A Note on Aggregating Human Capital Across Heterogeneous Cohorts

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January 30, 2012

* * * Preliminary and incomplete * * *

Abstract. We consider a general framework for computing the aggregate human capital stock under heterogeneity across population cohorts, distinguishing between aggregate human capital stocks in the whole population and in the labor force. Based on this framework, we find that the "macro-Mincer" (log-linear) relationship between aggregate human capital and average years of schooling obtains only in cases which are inconsistent with heterogeneity in years of schooling and based on empirically implausible demographic survival laws.

Keywords: human capital, aggregation, heterogeneity, Mincerian wage equation

JEL Classification Numbers: J24, O47.

1 Introduction

The recent literature has questioned the prevalent view that the log-linear Mincer (i.e., "micro-Mincer", cf. Mincer, 1974) relationship between individual wages (or human capital stocks) and years of schooling can be carried forward to country-level data on *aggregate* human capital stocks and *average* years of schooling (cf. Krueger and Lindahl, 2001, Bloom et al., 2004), from a number of standpoints:

- the "macro-Mincer" approach assumes perfect substitutability between unskilled and skilled labor (Pandey, 2008, Jones, 2011a, 2011b),
- it assumes that each individual's skill level can be summarized by a single number and thus there is no heterogeneity in tasks (Jones, 2011b),
- it considers years of schooling as an exogeneous variable and thus neglects individuals' optimal decisions on the duration of their schooling (Jones, 2011a),

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- it neglects the fact that maintaining a constant aggregate level of human capital in the society requires replacement investment, because human capital is embodied in people whose lifetimes are finite (Growiec, 2010).

Violation of any of the above assumptions has been shown to lead to significant departures from the baseline "macro-Mincer" relationship between the aggregate human capital stock and average years of schooling.

The objective of the current note is to add to the last line of criticisms of the "macro-Mincer" approach by emphasizing two important theoretical points, not addressed so far in the literature. First, based on a general framework for computing the aggregate human capital stock under heterogeneity across cohorts, building on Growiec (2010), we shall show that the "macro-Mincer" relationship between aggregate human capital and average years of schooling is generally *lost upon aggregation*. More precisely, we find that even if the cross-sectional "micro-Mincer" relationship does hold at the level of individuals, the "macro-Mincer" equation can be obtained only in very special cases. We proceed to explain that these cases are *inconsistent with heterogeneity*, insofar they require the aggregated individuals to have an equal number of years of schooling. Furthermore, in the case where individuals first attend school full time and then work full time, the "macro-Mincer" equation requires the demographical survival law to have the "perpetual youth" property (Blanchard, 1985), which is empirically implausible.

Second, we shall also demonstrate an important difference in aggegation results whether human capital stocks in the *whole population* or in the *labor force* are considered. In particular, the "macro-Mincer" relationship can only be obtained (under additional restrictions) for the latter case but not for the former. In the empirical literature (see e.g., Caselli and Coleman, 2006), the macro-Mincer approach is often applied to educational attainment of the whole population, though – or at least of the whole working-age population (which is somewhat closer to our definition). Our analysis strongly suggests that these concepts should not be used interchangeably.

In all our calculations, we shall maintain the assumption that skill levels are perfectly substitutable and there is no intra-cohort heterogeneity of tasks or skills. Hence, all heterogeneity considered here comes from the fact that people are born at different times, and gradually accumulate human capital across their lives. By making these assumptions, we attempt to isolate the effects coming from the heterogeneity of human capital due to demographics alone. Adding intra-cohort heterogeneity to the picture is left for further research.

2 Aggregation of human capital across population cohorts

2.1 Framework

We denote the current calendar time as t, and a person's age as τ. A person who is τ years old in year t must have thus been born at $t-\tau$. At time t, there is a continuum of mass $N(t)$ of individuals. Our results are obtained under the following assumptions.

