MODEL FOR DETERMINATION OF THE OPTIMAL RATE OF PROFIT TAXES OF DIFFERENT SECTORS OF ECONOMY

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Abstract.

A mathematical economic model was developed to determine the optimal rate of direct taxes for firms of different sectors of economy, which, in the conditions of absence of influence of inflation, results in optimal rates of direct taxes while maximizing government revenue, and provides incentives for firms to invest and undertake busines risks.

Keywords: profit tax, tax rates, sectors of economy, maximizing government revenue, Laffer curve

Introduction

Key element of management and attainment of economic and financial sustainability of firms in different sectors of economy under the emerging market conditions is the conduct of balanced research on taxation system. Here, it is particularly important to set a rate of direct taxes for firms that would take the maximum account of characteristics and value added of different sectors of economy. In doing so, it is necessary to develop a methodical approach to estimation and analysis of impact of tax rate on the results of firm performance for various sectors of economy. There are a large number of contemporary publications devoted to issues of taxation. However, there is an obvious scarcity of practical studies on taxation across sectors, particularly on identifying quantitative and qualitative characteristics of tax burden on firms for different sectors of economy.

Definition of the task

The profit of a firm for different sectors of economy is the sum of difference between the revenue from sales of goods in the market and expenditures during a specific period $[t_0, T]$ discounted by a ratio *r* [3]:

$$P^l = \int_0^T \left[p^l(t) Q^l(t) - C^l(t) \right] e^{-rt} dt . \qquad (1)$$

Here, $E^l(t)$ – are current expenditures that can be expressed as linearly dependent on the volume of products of a firm of sector *l* of economy as follows:

$$C^l(t) = k^l(t) Q^l(t) , \qquad (2)$$

where $p^l(t)$, $k^l(t)$ – are, respectively, the price and value of 1t of product of sector *l* of economy at a year *t*; $Q^l(t)$ – is the current level of products of sector *l* of economy at time *t*, which can be shown as:

$$Q^l(t) = \int_0^t n^l(\tau) q^l(\tau, t) d\tau . \qquad (3)$$

Here $n^l(\tau)$ – is the unit of capacity of sector *l* of economy (for oil and gas extraction – this is a number of wells, for power generation – it is construction of generation capacity, for agriculture – it is a plot of land, for manufacturing – it is a number of equipments, etc.), integrated into production at time - τ ; $q^l(\tau, t)$ - is the current level of production of sector *l* of economy, put into production at time - τ : $0 \le \tau \le t$.

From equations (1) - (3) we get:

$$P^l = \int_0^T \left[p^l(t) - k^l(t) \right] e^{-rt} \int_0^t n^l(\tau) q^l(\tau, t) d\tau dt . \qquad (4)$$

The type of taxation in each country of the world is such that usually according to legislation of any country, the tax is levied from annual profit. That is why it would be best to construct the model by discrete value for sector *l* of economy at time *t*, namely on annual basis:

$$P^l = \sum_{i=1}^{T} \frac{p_i^l - k_i^l}{(1+r)^{i-1}} \sum_{j=1}^{i} n_i^l q_{ij}^l . \qquad \overline{P}^l = (1 - x^l) \frac{p_i^l - k_i^l}{(1+r)^{i-1}} \sum_{j=1}^{i} n_i^l q_{ij}^l \qquad (5)$$

Let us assume that every ruble of profit earned by a firm is fully reinvested. In doing so, fixed capital is invested to add productive capacity. Let a firm of sector *l* of economy have initial capital in the amount - K^l . For simplicity of calculations, let us allow that $p_i^l - k_i^l = c^l = const$ and $r = 0$. Also, let us allow that the value of a unit of capacity of a firm of sector *l* of economy remains constant throughout the period in question: $w^l = const$.

