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What Does the Consumer Price Index Measure? Bias Estimates for Poland[1]

Abstract

The paper addresses the problem of bias in the measure of inflation as provided by the price index of consumer goods and services (CPI) in Poland. We estimated the size of the bias resulting from two sources: substitution effect (Boskin et al. 1996) and the application of plutocratic weights in index calculation (Prais, 1959). The study involved a comparison of the official consumer price index in Poland with superlative indices (Diewert, 1978) and the democratic index in 2005-2011 period. The survey did not identify an upward CPI bias and the findings indicate a slight understatement of the CPI stemming from both sources (respectively: 0.1 and 0.3 pp. per annum). A downward bias due to substitution effect is rather unusual. A deeper analysis pointed to two possible explanations to this phenomenon. On the one hand, overstatement may be absent due to frequent adjustments in the weights used for CPI calculation, which results in a better match between the index and the changes occurring in the consumption structure. On the other hand, it was proved that in the period analysed, there was a faster-than-CPI rise in the prices of those goods and services the demand for which is relatively inelastic, and a positive growth of households' real income has been observed over the recent decade. When looking into the "plutocratic gap", it was found that the CPI (plutocratic) index for Poland was lower than the democratic index. Such a result of the "plutocratic gap" survey is in line with the research conducted for other countries.

Keywords: inflation, CPI bias, Fisher index, Laspeyres index, democratic index

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Introduction

The price index of consumer goods and services (CPI) is commonly applied as the basic measure of inflation. It approximates the change in such consumption expenditure that ensures constant level of utility for households (COLI, Cost of Living Index; see e.g. ILO, 2004; Diewert, 1998). CPI calculation methodology in Poland is based on the Laspeyres formula with weights determined as the ratio of expenditure on a particular item to the total household expenditure in the previous year. The choice of this formula is justified with the ease of its calculation and publication. On the other hand, this method of construction and computation the index is not free from certain limitations. When collecting price data, it is difficult to keep up with changes in consumers' behaviour (i.e. the substitution of goods becoming more expensive with their cheaper equivalents), account for the changing quality of the purchased goods or for the arrival of new goods in the market in the period between the weight-setting and the price survey. Moreover, the construction of weights may lead to an overrepresentation of households with the highest consumption spending. Therefore, the CPI may be flawed with a certain bias and thus it may fail to fully reflect the changes in the cost of living.

The fact that this problem – by no means purely methodological, as it might seem – is given so much attention stems from the essential role of inflation in economic processes. Failure to measure inflation accurately with CPI may have a distorting influence on economic policy, including monetary policy.

The role of the CPI in monetary policy is particularly crucial in countries where central banks rely on direct inflation targeting. Between 1990 and 2010, approximately 10 developed countries and 15 developing ones embarked on this strategy (Svensson, 2010). The European Central Bank (ECB, 2004) hinges its policy on the harmonized index of consumer prices (HICP). The target HICP level should remain below, but close to 2%. The positive value of the target was directly justified with an eventuality of an upward bias in the HICP indices (ECB, 2004, p.53). This is in compliance with a widely accepted practice of central banks. As indicated by, among others, Kokoszczyński (2004), Wojtyna (2004) and Baranowski (2008), a potential upward bias of the CPI estimates is one of the purely technical reasons for maintaining a positive inflation target.

The reason why the CPI should reflect actual inflation as closely as possible is its ubiquity in the indexation of nominal quantities in the economy. Indexation is of particular relevance to social policy[2] and entrepreneurs' pricing decisions. In many theoretical models (e.g. Gali, Gertler 1999) looking into pricing mechanisms and examining nominal rigidities it is assumed that some prices in the economy are subjected to indexation with the current inflation rate.

For some time now, economists have been trying to identify the sources of the CPI bias as well as estimating its size. Research conducted to date shows that in stable, advanced economies the bias is usually small and positive[3], although some studies have also identified a downward bias of the CPI. A very large upward bias is

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[2] The CPI in Poland is used for, e.g. indexation, as stated in the Act on Retirement and Disability Pensions from Social Security Fund.

[3] The upward bias of the CPI overstates inflation in relation to its actual level.

observed in economies under transition, e.g. during rapid changes in the consumption structure. It should be noted that the size of the bias does not only depend on the country and period examined, but also on the research method.

In this paper, we estimate the CPI bias for Poland in the period 2005-2011, using 300 sub-indices of goods and services. Considering the restricted availability of data, the study only seeks to estimate the bias resulting from the substitution effect and from the application of plutocratic weights[4]. Research hypotheses posited here refer to those two types of bias. Based on theoretical premises (the index number theory) and empirical research for other countries, we expect to find an upward CPI bias due to substitution effect in Poland. On the other hand, the CPI bias stemming from weight calculation (plutocratic vs. democratic weights) is expected to be negative. This result would be in line with the available empirical research relating to other economies.

The paper consists of three parts. The first addresses potential sources of the CPI bias. The second part summarises the research into this phenomenon conducted in other countries. Finally, we discuss the calculation methods employed in the estimations of the CPI bias in Poland in the period 2005-2011 and present the results.

1. Sources of the CPI bias

Broadly speaking, imprecise estimation of price changes in an economy results from deficiencies in data collection methods and weight-based aggregation methods. The reasons why the CPI diverges from the true COLI are pondered by many researchers. The most frequently cited sources of the bias can be divided into two groups. The first relates to the choice of weights for the respective products in the price index. The other sources of the bias stem from the method to collect data on prices of products comprised by the index.

Bias resulting from the definition of weights:

- product substitution bias *(product bias)*

Product substitution is connected with changes in the relative prices of components in the CPI basket. The substitution effect occurs when consumers – who are price takers – respond to price changes by replacing the products and services relatively expensive with the ones that have become relatively cheaper.

The substitution bias in the CPI indices results from fixed weights adopted in index calculation (the Laspeyres index concept). An index computed in this manner will not account for changes in consumption occurring over the years, including those which result from the substitution of goods and services rising in price in the period for which the index is calculated. This means that the CPI computed with the Laspeyres formula may be biased. For this reason, economists attempt to approximate the CPI

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[4] In order to calculate other biases of the CPI it would be necessary to have, among others, information on the outlet where the price was recorded, a detailed description of the product surveyed and on the method of the CPI basket adjustment for change in the quality of the products.

with other indices regarded as the best approximation of the COLI (the so-called superlative, or ideal indices). Amongst those considered superlative, it is the Fisher index that is the most frequent choice. The difference between the Laspeyres index and an ideal index will indicate the size of the CPI bias attributable to product substitution.