Assumption 1 *Human capital of the representative τ years old individual born at time t is accumulated using the* linear *production function:*

$$\frac{\partial}{\partial \tau} h(t,\tau) = [\lambda \ell_h(t,\tau) + \mu \ell_Y(t,\tau)] h(t,\tau), \tag{1}$$

where $\lambda \geq 0$ denotes the unit productivity of schooling, and $\mu \geq 0$ denotes the unit productivity of on-the-job training (experience accumulation). $\ell_h(t,\tau) \in [0,1]$ is the fraction of time spent by an individual born at t and aged τ on formal education, whereas $\ell_Y(t,\tau) \in [0,1]$ is the fraction of time spent at work. We assume $\ell_h(t,\tau) + \ell_Y(t,\tau) \leq 1$ for all $t, \tau \geq 0$, and take $h(t,0) \equiv h_0 > 0$.

Even though the current framework singles out the time spent on education and work only, it can easily accomodate other uses of time, such as leisure or childrearing. In particular, we thus also allow for retirement. We shall say that these alternative possibilities are exercised when $\ell_h(t,\tau) + \ell_Y(t,\tau) < 1$.

Equation (1) can be easily integrated with respect to the individual's age, to yield the human capital stock of an individual born at t, aged τ:

$$h(t,\tau) = h_0 \exp\left[\lambda \int_0^\tau \ell_h(t,s)ds + \mu \int_0^\tau \ell_Y(t,s)ds\right]. \tag{2}$$

This is directly the "micro-Mincer" equation, signifying the log-linear relationship between the individuals' human capital and their cumulative stocks of education and work experience.

Assumption 2 *At every age $\tau \geq 0$, the individual may either survive or die. The unconditional survival probability is denoted by $m(\tau)$, with $m(0) = 1, \lim_{\tau\to\infty} m(\tau) = 0$ and with $m(\tau)$ weakly decreasing in its whole domain. The survival probability does not depend on calendar time t.*

Please note that by assuming the survival law to be independent of t, we exclude the possibility of declining mortality due to, e.g., progress in medicine. Accomodating this possibility is left for further research.

Assumption 3 *The age structure of the society (the cumulative density function) is stationary. At time t, there are $P(t,\tau) = bN(t-\tau)m(\tau)$ people aged τ in the population. The total population alive at time t is $N(t)$, with*

$$N(t) = \int_0^\infty P(t,\tau)d\tau = \int_0^\infty bN(t-\tau)m(\tau)d\tau. \tag{3}$$

The total labor force at time t is computed as

$$L(t) = \int_0^\infty P(t,\tau)\ell_Y(t-\tau,\tau)d\tau = \int_0^\infty bN(t-\tau)m(\tau)\ell_Y(t-\tau,\tau)d\tau. \tag{4}$$

By the virtue of the Law of Large Numbers, the above assumption implies that the aggregate birth rate b and death rate d are constant. This in turn implies a constant population growth rate, and thus $N(t) = N_0 e^{(b-d)t}$. In consequence, the shares of all vintages in the total population are indeed constant:

$$\frac{P(t,\tau)}{N(t)} = bm(\tau)\frac{N(t-\tau)}{N(t)} = bm(\tau)e^{-(b-d)\tau}, \qquad \text{independently of } t. \tag{5}$$

Furthermore, the death rate d is computed uniquely from the given survival law $m(\tau)$. If the birth rate times life expectancy at birth exceeds unity, then $b > d$ and thus the total population is growing. If this product is less than unity, then $b < d$ and thus the population is declining (for the derivation, please refer to Appendix A.6 in Growiec, 2010).

The first corollary from our assumptions is that, under stationary age structure, and assuming that time profiles of education and work are independent of calendar time t, i.e., $\ell_h(t,\tau) \equiv \ell_h(\tau)$ and $\ell_Y(t,\tau) \equiv \ell_Y(\tau)$, it must be the case that the human capital stock of an individual $h(t,\tau)$ depends only on her age τ, but not on the year when she was born, t. Despite exponential growth in each individual's human capital across time, we thus have $h(t,\tau) \equiv h(\tau)$ (see the discussion in Growiec, 2010).

