ACTIVE LEARNING AND OPTIMAL CLIMATE POLICY

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Abstract

This paper develops a climate-economy model with uncertainty, irreversibility and active learning. Whereas previous papers assume passive learning from one observation per period, or experiment with control variables to gain additional information, this paper considers active learning from research investment in improved observations. We restrict ourselves to improving observations of the global mean temperature. We find that the decision maker invests a significant amount of money in climate research, far more than the current level, in order to increase the rate of learning about climate change. This helps the decision maker take improved decisions. The level of uncertainty decreases more rapidly in the active learning model with research investment than in the passive learning model only with temperature observations. As a result, active learning reduces the optimal carbon tax. The greater the risk, the larger is the effect of learning. The method proposed here is applicable to any dynamic control problem where the quality of monitoring is a choice variable.

Key words

Climate policy; irreversibility; learning; active learning

JEL Classification

Q54; O3; C63

1 Introduction

It has long been known that the prospect of future affects current decisions, and in some cases it is optimal to experiment with policy so as to acquire information. In this paper, we consider investment in improved monitoring as an alternative route to improve information, and investigate the implications for climate policy.

There is considerable uncertainty about every aspect of climate change and climate policy. As the consequences of emission abatement decisions are irreversible or at least long-lived, the prospect of future learning materially affects optimal climate policy in the short-run. The previous literature assumes that new knowledge either arrives exogenously or arrives without cost in the form of new observations about the climate system. However, learning requires investment and can be decelerated or accelerated by monitoring and research. We here explore simultaneous decisions about learning and abatement.

Manne and Richels (1992), Peck and Teisberg (1993), Kolstad (1996a, b), Nordhaus and Popp (1997), Ulph and Ulph (1997), Gollier et al. (2000), and Webster (2002) incorporate exogenous learning in analyses of climate policy. In these studies, information arrives exogenously at some points in time, and thus learning is independent of the actions of the decision maker. Kelly and Kolstad (1999), Leach (2007), Webster et al. (2008), Jensen and Traeger (2013), Kelly and Tan (2013), Hwang et al. (2014), Lemoine and Traeger (2014) endogenize learning.[1] Uncertainty becomes smaller as the decision maker observes the system. As the state of the system, and thus the information it contains, depends on past decisions, the planner controls, to a certain extent, what is learned. However, in these models, learning is passive in that it is a by-product of decisions on the carbon tax. In a cartoon representation, there are two state variables (the atmospheric concentration of carbon dioxide, and the stock of knowledge) but only one control

[1] Karp and Zhang (2006) consider learning about damage costs in their theoretical paper.

variable (the carbon tax). Decisions are therefore suboptimal (Tinbergen, 1954). Here, we introduce a second control variable (i.e. research investment) in order to control learning and emissions separately.

In a seminal paper, Prescott (1972) shows that there is a tradeoff between control and information when there is uncertainty about the effect of a policy instrument. He finds it is optimal to sacrifice part of the current benefits of control in order to obtain information that improves future decisions – we refer to this as experimentation. Especially when uncertainty is large or when time horizon is long, experimentation becomes more important (MacRae, 1972; Bar-Shalom and Tse, 1976; Grossman et al., 1977; Wieland, 2000a; Beck and Wieland, 2002).[2]

The current paper follows the standard Bayesian approach.[3] In the early literature (e.g., Kendrick, 1972; Grossman et al., 1977), the decision maker has a prior belief on an uncertain parameter and she updates her belief using Bayes' Rule – we refer to this as passive learning. Later developments include multiple uncertainties (e.g., Wieland, 2000a), other dynamics than passive learning (e.g., Bertocchi and Spagat, 1998), time-varying parameters (e.g., Beck and Wieland, 2002), alternative utility functions (e.g., Johnson, 2007), and correlated information (e.g., Marcoul and Weninger, 2008). The model is typically solved using dynamic programming techniques with a grid search (e.g., Wieland, 2000a; Bond and Loomis, 2009).

The current paper differs from the literature. First and foremost, one of the control variables (i.e. research investment) is used exclusively to increase the speed of learning (i.e. the precision of climate

[2] In the literature, experimentation has been mainly investigated for monetary policy (e.g., Kendrick, 1982; Bertocchi and Spagat, 1993; Wieland 2000b; Beck and Wieland, 2002; Yetman, 2003). The other applications include a firm or consumer behavior (e.g., Grossman et al., 1977; Mirman et al., 1993), economic growth (e.g., Bertocchi and Spagat, 1998; Johnson, 2007), public election (e.g., Strulovici, 2010), job choice (e.g., Antonovics and Golan, 2012), and natural resource management (e.g., Marcoul and Weninger, 2008; Bond and Loomis, 2009). For asymptotic properties, see Taylor (1974), Easley and Kiefer (1988), and Agihon et al. (1991).

[3] For the non-Bayesian approach including dual control, see Bar-Shalom and Tse (1976), Kendrick (1982; 2005).

sensitivity estimation) – we refer to this as active learning. In previous papers, learning is a by-product of control (passive learning) or control is a by-product of learning (experimentation). Because learning is a control variable, we have to take the cost of learning into account, whereas most papers have costless learning. In an active learning model with research investment, gains from learning consist of improved decisions, whereas the costs of learning are the investment in observations and research. Balancing the gains and losses, the rate of learning is determined, together with the optimal carbon tax.

Existing papers on the decision making under uncertainty and learning about climate change assume that knowledge grows by one observation per year with constant precision (or observational errors) (e.g., Kelly and Kolstad, 1999; Leach, 2007; Webster et al., 2008; Kelly and Tan, 2013; Hwang et al., 2014). Instead, this paper considers additional learning through improved observations. Research investment in the global climate observational system increases the precision of temperature observations, lowering estimation errors for the equilibrium climate sensitivity.

The implementation of active learning in a climate economy model is worthwhile since decision makers make explicit efforts to gather information on uncertain variables. For instance, WMO and UNEP (2010) estimate that global annual expenditures on climate observations are about $4~6 billion. As a result, temperature observational errors have been substantially decreased (Kennedy et al. 2011). Likewise, decision makers make explicit efforts to promote research activities to learn about climate processes. The rate of learning depends on such efforts.

In order to solve our large model, we apply the simulation-based approximation method instead of a grid search. The model and computational methods of this paper are similar to those of Hwang et al. (2014), who in turn draw on Marliar and Marliar (2005) and Judd et al. (2011), except that research investment is introduced. This paper follows the tradition of Bayesian statistical decision theory which requires that uncertainty or partial ignorance can be represented as a probability distribution (DeGroot, 1970).

This paper proceeds as follows. Section 2 briefly describes the model. Section 3 illustrates the way of additional learning and our calibrations. Section 4 shows computational methods. Section 5 presents the main results of this paper and sensitivity analyses are given in Section 6. Section 7 provides conclusions.

2 A climate-economy model

2.1 Economy

The decision maker in our model chooses the rate of emissions control and research investment for each time period so as to maximize social welfare defined as in Equation (1). Gross output, net of damage costs and abatement costs, is allocated to consumption, research investment, and gross investment other than climate research. Reducing the computational burden, the savings rate is assumed to be exogenous. This assumption does not materially affect the results, since the savings rate does not change much for plausible model specifications.

$$\max_{\mu_t, R_t} \mathbb{E} \sum_{t=0}^{\infty} L_t \beta_t U(C_t, L_t) \tag{1}$$

$$C_t = \left(1 - \theta_1 \mu_t^{\theta_2}\right) \Omega_t Q_t - I_t - R_t \tag{2}$$

where $\mathbb{E}$ is the expectation operator, U is the utility function, C is consumption, L is population, μ is the rate of emissions control, I is gross investment (other than climate research), Ω is the damage function, Q

is gross output, R is investment in climate research, β is the discount factor, θ_1 and θ_2 are parameters. See Appendix A for the full model and the parameter values.[4]

The research capital stock accumulates as follows:

$$K_{R,t+1} = (1 - \delta_R)K_{R,t} + R_t \tag{3}$$

where K_R is the research capital stock, δ_R is the depreciation rate of research investment. For simplicity the research capital stock is assumed not to depreciate over time (δ_R=0).[5]

2.2 Temperature response model

Integrated assessment models usually use some energy balance models for temperature (Marten, 2011), which includes radiative forcing of the atmosphere and heat exchange between atmosphere and ocean through upwelling and diffusion (Baker and Roe, 2009).

If we assume that there are two boxes for temperature (the mixed layer and the deep ocean), the temperature response model becomes:[6]

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[4] Unlike DICE, the time step is annual with infinite time horizon in our model. In order to consider the effect of uncertainty more properly, we let the lower bound of consumption as low as possible and remove the upper bound of temperature increases. In addition, backstop technology is not considered in the model.

[5] This assumption does not affect the main results of this paper. For instance, even if the ordinary depreciation rate of the capital stock, namely 0.1 per year, is applied to the model, there is no significant difference in the results, except that research investment stabilizes at a certain low level (not zero) so as to compensate for the amount of depreciated research capital stock (results not shown).

