# Identifying SVARs with Sign Restrictions and Heteroskedasticity<sup>\*</sup>

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#### Abstract

This paper introduces a new method to identify structural vector autoregressions. The method combines the sign restrictions method with the identification through heteroskedasticity. I show that different volatility regimes of structural shocks can be used to strengthen the partial identification through sign restrictions. The method is applied to the identification of the monetary policy shocks. The standard sign restriction method is inconclusive about the sign of the response of output following a monetary policy shock. On the other hand, using the proposed method, the identified monetary policy shock lowers output significantly as predicted by theory.

*Keywords:* Structural Vector Autoregression, Heteroskedasticity, Sign restrictions, Monetary shocks, Partial identificationion.

JEL Classification: C3

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#### 1 Introduction

The identification of Structural Vector Autoregression models (SVAR) is still a central topic in macroeconomics. The identification still has to rely on subjective researches' assumptions. Therefore, SVAR model is considered empirically relevant only when the identification assumptions are seen as plausible by research community.

Faust (1998), Canova and Nicoló (2002) and Uhlig (2005) were first to identify SVAR models with uncontroversial sign restrictions that are based on robust predictions from theory and agreed upon by a majority of researchers. The uncontroversial nature of sign restriction has its price - sign restrictions only partially identify the model and the set of models consistent with sign restrictions is normally large. The large set of accepted models can result in a weakly identified SVAR model that offers mostly inconclusive predictions. An alternative approach was proposed by Rigobon (2003) who showed that the heteroskedasticity in the data can be used to identify the model. The identification is statistical and researcher does not need to impose any subjective assumptions. Nevertheless, the identification only works under the assumption that the variance of the structural shocks is varying in different regimes, but the structure of economy is fixed in time, which is seen as unrealistic by a large part of research community.

In this paper, I combine the sign restrictions methodology with the idea that heteroskedasticity is present in the data. The presence of heteroskedasticity helps to strengthen the identification through sign restrictions. The heteroskedasticity provides additional constraints on the structural parameters if we assume that the sign of the responses to the structural shocks is the same in different regimes. Therefore, the maintained assumption is much weaker than in case of purely statistical identification through heteroskedasticity, as we only have to assume that the sign of the responses to the structural shocks is the same in different regimes and not that the structure of the economy is fixed as assumed in Rigobon (2003).

The methodology is applied to study the response of output to a monetary policy shock. Similar to Uhlig (2005), I find that monetary policy shocks have an ambiguous effect on real GDP when using agnostic sign restrictions on prices and interest rate. However, I show that the identification can be considerably strengthen by using the methodology proposed in this paper. The methodology relies on two assumptions. First, the volatility of monetary shocks is assumed to vary over time. Second, it is assumed that the sign of the output response to a monetary policy shock is not changing over time. This additional agnostic assumptions lead to a more precisely identified model which predicts that output decreases significantly after the monetary shock as predicted by theory.

The rest of this paper is organized as follows. Section 2 presents the identification methodology. In section 3 the identification methodology is applied to simulated data. In section 4 the methodology is used to study the response of output to the monetary shock. Section 5 concludes.

## 2 Econometric methodology

A Structural Vector Auto-Regressive model (SVAR) can be written as:

$$y_t = A_0^{-1} A_1 y_{t-1} + A_0^{-1} A_2 y_{t-1} + \dots + A_0^{-1} A_K y_{t-K} + A_0^{-1} B \varepsilon_t,$$
(2.1)

where  $y_t$  is the  $N \times 1$  vector of endogenous variables, K is a finite number of lags, and the structural shocks  $\varepsilon_t$  are assumed to be white noise,  $\mathcal{N}(0, I_N)$ .  $A_0$  describes the contemporaneous relations between the variables, while matrices  $A_k$ ,  $k \in [1, 2, ..., K]$ describe the dynamic relationships. The diagonal matrix B contains the standard errors of the structural shocks.

The system (2.1) implies a following structural moving average representation,  $y_t = B(L)\varepsilon_t$ , where B(L) is a polynomial in the lag operator. The system in (2.1) cannot be estimated directly, but needs to be estimated in its reduced form:

$$y_t = A_1^* y_{t-1} + A_2^* y_{t-2} + \dots + A_P^* y_{t-K} + u_t,$$
(2.2)

where  $u_t = A_0^{-1} B \varepsilon_t$  and  $A_k^* = A_0^{-1} A_k$ .