Let us assume that the rate of profit tax of a firm in a sector *l* of economy is equal to x^l. The taxable income of a firm of sector *l* of economy earned each year is taxed at a rate x^l. The rest of the profit of the firm of sector *l* of economy ($\overline{P}_i^l$) is left for a firm, which it can freely dispose:

$$\overline{P}_i^l = (1 - x_i^l)(p_i^l - k_i^l)\sum_{j=1}^{i} n_i^l q_i^l \ . \qquad (6)$$

Taking into account the assumptions and equation (6) mentioned above, we can get the following recurrent relationship for a unit of capacity of a firm of sector *l* of economy put in use at year *i*:

$$n_i^l = \frac{c^l}{w^l}(1 - x^l)\sum_{k=1}^{i-1} n_k^l q_{ki}^l, \quad i = 2,...,T \ , \text{ где } \ n_1^l = \frac{K^l}{w^l} \ . \qquad (7)$$

Upon some modifications, we get the following relationship from (7) for a unit of capacity of a firm of sector *l* of economy put in use at year *i*, depending only on tax rate on profit *x*:

$$n_i^l = \sum_{k=0}^{i-1} a_i^l (1 - x^l)^k, \quad i = 1,...,T \ , \qquad (8)$$

Where

$$a_i^k = \left(\frac{c^l}{w^l}\right)^k \sum_{j=k}^{i-1} a_{jk-1}^l q_{jk-1}^l, \ k = 1,...,i-1; \ i = 2,...,T;$$

$$a_{10}^l = \frac{K^l}{w^l}; \ \ a_{jo}^l = 0, \ j = 2,...,T; \ \ a_{1k}^l = 0, \ k = 1,...,T$$

Cumulative unit of capacity for the entire forecast period of a firm of sector *l* of economy:

$$N^l(x) = \sum_{i=1}^{T} n_i^j = \sum_{i=1}^{T}\sum_{k=0}^{i-1} a_{1k}^l (1 - x^l)^k \ . \qquad (9)$$

As profit tax rate of a firm for each sector *l* of economy - x^l increases from zero to 100 percent, cumulative units of capacity $N^l(x^l)$ are gradually declining for the entire forecast period $[0, T]$, starting from its maximum at a point of zero tax rate $N^{*l} = N^l(0)$, and equals zero at 100 percent tax rate.

This is explained by declining investment capacity of firms at higher rates of profit tax.

Considering equation (8), we can modify relationship (5) by disclosing the sum and grouping the parameters and get a suitable form for taxable income of a firm of sector *l* of economy dependent on rate of profit tax – x^l :

$$P^l = c^l \sum_{i=1}^{T} \sum_{j=1}^{i} q_{ij}^l \sum_{k=0}^{i-1} a_{ik}^l (1 - x^l)^k \ . \tag{10}$$

Hence, recurrent formula (1) - (10) allows to identify correlation between a system of indicators of revenues, expenditures and profit of a firm in different sectors of economy dependent on various levels of tax rates and in doing so enables the identification of an effective level of taxation in firms.

Optimization of tax rate

First case. At the outset, in order to calculate necessary quantity assessments let us have a look at simple example. Let us assume that, amount of profit tax on any *l*-sector out of the *m* number of sectors of the economy under the consideration depends on its own x^l tax rate, i.e. $Tax^l = Tax^l(x^l)$. Famous American economist, Laffer, aggregating statistical data of different countries, discovered a rule, according to which a curve characterizing the relationship between tax burden and government revenue is hump-shaped and implies that declining tax rates may stimulate higher output, hence expand the tax base [1,2]. In other words, government will get more revenue at a lower rate as higher taxes crowd out output and deprive firms of cash flow and investments.

Hence direct taxes are becoming a more subtle and permanent source of income for the government. However, if set too high a direct tax rate can render much of output and investment projects economically less viable. In addition, there may be stronger incentives to shift into so-called “grey” or “shadow” economy, when practices like tax evasion, use of double accounting, etc. occur. In this case, it is not rare to see the volume of tax receipts decline in absolute terms in spite of rise in the tax rate. In the opposite case, when the tax rate is set too low, the government may not get the portion of taxes that could have been taken away from the firms without having a negative impact on the economy.

Summing up the above mentioned, we can say that according to conditions of Laffer curve, tax rates should be defined based on optimization conditionality of the curve that is characterized by government revenues and the rate of direct taxes.