Under the aforementioned stationarity assumptions, it also follows that the employment rate in the economy $\frac{L(t)}{N(t)}$ is independent of calendar time t, too:

$$\frac{L(t)}{N(t)} = \frac{\int_0^\infty bN(t-\tau)m(\tau)\ell_Y(\tau)d\tau}{N(t)} = \int_0^\infty be^{-(b-d)\tau}m(\tau)\ell_Y(\tau)d\tau. \tag{6}$$

Let us now place some restrictions on the considered stationary time profiles of education and work. We shall deal with three alternative, naturally understandable scenarios which can be considered as limiting cases of more general time profiles:

- **Case "S+W".** First attend school full time, until you reach S years of age, and then work full time until death:

$$\ell_h(t,\tau) = \begin{cases} 1, & \tau \leq S, \\ 0, & \tau > S, \end{cases} \qquad \ell_Y(t,\tau) = \begin{cases} 0, & \tau \leq S, \\ 1, & \tau > S. \end{cases} \tag{7}$$

- **Case "S+W+R".** First attend school full time, until you reach S years of age; then work full time, until you reach R years of age, then retire, and stay retired until death:

$$\ell_h(t,\tau) = \begin{cases} 1, & \tau \leq S, \\ 0, & \tau > S, \end{cases} \qquad \ell_Y(t,\tau) = \begin{cases} 0, & \tau \in [0,S] \cup [R,+\infty), \\ 1, & \tau \in (S,R). \end{cases} \tag{8}$$

- **Case "Fix".** Spend fixed fractions of time on schooling and work throughout your entire life:

$$\ell_h(t,\tau) \equiv \bar{\ell}_h, \qquad \ell_Y(t,\tau) \equiv \bar{\ell}_Y. \tag{9}$$

Please note that *years of schooling* are directly captured by S in the cases "S+W" and "S+W+R". On the other hand, there is no direct equivalent of S in the case "Fix". However, one can proxy years of schooling by the schooling intensity $\bar{\ell}_h$. We shall adopt this convention in our further considerations.

In accordance with Assumptions 1–3, we shall make use of the following definitions.

Definition 1 *The aggregate human capital stock of the whole population alive at time t is given by:*

$$H_{POP}(t) = \int_0^\infty P(t,\tau)h(t-\tau,\tau)d\tau. \tag{10}$$

Human capital stocks provided by individuals of all ages are perfectly substitutable. The average human capital stock in the population is $h_{POP}(t) = \frac{H_{POP}(t)}{N(t)}$.

Definition 2 *The aggregate human capital stock of the labor force working at time t is given by:*

$$H_{LF}(t) = \int_0^{\infty} P(t,\tau)\left(\ell_Y(t-\tau,\tau)h(t-\tau,\tau)\right)d\tau. \qquad (11)$$

Labor services provided by individuals of all ages are perfectly substitutable. The average human capital stock in the labor force is $h_{LF}(t) = \frac{H_{LF}(t)}{L(t)}$.

Let us now present our results under two specific survival laws $m(\tau)$, and then provide more general considerations relating to the (im)possibility and (im)plausibility of obtaining the "macro-Mincer" result, postulated in the related empirical literature.

2.2 Results under the "perpetual youth" survival law

Apart from Assumptions 1–3, let us now also assume the Blanchard's (1985) simple "perpetual youth" survival law $m(\tau) = e^{-d\tau}$, where d is directly the aggregate death rate. Under this condition, the stationary age structure satisfies $\frac{P(t,\tau)}{N(t)} = be^{-b\tau}$. The results are presented in Table 1.[1] We consider *average* human capital stocks instead of aggregate ones there, as they are – given the stationary age structure of the population – independent of calendar time t.

Table 1: Average human capital stocks under the "perpetual youth" survival law.

Case	$h_{POP}(t)$	$h_{LF}(t)$
S+W	$\frac{bh_0}{b-\lambda}\left(1-e^{(\lambda-b)S}\right) + \frac{bh_0}{b-\mu}e^{(\lambda-b)S}$	$\frac{bh_0}{b-\mu}e^{\lambda S}$
S+W+R	$\frac{bh_0}{b-\lambda}\left(1-e^{(\lambda-b)S}\right)$ $+\frac{bh_0}{b-\mu}\left(e^{(\lambda-b)S}-e^{(\lambda-\mu)S+(\mu-b)R}\right)$ $+h_0e^{(\lambda-\mu)S+(\mu-b)R}$	$\frac{bh_0}{b-\mu}\frac{e^{(\lambda-b)S}-e^{(\lambda-\mu)S+(\mu-b)R}}{e^{-bS}-e^{-bR}}$
Fix	$\frac{bh_0}{b-\lambda\bar{\ell}_h-\mu\bar{\ell}_Y}$	$\frac{bh_0}{b-\lambda\bar{\ell}_h-\mu\bar{\ell}_Y}$

In the case "S+W", $H_{LF}(t)$ is computed by aggregating individuals above the age S only, whereas $H_{POP}(t)$ is a sum of $H_{LF}(t)$, i.e., human capital of the workers (or equivalently, working-age population), and human capital of younger individuals who are still at school. In this case, the share of the working population is constant and equal to $\frac{L(t)}{N(t)} = e^{-bS}$.