The moving average representation of (2.2) is  $y_t = C(L)u_t$ . Therefore, the reduced form response function, C(L), is related to the structural impulse response function by  $B(L) = A_0C(L)$ . In other words, to identify the structural shocks and obtain the structural impulse responses,  $A_0$  ought to be identified.

Given  $S = A_0^{-1}B$ ,  $A_0$  is such that  $\Sigma_u = SS'$ , where  $\Sigma_u$  is the variance-covariance matrix of the reduced form errors. The decomposition  $\Sigma_u = SS'$  is not unique. For any H such that HH' = I, the matrix SH also satisfies this condition. In this case,  $SH(SH)' = SHH'S' = SS' = \Sigma_u$ . Therefore, starting from any arbitrary  $\tilde{S}$ , such that  $\Sigma_u = \tilde{S}\tilde{S}'$  (i.e. a Cholesky decomposition of  $\Sigma_u$ ), alternative decompositions can be found by post-multiplying by any H. The entire set of permissible impact matrices is infinite and the impact matrix cannot be identified uniquely from data.

### **2.1** $2 \times 2$ example

The proposed identification methodology is best described by a simple  $2 \times 2$  example.

Contemporaneous effects Dynamic effects Size of shocks  

$$A_0^{-1} = \begin{bmatrix} 1 & b \\ c & 1 \end{bmatrix} \qquad A_h = \begin{bmatrix} \alpha_{1,1}^h & \alpha_{1,2}^h \\ \alpha_{2,1}^h & \alpha_{2,2}^h \end{bmatrix} \qquad B = \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix}$$

Impact matrix  

$$A_0^{-1}B = \begin{bmatrix} \sigma_1 & b\sigma_2 \\ c\sigma_1 & \sigma_2 \end{bmatrix}$$

$$\Sigma_u = \begin{bmatrix} \sigma_1^2 + b^2\sigma_2^2 & c\sigma_1^2 + b\sigma_2^2 \\ c\sigma_1^2 + b\sigma_2^2 & c^2\sigma_1^2 + \sigma_2^2 \end{bmatrix}$$

The identification problem is that we want to identify impact matrix,  $A_0^{-1}B$ . In other words, we want to determine the values of four parameters,  $b, c, \sigma_1, \sigma_2$ . The values of parameters can be inferred from the estimated variance-covariance matrix of reduced form shocks,  $\hat{\Sigma}_u$ :

$$\Sigma_u = \begin{bmatrix} \sigma_1^2 + b^2 \sigma_2^2 & c \sigma_1^2 + b \sigma_2^2 \\ c \sigma_1^2 + b \sigma_2^2 & c^2 \sigma_1^2 + \sigma_2^2 \end{bmatrix} \qquad \qquad \begin{aligned} & \hat{\Sigma}_u = \begin{bmatrix} \hat{\Sigma}_{11} & \hat{\Sigma}_{12} \\ \hat{\Sigma}_{21} & \hat{\Sigma}_{22} \end{bmatrix} \end{aligned}$$

The variance-covariance matrix is symmetrical, implying we have only 3 equations for 4 parameters:

$$\begin{aligned} \sigma_1^2 + b^2 \sigma_2^2 &= \hat{\Sigma}_{11} & b, c, \sigma_1, \sigma_2 \\ c\sigma_1^2 + b\sigma_2^2 &= \hat{\Sigma}_{21} & 4 \text{ unknowns} \\ c^2 \sigma_1^2 + \sigma_2^2 &= \hat{\Sigma}_{22} & 3 \text{ equations} \end{aligned}$$

and therefore the system is system is not identified. A standard approach to identification is to assume a so-called exclusion restriction - in our example we would assume that bor c are zero. Then we could use three equations to determine the values of three free parameters. Alternatively, we can use heteroskedasicity of shocks to identify the model or only partial identify the parameters by using sign restrictions, for example.