[6] See Marten (2011) and Hwang et al. (2015) for more discussion on temperature response model and its implications.

$$T_{AT_{t+1}} = T_{AT_t} + \xi_1 \{ RF_{t+1} - (\eta/\lambda) T_{AT_t} - \xi_3 (T_{AT_t} - T_{LO_t}) \} \tag{4}$$

$$T_{LO_{t+1}} = T_{LO_t} + \xi_4 \{ T_{AT_t} - T_{LO_t} \} \tag{5}$$

where T_{AT} and T_{LO} are atmospheric and oceanic temperature changes, respectively, from 1900, $RF = \eta \ \ln(M_t/M_b)/\ln(2) + RF_{N,t}$ is radiative forcing, $RF_{N,t}$ is radiative forcing from other than greenhouse gas, λ is the equilibrium climate sensitivity, η, ξ_1, ξ_3, and ξ_4 are parameters.

The equilibrium climate sensitivity refers to the equilibrium global warming in response to a doubling of the atmospheric concentration of carbon dioxide, the major anthropogenic greenhouse gas (°C/2xCO_2). The probability distribution of the climate sensitivity is derived from the distribution of the total feedback factors (Roe and Baker, 2007), using

$$\lambda = \lambda_0/(1 - f) \tag{6}$$

where f is the total feedback factors which is assumed to be strictly less than 1, and λ_0 is the equilibrium climate sensitivity in a black body planet without any feedbacks.

The total feedback factors denote the aggregate impacts of physical factors such as water vapor, cloud, and albedo on radiative forcing in a way to magnifying the response of the climate system (Hansen et al., 1984). For instance, "[A] positive radiative forcing such as that due to an increase in CO_2 tends to increase temperatures, which tends to increase water vapor, which, in turn, produces a perturbation in the down welling long wave radiation that amplifies the original forcing" (Roe, 2009: 97).

This framework of feedback analysis is useful in the following reasons: 1) the total feedback factors are observable, unlike the climate sensitivity; 2) it is easy to apply Bayes' Theorem since the total feedback factors are assumed to be normally distributed; 3) the resulting climate sensitivity distribution has fat tails (Roe and Baker, 2007). Risk is fat-tailed if the probability density of an uncertain variable falls more slowly than exponentially in the tail (Weitzman, 2009).

Substituting Equation (6) for λ in Equation (4), rearranging, replacing radiative forcing with its components, and introducing temperature shock results in:

$$T_{AT_{t+1}} = (\zeta_1 f + \zeta_2)T_{AT_t} + \zeta_3 \ln(M_{AT_t}/M_b) + \zeta_4 T_{LO_t} + \zeta_5 RF_{N,t} + \varepsilon_{t+1} \tag{7}$$

where RF_N is radiative forcing from non-CO_2, M_{AT} is the carbon stock in the atmosphere, M_b(=596.4GtC) is the pre-industrial carbon stock in the atmosphere, ε is natural temperature shock (or natural variability) (Brohan et al., 2006; Webster et al., 2008), $\zeta_1 = \xi_1\eta/\lambda_0$, $\zeta_2 = 1 - \zeta_1 - \zeta_4$, $\zeta_3 = \xi_1\eta/\ln(2)$, $\zeta_4 = \xi_1\xi_3$, and $\zeta_5 = \xi_1$ are adjusted parameters.

Actual temperature increase is governed by Equation (7). *Observed* temperature increase however is also affected by observational errors (e.g., measurement errors and data coverage bias) as follows.

$$T_{AT_t}^{obs} = T_{AT_t} + \varepsilon_t^{obs} \tag{8}$$

where ε^{obs} denotes observational errors, T_{AT_t} is the actual temperature change (Equation 7).

3 Improved observations

3.1 Research investment and observational errors

The rate of learning about the climate sensitivity is sensitive to temperature shocks (Webster et al., 2008). As the standard error in the mean of temperature increases (decreases, respectively), the signal to noise ratio falls (grows, respectively), making it more difficult (easier, respectively) to detect the true state of the world. The standard error in the mean of temperature falls as the global climate observational system improves. For instance, as illustrated in Figure 1, global temperature observational errors, one of the components of temperature shocks, have decreased over time as the number of observational instruments such as weather stations has increased.[7]

In order to build a learning model with research investment, this paper assumes that the variance of natural temperature shock is constant over time. This is not unreasonable in that natural temperature shock such as natural variability is not controlled by the decision maker and the effect of climate change on natural temperature shock can be thought of as negligible at least for hundreds of years. Then the standard error in the mean of observed temperature is decomposed into two elements as follows:

$$\sigma_{\varepsilon,t}^2 = \sigma_{ob,t}^2 + \sigma_{natural}^2 \tag{9}$$

where σ_{ε}, σ_{ob}, and $\sigma_{natural}$ are the standard error in the mean of observed temperature, observational errors, and natural temperature shock, respectively.

[7] The quality of observations as well as the number of observations is important for the standard error in the mean of temperature. For instance, increasing the number of observational system does not necessarily improve the precision of the measurement of climate change if the quality of observations is limited. However for simplicity we refer the consideration of the quality of observations to future researches.

Broadly speaking, temperature's observational errors are linearly related to the reciprocal of the number of observational instruments (Jones et al., 1997; Brohan et al., 2006), at least in the relevant domain (see Figure 1). Assuming independence between sea surface temperature (SST) observational errors and land air temperature (LAT) observational errors, the total observational errors of global mean air temperature can be calculated as follows.

$$\sigma_{ob_t}^2 = \sum_j \omega_j \sigma_{ob_{j,t}}^2 = \sum_j \omega_j \left(\alpha_j / N_{o\,j,t} + \beta_j\right) \tag{10}$$

where $j \in \{l, s\}$ refers to each observation (l for LAT and s for SST), ω is the respective area of the land or the sea, N_o is the number of observational instruments, α and β are parameters.

For simplicity, we assume that observational errors approach zero as investment in the global temperature observational system increases arbitrarily large (β_j=0). Then Equation (9) leads to Equation (11), the channel through which research investment affects the uncertainty about temperature shocks:

$$\sigma_{\varepsilon,t}^2 = \omega_l\, c_l \alpha_l / \left(pK_{R_t}\right) + \omega_S c_s \alpha_s / \left\{(1-p)K_{R_t}\right\} + \sigma_{natural}^2 = a_R / K_{R_t} + \sigma_{natural}^2 \tag{11}$$

where K_R is the research capital stock for the global temperature observational system, $0 \le p \le 1$ is the proportion of money spent on land observations, $c_l \equiv pK_R/N_{0,l}$ and $c_s \equiv (1-p)K_R/N_{0,s}$ are the unit cost of LAT and SST observation, respectively, and $a_R \equiv \omega_l\, c_l \alpha_l / p + \omega_S c_s \alpha_s / (1-p)$.

3.2 Bayesian updating

The decision maker updates her belief on the total feedback factors using Bayes' Theorem as follows:

$$p(f|T_{AT}^{obs}) \propto p(T_{AT}^{obs}|f) \times p(f) \quad (12)$$

where $p(f)$ is the prior distribution, $p\left(T_{AT}^{obs}\middle|f\right)$ is the likelihood function, and $p\left(f\middle|T_{AT}^{obs}\right)$ is the posterior distribution.

The normal distribution of Roe and Baker (2007) with parameters $\overline{f_t}$ and v_t is used as the initial prior for the year 2005. Note that the parameters become endogenous state variables. The general techniques for Bayesian updating as discussed in DeGroot (1970) and Greenberg (2007) are applied. The resulting posterior mean and the posterior variance of the total feedback factors are:

$$\overline{f_{t+1}} = \frac{\overline{f_t} + \zeta_1 T_{AT_t}^{obs} H_{t+1}\, v_t / v_{\varepsilon,t}}{1 + \left(\zeta_1 T_{AT_t}^{obs}\right)^2 v_t / v_{\varepsilon,t}} \quad (13)$$

$$v_{t+1} = \frac{v_t}{1 + \left(\zeta_1 T_{AT_t}^{obs}\right)^2 v_t / v_{\varepsilon,t}} \quad (14)$$

where $\overline{f_t}$ and v_t are the mean and the variance of the total feedback factors, $v_{\varepsilon,t} = \sigma_{\varepsilon,t}^2$ is the variance of observed temperature shocks, and $H_{t+1} \equiv T_{AT_{t+1}}^{obs} - \zeta_2 T_{AT_t}^{obs} - \zeta_3 \ln(M_t/M_b) - \zeta_4 T_{LO_t} - \zeta_5 RF_{N,t}$.

The posterior distribution with parameters $\overline{f_{t+1}}$ and v_{t+1} of Equations (13) and (14) serves as the prior for the next time period. In this way the decision maker learns about the true value of the total feedback factors for each time period. Note that the parameters of the posterior distribution are affected by research investment through Equations (11, 14). The higher is research investment the lower is the variance of the total feedback factors.

For simulations, the initial values for $\overline{f_t}$ and v_t are assumed to be 0.65 and 0.13^2, respectively, following the current scientific knowledge (Roe and Baker, 2007). Since the total feedback factors are bounded above, the posterior distribution is derived first with the conjugate normal prior, and then an upper bound ($\overline{f_t} \leq 0.999$) is set for simulations. The upper bound corresponds to the climate sensitivity of 1,200°C/2xCO_2, which is far higher than any admitted values.