In the case "S+W+R", $H_{LF}(t)$ is computed by aggregating individuals aged between S and R only, whereas $H_{POP}(t)$ supplements this stock with the human capital of younger and older individuals. In this case, the share of the working population is fixed at $\frac{L(t)}{N(t)} = e^{-bS} - e^{-bR}$.

The case "Fix" has already been considered by Growiec (2010), who concentrated on $H_{POP}(t)$ and did not compute $H_{LF}(t)$. With a fixed share of time spent on work irrespective of individuals' age, the it is however clear that $H_{LF}(t) = \bar{\ell}_Y H_{POP}(t)$, so that the qualitative results for both aggregates are identical up to a multiplicative constant. Also, the share of the working population is naturally $\frac{L(t)}{N(t)} = \bar{\ell}_Y$, and thus $h_{LF}(t) = h_{POP}(t)$.

[1] In case $\lambda = b$, the formula $\frac{bh_0}{b-\lambda}\left(1-e^{(\lambda-b)S}\right)$ should be replaced by bh_0S in the $h_{POP}(t)$ column. Furthermore, if $\mu = b$ in the case "S+W+R" then the formula $\frac{bh_0}{b-\mu}\left(e^{(\lambda-b)S}-e^{(\lambda-\mu)S+(\mu-b)R}\right)$ should be replaced by $bh_0e^{(\lambda-b)S}(R-S)$.

To ensure that the aggregate human capital stock remains finite under the considered survival law, we must assume that $\mu < b$ in the case "S+W", and $\lambda\bar{\ell}_h - \mu\bar{\ell}_Y < b$ in the case "Fix".

As regards the derivation of the "macro-Mincer" equation, we note the following straightforward proposition.

Proposition 1 (Sufficient conditions for "macro-Mincer") *Let Assumptions 1–3 hold and assume the "perpetual youth" survival law. Then the "macro-Mincer" equation holds for the labor force (but not the whole population):*

- *under the "S+W" scenario,*
- *under the "S+W+R" scenario, but only if there is no on-the-job training ($\mu = 0$).*

The Mincerian exponent is then equal to the individual rate of return to education λ. Apart from these two cases, the "macro-Mincer" equation does not hold.

As we shall see shortly, under the "S+W" scenario, the "macro-Mincer" result actually *requires* the survival law to satisfy the Blanchard's (1985) "perpetual youth" property. Unfortunately, the "perpetual youth" survival law is highly implausible empirically: it implies that irrespective of age, individuals face the same unconditional probability of dying next year. According to empirical evidence (cf. e.g., Boucekkine et al., 2002), this is clearly not the case, not even approximately.[2]

One further important caveat here is that the "macro-Mincer" relationship obtained in the current section is inconsistent with heterogeneity in years of schooling. It is obtained for $H_{LF}(t)$ in the case "S+W", which requires that every worker (i.e., every person above the age S) has the same level of education, $h_0 e^{(\lambda-b)S}$. All potential differences in workers' human capital are driven by differences in work experience, which is gradually accumulated at work if $\mu > 0$. Even more strikingly, under the "S+W+R" scenario with $\mu = 0$, the "macro-Mincer" equation requires not only years of schooling to be equal across the population, but also the individuals' levels of work experience.

2.3 Results under fixed lifetimes

Let us now substitute the Blanchard's (1985) "perpetual youth" survival law with the assumption that individuals' lifetimes are deterministically fixed at T, i.e., $m(\tau) = 1$ for $\tau < T$ and $m(\tau) = 0$ for $\tau \geq T$, with $T > S$ and $T \geq R$. Under this condition, the age structure satisfies $\frac{P(t,\tau)}{N(t)} = be^{-(b-d)\tau}$ for $\tau < T$ and zero otherwise. The aggregate death rate d is related to the age T via the equality $T = \frac{\ln b - \ln d}{b-d}$. It is obtained that $b > d$ if and only if $T > 1/b$.