To see how we can identify the model through heteroskedasticity, assume that we have two volatility regimes with  $\Sigma \neq \Sigma^*$ . In this case we have six equations for six parameters:

$$\begin{aligned} \sigma_1^2 + b^2 \sigma_2^2 &= \hat{\Sigma}_{11} \quad \sigma_1^{*2} + b^2 \sigma_2^{*2} = \hat{\Sigma}_{11}^* \quad b, c, \sigma_1, \sigma_2, \sigma_1^*, \sigma_2^* \\ c\sigma_1^2 + b\sigma_2^2 &= \hat{\Sigma}_{21} \quad c\sigma_1^{*2} + b\sigma_2^{*2} = \hat{\Sigma}_{21}^* \quad 6 \text{ unknowns} \\ c^2 \sigma_1^2 + \sigma_2^2 &= \hat{\Sigma}_{22} \quad c^2 \sigma_1^{*2} + \sigma_2^{*2} = \hat{\Sigma}_{22}^* \quad 6 \text{ equations} \end{aligned}$$

The system is now exactly identified - using six equations we can exactly determine the values of six parameters. This simple example shows how we can identify why assuming heteroskedasticity.

The main advantage of identification through heteroskedasticity is that it is based on data and therefore does not demand subjective constraints on parameters. However, notice that in order to achieve identification through heteroskedasticity you have to assume that the contemporanous effects, b, c, do not change between the regimes. Namely, if we allow  $b, c \neq b^*, c^*$ , we still have 6 equations, but now 8 parameters, so again the system is not identified. The assumption that the structure of the economy does not change is an extremely strong assumption. For example, under this assumption the monetary policy in the US did not change in 80s.

The second alternative is to use set restrictions on the values of parameters and identify a set of models that is consistent with the data and those restrictions. This partial identification method became the most popular identification method, as restrictions are mostly uncontroversial sign restrictions that are based on robust predictions from theory.

The method is best understood as a set of solutions of equations under parameter constraint. In order to facilitate the presentation, assume  $\sigma_1 = 1$  and  $\sigma_2 = 1$  in our 2 × 2 example. This allows us to discard two equations of the system, which reduces to one equation,  $c + b = \hat{\Sigma}_{21}$ , and two unknowns, b and c.

Figure 1: Solutions of equations under sign restriction



For  $\hat{\Sigma}_{21} = 1$ , black line shows all possible solutions of b and c. Shaded area represents sign restrictions, b, c > 0. Bold black line shows all b, c consistent with the data and the prior beliefs. The sign restrictions method can give conclusive predictions only when the



Figure 2: All solutions consistent with the data in system (2.3)

set of models that is consistent with the data and the restrictions (bold black line in our graph) is small. Unfortunately, in practice only using uncontroversial sign restrictions frequently results in inconclusive predictions as the set of models consistent with those restrictions is large.

#### 2.2 Sign restrictions and heteroskedasticity

In this subsection I show how the heteroskedasticity of shocks can be used to strengthen the agnostic sign restrictions. Continuing with our example, assume two regimes,  $\Sigma = 1$ and  $\Sigma^* = 5$ . We assume the sign restriction on parameter b and b<sup>\*</sup>. We have two systems:

$$b + c = 1 \quad b^* + c^* = 5 b > 0 \qquad b^* > 0$$
(2.3)

The yellow plane in Figure 2 shows all solutions that is consistent with the data - all solutions of b + c = 1 and  $b^* + c^* = 5$  for  $b, b^*, c$  and  $c^*$ . The vertical axis shows the ratio  $c/c^*$  in order to present solutions on the 3-d graph.

Figure 3 further adds the sign restrictions, b > 0 and  $b^* > 0$ . The green plane present the solutions to the system in (2.3). We can notice that the imposition of the sign restrictions considerably reduces the space of models.

Finally, we can use the fact that those two system are somehow related. In an extreme case we could assume that the systems represent two equal economies, except for the difference in the variance of the structural shocks,  $b = b^*$  and  $c = c^*$ . Under this assumption, we would obtain a unique values of parameters - exact identification - as



Figure 3: All solutions consistent with the data and sign restrictions in system (2.3)

Figure 4: All solutions consistent with the data, sign restrictions and in system (2.3)



shown in subsection 2.1. However, as discussed above this assumption is strong and hard to defend.

