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1 Introduction

1.1 Motivation

Since the latter half of 1990s, it has been discussed that many developing countries are going to face serious population aging problem while these economies are still underdeveloped and not adequately prepared against aging yet in terms of institutional reform such as social security. As the global interdependence of national economies has been deepened, a socio-economic problem in one country comes to have significant influences on many other economies and its effect might easily spillover around the world. Objectives of our study are to analyze the impact of social security reforms in one region on other regions with special emphasis on trade and capital flows among multiple regions or economies.

This study presents a basic analysis on interregional cooperative framework which may offset negative effects of population aging and make it possible to take advantage of the so-called “population dividends” that are derived from the population structure with a large size of working population relative to the number of dependent population.

Using a prototype Overlapping Generations (OLG) model applied to three regions with different demographic structure, we conduct simulation analysis to examine effects of aging and pension reforms on the patterns of interregional trade, capital flows, savings and economic growth.

The simulation results revealed that, as previous works suggested, interregional capital movements between regions may play a significant role to moderate the impact of population aging and pension reforms. When contribution rate is increased in a pay-as-you-go (PAYG) pension system, its effect becomes just like the case of a tax increase. Savings may decrease so that the capital accumulation slows down, and consumption also may shrink. When the economy is open, relatively higher interest rate because of the scarce capital stock may induce foreign capital inflows and save consumption through growth effects. In addition, removing distortions in interregional trade, such as tariff and non-tariff barriers, would promote interregional adjustment in resource and capital allocations.

1.2 Review on Pension Reform

In this section, we will present a limited literature review on the pension reform studies which use overlapping generations model as their analytical tool. Overlapping generations (OLG) model which sets up consumption and saving decisions by multiple generations living in one period was first developed by Samuelson (1958) and sophisticated by Diamond (1965) and Lucas (1972). In the 1970 and 80’s, many of the simulation analysis on pension reforms
were conducted by econometric models with elaborated public finance sector. However, since
Auerback and Kotlikoff (1987) who introduced OLG consumers into a general equilibrium
framework, it is OLG models that have replaced econometric models and been widely used in
pension reform analysis. There are many existing researches on pension reforms using OLG
model. However, there are not many works conducted in a way using a forward-looking general
equilibrium model with overlapping generations type consumers in a multi-regional framework
(multiple regions OLG/GE model). Here, our review focuses only on existing researches by
multiple regions OLG/GE models.

One of few researches conducted by multiple regions OLG/GE models on pension reforms
is the work by Karam et al. (2010). Karam et al. (2010) used a large scale world macro-model
with stochastic elements, GIMF, developed by International Monetary Fund (IMF). It is a
multi-region model consisting of USA, EU, Japan, emerging Asian countries and the rest of
the world with two sectors in production, intermediate goods and final goods and with OLG
consumers who live 20 periods. Labor productivity declines by 5% every year, which makes
consumption patterns in a way of life-cycle type. Liquidity-constraint households account for
20% of households in industrial countries and 50% in Asia, which enhances the impact of fiscal
policy.

Using GIMF, Karam et al. (2010) analyzed three cases of pension reforms, i.e., raising con-
tribution rates, decreasing benefit levels and extending retirement age, applied to individual
region as well as the case that pension reforms are implemented in an internationally con-
certed way. Raising contribution rates led to a fall in disposable incomes, a contraction in
labor supply and a decrease in production on supply side. Consumptions decreased due to
a fall in disposable incomes, which resulted in a weaker demand on demand side. In case of
cutting benefits, household increased savings during working periods in anticipation for a fall
in post-retirement incomes, which strengthened capital accumulation and improved growth.
Extending retirement age in USA, EU and Japan encouraged labor to stay in the labor mar-
ket. As a consequence, labor supply increased in a short term. Despite a fall in savings and a
consequential smaller capital accumulation, GDP improved due to a larger consumption and
a better fiscal position. Based on those findings above, Karam et al. (2010) concluded that
extending a retirement age among three reform options is the most effective in view of its im-
 pact on growth while minimizing adverse and distortional impacts of reforms such as that on
labor market. Its impact on debt reduction was limited compared with the other two options.
Karam et al. (2010) also found that a synchronized concerted reform among multi-regions was
more effective than individual reforms. These observations are consistent to other researchers
and econometric researches such as Andersen (2008).