The results for this case are presented in Table 2.

Under the currently considered survival law where lifetimes are bounded, aggregate human capital is always finite. From Table 2, it should also be clear that under fixed lifetimes, reproducing the "macro-Mincer" equation is possible only if there is no on-the-job training ($\mu = 0$):

Proposition 2 (Sufficient conditions for "macro-Mincer") *Let Assumptions 1–3 hold and assume that the individuals have a fixed lifetime T. Then the "macro-Mincer" equation holds for the labor force (but not the whole population):*

[2] Though it might be an accurate description of survival laws for very poor, war-ridden regions or ancient-to-medieval times.

Table 2: Average human capital stocks under fixed lifetimes.

Case	$h_{POP}(t)$	$h_{LF}(t)$
S+W	$\frac{bh_0}{b-d-\lambda}\left(1-e^{(\lambda-(b-d))S}\right) + \frac{bh_0 e^{(\lambda-\mu)S}}{b-d-\mu}\left(e^{(\mu-(b-d))S}-e^{(\mu-(b-d))T}\right)$	$\frac{(b-d)h_0 e^{(\lambda-\mu)S}}{b-d-\mu}\frac{e^{(\mu-(b-d))S}-e^{(\mu-(b-d))T}}{e^{-(b-d)S}-e^{-(b-d)T}}$
S+W+R	$\frac{bh_0}{b-d-\lambda}\left(1-e^{(\lambda-(b-d))S}\right) + \frac{bh_0}{b-d-\mu}\left(e^{(\lambda-(b-d))S}-e^{(\lambda-\mu)S+(\mu-(b-d))R}\right) + \frac{bh_0}{b-d}\left(e^{(\lambda-\mu)S+(\mu-(b-d))R}-e^{(\lambda-\mu)S+\mu R-(b-d)T}\right)$	$\frac{(b-d)h_0}{b-d-\mu}\frac{e^{(\lambda-(b-d))S}-e^{(\lambda-\mu)S+(\mu-(b-d))R}}{e^{-(b-d)S}-e^{-(b-d)R}}$
Fix	$\frac{bh_0}{b-d-\lambda\bar{\ell}_h-\mu\bar{\ell}_Y}\left(1-e^{\lambda\bar{\ell}_h+\mu\bar{\ell}_Y-(b-d)T}\right)$	$\frac{bh_0}{b-d-\lambda\bar{\ell}_h-\mu\bar{\ell}_Y}\left(1-e^{\lambda\bar{\ell}_h+\mu\bar{\ell}_Y-(b-d)T}\right)$

- *under the "S+W" scenario with $\mu = 0$,*
- *under the "S+W+R" scenario with $\mu = 0$.*

The Mincerian exponent is then equal to the individual rate of return to education λ. Apart from these two cases, the "macro-Mincer" equation does not hold.

2.4 The case without on-the-job training

The case without on-the-job training ($\mu = 0$) has already stood out as a very specific case in our above calculations. It is no coincidence. Actually, we can straightforwardly generalize our above considerations, yielding the following proposition:

Proposition 3 (Sufficient condition for "macro-Mincer") *Let Assumptions 1–3 hold and assume $\mu = 0$. Then under the "S+W" and "S+W+R" scenarios, the "macro-Mincer" equation holds for the labor force $h_{LF}(t)$ regardless of the underlying survival law $m(\tau)$. The Mincerian exponent is equal to the individual rate of return to education λ.*

Proof. Using equations (5)–(6), under the "S+W" scenario we have:

$$\begin{aligned} h_{LF}(t) &= \int_0^\infty \frac{P(t,\tau)}{L(t)}\ell_Y(t-\tau,\tau)h(t-\tau,\tau)d\tau = \int_S^\infty h_0 e^{\lambda S} b e^{-(b-d)\tau} m(\tau)\frac{N(t)}{L(t)}d\tau = \\ &= h_0 e^{\lambda S}\frac{\int_S^\infty be^{-(b-d)\tau}m(\tau)d\tau}{\int_S^\infty be^{-(b-d)\tau}m(\tau)d\tau} = h_0 e^{\lambda S}. \end{aligned} \tag{12}$$