The novelty of this paper is to propose a less stringent assumption. Instead of assuming that parameters are exactly equal in the two regimes, we can assume that the parameters share the same sign in the two regimes. For example, instead of assuming that monetary policy did not change from 70s to 80s, we can assume that the sign of the response of output to a monetary policy shock was the same before and after 80s. The advantage is that we do not have to assume whether response was positive or negative, only that the sign of the response was the same over the two regimes.

The blue plane in Figure 4 presents the solutions to the system in (2.3) when we impose such restriction - we assume that c and  $c^*$  have equal sign in the two regimes.

We can notice that the blue plane is covers smaller space compared to the green plane that presents the solutions when only sign restrictions are imposed. This graphically shows that using heteroskedasticity and imposing "sign" restriction over the two regimes reduces the set of models consistent with the data and prior constraints. As is shown in the empirical section below, this approach can considerably strengthen the identification with sign restrictions.

#### 2.3 Estimation algorithm

The estimation procedure consists of three steps. In the first step, the reduced form VAR model is estimated on each subsample. In the second step, the subsample estimates are concatenated in one system. The QR method proposed by Rubio-Ramirez et al. (2010) is then used to rotate the concatenated system to obtain the models that are consistent with the data and the sign restrictions. In the third step, estimation uncertainty is taken into account. Formally, the steps are:

- 1. Estimate reduced-form VAR: Given a data sample m and a chosen number of lags,  $\hat{K}$ , a  $VAR(\hat{K})$  is estimated by Ordinary Least Squares (OLS) to obtain an estimate of autoregressive coefficients  $\hat{A}_m^*(L)$ , the corresponding reduced form impulse response function,  $\hat{C}_m(L)$  and of the variance-covariance of reduced form errors,  $\hat{\Sigma}_m$ , for each subsample m.
- 2. Identification: The first step is to construct the concatenated large system from the subsample estimates:

$$\widehat{\boldsymbol{\Sigma}} = \begin{pmatrix} \widehat{\Sigma}_1 & 0 & \cdots & 0 \\ 0 & \widehat{\Sigma}_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \widehat{\Sigma}_M \end{pmatrix} \qquad \widehat{\boldsymbol{C}}(\boldsymbol{L}) = \begin{pmatrix} \widehat{C}_1(L) & 0 & \cdots & 0 \\ 0 & \widehat{C}_2(L) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \widehat{C}_M(L) \end{pmatrix} \quad (2.4)$$

C(L), is related to the structural impulse response function via  $B(L) = A_0 C(L)$ and reduced form errors from subsample m,  $u_{m,t}$ , are related to structural shocks as  $u_{m,t} = A_{0,m}^{-1} B_m \varepsilon_{m,t}$ . We can construct the system-wide impact matrix:

$$\boldsymbol{S} = \begin{pmatrix} A_{0,1}^{-1}B_1 & 0 & \cdots & 0 \\ 0 & A_{0,2}^{-1}B_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & A_{0,M}^{-1}B_M \end{pmatrix}$$
(2.5)

and the system wide impact matrix must satisfy:  $\Sigma = SS'$ .

- (a) The initial estimate of  $\hat{S}$  is obtained by a Cholesky decomposition of the system wide variance-covariance matrix of reduced form errors,  $\hat{\tilde{S}} = chol(\hat{\Sigma})$ , giving an initial estimate of the impulse response function is  $\hat{\tilde{B}}(L) = \hat{C}(L)\hat{\tilde{S}}$ .
- (b) A  $nM \times nM$  matrix  $\boldsymbol{P}$  is drawn from standard normal distribution,  $\mathcal{N}(0,1)$ and the QR decomposition of  $\boldsymbol{P}$  is derived. Note that  $\boldsymbol{P} = \boldsymbol{Q}\boldsymbol{R}$  and  $\boldsymbol{Q}\boldsymbol{Q'} = \boldsymbol{I}$ .
- (c) The initial estimate of the impulse response function is post-multiplied by Q, to obtain a candidate impulse response function  $\widehat{B}(L) = \widehat{C}(L)\widehat{\widetilde{S}}Q$ , where:

$$\widehat{\boldsymbol{B}}(\boldsymbol{L}) = \begin{pmatrix} \widehat{B}_1(L) & 0 & \cdots & 0 \\ 0 & \widehat{B}_2(L) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \widehat{B}_M(L) \end{pmatrix}$$
(2.6)

and  $\widehat{B}_m(L)$  is the candidate impulse response function in a subsample m.