3.3 Calibration

Instead of estimating all the parameters in Equation (11), this paper estimates only a_R and $\sigma^2_{natural}$. To this end, first, global expenditures on temperature observations are estimated. Currently, global mean LAT is calculated from the records of each country's weather stations and global mean SST is calculated from the reports of observational platforms such as ships, drifting buoys, and moored buoys (Kennedy et al., 2011). Thus we multiply the number of observational instruments and the unit cost of each instrument (see Table 1). Annual operational costs for temperature observational instruments are estimated to be about \$450 million in 2005.[8] The total installation costs for all the existing instruments are about \$500 million.[9] Second, $\sigma^2_{natural}$ is calculated as the difference between the total variance of temperature shocks (=0.10^2) estimated by Tol and De Vos (1998) and the variance of observational errors (=0.06^2) obtained from the HadCRUT4 dataset (Morice et al., 2012). Then it is not unreasonable to assume that the current

[8] For comparison, the United States spent \$140 million on *in-situ* climate observations in 2010 (submission of USA to UNFCCC/SBI 35). WMO and UNEP (2010) estimate that annual global expenditures on climate observations are about \$4~6 billion. Douglas-Westwood (2006) estimate that the total costs of ocean observations are \$402 million in 2005. Their estimates are not directly comparable to this paper, however, because their estimates include all kinds of observations besides temperature, such as precipitation, wind, ice, as well as satellite observations.

[9] This number is small compared to the world economy. For instance, the initial value for the global capital stock (in 2005) is \$137 trillion in the original DICE model. Thus the research investment in climate observations has a negligible effect on the growth path of the world economy.

research capital stock (K_{R_0} = \$950 million) produces the current variance of temperature shocks ($\sigma^2_{\varepsilon,0}$=0.10^2) through Equation (11). Therefore, a_R=\$3.42 million.[10]

As shown in Figure 2, our parameterizations imply that the variance of temperature shocks decreases (increases) as the research capital stock increases (decreases). If there is no change in the research capital stock the variance of observational errors (in turn, the variance of temperature shocks) remains the same.

Finally, we assume that the decision maker does not make an effort to reduce the variance of observational errors if she thinks there has been enough learning.[11] More specifically, we set R_t=0 if $\sigma^2_{\varepsilon,t} - \sigma^2_{natural} < \omega_c$, where ω_c reflects the level of satisfaction of the decision maker about the magnitude of learning. Note that $\sigma^2_{\varepsilon,t}$ is always higher than $\sigma^2_{natural}$ in our model although $\sigma^2_{\varepsilon,t}$ becomes close to $\sigma^2_{natural}$ as the research capital stock increases. Put differently, $\sigma^2_{natural}$ is the lower bound for the variance of temperature shocks in the model. From Equations (3) and (11) this assumption serves as an upper bound of research investment ($R_t \leq a_R/\omega_c - (1-\delta_R)K_{R,t}$). For instance, with ω_c=10^{-5} and the above parameterizations (i.e., a_R=\$3.42 million, $K_{R,0}$=\$950 million, δ_R=0), the upper bound of the initial research investment is about \$341 billion. The upper bound is sensitive to the cost estimates (a_R), the decision maker's satisfaction about the magnitude of learning (ω_c), and the level of the research capital stock (see Section 6.1).

[10] These calibrations assume that operational costs are included in the research capital stock, for simplicity. An alternative is to explicitly represent operational costs in the model, but this does not affect the main results of this paper (results not shown).

[11] As illustrated below and in Section 6.1, this assumption is useful for setting the upper bound of annual research investment. Roughly, we can think of this as a kind of budget constraint.

4 Computational methods

In order to solve the active learning model, the dynamic programming method proposed by Maliar and Maliar (2005) is applied. For more details on the solution algorithm see Hwang (2014). The problem is reformulated in a recursive way as:

$$W(\boldsymbol{s}_t, \boldsymbol{\theta}_t) = \max_{\boldsymbol{c}_t} [U(\boldsymbol{s}_t, \boldsymbol{c}_t, \boldsymbol{\theta}_t) + \beta \mathbb{E}_t W(\boldsymbol{s}_{t+1}, \boldsymbol{\theta}_{t+1})] \tag{15}$$

$$W(\boldsymbol{s}_t, \boldsymbol{\theta}_t) \approx \sum_{n=1}^{N} \psi(\boldsymbol{s}_t, \boldsymbol{\theta}_t; \boldsymbol{b}_{\boldsymbol{n}}) \tag{16}$$

where W is the value function starting from period t, $\boldsymbol{c}$ is the vector of control variables (μ, R), $\boldsymbol{s}$ is the vector of state variables (K, K_R, M_{AT}, M_U, M_L, T_{AT}, T_{LO}, $\bar{f}$, v, L, A, σ), M_U and M_L are the carbon stocks in the upper ocean and the lower ocean, respectively, σ is the emissions-output ratio, $\boldsymbol{\theta}$ is the vector of uncertain variables (f, ε), ψ is the basis function, and $\boldsymbol{b}$ is the vector of coefficients for the basis function.

The solution algorithm is summarized as follows. First, approximate the value function with a flexible basis function. Second, derive the first order conditions for optimal policy rules. Third, choose an initial guess on the coefficients $\boldsymbol{b}$ of the basis function: $\boldsymbol{b}^{(0)}$. Fourth, simulate a time series of variables satisfying the first order conditions, transitional equations, and boundary conditions with the initial guess $\boldsymbol{b}^{(0)}$.[12] Fifth, calculate the left hand side and the right hand side of the Equation (15) using the simulated time series, and then find $\boldsymbol{b}$ that minimizes the difference between them: $\widehat{\boldsymbol{b}}$.[13] Sixth, update the initial

[12] The simulation length is set at 1,000 years. Longer horizons do not affect the main results of this paper.

[13] The Gauss-Hermite integration is applied for the expectation in Equation (15) with 10 integration nodes. Higher number of nodes does not affect the main results of this paper.

guess $\boldsymbol{b}^{(0)}$ using a pre-specified updating rule: $\boldsymbol{b}^{(1)}$. Seventh, iterate the above process with the new guess $\boldsymbol{b}^{(1)}$ until the value function converges. [14]

Accounting for random realizations of the uncertain variables, the model is run 1,000 Monte Carlo simulations and the average of all simulations is presented in Sections 5 and 6. For additional results, see Appendix B. The true value of the total feedback factors is set at 0.6 (which corresponds to the equilibrium climate sensitivity 3°C/2xCO_2, the most likely value according to the current scientific knowledge (See Stocker et al., 2013) throughout the results in this paper. The models are also simulated with a different true value of the total feedback factors and different initial beliefs, but the general implications of these simulations do not change (results not shown).

5 Research investment and climate policy

5.1 The rate of learning

Figure 3 shows the evolutions of the climate sensitivity distribution. For comparison, the results of the learning model where learning takes place only from instrumental temperature observations (with the constant variance of temperature shocks) are also presented. Table 2 shows the corresponding probability of high temperature increases of each case. As expected, the mean parameter $\bar{f}$ converges to the pre-specified true value and the variance parameter v approaches – but never reaches – zero over time. The rate of learning, measured as the reduction in the (simulated) coefficient of variation of the climate sensitivity, is higher under active learning with research investment than under passive learning only from temperature observations. It takes 45 years for the coefficient of variation to be reduced to a half level for improved observations whereas it takes 51 years in the passive learning model. This is because by

[14] The maximum tolerance level is set at 10^{-4}.

construction, learning in this paper constitutes an additional way to produce information. The probability density in the upper tail of the climate sensitivity distribution shrinks faster for active learning than for passive learning. Therefore the probability of high temperature increases is higher in the active learning model than in the passive learning model. For instance, the probability of temperature increases higher than 10℃ for active learning is more than 20 times higher than the one for passive learning case.

For comparison, the learning time for 50% reduction in the coefficient of variation of the climate sensitivity is about 60~70 years in Webster et al. (2008) when the prior similar to the current paper is used (see Figure 10 of their paper). The rates of learning in Kelly and Kolstad (1999), Leach (2007), and Kelly and Tan (2013) are not directly comparable to the current paper since they define learning differently from ours: learning takes place in their models when the mean of the uncertain variable becomes statistically close (e.g., the significance level of 0.05) to the pre-specified true value.

5.2 Optimal research investment and carbon tax

The optimal level of investment in climate research is much higher than the current level of annual expenditures. For instance, the initial level of investment in the global climate observational system is about $340 billion per year, compared to the current level of about $450 million per year in 2005. These results confirm that the benefits of learning are far greater than the costs of learning (Keller et al., 2007a,b; Baehr et al., 2008).

After the initial peak, research investment decreases rapidly (see Figure 4). This reflects the point that early investment to reduce uncertainty is more beneficial because (1) it benefits from a longer future and (2) knowledge saturates in our model specifications. Note that research investment is here mostly in equipment rather than specialist personnel, so that a rapid scaling-up and –down is feasible. These results imply that as the cost of learning is much lower than the benefit of learning, the optimal decision is to make uncertainty as low as possible.