- New goods bias

This type of bias is the effect of a lagged introduction of new goods to the CPI basket[5]. Due to the lag, change in the price of some goods is reflected in CPI long after the product became popular. Moreover, the price of such a product upon its inclusion in CPI is lower than its price at market launch[6]. Hence, the CPI does not reflect substantial reductions in the prices of new products as they become commonly used.

The remaining sources of bias:

- Quality bias

One of the key problems in CPI calculation is how to account for changes in the quality of examined consumer goods. By definition, CPI seeks to measure changes in the prices of goods and services under the assumption that their characteristics have remained unchanged in relation to the base period. In reality however, the products covered by the index do evolve: they may be withdrawn from the market or replaced with a new, enhanced version. In the former case, the price collectors should replace the good with one of similar properties. The latter calls for the application of quality adjustment techniques.

Most of the research on this topic suggest that quality adjustment techniques tend to underestimate changes in the quality of a product or service, and so, even if they have been applied, the CPI remains overstated (Ducharme, 2000). In particular, the problem concerns products which are subject to frequent technological changes. On the other end of the spectrum, there exist groups of products with an estimated negative bias. Schultz (1995) indicates that this may be the case of goods not affected by fast technological development (e.g. clothing).

- Outlet bias

This type of bias results from substitution triggered by price differences between individual points of sale. The CPI bias stems, in this case, from ignoring differences in the prices of the same item at various outlets. As a result, the measurement method usually does not capture changes in distribution channels, in particular, the growing popularity of large shops offering goods at discounted prices and the rise in the number of purchases made via electronic channels (the Internet). Seen from this

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[5] For example, in Poland mobile telephone fees were taken into account as late as 2006.

[6] According to the product lifecycle theory, at the initial phase of the cycle (*introduction*), the price of a new product is relatively high, although, depending on the selected marketing strategy, the business may opt for low prices at that stage (this is particularly the case of businesses setting out to capture price-sensitive consumer segments). In the subsequent phases (*growth, maturity*) the price is gradually lowered in response to the emergence of competitive products; also, promotions and discounts are introduced (Kotler, 2005).

vantage, the CPI is not sufficiently sensitive to price discounts available only at selected outlets or distribution channels.

The discrepancy between points of sale included in the price survey and the actual consumer habits has been thoroughly examined on the example of the Czech Republic (Filer, Hanousek, 2003). The discrepancies are particularly prominent in two product groups: food and non-alcoholic beverages, clothing and footwear. The authors point out that only 10% of the prices in the food and beverage group are recorded at hypermarkets. At the same time, independent market studies show that purchases made at hypermarkets account for approx. 20% of the overall spending on commodities in this group. A similar relationship is observed in the other of the two groups, clothing and footwear: 10% of the goods are purchased at hypermarkets, while only 5% of the prices are recorded there.

Index mismatch resulting from the difference in product prices due to unrepresentative channels of distribution results in a CPI bias estimated at approx. 0.1-0.7 pp (Boskin *et al.* 1996, Hanousek, 2003).

- Formula bias

The CPI may be biased as a result of the calculation method applied at the lowest level of data aggregation. Whenever a price index for a certain (representative) good is computed as an arithmetic mean of price indices at the individual outlets, the index will prove overstated. On the other hand, if first the mean price of the (representative) good at all outlets is calculated for a given period and then related to the mean price of this good in the previous period, the index should be unbiased (Ducharme, 2000). In Poland, the price index for the (representative) good is computed as the ratio of its average monthly price to its average price in the base period. Hence, we can infer that the CPI in Poland is free from formula bias.

On the other hand, as a result of applying the arithmetic mean, higher prices recorded may be overrepresented in the final result. This is why in some countries the arithmetic mean is being abandoned in favour of the geometric mean.

The "plutocratic gap"

Apart from the sources of bias mentioned above, the methodology for CPI calculation faces the problem of assigning weights to the respective households. We did not mention this issue among bias sources listed above, since it is difficult to consider it in terms of error. The democratic index described in the subsequent section of this article is just an alternative to the commonly used CPI.

Prais (1959) stated that the CPI can be interpreted as the weighted average of price indices of respective households, with the weights determined by the share of a household's spending in the total expenditure by all households. This method of averaging indices results in under-representation of the lower-spending households. Consumption patterns of households with higher overall spending are more strongly reflected in the CPI – hence the term "plutocratic". In contrast, the democratic index is based on weights defined with the use of the arithmetic mean of households' expenditure on a given item. If consumption structure changes in step with total

spending level, then using the plutocratic index involves a bias called the "plutocratic gap" in literature (Ley, 2005). The sign of the bias depends on the distribution of price changes within various commodity groups. For instance, the plutocratic index is lower than inflation expressed in democratic index terms if the price of goods purchased by poorer households rises faster than the other goods.

The weakness of the plutocratic index lies in the fact that it measures change in the price of a consumption basket, which is dominated by higher-income households. As Prais (1959) indicates, one of the methods to minimise this bias is to opt for the aforementioned democratic index; another one involves the use of weights representing a median household. According to Prais, the index constructed as a result of these modifications will be more accurate in capturing changes in the cost of living incurred by a typical household.

Thus, the problem of the plutocratic index is not a merely statistical one. Its application has a strong influence on economic policy, which is why the choice of the plutocratic vs. democratic index may spark heated public debate.

To summarise the various sources of the CPI bias, it is necessary to point out that by choosing one of the superlative indices we opt for a certain approach to the calculation of the price index (this issue is elaborated on in the subsequent part of the article). The choice between the democratic and plutocratic index is subjective and depends on the purpose for which the index is computed. By employing plutocratic weights, we implicitly perceive processes from the macroeconomic perspective (e.g. monetary policy). On the other hand, an index based on democratic weights focuses more on social developments (e.g. in social policy). Thus, it may be concluded the two indices are complementary to each other and provide two different perspectives on inflation.

2. CPI bias estimates for selected countries

The problem of consumer price indices being measured with a bias has been the subject of numerous research studies in various countries. The findings of these efforts, compiled in Table 1, point to an upward bias on nearly every consumer price index examined.

An important contribution to the advancement of research on the CPI bias was made by the Boskin Commission Report (1996). The conclusions of the Boskin Commission followed a detailed review of the bias of the respective components of the CPI in the USA. Based on this, the Commission estimated the bias of the inflation index in the years 1995-96. At the time of the research, CPI weights were based on data collected between 1982 and 1984, which meant a lag in estimating consumption structure of up to 14 years. In the course of the study, the CPI in the USA was found to be biased by approx. 1.1 pp, of which 0.5 pp was due to substitution (product – 0.4 pp. outlet – 0.1 pp) and the remaining 0.6 pp was attributable to quality and new product bias. Following the publication of the Boskin Commission Report, the frequency of weight calculation in the United States was significantly increased.