Another work on pension reform by a large scale world model is one by INGENUE model
built and developed by INGENUE Team (2001). The INGENUE covers EU, North America
block, Japan, Asia (China, Korea and Russia) and the rest of the world (India and Latin America) with 15 generations (one generation is 5 years) producing one goods and one financial asset, was applied to analyze the impact of three reform options and obtained almost the same outcomes. Firstly, reducing benefits raise savings during working period and increases capital accumulation as observed in Karam et al. (2010). Raising contributions discouraged savings and led to negative impact on growth in comparison with base case scenario. In case of extending a retirement age, savings fell in a short-term due to shortened post-retirement period and increased life time income, a relative share of the working age population gradually increases, which pushed up demand for capital and increased capital accumulation. Additionally, as aging advances, those countries with generous pension system tend to import capita from abroad, which mitigates the adverse impact of pension reform.

There are some researches on pension reform by a small open economy model. Nickel et al. (2008) used two country OLG/GE model with one good to analyze pension reform in EU area. Since this is an open-economy model, interest rate is given. The findings by Nickel et al. (2008) are almost the same as Karam et al. (2010) and INGENUE Team (2001). They concluded that while they address advantages of extending a retirement age, their policy recommendation is a combination of three reforms in accordance to the character of the economy.

Shimasawa and Oguro (2011) also analyzed pension reform with special emphasis on the difference of aging using a small open OLG/GE model. They found that the different speed of aging between two countries affects direction and size of the capital flows, which could mitigate the adverse impact of pension reform. They also found that the impact of pension reform is more significant in case of an open economic model than a closed economic model because capital can be more efficiently allocated in an open economic model than a closed economic model. A similar finds are observed elsewhere. Heer and Irman (2008) showed that it is the difference in speed of aging among countries rather than aging itself that accelerate capital flows among countries.

Heer and Irman (2008) investigated the impact of pension reform in the USA with endogenous growth. The agents start dying at a constant rate and everyone in one generation die at 75. The model produces two goods with endogenous labor-saving technology. In this model, productivity is endogenously determined by labor-saving innovation investment by firms. Therefore, as aging advances, labor becomes scarce, which raises wage rate relative to interest rate. Relatively expensive wage rate induces firms to undertake labor-saving investment, which contributes to economic growth. In this model, saving rates increases as a consequence of aging.

Heer and Irman (2008) obtained a very different outcome in case of extending a retirement age while the outcomes were the same for the other two reforms. Because of shortened post-retirement period as a result of extending a retirement age, an incentive for savings is
undermined. The impact of a lengthened working period outweighed a fall in savings and contributed to growth. Among three reform options, cutting benefits is the most desirable according to their conclusion.

Observing past analysis, there are some areas for improvement. Firstly, the assumption for fixed coverage of the pension system is reasonable in developed countries while it is not in developing countries since coverage of the pension system in total labor force is still very low and expected in future to increase in developing countries. Secondly, an extension of life expectancy which is widely observed in developing countries is not incorporated in the model. Thirdly, not many models used in empirical analysis is equipped with bequest while it is in theoretical analysis. Fourthly, many of the existing model used a perfect capital mobility while developing countries gradually started liberalizing their capital account in the 1990’s. In fact, still many of them have not completely liberalized. Now, China and India account for a significant share in the global economy. Therefore, cautions have to be exercised for the assumption of perfect capital mobility.

2 Model

In this section, we outline the major assumptions and the structure of the forward-looking, multi-regional growth model with overlapping generations, developed for this study.

2.1 Major Assumptions

There are five major assumptions we made, and they are defining features of the model.

■Forward-Looking, Multi-Regional Growth Model Our modeling framework is on a dynamic multi-regional growth model with overlapping generations, which is based on the Samuelson-Diamond type neoclassical growth theory. The global economy is divided into three regions, and they are assumed to be at different phase of population aging process. Industries are aggregated into a single sector. Economic growth is led by exogenous growth of labor and total factor productivity (TFP). When the global economy is in a steady state, economic growth rates should be equalized among three regions.