Using equations (5)–(6) again, under the "S+W+R" scenario we have:

$$\begin{aligned} h_{LF}(t) &= \int_0^\infty \frac{P(t,\tau)}{L(t)}\ell_Y(t-\tau,\tau)h(t-\tau,\tau)d\tau = \int_S^R h_0 e^{\lambda S} b e^{-(b-d)\tau} m(\tau)\frac{N(t)}{L(t)}d\tau = \\ &= h_0 e^{\lambda S}\frac{\int_S^R be^{-(b-d)\tau}m(\tau)d\tau}{\int_S^R be^{-(b-d)\tau}m(\tau)d\tau} = h_0 e^{\lambda S}. \blacksquare \end{aligned} \tag{13}$$

The above result is driven by two crucial facts. First, the "S+W" and "S+W+R" scenarios require all working individuals to have the same number of years of schooling. They are therefore inconsistent with heterogeneity in years of schooling among the working population. Second, the assumption $\mu = 0$ (absence of on-the-job training) implies that all working individuals also have the same *human capital level*. The aggregation is thus done across entirely homogeneous population cohorts. In such a situation, it is no surprise that the Mincerian relationship between human capital and years of schooling is directly transferred from the individual to the aggregate level.

2.5 Necessary conditions for the "macro-Mincer" equation

Let us now ask the converse question: for which survival law $m(\tau)$ will the "macro-Mincer" equation be recovered from the micro-level Mincerian relationship. Growiec (2010) has already addresed this question for the scenario "Fix", showing that it is not possible unless the survival function depends on $\bar{\ell}_h$ in one crucial and arguably implausible way. For the case "S+W", as we have just shown above, however, that is possible at the level of the labor force, if the survival law satisfies the "perpetual youth" property. Furthermore, if one disregards on-the-job training (by assuming $\mu = 0$), then this result also follows in the "S+W+R" scenario and under a wide range of survival laws. In that case, however, all individuals in the labor force share exactly the same human capital level $h_0 e^{\lambda S}$, and it is precisely this homogeneity that drives the result.

It turns out that if $\mu > 0$, so there is some heterogeneity in human capital across working cohorts, then the "macro-Mincer" result can be reproduced under the "S+W" scenario *only* in the "perpetual youth" case. The following proposition holds.

Proposition 4 (Necessary conditions for "macro-Mincer") *Let Assumptions 1–3 hold, and assume that the "macro-Mincer" equation holds for the labor force, with $\mu \in (0, b)$. Then, if the individuals stay at school until the age S and then they work full-time until death, the survival law must be $m(\tau) = e^{-d\tau}$, i.e., it must satisfy the "perpetual youth" property. The implied Mincerian exponent is equal to the individual rate of return to education λ.*

Proof. Upon aggregation, we have:

$$h_{LF}(t) = \int_S^\infty h(t-\tau,\tau)\frac{P(t,\tau)}{L(t)}d\tau = h_0 e^{(\lambda-\mu)S}\frac{\int_S^\infty be^{(\mu-(b-d))\tau}m(\tau)d\tau}{\int_S^\infty be^{-(b-d)\tau}m(\tau)d\tau}. \tag{14}$$

We shall use the notation:

$$\varphi(S) = \frac{\int_S^\infty be^{(\mu-(b-d))\tau}m(\tau)d\tau}{\int_S^\infty be^{-(b-d)\tau}m(\tau)d\tau} \tag{15}$$

which implies $h_{LF}(t) = \varphi(S)\cdot h_0 e^{(\lambda-\mu)S}$. Since all $\tau \geq S$ in the considered integrals, it is easily verified that for all $S \geq 0$, $\frac{\varphi(S)}{e^{\mu S}} > 1$. Furthermore, applying the l'Hôpital's rule twice, we obtain:

$$\begin{aligned} \lim_{S\to+\infty}\frac{\varphi(S)}{e^{\mu S}} &= \lim_{S\to+\infty}\frac{\int_S^\infty e^{(\mu-(b-d))\tau}m(\tau)d\tau}{e^{\mu S}\int_S^\infty e^{(-(b-d))\tau}m(\tau)d\tau} = \\ &= \frac{1}{1-\frac{\mu\int_S^\infty e^{-(b-d)\tau}m(\tau)d\tau}{m(S)e^{-(b-d)S}}} = \frac{1}{1+\frac{\mu}{\frac{m'(S)}{m(S)}-(b-d)}}. \end{aligned} \tag{16}$$

Equation (16) will be useful later.