The work of the Commission was followed up, though with a much narrower focus in Gordon, van Goethem, 2005. The study spanned the period 1914-2003. The findings

related to one of the CPI components – housing rental prices, which had not been covered by the Boskin Report. According to the authors, housing rental prices are understated by 1 pp. In the light of the study, the original Boskin Commission findings should be revised downwards by 0.2 pp.

What is remarkable amongst the figures listed in Table 1, is the high estimates of the CPI bias in the Czech Republic (Filer, Hanousek, 2003). The overstatement of the CPI in the Czech Republic under transition (1990-1999) amounted to 4.4 and 4.8 pp. on an annual basis, depending on the sub-period examined. The difference is the effect of the different scale of bias due to product substitution. The new goods and quality change bias remained unchanged in the sub-periods at 3.9 pp.

Amongst numerous sources of the CPI bias, the authors point to long periods of using unchanged weights. Surveys of consumption structure quickly become outdated and, as data show, the lag between the period in which consumption structure is recorded and the time when the weights are applied can be as long as 15 years (Crawford, 1998, Boskin et al., 1996). This enhances the resultant bias, particularly in terms of product substitution, which involves fast changes in consumption structure.

The research cited so far pointed to an upward CPI bias. The exception to the rule is a study based on the bias estimation method involving inflation perceived by consumers (Nordhaus, 1998) and estimates of the democratic index (Prais, 1959 and Ruiz-Castillo et al., 2002).

Nordhaus (1998) proposed a subjective method of the CPI bias examination. His research covered the period 1968-1994 and involved a comparison of households' real income deflated with CPI, with a subjective assessment of the change in the household's financial situation. It is assumed that CPI is not biased if the real income changes at the same pace as the self-assessed economic status.

Nordhaus' method was applied in subsequent research studies on many occasions. (Marini et al., 2007, Giovane i Sabbatini, 2008). In one of the studies, Giovane and Sabbatini (2008) indicate a vast understatement of the HICP inflation in Italy in the years 2002-2004. This was related to the adoption of the common EMU currency. Italy represents a particularly acute manifestation of this problem, whereas in the remaining countries which adopted the euro, the gap between the perceived and announced inflation was rather small and temporary. According to the authors, in Italy it persisted at a very high level for 2 years.

Bias estimates based on households' subjective assessment of their financial situation are criticized on account of distortions in perceived inflation caused by multiple factors independent of the CPI measurement method. One point raised by the critics is that consumers tend to perceive inflation through the lens of the most frequently purchased goods and services, as well as those which have seen the sharpest price increases (Łyziak, Stanisławska, 2008, Pike et al., 2008). This leads to an overstatement of the perceived price growth in comparison to the actual inflation. Thus, any conclusions concerning the price index bias based on this research method should be drawn with utmost caution, bearing in mind the problems in consumers' subjective assessment of inflation mentioned above.

A detailed overview of the studies referred to is presented in Table 1.

Table 1 Overview of research studies on the consumer price index bias

Country, period examined	Index bias	Average inflation y/y	Source
USA, 1995-1996	CPI overstates inflation by a total of 1.1 pp. (0.8-1.6), out of which: product bias accounts for 0.4 pp., outlet bias for 0.1 and quality bias or new goods bias for 0.6 pp. The weights applied during the period examined dated back up to 14 years before.	2.9%	Boskin *et al.* (1996)
USA, 1914-2003	Housing rental index bias (CPI component) amounts to approx. -1 pp.	3.4%	Gordon, van Goethem (2005)
USA, 1968-1994	CPI overstates inflation by 1.5 pp. (subjective method)	5.7%	Nordhaus (1998)
United Kingdom 1995-1996	RPI overstated inflation by 0.35 – 0.8 pp., including product bias of 0-0.1 pp. Weights adjusted annually.	approx. 3%	Cunningham (1996)
United Kingdom, 1996-2004 and Italy, 1990-2004	In the UK, no bias was observed. In Italy before the euro area accession, HICP did not display a bias; however between 2002 and 2004, HICP underestimated inflation versus the level perceived by consumers by 6 pp. (subjective method)	UK: 1.3% Italy: 2.3%	Marini *et al.* (2007)
Canada 1962-1994	Overall CPI overstatement of up to 0.7 pp. due to substitution of 0.2 pp. (including product substitution of 0.1 pp.), new products quality change: 0.5 pp. Weights updated every 4 years.	5.3%	Crawford (1998)
Spain 1992-1998	Survey of the "plutocratic gap". Overall bias: from -0.12 pp. (understatement) to +0.03 pp. (overstatement).	6.8%	Ruiz-Castillo *et al.* (2002)
Czech Republic 1990-1999	Overall, CPI overstated actual inflation by +4.4 to +4.8 pp. on an annual basis.	approx. 10%	Filer, Hanousek (2003)
France 1995-1996	Total overstatement: from 0.10 to 0.25 pp., including product substitution by 0.05-0.10 pp. Weights adjusted annually.	2.0%	Lequiller (1997)
Japan 1982-2002	CPI bias calculated as the difference between the Laspeyres index (the announced CPI) and the Fisher index (a superlative index): 0.03 pp.	0.8%	Unayama (2004)
Portugal 1983-1991	Average bias, calculated as the CPI relation to Fisher and Tornqvist indices amounted to about 0.01 pp.	14.4%	Neves, Sarmento (1997)
Tanzania 2003-2006	CPI understated inflation measured with the plutocratic index by an average of 2.7 pp.	5.6%	Ngasamiaku, Mkenda (2009)

Source: own compilation

3. CPI bias estimates for Poland

Our CPI bias estimation survey is conducted in two stages. The first stage deals with product bias, the second with the "plutocratic gap". The results in each case are preceded by fairly detailed description of the methodology.

3.1   Substitution bias

As mentioned in the second part of the article, index bias due to substitution may result from the index formula applied, which is based on a pre-defined system of weights. This raises the question of which weight system should be used, and hence, what index calculation formula should be employed.