■Perfect Competition The model follows essentially the neoclassical growth theory, and thus its solution can be regarded as the result of perfect competition. This is one straight-forward implication from the model. Since perfect competition is hardly realizable in actual economies, the simulation results may be interpreted as giving only a potential picture of a hypothetical economy under conditions of perfect competition, on the basis of which we can abstract fundamental determinants of economic growth. When one assumes monopolistic or oligopolistic
scale economies in the model, the impact of policy changes may be amplified.

**Primary Factors**  
Labor is assumed to be immobile beyond the regional boundary. In contrast, investment capital can flow across regions (foreign capital inflow/outflow), and its flow is determined so as to balance each region’s current accounts. It is assumed that the representative consumers in every region receives factor income from domestic firms, and that they then invest a fraction of their income through the interregional capital market. In addition, note that full employment of labor is assumed and plays an important role in policy simulation*1.

**Dynamic Consistency**  
The agents’ intertemporal behavior is assumed to be rational, so that the entire system of prices over time is internally consistent. This is because the model calculates variables of all the periods at the same time. Consumption and investment are determined on the basis not of what happened in the past, but of the assumed future conditions of technology, preference, and policy change. Changes in the future exogenous variables can affect present endogenous variables.

**Discrete Time Formulation**  
For the purpose of numerical implementation, the intertemporal problem is formulated in discrete time. Discounting in discrete time requires a dating convention. In order to keep derivation and calibration as simple as possible, all transactions are assumed to take place at the end of the period (while decisions are made or planned at the beginning of the period).

### 2.2 Basic Structure

We describe the basic structure of the model used in this study, focusing on the dynamic side of the model. Our model is an extension of a typical comparative static global trade model, such as that developed by Hertel (1997), with forward-looking properties and overlapping generations. In the following description, subscripts $s$ and $t$ respectively denote age of a representative consumer and time period.

**Firm**  
There assumed to be a competitive firm in each region, which produces a single product. Production and factor inputs are all determined endogenously so that resources are optimally used by maximizing net income. Factor substitutability is assumed among labor and capital inputs. Nested structure of factor inputs in production is assumed, and technology is based on constant returns to scale. Given the initial capital stock, interregional rate of return and prices of primary factors, composite investment good, and output, and then the dynamic decision problem of firm is to choose a time path of investment that will maximize the value

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*1 Itakura et al. (2003) suggest that the investment capital may flood into a particular region in the wake of trade-related policy change under the full employment assumption.
of the firm, defined as the discounted sum of temporal net cash flow yielded in every period. Investment comprises raw capital, and is equipped to form the capital stock of each region. Since this is a long-term model, inventory is included in investment.

A firm’s optimization problem can be divided into two stages. While the production segment chooses the volume of output and factor inputs to maximize profit, the investment segment makes dynamic investment plan to maximize the value of the firm. The producer’s optimization problem can be expressed as follows:

$$\text{Max } \Pi_t = p_tY_t - w^K_t K_t - w^L_t L_t$$

subject to

$$Y_t = \phi K_t^\alpha L_t^{1-\alpha}$$

(1)

where \(p_t\) is consumer price of output, \(w^K_t\) is capital rental rate, \(w^L_t\) is labor wage rate, \(K_t\) is capital input, \(L_t\) is labor input, \(t\) is time period, \(Y_t\) is output, \(\alpha\) is share of capital input in production function, \(\Pi_t\) is profit, and \(\phi\) is unit coefficient in production function.

First order conditions derived from this producer’s optimization problem formulate the static side of the model. Using the conditions, we obtain the following relations:

$$w^K_t = \alpha p_t^Y \frac{Y_t}{K_t}$$

(2)

$$w^L_t = (1-\alpha) p_t^Y \frac{Y_t}{L_t}$$

(3)

$$p_t^Y = p_t$$

(4)

where \(p_t^Y\) is producer price of output.