Nordhaus and Popp (1997) estimate that the value of information on the climate sensitivity is \$6.9 ~ 11.7 billion with a discrete uncertainty representation (i.e., 5 states of the world: mean, ± 1 standard deviation, ± 2 standard deviation) and exogenous learning. The value of information in their model is calculated as the difference in expected utility between instant learning and learning in 50 years. Peck and Teisberg (1993) estimate that the value of information on the climate sensitivity is \$148 billion with a discrete uncertainty (i.e., 3 states of the world: 1, 3, 5°C/2xCO_2) and exogenous learning. The value of information in their model is calculated as the difference in expected utility between instant learning and no-learning. If learning in 40 years is considered, the value of information is \$24 billion. Keller et al. (2007a) estimate that the value of information associated with early detection of changes in the North Atlantic meridional overturning circulation (MOC) is a number in the tens of billions of dollars, which is far higher than the cost of MOC observation systems (tens of millions of dollars, see Baehr et al., 2007). Keller et al. (2007b) estimate that the value of information about climate sensitivity (with 3 states of the world) is about \$10 billion, but they also find that the value of information increases substantially if there are climate thresholds. For instance, if there is a temperature limit of 2.5°C the value of learning about the climate sensitivity is \$800 billion. Baker and Solak (2010) estimate that optimal level of R&D investment in energy technology under uncertainty about climate damages is on the order of tens of billions of dollars.

It is well known that fat-tailed risk substantially increases the stringency of climate policy (Tol, 2003; Weitzman, 2009). Since the current paper deals with fat-tailed risk it is not surprising that the benefit of learning is greater in our model than in the literature.

Learning substantially reduces the effect of fat-tailed risk since learning is faster in the tail. As a result, policy recommendations such as carbon tax as shown in the next paragraph would be substantially different when there is a possibility of learning. The optimal carbon tax is calculated as a Pigovian tax as in the original DICE model (Nordhaus, 2008). As expected, the optimal carbon tax is highest for the uncertainty model and is lowest for the deterministic model (see Figure 5 and Table 3). Passive learning lowers the optimal carbon tax and active learning further reduces the carbon tax.

6 Sensitivity analysis

6.1 Cost of learning

As shown in Figure 6, the level of research investment increases as the cost of learning increases. After the initial peak the research investment decreases to a low level, because knowledge saturates fast. This is intuitive in that the rate of variance-reductions from climate research ($\partial v_{t+1}/\partial K_{R_{i,t}}$) diminishes as the research capital accumulates. Put differently, after the initial peak more effort is required for further variance-reductions: one unit of variance-reduction becomes more expensive over time.

If the decision maker wants more (less) precise observations, the amount of money spent on the global observational system should be increased (decreased). For instance, if the decision maker sets a criterion that ω_c=10^{-6} (10^{-4}, respectively), instead of 10^{-5} as in the reference case, the level of investment is \$3.4 trillion (\$34 billion, resp.) for the improved observations model. The initial level of investment does not grow as much as the upper bound if the cost of learning is high. For instance, the upper bound of investment is about \$34 trillion for the case with 100 times the base cost, but the optimal investment is about \$4.5 trillion.

6.2 Damage functions

The effect of fat-tailed risk on climate policy is sensitive to the assumed shape of the damage function (Hwang et al., 2013). Table 3 presents the results when the damage function of Weitzman (2012), a highly reactive damage function, is applied. For comparison the results for the deterministic, uncertainty, and passive learning are also presented. The effect of the fat tail on climate policy is clear when the damage function of Weitzman (2012) is applied (see the results of the uncertainty model), but is greatly reduced when passive learning is introduced. Active learning further enhances this.

6.3 True values of climate sensitivity

We have assumed that our knowledge converges to the true value of climate sensitivity over time. However the true value is not known with certainty. This subsection investigates how sensitive our main results to the different true values of climate sensitivity.[15] Figure 7 shows some results. We observe that the higher the true value of climate sensitivity, the fatter the right tail of climate sensitivity distribution. As expected, the corresponding temperature changes and optimal carbon tax are higher (lower, respectively) for the higher (lower, resp.) true value cases.

Optimal research investment does not change much according to the true value of climate sensitivity: early investment to reduce uncertainty as much as possible (results not shown). One of the reasons is that the rate of learning about climate sensitivity is slow (see Section 5). Put differently, it takes a long time (a hundred years or more) for our knowledge to approach to the true value of the climate system. For instance, the probability of climate sensitivity being higher than 4.5℃/2xCO2 in 2055 in our model specifications is about 4.2% when the true value of climate sensitivity is 2℃/2xCO2.

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[15] Considering computational burden, we restrict ourselves to sensitivity analysis. For a full uncertainty analysis on the true value of climate sensitivity with a passive learning model, see Hwang et al. (2014).

6.4 Limits to learning

As mentioned above, there are limits to learning. The variance of the components of temperature shocks other than measurement error remains fixed at 0.08^2 for the reference case in Section 5. This serves as the lower bound for the variance of temperature shocks. The sensitivity of the optimal carbon tax to this lower bound is shown in the top left panel of Figure 8. The optimal carbon tax decreases if the lower bound of temperature shocks decreases. Put differently, the higher is the magnitude of learning, the lower is the optimal carbon tax.

7 Conclusions

This paper investigates the impact of active learning on optimal climate policy. More specifically, learning about the climate sensitivity from investing in improved observations has been introduced into an integrated assessment model (IAM). The decision maker reduces the uncertainty about climate change through significant investment in climate research, two orders of magnitude greater than the current level of expenditures. This helps the decision maker make better decisions on climate policy. The level of uncertainty decreases more rapidly with active learning model than with passive learning. As a result, the optimal carbon tax is lower for active learning than the carbon tax with passive learning, which in turn is lower than the carbon tax without learning. The effect of learning is more pronounced as tail risk fattens.

This paper is the first to introduce active learning into an integrated assessment model of climate and the economy. Applying alternative ways of learning would help to understand the role of learning further. This paper considers investments in improved monitoring of the global mean temperature. Other options include investment in climate research, which would sharpen the prior, and reconstructions of past temperature, which would increase the number of observations in the likelihood. However general conclusion would not change: 1) as the effect of uncertainty grows, learning plays a more significant role;

2) as long as the cost of learning is lower than the benefit of learning, it is optimal for the decision maker to invest in learning; 3) earlier investment in climate research is more beneficial than later investment; and 4) learning reduces the optimal carbon tax.

Active learning apart, this paper closely follows Nordhaus' DICE model. Other specifications should be explored, including alternative utility functions (e.g., Sterner and Persson, 2008), different mitigation cost and climate impact functions, and other economic growth models. Perhaps more importantly, we here posit a true value of the climate sensitivity, rather than a PDF, which exaggerates the effects of learning. We only consider one uncertain parameter, which suppresses both the effect of uncertainty and the cost of learning. We study the case of a global planner. Kolstad and Ulph (2008; 2011) show that uncertainty enhances cooperation. This would imply that active learning fosters free-riding. All these matters are deferred to future research.

Active learning by improved monitoring also applies to other areas of public policy. Learning by experimentation with policy variables is informative for issues with a short characteristic life time – monetary policy, for instance – but less so for issues that span long periods – besides climate change, pensions and structural unemployment come to mind. The method proposed here applies to any area in which knowledge of the response to policy is imperfect partly due to imperfect monitoring.

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References

Agihon, P., Bolton, P., Harris, C., and Tullien, B., 1991. Optimal learning by experimentation. The Review of Economic Studies 58(4), 621-654.

Antonovics, K., and Golan, L., 2012. Experimentation and job choice, Journal of Labor Economics 30(2), 333-366.

Baehr, J., Keller, K., and Marotzke, J., 2008. Detecting potential changes in the meridional overturning circulation at 26 N in the Atlantic, Climatic Change 91, 11-27.

Baker, E. and Solak, S., 2010. Climate change and optimal energy technology R&D policy, European Journal of Operational Research, 213(2), 442-454.

Bar-Shalom, Y., and Tse, E., 1976. Caution, probing, and the value of information in the control of uncertain systems. Annals of Economic and Social Measurement 5(3), 323-337.

Beck, G.W. and Wieland, V., 2002. Learning and control in a changing economic environment, Journal of Eocnomic Dynamics and Control 26, 1359-377.

Bertocchi, G. and Spagat, M., 1993. Learning, experimentation, and monetary policy. Journal of Monetary Economics 32, 169-183.

Bertocchi, G. and Spagat, M., 1998. Growth under uncertainty with experimentation. Journal of Economic Dynamics and Control 23, 209-231.

Bond, C.A., and Loomis, J.B., 2009. Using numerical dynamic programming to compare passive and active learning in the adaptive management of nutrients in shallow lakes. Canadian Journal of Agricultural Economics 57, 555-573.

Brohan, P., Kennedy, J.J., Harris, I., Tett, S.F.B., and Jones, P.D., 2006. Uncertainty estimates in regional and global observed temperature changes: A new data set from 1850. Journal of Geophysical Research, 111, D12106.

Cai, Y., Judd, K.L., and Lontzek, T.S., 2012. Open science is necessary. Nature Climate Change 2(5), 299.

Cosimano, T.F., 2008. Optimal experimentation and the perturbation method in the neighborhood of the augmented linear regulator problem. Journal of Economic Dynamics and Control 32, 1857-1894.

DeGroot, M.H., 1970. Optimal statistical decisions. McGraw-Hill, Inc.

Detrick, R., Frye, D., Collins, J., Gobat, J., Grosenbaugh, M., Petitt, R., Plueddeman, A., der Heydt, K., Wooding, F.B., and Orcutt, J., 2000. DEOS moored buoy observatory design study. Woods Hole Oceanographic Institution Technical Report, August.

Douglas-Westwod, 2006. Global markets for ocean observation systems, Final OOS report.