The current proposition refers to the functional specifications of $m(\tau)$ for which $\varphi(S) = Ge^{HS}$ for some $G > 0$ and $H \in \mathbb{R}$. Before we address this issue directly, let us formulate its important corollary. Namely, assuming this functional relationship, it is found that

$$\lim_{S\to+\infty} \frac{\varphi(S)}{e^{\mu S}} = \lim_{S\to+\infty} Ge^{(H-\mu)S}. \tag{17}$$

Since $\frac{\varphi(S)}{e^{\mu S}} > 1$ for all $S \geq 0$, then it must be the case $H \geq \mu$. Furthermore, one must set $G > 1$ so that $\varphi(0) > 1$. The cases $H > \mu$ and $H = \mu$ should be addressed separately.

Consider first the case $H > \mu$. In such case we obtain $\lim_{S\to+\infty} Ge^{(H-\mu)S} = +\infty$. Coupled with equation (16), this implies:

$$\lim_{S\to+\infty} \frac{m'(S)}{m(S)} = b - d - \mu. \tag{18}$$

We shall now pass to the central part of the proof. Positing $\varphi(S) = Ge^{HS}$ and rearranging yields:

$$\int_S^\infty be^{(\mu-(b-d))\tau} m(\tau)d\tau = Ge^{HS} \int_S^\infty be^{-(b-d)\tau} m(\tau)d\tau. \tag{19}$$

Equation (19) is a *functional identity* and thus it holds for all $S \geq 0$. It is also possible to differentiate both sides of (19) with respect to S. Doing this twice and rearranging terms, we obtain:

$$\frac{m'(S)}{m(S)} = \frac{(\mu - H - b + d)e^{(\mu-H)S} - G(d - b + H)}{G - e^{(\mu-H)S}}. \tag{20}$$

Coupling (18) with (20) and using the assumption $H > \mu$, we obtain:

$$\lim_{S\to+\infty} \frac{m'(S)}{m(S)} = b - d - H = b - d - \mu, \tag{21}$$

and thus $H = \mu$, a contradiction. The case $H > \mu$ is thus ruled out. The only remaining possibility is $H = \mu$.

Let us consider this remaining possibility. Inserting the condition $H = \mu$ into (20) and simplifying we obtain:

$$\frac{m'(S)}{m(S)} = -\frac{(G-1)(-b+d) + G\mu}{G-1} = (b - d) - \frac{G\mu}{G-1}. \tag{22}$$

Solving this differential equation for $m(S)$ and using the border condition $m(0) = 1$, we obtain the only survival law $m(\tau)$ consistent with the "macro-Mincer" formulation:

$$m(S) = \exp\left(\left((b - d) - \frac{G\mu}{G-1}\right) S\right), \qquad \forall\ (S \geq 0). \tag{23}$$

Please note that this survival law is exponential and thus has the "perpetual youth" property. Let us now make the parametrization of $m(\tau)$ in equation (23) consistent with its interpretation, i.e.

ensure that the implied death rate is indeed equal to d. Under a stationary age structure, this is achieved by checking the following demographic identity:

$$N(t)=\int_{-\infty}^{t} bN(s)m(t-s)ds = N_0 e^{(b-d)t}. \tag{24}$$

From (23) and (24) it follows that

$$\int_{-\infty}^{t} b\exp\left(\left(\frac{G}{1-G}\right)\mu(t-s)\right)ds = 1. \tag{25}$$

Computing the last integral reveals that $G=\frac{b}{b-\mu}$. Plugging this into (23), we obtain $m(\tau)=e^{-d\tau}$. Also, $\varphi(S)=\frac{b}{b-\mu}e^{\mu S}$ and thus $h_{LF}(t)=\frac{bh_0}{b-\mu}e^{\lambda S}$ so that the "macro-Mincer" equation holds with the Minincerian exponent λ. ■

3 Concluding remarks

To be written.

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