The invester’s dynamic decision making can be expressed as follows:

$$\text{Max } VF = \sum_{t=0}^{T} \left[ \left( \frac{1}{1+r} \right)^t \left( w^K_t \hat{K}_t - p_t I_t \right) \right] + \left( \frac{1}{1+r} \right)^T (1+\gamma) p^{KT} \hat{K}_T$$

subject to

$$\hat{K}_t = \hat{K}_0 (t = 0)$$

$$= I_{t-1} + (1-\delta) \hat{K}_{t-1}$$

$$I_T = (\gamma + \delta) \hat{K}_T$$

(5)

(6)

where \(I_t\) is fixed capital formation, \(\hat{K}_t\) is capital stock, \(\hat{K}_0\) is initial capital stock, \(p^{KT}\) is marginal value of post-terminal stock of capital, \(r\) is interest rate, \(T\) is terminal period, \(VF\) is value of the firm, \(\gamma\) is population growth rate, and \(\delta\) is physical depreciation rate.

The second term of the right hand side of the objective function is the present value of the firm at the terminal period \((t = T)\), which must be zero when \(T \to \infty\) as the transversality condition. The transversality condition places a limit on borrowing and ensures that the maximand is bounded. This value of capital is returned to the household at the end of the
terminal period, and finance the series of final consumption after the time period \( t = T + 1 \) as the non-human wealth along with the human wealth. Equation (6) is the stationary state condition, which is utilized instead of the transversality condition.

First order conditions derived from this inveter’s optimization problem formulate the dynamic side of the model:

\[
\begin{align*}
 p^K_t &= w^K_t + (1 - \delta) \frac{p^K_{t+1}}{1 + r} \quad (t \neq T) \\
p^K_T &= w^K_T + (1 - \delta) p^{KT} \\
p_t &= \frac{p^K_{t+1}}{1 + r} \quad (t \neq T) \\
p_T &= p^{KT} \quad (t = T)
\end{align*}
\]

where \( p^K_t \) is marginal value of capital stock.

**Household** Given the interregional rate of return, composite price of consumption good, and regional wealth, the representative consumer in each region chooses a time path of savings that will maximize her/his discounted utility of the temporal sequence of aggregated consumption. The utility function is homogenous and additively separable with constant elasticity of marginal utility. The utility is discounted by the consumer’s positive and constant rate of time preference. Since the financial claims are perfect substitutes \textit{ex ante}, we cannot uniquely determine the individual consumer’s optimal portfolio shares. However, since the goods are imperfect substitutes, interregional capital market equilibrium conditions define the foreign borrowings/lending for each region endogenously. The model treats capital flows as a mirror image to the balance of trade, adjusted for debt-service payment/receipt, and the stream of debt-service payment/receipt arising from an increase in foreign borrowings/lending is incorporated into the representative consumer’s decision making. Without uncertainty and with efficient capital markets, financial assets among regions earn the same anticipated rate of return.

A representative consumer’s optimization problem can be expressed as follows:

\[
\begin{align*}
\text{Max } u &= \sum_{s=0}^{S} \left\{ \left( \frac{1}{1 + \rho} \right)^s \frac{1 - \theta}{1 - \theta} \right\} \\
&\quad + \left( \frac{1}{1 + r} \right)^S \lambda_t^S b_s (s = S) + \left( \frac{1}{1 + r} \right)^S \lambda_T^T a_{sT} \quad (t = T) \\
\text{s.t. } a_{st} &= \pi_{s0} \\
&= \left( \frac{1}{1 + \gamma} \right)^S \bar{b} \quad (s = 0) \\
&= (1 + r) a_{s-1t-1} + \left( 1 - \tau^F \zeta^F - \tau^P \zeta^P \right) w^L_{t-1} \pi_{s-1t-1} - p_{t-1} c_{s-1t-1} \quad (0 < s \leq x + 1)
\end{align*}
\]
\[(1 + r) (a_{s-1t-1} - a_{s-1t-1}^F) + \xi_{t-1} a_{s-1t-1}^P - p_{t-1} c_{s-1t-1} \quad (s = x + 2) \]
\[(1 + r) a_{s-1t-1} + \xi_{t-1} a_{s-1t-1}^P - p_{t-1} c_{s-1t-1} \quad (x + 2 < s \leq S) \]
\[b = (1 + r) a_{St} + \xi_{t} a_{St}^P - p_{t} c_{St} \quad (s = S) \]
\[a_{st} = \left(\frac{1}{1 + \gamma}\right)^{s} \cdot \frac{b}{(a_{st})} \quad (t = T, \ s = 0) \]
\[(1 + r) a_{s-1T} + (1 - \tau_F \zeta^F - \tau_P \zeta^P) w_T \pi_{s-1T} - p_T c_{s-1T} \quad (t = T, \ 0 < s \leq x + 1) \]
\[(1 + r) (a_{s-1T} - a_{s-1T}^F) + \xi_{T} a_{s-1T}^P - p_T c_{s-1T} \quad (t = T, \ s = x + 2) \]
\[(1 + r) a_{s-1T} + \xi_{T} a_{s-1T}^P - p_T c_{s-1T} \quad (t = T, \ x + 2 < s \leq S) \]