The literature of the index number theory presents several approaches to selecting the price index formula. The first approach[7] proposes using a fixed-structure consumer spending basket in the calculation (Kokoszczyński, 2004). Since the index is meant to reflect the overall price change between the base and the current period, the weights should correspond with the expenditure structure of either of these. The indices calculated on this principle include:

the Laspeyres index:

$$P_t^L = \frac{\sum_{i=1}^{n} p_{i,t} \cdot q_{i,b}}{\sum_{i=1}^{n} p_{i,b} \cdot q_{i,b}} \tag{1}$$

and the Paasche index:

$$P_t^P = \frac{\sum_{i=1}^{n} p_{i,t} \cdot q_{i,t}}{\sum_{i=1}^{n} p_{i,b} \cdot q_{i,t}} \tag{2}$$

where p_i denotes the price of good i, q_i stands for the quantity of good i in the basket, while the t and b subindices refer to, respectively, the current and the base period.

Due to their simple construction, the Laspeyres and Paasche indices are broadly applied in price index calculation[8]. It should be noted, however, that the changes in consumption structure in response to price movements, cause both of the indices to demonstrate a certain bias. The Laspeyres index, based on the consumption structure observed in the period preceding the survey, may overstate actual inflation. On the other hand, the Paasche index may understate it, as the weights refer to consumption structure already adjusted in response to price changes. It follows from the index number theory that in case of a negative correlation between the price and the quantity of goods purchased (a theoretically supported demand response to a price change), the Laspeyres index will yield higher values than the Paasche index. Under a positive correlation, the reverse is true (ILO, 2004).

Since the Laspeyres index may overstate, and the Paasche index understate inflation, it can be inferred that value of the COLI index will lie between those two indices. This gave rise to a proposal that the two indices should be averaged, or the weights should be calculated as a mean (whether arithmetic or geometric) of weights derived from the consumption structure of, respectively, the current and the base

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[7]Called the "fixed basket approach" in literature (Diewert 2002).
[8] The CPI is an example of a Laspeyres index, while the PPI is a Paasche index.

period. Such averaging of consumption structure is typical for superlative indices (Boskin 1996). Examples of those indices include (formulae based on: Kokoszczyński 2004 p. 139 and subsequent pages, also see Białek, 2010):

- The Fisher index, computed as a geometric mean of the Laspeyres and Paasche indices:

$$P_t^F = \overline{P_t^L \cdot P_t^P} \tag{3}$$

- the Marshall-Edgeworth index, based on the arithmetic mean of weights from the base and current periods:

$$P_t^M = \frac{\sum_{i=1}^n p_{i,t} 0.5(q_{i,b}+q_{i,t})}{\sum_{i=1}^n p_{i,b} 0.5(q_{i,b}+q_{i,t})} \tag{4}$$

- the Walsh index, based on a geometric mean of weights from the base and current periods:

$$P_t^W = \frac{\sum_{i=1}^n p_{i,t} \overline{q_{i,t} \cdot q_{i,b}}}{\sum_{i=1}^n p_{i,b} \overline{q_{i,t} \cdot q_{i,b}}} \tag{5}$$

It should be noted, on the other hand, that consumer indices based on the current-period consumption structure may be announced with a considerable lag. This undercuts their utility in e.g. the conduct of monetary policy.

Another approach is the axiomatic approach (ILO, 2004). It rests on the assumption that an ideal index should possess a range of characteristics[9]. The indices which lack certain basic – or should we say fundamental – features should be rejected, as they may display undesirable properties (Diewert, 1995). When analysed in terms of compliance with the respective characteristics, the indices can be ranked from the "best" to the "worst" ones. One should bear in mind that there is no agreement among researchers as to which qualities are "best". Thus, the axiomatic approach may lead us to more than one superlative price index. The literature of the subject lists many properties, among which the following can be considered the most relevant (Kokoszczyński, 2004):

1. The index equals one if prices in period t and the base period are the same.
2. The proportionality test – if all prices in period t are multiplied by a positive number p, the index will amount to p. If all prices in the base period are multiplied by a positive number p, the index will amount to $1/p$.
3. The time reversal test– if the base period is changed around with period t, the resultant index should equal the reciprocal of the original index.
4. The monotonicity test – if price $p1>p2$ then the index based on the higher prices will be higher than the index based on the $p2$ prices (assuming that prices in the base period and the weights are constant).
5. The condition of index consistency in multi-stage aggregation of individual indices.

The index displaying most of the properties listed above is the one constructed by Fisher. The Paasche and Laspeyres indices fail the time reversal test. On the other

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[9] The work on the properties of the respective indices was pioneered by Fisher (1992).

hand, both boast a very useful quality: the consistency in multi-stage aggregation of individual indices. The other indices sometimes referred to as superlative, such as Walsh or the Marshall-Edgeworth indices, also fail to display all the listed properties. In sum, it seems that from the perspective of the axiomatic approach, Fisher's is the best index.

Another method of choosing the index is the stochastic approach (weighted or unweighted, ILO, 2004). It is based on the assumption that any price relationship can be regarded as an approximation to the inflation index. Indices based on the stochastic unweighted approach were proposed by Carli[10] and Jevons. The latter is expressed by the following formula:

$$P_t^J = \prod_{i=1}^{n} \sqrt[n]{\frac{p_{i,t}}{p_{i,b}}} \tag{6}$$

However, researchers often emphasise the weakness of these indices resulting from the assumption that every price relationship is of equal importance. In the weighted approach, it is assumed that each price relation should be attributed a weight corresponding to its economic significance. An index based on this assumption is the Tornqvist-Theil index:

$$\ln p_t^T = \sum_{i=1}^{n} \frac{1}{2} \left(s_{i,t} + s_{i,b} \right) \ln \frac{p_{i,t}}{p_{i,b}} \tag{7}$$

where:

$$s_{i,t} = \frac{q_{i,t} \cdot p_{i,t}}{\sum_{k=1}^{n} p_{k,t} \cdot q_{k,t}} \ , \quad s_{i,b} = \frac{q_{i,b} \cdot p_{i,b}}{\sum_{k=1}^{n} p_{k,b} \cdot q_{k,b}}$$

As it possesses many relevant qualities, the Tornqvist-Theil index is deemed to be the superlative index based on the weighted stochastic approach[11]. On the other hand, it should be applied with caution whenever individual price relations substantially deviate from the average level. In such case, the value of the index will approach zero or infinity, respectively.

Yet another approach to constructing a superlative index is based on economic theory. It was developed by Konüs (1939) and rests on the assumption of consumers' optimising behaviour. With a given price vector p^t faced by consumers in period t, it is assumed that the basket of goods q^t purchased by them complies with their utility function f (or the preferences). This approach assumes that the basket of goods purchased is determined by the consumer preferences and the prices encountered in a given period.