Easley, D. and Kiefer, N.M., 1988. Controlling a stochastic process with unknown parameters. Econometrica 56(5), 1045-1064.

Gollier, C., Jullien, B., and Treich, N., 2000. Scientific progress and irreversibility: An economic interpretation of the precautionary principle. Journal of Public Economics 75**,** 229-253.

Greenberg, E., 2007. Introduction to Bayesian econometrics. Cambridge: Cambridge University Press.

Grossman, S.J., Kihlstrom, R.E., and Mirman, L.J., 1977. A Bayesian approach to the production of information and learning by doing. The Review of Economic Studies 44(3), 533-547.

Hansen, J., Lacis, A., Rind, D., Russell, G., Stone, P., Fung, I., Ruedy, R., and Lerner, J., 1984. Climate sensitivity: Analysis of feedback mechanisms. Geophysical Monograph Series 29, 130-163.

Horowitz, J. and Lange, A., 2014. Cost-benefit analysis under uncertainty: A note on Weitzman's dismal theorem. Energy Economics 42, 201-203.

Hwang, I.C., Reynès, F., Tol, R.S.J., 2013. Climate policy under fat-tailed risk: An application of DICE. Environmental and Resource Economics 56(3), 415-436.

Hwang, I.C., Reynès, F., Tol, R.S.J., 2014. The effect of learning on climate policy under fat-tailed uncertainty. MPRA Working Paper No. 53681.

Hwang, I.C., Tol, R.S.J., Hofkes, M., 2015. Fat-tailed risk about climate change and climate policy. Unpublished manuscript. Available upon request.

Hwang, I.C., 2014. A recursive method for solving a climate-economy model: Value function iterations with logarithmic approximations. Munich Personal RePEc Archive Working Paper No. 54782.

Jensen, S., Traeger, C.P., 2013. Optimally climate sensitive policy under uncertainty and learning. Unpublished manuscript.

Johnson, T.C., 2007. Optimal learning and new technology bubbles. Journal of Monetary Economics 54, 2486-2511.

Jones, P.D., Osborn, T.J., and Briffa, K.R., 1997. Estimating sampling errors in large-scale temperature averages. Journal of Climate 10, 2548-2568.

Judd, K.L., Maliar, L., and Maliar, S., 2011. Numerically stable and accurate stochastic simulation approaches for solving dynamic economic models. Quantitative Economics 2, 173-210.

Karp, L. and Zhang, J., 2006. Regulation with anticipated learning about environmental damages. Journal of Environmental Economics and Management 51, 259-279.

Keller, K., Deutsch, C., Hall, M.G., and Bradford, D.F., 2007a. Early detection of changes in the North Atlantic meridional overturning circulation: implications for the design of ocean observation systems. Journal of Climate 20, 145-157.

Keller, K., Kim, S.R., Baehr, J., Bradford, D.F., and Oppenheimer, M., 2007b. What is the economic value of information about climate thresholds? Human-Induced Climate Change: an Interdisciplinary Assessment, Schlesinger,M., Kheshgi, H., Smith, J., De La Chesnaye, F., Reilly, J.M., Wilson, T., and Kolstad, C. (eds).Cambridge University Press: Cambridge 343-354.

Kelly, D.L. and Kolstad, C.D., 1999. Bayesian learning, growth, and pollution. Journal of Economic Dynamics and Control 23, 491-518.

Kelly, D.L. and Tan, Z., 2013. Learning and Climate Feedbacks: Optimal Climate Insurance and Fat Tails. University of Miami Working Paper.

Kendrick, D., 1982. Caution and probing in a macroeconomic model. Journal of Economic Dynamics and Control 4, 149-170.

Kendrick, D.A., 2005. Stochastic control for economic models: Past, present and the paths ahead. Journal of Economic Dynamics and Control 29, 3-30.

Kennedy, J.J., Rayner, N.A., Smith, R.O., Parker, D.E., and Saunby, M., 2011. Reassessing biases and other uncertainties in sea surface temperature observations measured in situ since 1850: 2. Biases and homogenization. Journal of Geophysical Research 116, D14104.

Kent, E., Hall, A.D., and Leader VOSClim Task Team, 2000. The voluntary observing ship (VOS) scheme. American Geophysical Union, 2000 Florida Ave., N. W. Washington DC 20009 USA.

Kolstad, C.D., 1996a. Learning and stock effects in environmental regulation: The case of greenhouse gas emissions. Journal of Environmental Economics and Management 31, 1-18.

Kolstad, C.D., 1996b. Fundamental irreversibilities in stock externalities. Journal of Public Economics 60, 221-233.

Kolstad, C.D. and Ulph, A., 2008 Learning and international environmental agreements. Climatic Change 89,125–141.

Kolstad, C.D. and Ulph, A., 2011. Uncertainty, Learning, and Heterogeneity in International Environmental Agreements. Environmental Resources Economics 50, 289-403.

Leach, A.J., 2007. The climate change learning curve. Journal of Economic Dynamics and Control 31, 1728-1752.

Lemoine, D. and Traeger, C., 2014. Watch Your Step: Optimal Policy in a Tipping Climate, AEJ-Economic Policy 6(1), 137-166.

MacRae, E.C., 1972. Linear decision with experimentation. Annals of Economic and Social Measurement 1(4), 437-447.

Maliar, L. and Maliar, S., 2005. Solving nonlinear stochastic growth models: Iterating on value function by simulations. Economics Letters 87, 135-140.

Manne, A. and Richels, R.G., 1992. Buying greenhouse insurance: the economic costs of carbon dioxide emission limits. The MIT Press.

Marcoul, P., and Weninger, Q., 2008. Search and active learning with correlated information: Empirical evidence from mid-Atlantic clam fishermen. Journal of Economic Dynamics and Control. 32, 1921-1948.

Marten, A., 2011. Transient temperature response modeling in IAMs: The effects of over simplification on the SCC. Economics: The Open-Access, Open-Assessment E-Journal 5, 2011-18.

Mburu, D.N., 2006. Possibilities for expansion of surface weather observing systems in east Africa. WMO publications IOM-94-TECO2006 1(6).

McKitrick, R., 2011. A simple state-contingent pricing rule for complex intertemporal externalities. Energy Economics 33, 111-120.

Meldrum, D., Wallace, A., Rolland, J., Burnett, W., Lumpkin, R., Niller, P., Viola, H., Charpentier, E., and Fedak, M., 2010. Data buoy observations: The status quo and anticipated developments over the next decade. Proceedings of OceanObs 9.

Millner, A., 2013. On welfare frameworks and catastrophic climate risks. Journal of Environmental Economics and Management 65, 310-325.

Mirman, L.J., Samuelson, L., and Urbano, A., 1993. Monopoly experimentation. Intenational Economic Review, 34(3), 549-563.

Morice, C.P., Kennedy, J.J., Rayner, N.A., and Jones, P.D., 2012. Quantifying uncertainties in global and regional temperature change using an ensemble of observational estimates: The HadCRUT4 data set. Journal of Geophysical Research 117, D08101.

Nordhaus, W.D., 1994. Managing the Global Commons: The Economics of Climate Change. The MIT Press.

Nordhaus, W.D., 2008. A question of balance: Weighing the options on global warming policies. New Haven and London: Yale University Press.

Nordhaus, W.D., 2011. The economics of tail events with an application to climate change. Review of Environmental Economics and Policy 5(2), 240-257.

Nordhaus, W.D. and Popp, D., 1997. What is the value of scientific knowledge? An application to global warming using the PRICE model. The Energy Journal 18, 1-46.

North, S., 2007. Report of the task team on satellite communications costs. SOT-IV. 16-21 April 2007, Geneva, Switzerland.

Peck, S.C. and Teisberg, T.J., 1993. Global warming uncertainties and value of information: An analysis using CETA. Resource and Energy Economics 15, 71-97.

Pindyck, R.S., 2011. Fat tails, thin tails, and climate change policy. Review of Environmental Economics and Policy 5, 258-274.

Prescott, E.C., 1972. The multi-period control problem under uncertainty. Econometrica, 40(6), 1043-1058.

Roe, G.H. and Baker, M.B., 2007. Why is climate sensitivity so unpredictable? Science 318(5850), 629-632.

Roe, G.H., 2009. Feedbacks, timescales, and seeing red. Annual Review of Earth and Planetary Sciences 37, 93-115.

Sterner, T. and Persson, U.M., 2008. An even sterner review: Introducing relative prices into the discounting debate. Review of Environmental Economics and Policy 2, 61-76.

Stocker, T.F., Qin, D., Platter, G., Tignor, M.M.B., Allen, S.K., Boschung, J., Nauels, A., Xia, Y., Bex, V., and Midgley, P.M., 2013. Summary for policymakers. In Climate change 2013: The physical science basis. Working Group 1 Contribution to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change.

Strulovici, B., 2010. Learning while voting: Determinants of collective experimentation. Econometrica 78(3), 933-971.

Taylor, J.B., 1974. Asymptotic properties of multiperiod control rules in the linear regression model. International Economic Review 15(2), 472-484.

Tinbergen, J., 1954. Centralization and decentralization in economic policy. Amsterdam, North-Holland.

Tol, R.S.J., 2003. Is the uncertainty about climate change too large for expected cost-benefit analysis? Climatic Change 56(3), 265-289.

Tol, R.S.J. and De Vos, A.F., 1998. A Bayesian statistical analysis of the enhanced greenhouse effect. Climatic Change 38(1), 87-112.