where \(a_{st}\) is per capita asset holding, \(\pi_{st}\) is initial asset holding, \(a_{st}^F\) is per capita pension reserve (fully funded), \(a_{st}^P\) is per capita contribution record (PAYG), \(b\) is per capita bequest, \(c_{st}\) is per capita consumption, \(s\) is age of a consumer, \(S\) is terminal age, \(u\) is utility level of a consumer, \(x\) is compulsory retirement age, \(\lambda_{st}^T\) is marginal utility from post-terminal holding of asset, \(\lambda_{st}^S\) is marginal utility from leaving bequest, \(\xi_{t}\) is replacement ratio (pay-as-you-go), \(\zeta^F\) is coverage ratio (fully funded), \(\zeta^P\) is coverage ratio (pay-as-you-go), \(\theta\) is intertemporal elasticity of substitution, \(\pi_{st}\) is productivity of a labor, \(\rho\) is time preference, \(\tau_F\) is contribution rate (fully funded), and \(\tau_P\) is contribution rate (pay-as-you-go).

Similar to the firm's investment segment, first order conditions derived from the above optimization problem formulate the savings side of the dynamics in the model:

\[\left(\frac{1}{1 + \rho}\right)^{s} \cdot \lambda_{s+1t+1}^{\theta} = \left(\frac{1}{1 + r}\right)^{s+1} \lambda_{s+1t+1}^{\theta} + \lambda_{s+1t+1}^{T} p_t \quad (s \neq S, \ t \neq T) \]
\[= \left(\frac{1}{1 + r}\right)^{s} \lambda_{s+1}^{T} p_t \quad (s = S) \]
\[= \left(\frac{1}{1 + r}\right)^{s+1} \lambda_{s+1T}^{T} p_t \quad (t = T) \]
\[\lambda_{st} = \lambda_{s+1t+1} \quad (s \neq S, \ t \neq T) \]
\[= \lambda_{s+1}^{T} (1 + r) \quad (s = S) \]
\[= \lambda_{s+1T} \quad (t = T) \]

where \(\lambda_{st}\) is marginal utility of income. The rate of return \(r\) is determined by the opportunity cost of savings, which in this study is the cost of foreign borrowings.

**Pension System** The model includes two kinds of pension system, Fully Funded (FF) and Pay-As-You-Go types. The former type is like a compulsory saving system for post-retirement
periods. In our model, the reserved FF pension is assumed to be returned in a lump sum to the pensioner in the first period of her/his post-retirement life. The FF pension is reserved as follows:

\[
a^F_{st} = \pi^F_{s0} \quad (t = 0)
\]

\[
= 0 \quad (s = 0)
\]

\[
= (1 + r) a^F_{s-1t-1} + \tau^F \xi^F w^L_{t-1} \pi_{s-1t-1} \quad (0 < s \leq x + 1)
\]

(13)

where \(\pi^F_{s0}\) is initial pension reserve (FF). Note that the variable \(a^F_{st}\) does not exist for the periods after the fund is returned.

The PAYG type is just like income taxes that would be directly transferred to older generations living in the same period. On the other hand, the contributions of each representative consumer in every period are all recorded and utilized to determine the replacement ratio for every consumer. The contribution record can be expressed as:

\[
a^P_{st} = \pi^P_{s0} \quad (t = 0)
\]

\[
= 0 \quad (s = 0)
\]

\[
= (1 + r) a^P_{s-1t-1} + \tau^P \xi^P w^L_{t-1} \pi_{s-1t-1} \quad (0 < s \leq x + 1)
\]

\[
= (1 + r) a^P_{s-1t-1} \quad (x + 1 < s \leq S)
\]

(14)

where \(\pi^P_{s0}\) is initial contribution record (PAYG). Based on \(a^P_{st}\), the replacement ratio \(\xi\) is derived to balance the receipts and disbursements of the fund every period.