The approach assumes that the consumer has well-defined preferences regarding the desired basket of goods. It is further assumed that each combination of preferred goods is represented by a continuous, non-decreasing, concave utility function. Thus, if the inequality $f(q^1) > f(q^0)$ is true, it follows that the consumer prefers the

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[10] The Carli index does not satisfy the time reversal condition.
[11] Meets, among others, the time reversal condition. See ILO (2004) page 303.

q^1basket to the q^0basket. By additionally assuming that the consumer minimises the cost of attaining this utility in the current and base periods, we can observe that the true cost of living index is contained between the Laspeyres and the Paasche indices[12]. If we further assume that the utility function is one of homothetic preferences, it can be proved that in terms of this approach, the indices constructed by Walsh, Fisher and Tornqvist-Theil are superlative.

Given the problems and issues above, many regard the Fisher index[13] as the closest approximation to actual inflation (Crawford, 1998). Diewert (1998) indicates a range of characteristics displayed by the Fisher index which make it a good reflection of the actual COLI. However, Reinsdorf and Triplett (2009, p. 21) highlight certain weaknesses of the tests which are supposed to evidence the superiority of the Fisher index over other indices: the tests are selected in an arbitrary manner and favour characteristics which the Fisher index happens to possess. Besides, they do not take into account aggregation consistency, which, according to the authors, should characterise an ideal index. In spite of these reservations, the authors fail to propose any other index that would have an advantage over Fisher's.

In spite of its many strengths, in practice it is difficult to apply the Fisher index in measuring inflation. This is because it requires data on the current structure of households' consumption, which are not available at the time of index calculation. Thus, the unbiased Fisher index would have to be announced, as already mentioned, with a considerable lag.

In effect, national statistical authorities publish indices constructed with weights based on past consumption patterns (the Laspeyres indices)[14], which can distort the index number. In order to recognise the size of the bias resulting from the substitution of goods rising in price by their cheaper equivalents, one can compare a price index calculated in line with the Laspeyres methodology for a given year with the superlative one.

The methodology used by the Central Statistical Office of Poland (CSO) to calculate the CPI is based on the Laspeyres formula. The weight system is defined by the structure of expenditures incurred by the households in the base year. Thus, it is a system of weights based on the value of goods and services purchased, computed as the product of their quantity and price. With an eye on significant shifts in consumption structure, CSO has been adjusting its weight system on an annual basis since 1990 (CSO 2009)[15].

In order to precisely estimate the difference between the individual indices regarded as superlative, we used highly disaggregated single-base consumer price indices,

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[12] This property was first observed by Konüs (1939).

[13] In terms of the fixed-basket, axiomatic and economic approaches, it is an ideal index.

[14] The lag varies from one to around a dozen years across countries. In Poland consumption expenditure data are obtained from previous years' surveys of household budgets.

[15] While comparing indices, it is important to account for changes in computing the individual indices. For example, in 2011 CSO introduced changes to the recording and weighting of seasonal products (vegetables, fruit, fish, clothing and footwear). According to CSO estimates, this change resulted in a rise in the CPI in May by 0.1-0.2 pp.

together with a system of weights based on the structure of household expenditure[16]. This involved approximately 300 indices of goods and services (the so-called elementary groups) and a system of weights valid for a given year. Next, these were applied in the calculation of the respective superlative indices in line with the formulas (2) – (7) presented in the earlier part of the paper.

To estimate the CPI bias in Poland, we examined the difference between the published index and indices regarded as the best approximation of the COLI. To this end, the consumer price index in the subsequent years was estimated with the formulae proposed by Paasche, Fisher, Marshall-Edgeworth, Walsh, Jevons and Tornkvist-Theil. Next, difference between the superlative indices and the CPI inflation figures were calculated. Our attempt to estimate the substitution effect in Poland was based on price indices for the years 2005-2011.

The results of the calculations are presented in Table 2.

Table 2 Price index and the estimated product bias (the difference between the CPI and the Fisher index)

Year	bias (CPI-Fisher)	announced CPI	Paasche index	Fisher index	Marshall-Edgeworth index	Walsh index	Jevons index	Tornqvist-Theil index
2005	-0.1	102.1	102.4	102.2	102.2	101.8	101.4	102.1
2006	0.0	101.0	101.1	101.1	101.0	101.0	100.7	100.9
2007	0.0	102.5	102.5	102.5	102.5	102.5	101.6	102.3
2008	0.0	104.2	104.3	104.3	104.3	104.3	103.1	104.1
2009	-0.1	103.5	103.6	103.5	103.5	103.5	102.5	102.9
2010	-0.1	102.6	102.7	102.6	102.6	102.6	102.3	102.5
2011	-0.2	104.3	104.6	104.4	104.3	104.2	103.2	104.2
average bias	-0.1		-0.1	-0.1	0.0	0.0	0.8	0.2

Source: own calculations based on CSO data.
The calculations were carried out with greater accuracy than presented in the table above. The differences, if any, result from rounding. The average bias was calculated based on the differences between the annual indices in the respective months during the years 2005-2011.

The findings of our survey are somewhat ambiguous[17]. Some of the superlative indices (Jevons and Tornkvist-Theil) indicated an overstatement of the Polish CPI. The Marshall-Edgeworth and Walsh indices showed the CPI to be equal to the superlative index. Finally, the Fisher index has identified an understatement of the Polish CPI.

The largest CPI bias (amounting to 0.8 pp.) was established when using the Jevons index. However, given the reservations[18] concerning this index, it should not be treated as the best approximation of the COLI. Finally, the Walsh, Marshall - Edgeworth and Fisher indices did not identify any significant bias (-0.1 – 0 pp.)

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[16] Data on household spending structure are derived from annual household budget surveys and provide the basis for the constructing a system of weights employed in the calculation of the CPI in Poland. The weight system for a given year is based on the previous year's household budget survey. *Ceny w gospodarce narodowej 2008* [Prices in the national economy 2008], *CSO (2009).*

[17] In his study (2006), Hill proved that only the Fisher index will always be contained between the Paasche and the Laspeyres indices. At the same time, he proved that in spite of the fact that superlative indices approximate each other to the second order, it does not follow that they render numerically identical results. In reality, the differences between individual indices may be large.

[18] As its greatest weakness it is indicated that the index does not take into account the weight of the individual goods in the consumer basket.

In the next section of our study, we compare the official CPI with the index calculated according to the Fisher method. There are several reasons for this. Diewert (1998) states that the Fisher index meets the largest number of requirements set by the axiomatic approach; at the same time, it complies with the revealed preference theory[19]. Secondly, the literature of the subject points to the Fisher index as the best approximation of the COLI (ILO, 2004, p. 316). Yet, another reason lies in methodological considerations. Most surveys cited in the third part of this paper are based on comparisons of the Laspeyres index with the Fisher index (see Boskin *et al.,* 1996, Crawford, 1998; Filer and Hanousek, 2003; Rossiter, 2005; Unayama, 2004). By employing the same methodology, we ensure that the findings obtained for Poland are comparable to those for other countries.