Ulph, A. and Ulph, D., 1997. Global warming, irreversibility and learning. The Economic Journal 107, 636-650.

Webster, M., 2002. The curious role of learning in climate policy: Should we wait for more data? The Energy Journal 23(2), 97-119.

Webster, M., Jakobovits, L., and Norton, J., 2008. Learning about climate change and implications for near-term policy. Climatic Change 89, 67-85.

Weitzman, M.L., 2009. On modeling and interpreting the economics of catastrophic climate change. Review of Economics and Statistics 91(1), 1-19.

Weitzman, M.L., 2011. Fat-tailed uncertainty in the economics of catastrophic climate change. Review of Environmental Economics and Policy 5(2), 275-292.

Weitzman, M.L., 2012. GHG targets as insurance against catastrophic climate damages. Journal of Public Economic Theory 14(2), 221-244.

Wieland, V., 2000a. Learning by doing and the value of optimal experimentation. Journal of Economic Dynamics and Control 24, 501-534.

Wieland, V., 2000b. Monetary policy, parameter uncertainty and optimal learning. Journal of Monetary Economics 46, 199-228.

WMO and UNEP, 2010. Implementation plan for the global observing system for climate in support of the UNFCCC (2010 update). GOOS-184 GTOS-76 WMO-TD No. 1523.

Yetman, J., 2003. Probing potential output: Monetary policy, credibility, and optimal learning under uncertainty. Journal of Macroeconomics 25, 311-330.

Appendix A: The full model

The list of variables and parameters are given in Tables A.1 and A.2.

$$\max_{\mu_t, R_t} \mathbb{E} \sum_{t=0}^{\infty} L_t \beta_t U(C_t, L_t) \tag{A.1}$$

$$C_t = \left(1 - \theta_1 \mu_t^{\theta_2}\right) \Omega_t Q_t - I_t - R_t \tag{A.2}$$

$$K_{R,t+1} = (1 - \delta_R) K_{R,t} + R_t \tag{A.3}$$

$$K_{t+1} = (1 - \delta_k) K_t + I_t \tag{A.4}$$

$$M_{AT\,t+1} = (1 - \mu_t) \sigma_t Q_t + E_{LAND_t} + \delta_{AA} M_{AT\,t} + \delta_{UA} M_{U\,t} \tag{A.5}$$

$$M_{U\,t+1} = \delta_{AU} M_{AT\,t} + \delta_{UU} M_{U\,t} \tag{A.6}$$

$$M_{L\,t+1} = \delta_{UL} M_{U\,t} + \delta_{LL} M_{L\,t} \tag{A.7}$$

$$T_{AT_{t+1}} = T_{AT_t} + \xi_1 \left\{ \eta \ \ln(M_t / M_b) / \ln(2) + RF_{N,t} - \eta T_{AT_t} / \lambda - \xi_3 (T_{AT_t} - T_{LO_t}) \right\} + \varepsilon_{t+1} \tag{A.8}$$

$$T_{AT_t}^{obs} = T_{AT_t} + \varepsilon_t^{obs} \tag{A.9}$$

$$T_{LO_{t+1}} = T_{LO_t} + \xi_4 \left\{ T_{AT_t} - T_{LO_t} \right\} \tag{A.10}$$

$$\overline{f_{t+1}} = \frac{\overline{f_t} + \zeta_1 T_{AT_t}^{obs} H_{t+1} \, v_t / v_{\varepsilon,t}}{1 + \zeta_1^2 {T_{AT_t}^{obs}}^2 \, v_t / v_{\varepsilon,t}} \tag{A.11}$$

$$v_{t+1} = \frac{v_t}{1 + \zeta_1^2 {T_{AT_t}^{obs}}^2 \, v_t / v_{\varepsilon,t}} \tag{A.12}$$

$$v_{\varepsilon,t} = \alpha_{R_1} / K_{R_t} + \sigma_{others}^2 \tag{A.13}$$

where $\mathbb{E}$ is the expectation operator given information at point in time t (annual).

Appendix B: Additional results

Figure B.1 is the results for improved observations (the reference case in Section 5). Each figure is the average of 1,000 Monte Carlo simulations. This figure shows how each variable evolves over time. For instance the rate of emissions control gradually increases during the first 2~3 centuries and then reaches at one (full abatement). The carbon stock gradually decreases after the rate of emissions control becomes one. Atmospheric temperature follows the same pattern with a time lag. The maximum temperature increases (from 1900) are less than 4°C (in the early 22nd century) for all the cases. There is an initial peak in research investment and then the level of research investment becomes trivial. Consumption and gross investment (other than research investment) grows continuously since our model is based on the DICE model, which represents continuous economic growth.

Figure B.2 shows the results of all runs. Almost all variables have high variation but it is less severe than the one for the learning model only with temperature observations. This is because there is additional learning in each learning model.

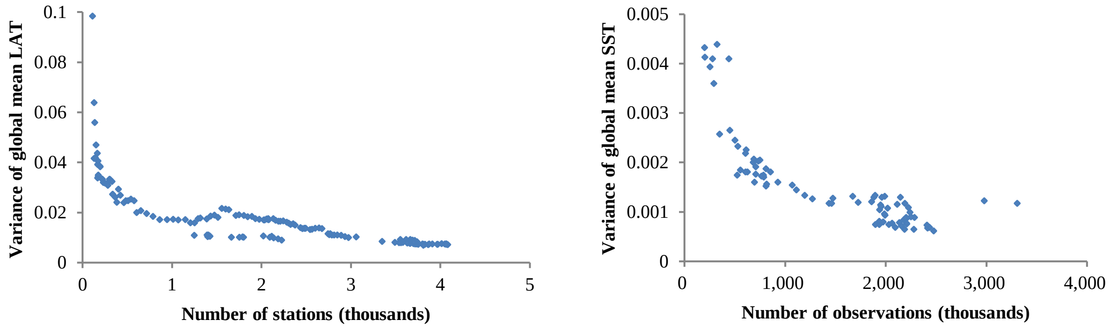

Figure 1 Uncertainty about global mean temperature (Left): The variance of global mean land air temperature (LAT) 1850-2006 (CRUTEM3, Brohan et al., 2006) as a function of the number of weather stations used to estimate the global mean temperature. **(Right):** The variance of global mean sea surface temperature (SST) 1925-2006 (HadSST3, Kennedy et al., 2011) as a function of the number of observations used to estimate the global mean temperature. The data were obtained from John Kennedy (personal communication).

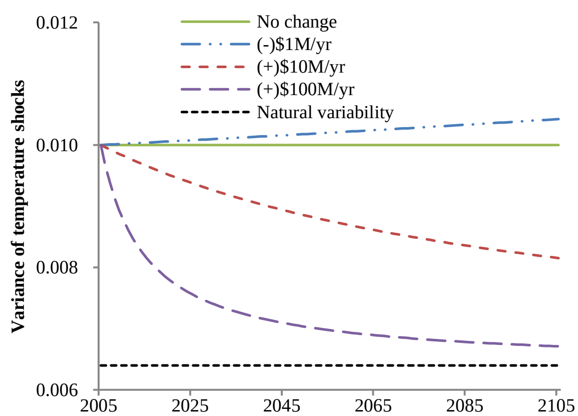

Figure 2 Hypothetical learning dynamics No change refers to the case where the research capita stock remains the same as in the initial year. + $X/yr (respectively, - $X/yr) refers to the case where the research capital stock increases (resp., decreases) $X every year from the initial level.

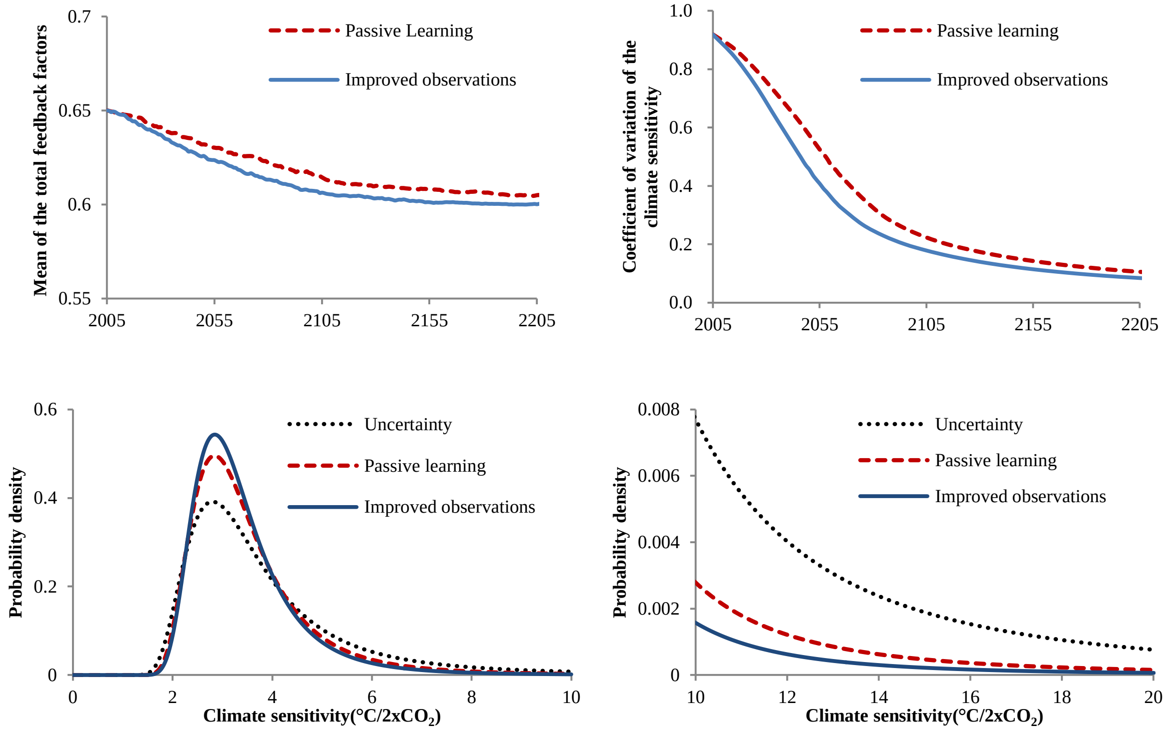

Figure 3 Climate sensitivity distribution (Top left): The mean of the total feedback factors **(Top right):** The (simulated) coefficient of variation of the climate sensitivity. **(Bottom left):** Climate sensitivity distribution in 2055 (0~10°C/2xCO_2). **(Bottom right):** Climate sensitivity distribution in 2055 (10~20°C/2xCO_2).

