{\partial r}{\partial B^i} M^i.$$  

(A-17)

With (A-9), we have

$$F_B^i = 1 - w^i \frac{\partial L^i}{\partial B^i} + \tilde{w}^i \frac{\partial L^i}{\partial K^i} \bigg|_L \frac{\partial K^i}{\partial B^i}.$$  

(A-18)

With help of

$$\frac{\partial L^i}{\partial B^i} = \frac{\partial L^i}{\partial B^i} \bigg|_L + \frac{\partial L^i}{\partial K^i} \bigg|_L \frac{\partial K^i}{\partial B^i}$$

(A-19)

we finally derive (27).

### 5.6. Derivation of the first-order conditions (34) - (36)

From (31) and (32) we derive:

$$T^i = \frac{\partial r/\partial T^i}{\partial K^i/\partial T^i} M^i - (\tilde{w}^i + V^i) \frac{\partial L^i}{\partial K^i/\partial T^i} = \frac{\partial r/\partial V^i}{\partial K^i/\partial V^i} M^i - (\tilde{w}^i + V^i) \frac{\partial L^i}{\partial K^i/\partial V^i}.$$  

(A-20)

Observe from the comparative statics results that

$$\frac{\partial r/\partial T^i}{\partial K^i/\partial T^i} = \frac{\partial r/\partial V^i}{\partial K^i/\partial V^i}$$  

(A-21)
and

\[ \frac{\partial L^i}{\partial T^i} = \frac{\partial L^i}{\partial V^i}. \]  

(A-22)

As a consequence, we derive (34) and (35) in the equilibrium. Inserting both in (33), and from the observation that (15) can be presented as (A-9), one obtains (36):

\[ F_B^i = 1 + \left[ (F_{KK}^i + F_{KL}^i \frac{\partial L^i}{\partial K^i} \bigg|_{L} \right] \frac{\partial K^i}{\partial B^i} M^i + \frac{\partial r}{\partial B^i} M^i = 1. \]  

(A-23)

References