Let us formulate the task of optimization of tax burden of firms representing various sectors of economy as follows:

We need to maximize the sum of tax collections to budget calculated as a percentage of profit of firm of sector *l* of economy during a period of $[0, T]$ dependent on tax rate – x^l taking into account the characteristics of different sectors of economy:

$$Tax^l(x^l) = c^l x^l \sum_{i=1}^{T}\sum_{j=1}^{i} q_{ij}^l \sum_{k=0}^{i-1} a_{ik}^l (1-x^l)^k \rightarrow \max . \quad (11)$$

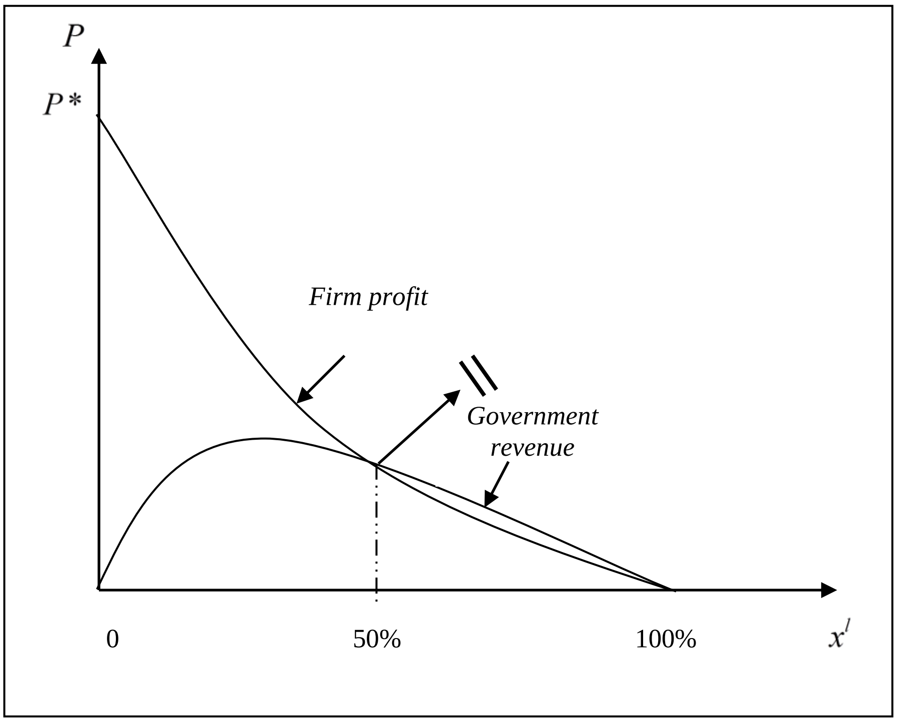
Given the following natural constraints of the key parameter of the profit tax rate - x^l :

$$0 \le x^l \le 1. \quad (12)$$

Hence, the task of identifying the optimal rate of direct taxes for firms of different sectors of economy (11) – (12) is to find the key parameter x^l given the condition (12) is met, when the sum of all taxes from the firm profits (11) would be at maximum.

Research on the type of question and solutions to it

The analysis of change in function characterizing profits of different sectors of economy (10) and tax receipts of the state budget (government revenue) (11) dependent on tax rate in the interval of $[0,1]$, demonstrates that as profit tax rate - x^l increases from zero to 100 percent profit of firm for the entire forecast period of $[0,T]$, starting from its maximum that is reached at the point of zero tax rate $P^{*l} = P^l(0)$, it starts gradually going down and equals to zero at 100 percent tax rate. And in process of growth of profit tax rate - x^l from zero to 100 percent tax receipts will grow from zero to some maximum level, and then to decrease to zero – Graph 1.



Graph 1. Change of profit of a firm of *sector l of economy* (P^l) and government revenue (Tax^l) from tax rate – x^l.

The obtained results show that the solution of the problem (11)-(12) is fully consistent with rules expressed by Laffer curve, and thus it is fair to make the following statements:

Statement 1. The points of the curve associated with the profit of a firm of sector *l* of economy (10) and tax receipts to state budget (11) are crossed by points associated with 50 percent direct tax rate and at this point the net profit of a firm is equal to government revenue.