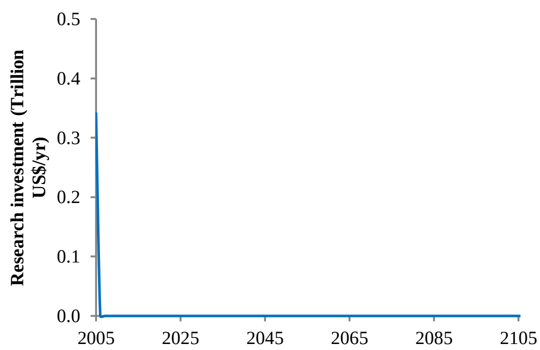

Figure 4 Research investment

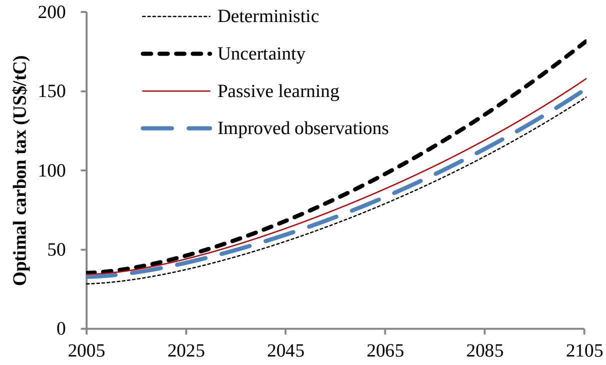

Figure 5 The optimal carbon tax

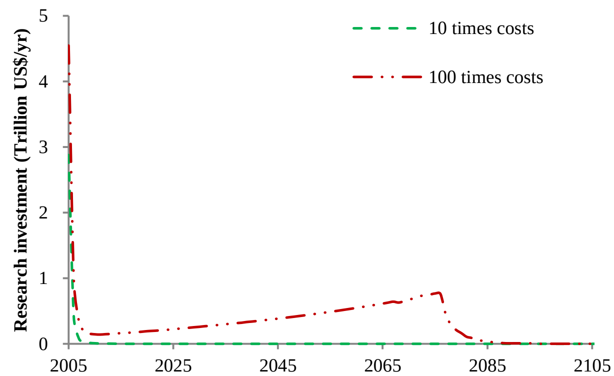

Figure 6 Research investment (sensitivity analysis)

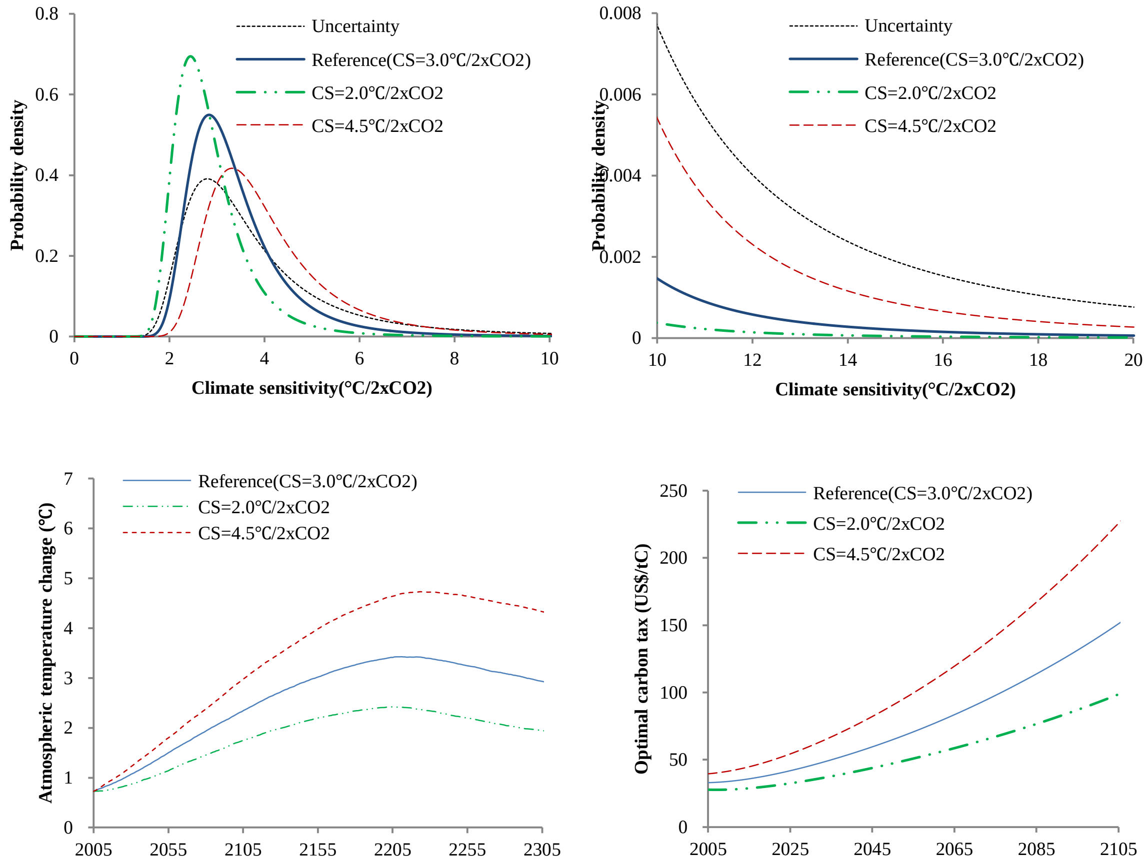

Figure 7 Sensitivity analysis (true values of climate sensitivity) (Top panels): Climate sensitivity distribution in 2055 **(Bottom left):** Atmospheric temperature **(Bottom right):** The optimal carbon tax

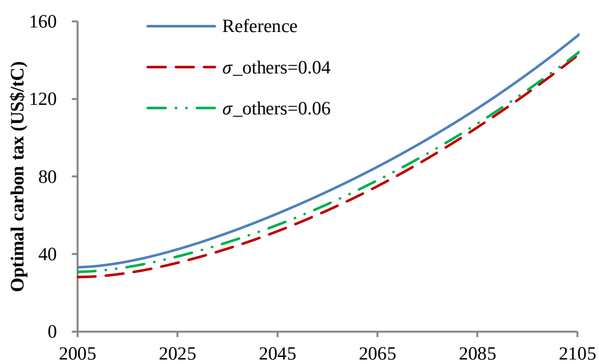

Figure 8 Sensitivity analysis (limits to learning)

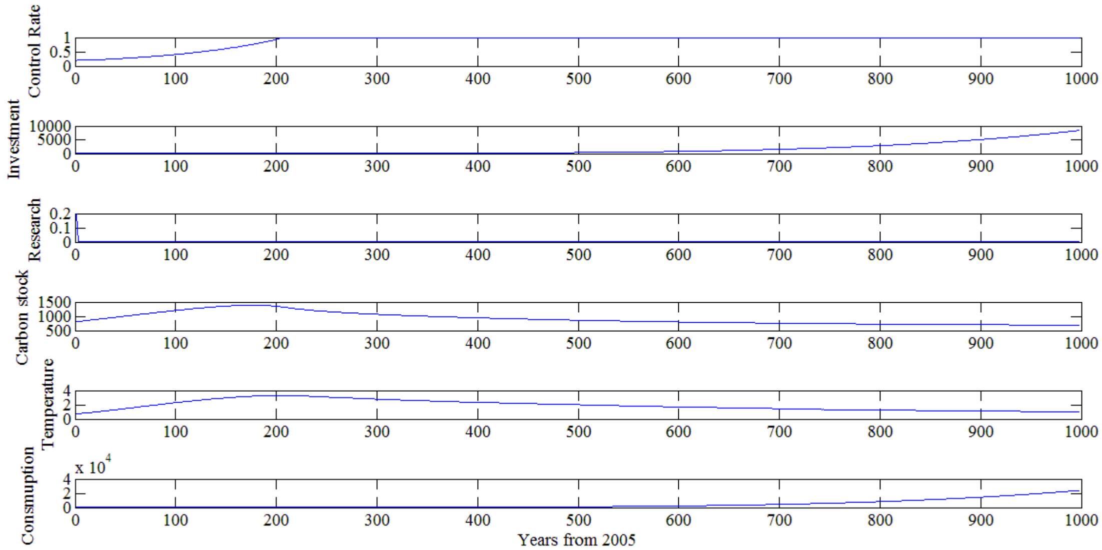

Figure B.1 Additional results (improved observations) The units for investment, research investment, the carbon stock, temperature increases, and consumption are $1,000 per person, trillion dollars, GtC, °C, and $1,000 per person, respectively.