The calculations we conducted reveal an average CPI understatement of 0.1 pp in the period analysed (2005-2011), ranging from 0.2 pp. to 0.0 pp. A review of the available literature on CPI bias due to product substitution suggests that the CPI is overstated in most countries. Since our result proved so different to the those for other countries and surprising in terms of economic intuitions, we embarked on an attempt to identify the factors which might have contributed to it.

The analysis of relative price changes in the years 2005-2011 showed a larger-than-CPI rise in the prices of goods and services characterised by relatively rigid demand. This group comprises food, energy (electric power, gas, heat and engine fuels) as well as dwelling services[20]. The consumption of goods and services in this group is never significantly reduced even in the face of considerable price rises. When consumption structure is rigid, given the definition of the weights (value-based), there exists a positive relationship between price growth and change in weights – in contrast to the effect generated by the product bias. In this situation, the price index calculated according to the Fisher formula is higher than the index calculated with the Laspeyres methodology. The positive growth in households' disposable income over the last decade indicates a real-terms improvement in their financial standing. Thus we can propose that households, in spite of rising prices of consumer goods and services may not opt for cheaper substitutes (as they can afford more expensive goods), but will stick to old consumption patterns. The results we obtained (the Fisher index value being higher than Laspeyres') may confirm this hypothesis.

In most countries, weights are updated every 4 to 15 years. In those countries research finds the CPI to be overstated on account of substitution bias. In Poland, on the other hand, the structure of weights is adjusted annually, which may be the reason why our study did not identify any CPI bias attributable to this factor.

3.2 The democratic index

As mentioned in the second part of this paper, price index bias also depends on the manner in which household expenditure is averaged in the calculation of weights for individual goods in the consumer basket. The CPI as announced in Poland is a

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[19] Diewert (1998). See also: the economic approach.

[20] Hamilton (2001) analysed demand for food in the United States, using unitary data obtained from the surveys of household budgets in the years 1974-1991. The findings show that the relative price of food did not have a significant bearing on the share of food expenditure in the overall household budget.

plutocratic one with the weights for the respective goods and services computed as a weighted average of the indices of individual households.

This paper seeks to estimate the "plutocratic gap" by constructing an alternative index of consumer prices which attaches equal weights to all the households.

The weight of a given good i in the plutocratic index is defined as a share of spending on that good in the total expenditure by all the households. It can be illustrated with the following formula:

$$w_i^P = \frac{\sum_{k=1}^{K} e_{i,k}}{\sum_{i=1}^{I} \sum_{k=1}^{K} e_{i,k}} \tag{8}$$

where:

w_i^P – the plutocratic weight of a particular good i,

$e_{i,k}$ – the expenditure of k-th household on a particular good i,

The democratic index in turn is based on weights which can be described with the following formula:

$$w_i^D = \frac{1}{K} \sum_{k=1}^{K} \frac{e_{i,k}}{\sum_{i=1}^{I} e_{i,k}} \tag{9}$$

According to Ley (2005), there may be three contributions to the difference between the plutocratic and the democratic index. The first one is a difference in the distribution of expenditure across households. Next, the spending structure of individual agents within the respective expenditure brackets is different. The third important factor is the various behaviour of prices of the individual goods and services.

The difference between the plutocratic and democratic indices can be interpreted as the "plutocratic gap". This should not be perceived as an error in estimating the CPI. Any evaluations concerning the superiority of either index in representing consumer price movements depend on the purposes for which the index is used (see Ruiz-Castillo *et al.*, 1999 and Ley, 2005). The sign of the "plutocratic gap" will depend on the changes in relative prices of individual goods and services. The plutocratic index attaches more weight to luxury goods than to normal goods because the former hold a larger share in the overall spending. Hence, should prices of luxury goods rise more slowly than those of normal goods, the plutocratic index will turn out to be lower than the democratic one, yielding a negative "plutocratic gap". This implies that inflation affecting lower-income households will be understated as compared to inflation calculated on the basis of democratic weights.

Ruiz-Castillo *et al.* (1999) deliberate on the political consequences of using one or the other index. The central question posed by the authors is – which measure should be used for indexation of incomes, social transfers or tax brackets. Most countries opt for the CPI as a deflator for these values, which means that if the "plutocratic gap" is negative, the extent of indexation is smaller than it would be with the democratic index.

As Ley (2005) states, from the point of view of more affluent households, one of the most important values indexed with the CPI are tax brackets[21]. Wherever inflation has a stronger impact on better-off households, indexation with the plutocratic CPI would mean the need to pay higher taxes (the adjustment of tax brackets would be smaller than the inflation experienced by this social group). In the converse situation (should the "plutocratic gap" be negative), CPI-based adjustment of tax brackets would result in relatively understated taxes to be paid by this group.

Apart from the above issues, both Prais (1959) and Ley (2005) point to a certain advantage of the plutocratic CPI over the democratic one. The CPI's strength lies in its simplicity of calculation and consistency with the deflators computed on the basis of the national accounts. The plutocratic index – unlike the democratic one – can provide the answer to the question how much the national income should rise in order to offset the observed price change. In addition, Prais (1995) points to the information noise that could be generated if more than one price index were to be published.

The estimation of the superlative indices incidentally showed the fastest rise in the prices of goods the demand for which was inelastic. These are the normal goods, whose share in the total expenditure is higher in the lower-income households. The rise in their relative prices should lead to the opening of a negative "plutocratic gap".