Statement 2. The functions (11) are positively concave in interval $[0,\ 1]$ and, consequently, the optimization problem (11) - (12) has only one extreme at $x^l \in [0,1]$, which gives the function (11) the global maximum $x^l = x^{*l}$:

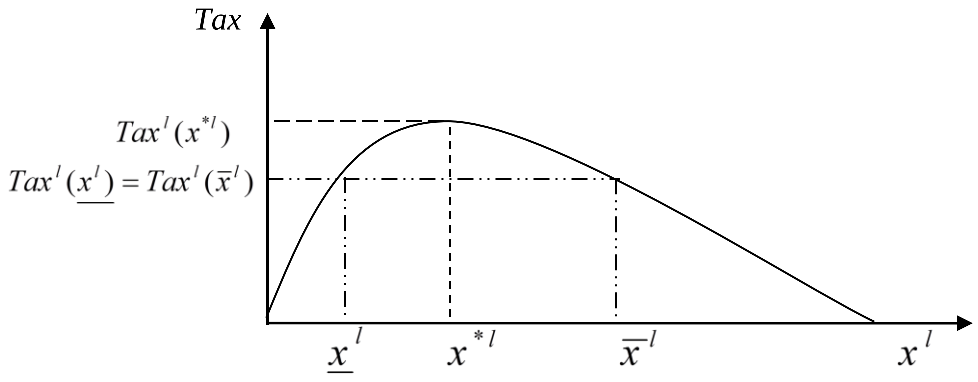
$$Tax^l(x^{*l}) = \max_{x^l \in [0,1]} Tax^l(x^l), \tag{13}$$

But, it also has two lowest values, which occur at the bounds of interval $[0,1]$:

$$\min_{x^l \in [0,1]} Tax^l(x^l) = Tax^l(0) = Tax^l(1). \tag{14}$$

In other words, function (11) is strictly monotonously increasing at $x^l \in [0, x^{*l})$ and strictly monotonously decreasing at $x^l \in (x^{*l}, 1]$, and at each interval there is only one point, the functional values of which are equal, i.e. given $\underline{x}^l \in [0, x^{*l})$ and $\bar{x}^l \in (x^{*l}, 1]$ - Graph 2:

$$Tax^l(\underline{x}^l) = Tax^l(\bar{x}^l). \tag{15}$$



Graph 2. Characteristics of changes in government revenue (Tax^l)

Statement 3. The point $x^l = x^{*l}$, giving the function (11) the maximum in the interval of [0,1], is not at the crossing point of curves of profit function of a firm of sector l of economy and of government revenue, i.e. always $x^{*l} \leq 50\%$.

Summing up the obtained results of equations (11) - (12), it can be concluded that in the conditions of absence of influence of inflation, the best economic effect is achieved not when the tax rates are the lowest, but when the rates are optimal (13), which is where the point $x^l = x^{*l}$, giving function (11) a maximum in the interval of [0,1].

To study some possible effects of parameters on the level of taxes, profit as well as the evaluation of practical application, a series of estimations were made on the development of reserves using the presented models (1) – (12). The estimations are carried out based on a pre-developed program, according to which the optimization problem is solved through iterations. To solve optimization problems (11) – (12), computer software was developed, which can be used to carry out hypothetical, multivariant, calculating experiments.

Second case. For nay economic sector the volume of profit tax depends on tax rate of all economic sectors under consideration, i.e. $Tax^l = Tax^l(x^1, x^2, ..., x^m)$. In such case (11) function characterizing profit tax is found as the form of multiplicative multinominal possessing sufficient complicated form in respect to its nominals $(1 - x^l), l = 1,2,...,m$. Solution of examples (11)-(12) is done via applying analytic procedures, and by repeating procedures for $x^1, x^2, ..., x^m$ tax rates aboveshown assessments remain valid.

We should note that, by considering inter-sectoral relations, parameters characterising each sector included in $Tax^l(x^1, x^2, ..., x^m)$ funksion are calculated on the basis of balanced prices model designed on the basis of intersectoral (output-input) balance of Leontyev.

Conclusions

A new model is proposed to identify optimal rate of direct taxes for firms of different sectors of economy allowing to determine the optimal tax rate in the conditions of absence of influence of inflation, which provides for maximal revenue for government and incentives for firms to work, invest and take business risks, thus, expanding the national output and income.

Bibliography

[1] Arthur B. Laffer, "The Laffer Curve: Past, Present, and Futhure", Excecutive Summary, Backgrounder, No 1765, June 1, 2004.

[2] Jude Wanniski, "Taxes, Revenues, and the `Laffer Curve,'" The Public Interest, Winter 1978.

[3] Валиев В.М., Пашаев Т.Р. Модель оптимизации налоговой нагрузки предприятий нефтегазодобывающего комплекса. Известия НАН АР, серия физика-техника и математика, том XXVI, №2, 2006, стр.179-184.