- (d) The steps 2b-2c are repeated until the candidate impulse responses,  $\widehat{B}(L)$ , satisfy the identifying restrictions within each sample and identifying restrictions between each sample.
- 3. Estimation uncertainty: to account for estimation uncertainty, we repeat 1000 times steps 1-2, each time with a new artificially constructed data,  $Y_m^*$ . To construct bootstrapped data, I use re-sampling of errors for each subsample separately. The new bootstrapped data sub-sample is constructed recursively as  $y_{m,t}^* = \widehat{A}_{m,1}^* y_{m,t-1}^* + \dots + \widehat{A}_{m,N} y_{m,t-N}^* + \widehat{u}_{m,t}^*$ , starting from initial values  $[y_{m,0}, \dots, y_{m,N-1}]$ .  $\widehat{A}_{m,n}^*$  are the estimated reduced form autoregressive coefficients and  $\widehat{u}_{m,t}^*$  are drawn randomly, with replacement, from the estimated reduced form errors,  $\widehat{u}_{m,t}$ .

The point estimates and confidence bands are given by the median and relevant percentiles of the distribution of retained impulse response functions over each subsample.

### 3 Simulation example

In this section I show how the proposed identification method works on the simulated data. For illustrative purposes, assume the following parameterization of (2.1):

Dynamic effects	$Contemporaneous \ effects$	Size of shocks
$A_1 = \left[ \begin{array}{rrrr} 0.4 & 0.2 & 0.3 \\ 0.2 & 0.4 & 0.2 \\ 0.2 & 0.2 & 0.4 \end{array} \right]$	$A_0^{-1} = \begin{bmatrix} 0.3 & -0.3 & 0.3 \\ 0.2 & 0.2 & -0.2 \\ 0.3 & -0.3 & -0.3 \end{bmatrix}$	$B = \left[ \begin{array}{rrrr} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right]$
Dynamic effects	$Contemporaneous \ effects$	-
$\begin{bmatrix} 0.4 & 0.2 & 0.3 \end{bmatrix}$	$A_0^{*-1} = \begin{bmatrix} 0.3 & -0.3 & 0.3\\ 0.6 & 0.7 & -0.1\\ 0.3 & -0.3 & -0.3 \end{bmatrix}$	$\begin{bmatrix} 5 & 0 \end{bmatrix}$
$A_1^* = \begin{bmatrix} 0.2 & 0.4 & 0.2 \end{bmatrix}$	$A_0^{*-1} = \left  \begin{array}{cc} 0.6 & 0.7 & -0.1 \end{array} \right $	$B^* = \left[ \begin{array}{ccc} 0 & 1 & 0 \end{array} \right]$
$\left[\begin{array}{ccc} 0.2 & 0.2 & 0.4 \end{array}\right]$	$\begin{bmatrix} 0.3 & -0.3 & -0.3 \end{bmatrix}$	

where \* stands for the parameterization in the second regime. The assumption is that the volatility of the first shocks changes between the regimes (heteroskedasticity). Contrary to a standard approach, where it is assumed that both  $A_1$  and  $A_0^{-1}$  are fixed in different regimes, here it is assumed that contemporaneous effects,  $A_0^{-1}$ , differ in different regimes. The dynamic effects,  $A_1$ , are kept fixed just for convenience.

To identify the shocks the following sign restrictions are imposed on impact:

Table 1: Sign restrictions

	shock 1	shock 2	shock 3
variable 1	+	-	+
variable 2	+	+	-
variable 3	(+)	(-)	-

As explained above, we can use the heteroskedasticity of shocks to strengthen identification when we do not want to impose a specific sign restriction. Instead we can assume that the direction of the response of variable is unknown, but its sign does not change between the regimes. For example, in empirical section below I assume that the response of output to the monetary policy shock is unknown, but the sign of its response is the same over the whole sample.