In order to measure the "plutocratic gap", indices were calculated according to formulae (8) and (9).The findings are presented in Fig. 2. The chart depicts the CPI development in two versions: a plutocratic index (the index of consumer goods and services prices as announced by CSO) and a democratic one.
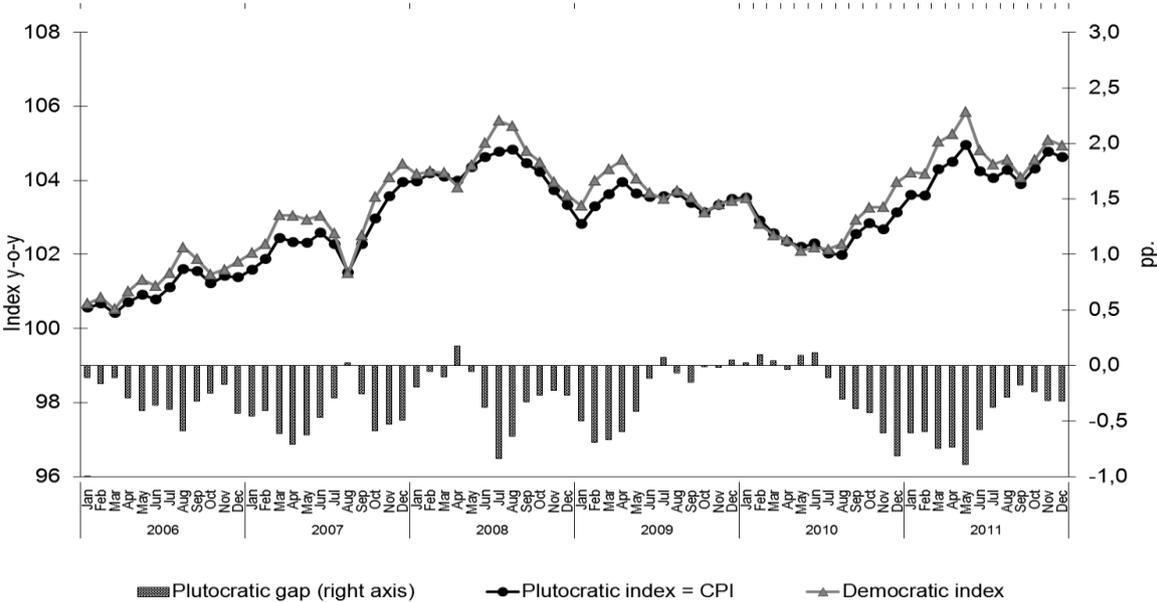

Fig. 1. Democratic and plutocratic indices of consumer goods and services.

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[21] In Poland, tax brackets are adjusted upwards in line with the index of average monthly wage growth (Personal Income Tax Act of 26 July 1991).

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The "plutocratic gap" is observed in the periods when the two indices diverge. This confirms the hypothesis of different consumption structures of households at different levels of overall spending. In the period under analysis, the democratic index was higher than the plutocratic one (on average by 0.3 pp). On the other hand, the negative sign of the "plutocratic gap" confirms the conclusions drawn from the analysis of the superlative indices concerning a faster growth of prices of goods characterised by inelastic demand. The higher Fisher and democratic indices may be a sign of normal goods prices in Poland rising faster than those of other goods.

It is worth noting that, except for a few periods, the "plutocratic gap" remains at a relatively constant level, which may facilitate a possible adjustment of the CPI[22].

The study of both kinds of bias is summarised in Table 3. It presents the annual index of consumer goods and services as well as the estimated product bias and the "plutocratic gap".

Table 3 Estimated CPI biases 2005-2011.

Year	announced CPI	product bias (CPI-F)	"plutocratic gap" (CPI-democratic)
2005	102.1	-0.1	
2006	101.0	0.0	-0.3
2007	102.5	0.0	-0.4
2008	104.2	0.0	-0.3
2009	103.5	-0.1	-0.3
2010	102.6	-0.1	-0.2
2011	104.3	-0.2	-0.5
average		-0.1	-0.3

Conclusions

In our paper, we presented an analysis of sources of potential CPI bias related to the weights applied in Poland. We also carried out empirical tests of the data for two kinds of bias: product and service substitution effect as well as the "plutocratic gap".

Our classification of bias sources is widely used in literature, facilitating reference to numerous theoretical and empirical studies relating to other countries.

Given the available information on the CPI calculation method in Poland, bias estimations in our study referred only to product bias and the bias resulting from the "plutocratic gap". The first part of the paper is based on a comparison of the Laspeyres index and superlative indices: the Paasche, Fisher, Marshall-Edgeworth, Walsh, Jevons and Tornqvist-Theil indices. The estimation was aimed to examine how changes in consumption structure influence the CPI.

The second part of our study of inflation index bias involved a comparison of the CPI with an index calculated using democratic weights. The "plutocratic gap", although presented as a kind of bias, is not considered an error in CPI estimation. One should, however, bear in mind the consequences of constructing an economic policy around

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[22] The democratic index may constitute a supplement to the price indices announced by CSO with regard to various social groups, i.e. employees, farmers, pensioners etc. In Poland the index of consumer prices for households of pensioners (if it exceeds the overall CPI) is used in the indexation of retirement and disability pensions (Act on Retirement Pensions and Disability Pensions from Social Security Fund of 17 December 1998).

the plutocratic index. Among its merits, we could mention the simplicity of calculation and consistency with the deflators estimated on the basis of the national accounts. On the other hand, the plutocratic index may have a distorting impact on social and fiscal policy as it underrepresents households with lower spending.

The findings of the survey, covering the period 2005-2011, did not confirm the hypothesis of a positive CPI bias existing in Poland due to product substitution. The announced CPI in the subsequent years is lower than Fisher index, which accounts for the substitution effect (by 0.1 pp). On the other hand, the provisional hypothesis of a downward CPI bias due to the "plutocratic gap" was confirmed: the democratic index we computed is higher than the plutocratic one by an average of 0.3 pp.

It is worth comparing these findings with the results of research into the consumer price index bias in other countries. In advanced economies, an overstatement of these indices due to the substitution effect is generally identified (amounting to a maximum of +1.5 pp. in annual terms). Exceptions occur in economies under transition, e.g. in periods of rapid changes in the consumption structure. For example, in the Czech Republic during the systemic transition, the CPI exceeded markedly the value calculated according to the Fisher formula – up to + 4.8 pp annually. The estimations of the "plutocratic gap" for Poland, on the other hand, showed an understated CPI, which was also found in the surveys of price indices for Spain and Tanzania. Thus, the results of the calculations are affected by the choice of the country, period to be examined and the applied method of bias estimation.

The absence of a positive substitution bias in Poland is a rare result in the context of the research overview presented in the earlier part of the study. The probable cause behind the absence of an upward bias due to substitution effect in Poland is frequent weight adjustment. In Poland, in contrast to many countries, it takes place every year, significantly reducing the risk of changes in the consumption structure being overlooked.

What should be noted at the same time, is the negative correlation of the inflation index bias and the frequency of weight adjustment. Countries where the weights are adjusted less frequently are characterised by a larger bias attributable to product substitution (see the Boskin Report against the background of the survey of France or Great Britain. This conclusion is quite often encountered in the literature (see Ducharme et al., 1997). Another hypothesis to explain the negative CPI bias with respect to product substitution effect is the faster rise in the prices of goods and services the demand for which is rigid. As households become more affluent, they may remain indifferent to increases in product prices and may choose not to switch to cheaper substitutes.