**Interregional Trade** The product of the firm in every region is not treated as homogeneous across countries but as imperfect substitute for that of another (Armington assumption). This assumption is necessary to accommodate cross hauling (the phenomenon of a country both importing and exporting the same product at the same time). This is inconsistent with the traditional Hecksher-Ohlin trade model, which is based on the premise of homogeneous products. The model adopts a transaction system similar to the GTAP model, presented by Hertel (1997), to note the interregional trade.

**Equilibrium Conditions** To arrive at a solution, both the intertemporal and within-period general equilibrium conditions have to be satisfied simultaneously. At every point in time, the usual general equilibrium conditions require that: (i) material balance in the demand and supply of all goods in the economy holds; (ii) the demand for total labor equals its supply; (iii) global-wide total of savings equals total investment\(^2\).

The intertemporal conditions ensure that future prices and quantities are fully anticipated and factored into the behavior of investment and consumption. They also guarantee that the

\(^2\) This condition can be dropped because of the Walras’ law. The counterpart variable of the condition (iii) is interregional rate of return. Every agent in every region faces this identical rate of return in the model.
path towards a new stationary state is unique. A sufficient condition is that the discount rate and the rate of time preference are positive and greater than the balanced-growth rate by the terminal period. To solve a growth model that has an infinite time horizon, we follow the usual procedure of imposing stationary state conditions at some future terminal period. On the investment side, the required condition is that investment is equal to the physical depreciation rate plus exogenously given post-terminal growth rate times capital stock. At the same time, current account is in equilibrium that debt-service payment/receipt is equal to the net exports/imports. Since the stream of debt-service payment/receipt is incorporated into the household’s decision making, as noted in the part of the household’s utility maximization, this condition functions like the so-called No Ponzi-Game condition for the consumer’s dynamics. As long as the terminal conditions are satisfied, the sums of various infinite series pertaining to the investment equation and the consumption function will be finite and well defined.

### Choice of the Terminal Period
As variables of different time-periods are interdependent, the computation burden is much larger than that for models which calculate solutions period by period (recursively dynamic or backward-looking models). Moreover, extensions of the calculation horizon increase calculation difficulty more than proportionally, and expansions of models with respect to the number of sectors or regions are more difficult. Because of these difficulties and a limited amount of computational resources, we set the terminal period at \( T = 50 \) making one period spans 20 years.

### Software
The model is formulated as a Mixed Complementarity Problem (MCP) and solved by “PATHC” of the General Algebraic Modeling System (GAMS). MCP is a set of Kuhn-Tucker conditions derived from certain optimization problems.

## 3 Simulation Scenarios
The model will be simulated on three scenarios. The first scenario is to see what happens to economic growth of individual region and to capital flows among regions if aging in one region with PAYG pension system advances faster than other two regions. This scenario reflects current situation of aging in the world where an industrial region equipped with PAYG is aging faster than other regions. The different speed of aging among regions will generate different saving ratios and, therefore, is projected to affect international capital flows and individual growth.

The second one is to simulate the impact of newly introducing PAYG pension system into one

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*3 Exports and imports do not balance in a stationary state in this study.

*4 Brooke et al. (1992).
region. While all of current industrial countries are already equipped with PAYG, introduction of PAYG are under way in many developing countries which are aging fast. Introducing PAYG in one region will affect direction of capital flows and growth in individual region as a consequence.

The third scenario is about pension reform by one region which has already introduced PAYG. In this case, pension reform which switches from PAYG to FF pension system will be analyzed. The advantages of FF compared with PAYG lie in its fiscally neutral feature and its possible contribution to capital accumulation. On the other hand, financial burden for those generations immediately after the reform from PAYG to FF is expected greater than other generations since they will have to continue paying to support pensioners under the old PAYG system and pay for their own FF pension system.

4 Simulation Results and Summary

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References


and Development.