In the present illustrative example we can, for example, drop sign restrictions in the brackets in Table 1 and only assume that the direction of responses is the same in the two assumed regimes. Figure 5 illustrates how much we gain from using heteroskedasticity

Figure 5: The IRFs from illustrative example



Standard identification

Figure 6: Standard identification + heteroskedasticity



The figure shows the IRFs to the first shock in the first sample from the illustrative example. The median blue line represents the median of the distribution of the retained IRFs that are consistent with the imposed restrictions. The red dotted line represents the 2.5th and 97.5th percentile of the distribution of retained IRFs. The IRFs are normalized by the standard deviation. The black dotted lines with circles represent true IRFs.

when identifying structural shocks.<sup>1</sup> The first panel shows IRFs when we use a standard approach and we identify the three shocks using the sign restrictions from Table 1.

We can notice that imposing an additional cross-regime restriction that the sign of the response for the third variable is the same across the two regimes for the first and second shock strengthens the identification. The response of the third variable to the first shock is insignificant when only using standard sign restrictions, while adding crossregime restrictions the response is more precisely estimated. The estimates now confirm that the third variable increases after the first shock as implied by the simulation model.

<sup>&</sup>lt;sup>1</sup>In simulations the reduced form VAR coefficients are imposed and not estimated. This implies that uncertainty is related only to identification uncertainty and not parameter uncertainty.



Figure 7: The IRFs under a standard identification - one regime

The figure shows the IRFs to the monetary shock. The median blue line represents the median of the distribution of the retained IRFs that are consistent with the imposed restrictions. The red dotted line represents the 16th and 84th percentile of the distrubution of retained IRFs. The IRFs are normalized by the standard deviation, so that the identified monetary shock leads to a 100 basis points increase in the federal funds rate.

## 4 Monetary shocks

In this section I study the response of output following a monetary shock by applying the proposed identification method.

#### 4.1 Data and the reduced-form model

The baseline dataset includes only three variables from the dataset used in Uhlig (2005) who in turn follow Bernanke and Mihov (1998). The baseline contains monthly interpolated series of real GDP, the GDP deflator and the federal funds rate for the U.S.. I use the same sample as used by Uhlig (2005) running from January 1965 until December 2003. The regime specific VARs are estimated in log-levels, except for the federal funds rate, with a constant and two lags.

#### 4.2 Results

I first start with a standard approach taken in the literature. The VAR is estimated on the whole sample. The monetary shock is identified by assuming that federal funds rate increases and GDP deflator decreases following the monetary shock, while the response of the GDP is unrestricted.<sup>2</sup>

Figure 7 present the results of this standard approach. Similar to what is find by Uhlig (2005), I find that the response of output is insignificant and if anything, the median IRF even suggest that output increases after the monetary shock. The output significantly decreases only after two years following the monetary shocks.

<sup>&</sup>lt;sup>2</sup>Following Uhlig (2005), responses are restricted for 6 months.

Figure 7 presents the results when we identify the monetary policy shock using the methodology proposed in this paper. Specifically, I assume two regimes that are separately estimated over two subsamples that are obtain by spliting the whole sample on half. The first subsample contains data from January 1965 until December 1981 and the second sample contains data from January 1982 until December 2003. Further, as in the baseline exercise, I do not restrict the response of output, but I assume that the sign of the response of output has the sa, sign in both regimes.

Figure 8: The IRFs: sign restrictions + heteroskedasicity - two regimes, first regime



Two regimes, first regime

The figure shows the IRFs to the monetary shock. The median blue line represents the median of the distribution of the retained IRFs that are consistent with the imposed restrictions. The red dotted line represents the 16th and 84th percentile of the distrubution of retained IRFs. The IRFs are normalized by the standard deviation, so that the identified monetary shock leads to a 100 basis points increase in the federal funds rate.

The upper panel in Figure 7 shows responses in the first subsample, the middle panel

shows the responses in the second subsample and the last panel shows the average response over the whole sample (mean of subsample responses). The additional restrictions considerably improve identification, as output significantly lowers after the monetary shock in both subsamples and on average.

## 5 Conclusion

This paper introduced a new method to identify the SVAR models. The method combines the partial identification methodology with the identification through heteroskedasticity. On the one hand, this method allows to relax a stringent assumption that the structure of SVAR model is fixed in different regimes that is needed when identifying SVAR purely through heteroskedasticity. On the other hand, the imposition of additional constraints strengthens the partial identification, providing more precise estimates.

The advantage of this method over a standard sign restrictions was used to study the effects of monetary policy shocks on output. The

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