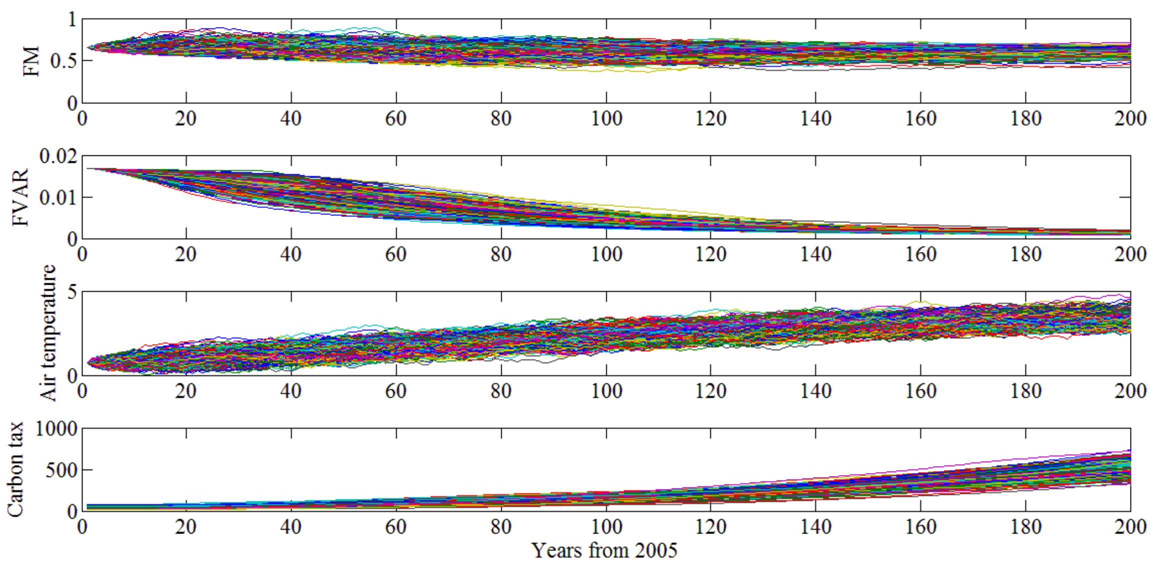

Figure B.2 Additional results (all simulations) (Top): The mean of the total feedback factors **(Upper middle):** The variance of the total feedback factors **(Lower middle):** Temperature increases (relative to 1900) **(Bottom):** The optimal carbon tax (US$/tC)

Table 1 Global temperature observational system in 2005

		Number of instruments / observations (thousands)	Unit cost (1,000US$)			
			Installation		Operation (per year)	
			Low	High	Low	High
LAT	Weather station	3,455	40		60	
SST	Number of instruments					
	VOS	5,429			4	55
	Drifting Buoy	1,267			4.5	7.8
	Moored Buoy	194	1,150	2,700	200	500
	Number of Observations					
	VOS	1,169				
	Drifting Buoy	1,632			0.00023	
	Moored Buoy	179				
	Sum	2,980				

Note: The number of land weather stations is the one used for building the database CRUTEM4 (Kennedy et al., 2011). The number of voluntary observing ships, drifting buoys, and moored buoys are available at www.bom.gov.au/jcomm/vos and www.aoml.noaa.gov/phod/dac. The unit cost for land weather station is drawn from Mburu (2006). The unit costs for voluntary observing ships, drifting buoys, and moored buoys follow Kent et al. (2010), Meldrum et al., (2010), and Detrick et al., (2000), respectively. The unit cost for data transmission using satellite communication systems is about \$0.23 per observation (North, 2007). The number of SST observations is drawn from Kennedy et al. (2011).

Table 2 The probability of high temperature increase

	2055		2105	
	Passive learning	Improved observations	Passive learning	Improved observations
Probability of climate sensitivity > 4.5℃/2xCO2	0.158	0.120	0.050	0.019
Probability of climate sensitivity > 6℃/2xCO2	0.053	0.032	0.006	0.001
Probability of climate sensitivity > 10℃/2xCO2	0.009	0.004	1.665E-04	7.962E-06

Table 3 The optimal carbon tax in 2015 (US$/tC)

	Deterministic f=0.6	Uncertainty $\bar{f}$=0.65, σ_f=0.13	Passive learning $\mathbb{E}_0 f$=0.65, σ_f=0.13	Active learning $\mathbb{E}_0 f$=0.65, $\mathbb{E}_0\sigma_f$=0.13
DICE damage function	32.0	39.0	37.6	35.8
Weitzman's damage function	37.7	201.2	56.4	43.2

Table A.1 Variables

U	Utility function	$=(C_t/L_t)^{1-\alpha}/(1-\alpha)$
C_t	Consumption	$=\left(1-\theta_1 \mu_t^{\theta_2}\right)\Omega_t Q_t - I_t - R_{i,t}$
μ_t	Emissions control rate	Control variable
R_t	Investment in climate research	Control variable
K_t	Capital stock	K_0=$137 trillion
$K_{R,t}$	Research capital stock	$K_{R,0}$=$950 million
$M_{AT\,t}$	Carbon stocks in the atmosphere	$M_{AT\,0}$=808.9GtC
$M_{U\,t}$	Carbon stocks in the upper ocean	$M_{U\,0}$=18,365GtC
$M_{L\,t}$	Carbon stocks in the lower ocean	$M_{L\,0}$=1,255GtC
T_{AT_t}	Atmospheric temperature deviations	T_{AT_0}=0.7307°C
T_{LO_t}	Ocean temperature deviations	T_{LO_0}=0.0068°C
$\bar{f}_t$	Mean of the total feedback factors	$\bar{f}_0$=0.65
v_t	Variance of the total feedback factors	v_0=0.13^2
Ω_t	Damage function	$=1/(1+\kappa_1 T_{AT_t}+\kappa_2 T_{AT_t}^{\kappa_3}+\kappa_4 T_{AT_t}^{\kappa_5})$
Q_t	Gross output	$=A_t K_t^{\gamma} L_t^{1-\gamma}$
I_t	Investment in general	$=sQ_t\Omega_t$
A_t	Total factor productivity	Exogenous
L_t	Labor force	Exogenous
σ_t	Emission-output ratio	Exogenous
$RF_{N,t}$	Radiative forcing from non-CO_2 gases	Exogenous
E_{LAND_t}	GHG emissions from the sources other than energy consumption	Exogenous
ε_t	Temperature shocks	Stochastic
$v_{\varepsilon,t}$	Variance of observed temperature shocks	$v_{\varepsilon,0}$=0.1^2

Note: The initial values for the state variables and the evolutions of the exogenous variables are from Cai et al. (2012), except for the research capital stock. The initial research capital stock does not affect the main results of this paper unless it is far higher than the default values. The lower bounds of the economic variables such as consumption, the capital stock, and gross world output are set to $0.001 per person per year in this paper. In addition, there are no upper bounds of temperature increases.

Table A.2 Parameters

λ	Equilibrium climate sensitivity	$=\lambda_0/(1\text{-}f)$
f	True value of the total feedback factors	0.6
λ_0	Reference climate sensitivity	1.2°C/2xCO_2
s	Savings rate	0.245
α	Elasticity of marginal utility	2
ρ	Pure rate of time preference	0.015
γ	Elasticity of output with respect to capital	0.3
δ_k	Depreciation rate of the capital stock	0.1
δ_R	Depreciation rate of research investment	0
κ_1, κ_2, κ_3, κ_4	Damage function parameters	κ_1=0, κ_2=0.0028388, κ_3=2, κ_4=κ_5=0
θ_1, θ_2	Abatement cost function parameters	θ_1=0.0561, θ_2=2.887
δ_{AA}, δ_{UA}, δ_{AU}, δ_{UU}, δ_{UL}, δ_{LL}, ξ_1, ξ_3, ξ_4, η	Climate parameters	δ_{AA}=0.9810712, δ_{UA}=0.0189288, δ_{AU}=0.0097213, δ_{UU}=0.005, δ_{UL}=0.0003119, δ_{LL}=0.9996881, ξ_1=0.022, ξ_3=0.3, ξ_4=0.005, η=3.8
α_R	Learning parameters	$3.42 million
σ^2_{others}	Learning parameters	0.0064
M_b	Pre-industrial carbon stock	596.4GtC
ω_c	Parameter reflecting satisfaction of the decision maker with the magnitude of learning	10^{-5}
$v_{\varepsilon,0}$	Initial value of the variance of observed temperature shocks	0.10^2

Note: The parameter values for climate parameters are from Cai et al. (2012). The parameter value for λ_0 is from Roe and Baker (2007). The other parameters are from Nordhaus (1994; 2008) except for the learning parameters.