Considering the narrow scope of the study we conducted against the background of many possible bias sources, one cannot unequivocally state that the consumer price index announced in Poland is understated. The study does not allow us to rule out possible disturbances to accurate estimation of the of cost-of-living index (COLI) when the CPI is applied.

References

Alchian A.A., Klein, B. (1973), *On A Correct Measure of Inflation*, Journal of Money, Credit and Banking", Vol. 5, No. 1, pp. 173-191.

Baranowski P. (2008), *Problem optymalnej stopy inflacji w modelowaniu wzrostu gospodarczego*, Wydawnictwo Biblioteka.

Białek J. (2010), *Uogólnione indeksy agregatowe*, Wiadomości Statystyczne No 10.

Boskin M.J., Dulberger E.R., Gordon R.J., Griliches Z., Jorgenson D. (1996) *Toward a More Accurate Measure of the Cost of Living*, Final Report to the Senate Finance Committee from the Advisory Commission to Study the Consumer Price Index.

Ceny w gospodarce narodowej w 2008 r., Warszawa 2009, GUS.

Consumer Price Index Manual: Theory and Practice ILO, 2004.

Crawford A., Fillion J-F., Lafleche T. (1997) *Is the CPI a Suitable Measure for Defining Price Stability?,* Bank of Canada Working Paper.

Crawford A. (1998) *Measurement Biases in the Canadian CPI: An Update*, Bank of Canada Review, Spring, pp. 38–56.

Cunningham A.W.F. (1996) *Measurement biases in price indexes: an application to the UK's RPI*, Bank of England, Working Paper Series 47.

Diewert W.E. (1978) *Superlative Index Numbers and Consistency in Aggregation*

Diewert W.E. (1995) *Axiomatic and economic approaches to elementary price indexes,* NBER Working Paper Series, No. 5104.

Diewert W.E. (1998) *Index Number Issues in the Consumer Price Index,* Journal of Economic Perspectives, Vol. 12, No. 1, pp. 47-58.

Diewert W.E. (2002)*Harmonized Indexes of consumer prices: their conceptual foundations*, ECB Working Paper Series.

Ducharme L.M. (red.), (1997) *Bias in the CPI: Experiences From Five OECD Countries,* Catalogue No. 62F0014MPB, No. 10, Prices Division, Statistics Canada.

Ducharme L.M. (2000) *The Canadian CPI and the bias issue: present and future outlooks*, Estadistica Espanola, Vol. 42, No. 145, pp. 25-41.

European Central Bank (2004), *Monetary Policy of the ECB.*

Filer R.K., Hanousek J. (2003) *Inflationary bias in middle to late transition Czech Republic*, Economic Systems 27, pp. 367-376.

Fisher I. (1922), *The making of index number*, Houghton Mifflin Company, Boston and New York.

Gali J., Gertler M. (1999), *Inflation dynamics: A structural econometric analysis*, Journal of Monetary Economics, Vol. 44.

Giovane P., Sabbatini R. (2008) *Perceived and measured inflation after the launch of the euro: explaining the gap in Italy* [in:] P. Del Giovane , R. Sabbatini (eds.) "The Euro, Inflation and Consumer's Perceptions: Lessons from Italy", Springer.

Gordon R.J., van Goethem T. (2005) *A Century of Housing Shelter Prices: Is There a Downward Bias in the CPI?*, NBER Working Paper 11776.

Hamilton B.W. (2001) *Using Engel's Law to Estimate CPI Bias*, The American Economic Review, Vol. 91.

Hill R.J. (2006), *Superlative index numbers: not all of them are super*, Journal of Econometrics 130 pp. 25-43

Kokoszczyński R. (2004) *Współczesna polityka pieniężna*, PWE.

Konüs A.A.(1939) *The Problem of the True Index of the Cost of Living*, Econometrica, Vol. 7, No. 1, pp. 10-29.

Kotler P., (2005) *Marketing*, Wydawnictwo Rebis.

Lequiller, F. (1997) *L'indice des prix à la consommation surestime-t-il l'inflation?,* Economie et statistique, N°303, pp. 3-32.

Ley E., (2005) *Whose Inflation? A characterization of the CPI plutocratic gap,* Oxford Economic Papers 57, pp. 634-646.

Łyziak T., Stanisławska E. (2008) *Consumer Inflation Expectations in Europe: Some Cross-country Comparisons*, Bank i Kredyt Vol. 39, No. 9, pp. 14-28.

Marini G., Piergallini A., Scaramozzino P. (2007) *Inflation bias after the Euro: evidence from the UK and Italy*, Applied Economics, Vol. 39, No. 4, pp. 461-470.

Ngasamiaku A., Mkenda W. (2009) *An Analysis of Alternative Weighting System on the National Price Index in Tanzania: The Implication to Poverty Analysis*, Botswana Journal of Economics, Vol. 6, No. 10, pp. 50-70.

Narodowy Bank Polski (2003), *Strategia polityki pieniężnej po 2003 roku*.

Neves P.D., Sarmento L. M. (1997) The substitution bias of the consumer price index, Banco de Portugal, Economic bulletin, June.

Nordhaus W.D. (1998) *Quality Change in Price Indexes,* Journal of Economic Perspectives, Vol. 12, pp. 59-68.

Pike, R., Marks C. Morgan D., (2008), *Measuring UK inflation,* Economic & Labour Market Review, Vol. 2, No. 9.

Prais S.(1959), *Whose Cost of Living*, Review of Economic Studies, No, 26, pp. 126-134.

Svensson L.E.O (2010) *Inflation Targeting*, Working Paper No, 16654, NBER.

Reinsdorf M, Triplett J.E (2009) *A Review of Reviews: Professional Thinking About the Consumer Price Index* [in:] "Price Index Concepts and Measurement", NBER.

Ruiz-Castillo J. Ley E., Izquierdo M., (2002) *The Laspeyres bias in the Spanish consumer price index*, Applied Economics, Vol. 34, No. 18, pp. 2267-76.

Rossiter J. (2005), *Measurement Bias in the Canadian Consumer Price Index*, Bank of Canada Working Paper 2005-39

Unayama T. (2004) *Upward bias in the consumer price index under the zero-inflation economy,* Economic Letters 85,pp. 139–144.

Wojtyna A. (2004), *Szkice o polityce pieniężnej*, PWE.

White W.R. (2006), *Is price stability enough?*, BIS Working Paper No. 205.