Long-Term Growth and Persistence with Endogenous Depreciation: Theory and Evidence*

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Abstract

Previous research has shown a strong positive correlation between short-term persistence and long-term output growth as well as between depreciation rates and long-term output growth. This evidence, therefore, contradicts the standard predictions from traditional neoclassical or AK-type growth models with exogenous depreciation. In this paper, we first confirm these findings for a larger sample of 101 countries. We then study the dynamics of growth and persistence in a model where both the depreciation rate and growth are endogenous and procyclical. We find that the model's predictions become consistent with the empirical evidence on persistence, long-term growth and depreciation rates.

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1 Introduction

Empirical evidence on the persistence of output fluctuations shows large differences across countries. Using quarterly GNP data for the Group of Seven (G7) countries, Campbell and Mankiw (1989) find important differences in the estimates of persistence. Consistent with this evidence, Cogley (1990) reports significant differences in the variability of the permanent component of output in a similar sample of countries. Further, Fatás (2000) finds that there is a positive and significant correlation between the degree of persistence of short-term fluctuations and long-term average growth rates for a sample of countries that includes the G7 countries and eight additional OECD countries. Figure 1 extends the results by Fatás (2000) for the G7 countries by plotting the degree of persistence of the GDP series against long-term average per capita output growth for a broad sample of 101 countries over the period 1970-2008.[1]

[Insert Figure 1 about here]

The degree of persistence is computed using Cochrane (1988)'s variance ratio with a window of five years. To construct it, we employ heteroskedasticity robust standard errors and correct for small-sample bias in the variance following the procedure outlined in Campbell, Lo and Mackinley (1997). The variance ratio is a measure of the extent to which annual fluctuations are trend reverting and, in turn, a measure of the permanent impact of business cycles on trend output. As shown in Figure 1, there is a clear positive correlation between the persistence of output fluctuations and long-term output growth. The cross-section regression provides evidence of a statistically significant (at the 1% level) positive coefficient on long-term average growth. This indicates that the greater the growth rate of an economy, the larger the permanent effect of cyclical fluctuations on trend output.

In standard RBC models cyclical fluctuations are simply deviations around an *exogenous* trend driven by the state of technological progress. In these models, there is

[1] The annual real GDP series employed throughout the paper are expressed in constant 2000 US$ and were retrieved from the *World Development Indicators* of the World Bank (2010).

no correlation between persistence and long-term output growth.[2] As noted by Fatás (2000), however, the significantly positive correlation between short-term persistence and long-run growth is consistent with RBC models with *endogenous* productivity shocks. In these models the degree of short-term persistence captures the extent to which cyclical fluctuations affect technological progress, which endogenously determines long-term growth.[3] Using a standard AK model, Fatás shows that a positive correlation between persistence and growth can be obtained when the stochastic nature of the trend is endogenous.

The standard AK growth model considers the rate of depreciation as a constant and assumes that capital services are a fixed proportion of the existing capital stock, as is usual in the growth literature. In this setting, the marginal cost of capital utilization is zero, which implies an optimal capital utilization rate equal to one. The empirical evidence on depreciation and capital utilization rates, however, is not consistent with this assumption.[4] In fact, the empirical evidence on depreciation and capital utilization rates across countries documents: (i) large differences in cross-country utilization rates, (ii) a positive correlation between capital utilization and per capita income, and (iii) a positive correlation between the depreciation and long-term average per capita income growth rates.[5]

[2] This is because all GDP series would be characterized by a random walk with a drift, which would render a variance ratio equal to one for all countries in the sample.

[3] Provided that the amount of resources allocated to growth varies procyclically, temporary shocks produce permanent effects on output.

[4] Using aggregate US manufacturing data, Epstein and Denny (1980) and Kollintzas and Choi (1985) provide evidence against the standard assumption of a constant depreciation rate. Abadir and Talmain (2001) estimate time-varying depreciation rates for Canada, Germany, Japan and the UK. In addition, Foss (1981), Orr (1989) and Beaulie and Mattey (1998) find upward trends for the capital utilization rate in the US.

[5] Using data for Pakistan, South Corea and the US, Kim and Watson (1974) find that the rate of capital utilization increases with per capita income. The same result is found by Mayshar and Halevy (1997) for a sample of 24,000 companies in ten European countries. Anxo et al. (1995) provide evidence of a large variation in utilization rates across Europe as well as much higher utilization rates in US manufacturing industries than in Europe. Finally, using cross-sectional data of 85 countries from the

Figure 2 extends the existing evidence on the positive relationship between output growth and the depreciation rate for our sample of 101 countries for which data on depreciation rates were available over the period 1970-2008. As in Figure 1, we use the growth rate of real per capita GDP averaged over the period 1970-2008 as a measure of long-term output growth.

[Insert Figure 2 about here]

In line with Gylfason and Zoega (2001a)'s results, there is a highly statistically significant positive coefficient on the depreciation rates, which supports the existence of a positive link between both series. This evidence, however, is in sharp contrast with the theoretical predictions of standard *exogenous* growth models, where the depreciation rate negatively affects long-run levels and short-run growth rates but not their long-run output growth rates. The endogeneity of growth, however, is not sufficient to generate the observed positive correlation between the depreciation rate and long-term average output growth. In fact, in a traditional AK-type model with exogenous depreciation rate –as the one outlined in Fatás (2000)– the growth rate of output is *negatively* related to the *exogenous* depreciation rate. Hence, the evidence exhibited in Figure 2 (which will be shown to be further reinforced by the significantly positive link between output growth and depreciation rates in the dynamic panel data estimations shown in Section 2) appears to be inconsistent with the AK model with exogenous depreciation.

The aim of this paper is to provide a theoretical explanation for the aforementioned cross-country positive correlation between short-term persistence and long-term growth by developing a model that is also consistent with the cross-country empirical evidence on depreciation and capital utilization rates. We are aware of no previous attempt in the literature to reconcile empirical evidence with theoretical predictions on persistence, long-term growth and depreciation rates. The basic idea of the model is to introduce the optimal choice of capital utilization in an otherwise standard AK-

World Bank averaged over the period 1965-1998, Gylfason and Zoega (2001a) find a positive correlation between the depreciation rate and per capita income growth.

type endogenous growth model and, unlike the existing growth literature, to treat the depreciation rate of capital as an *endogenous* variable. In particular, we will treat the rate of depreciation as an increasing function of the capital utilization rate. As we will show in the theoretical AK model with endogenous capacity utilization developed in Section 3, output fluctuations become persistent as long as the amount of resources allocated to growth (in our case physical capital accumulation and the degree of capacity utilization) varies procyclically. Further, consistent with the empirical evidence, the AK growth model with endogenous capacity utilization predicts a positive relationship between the depreciation rate and long-term output growth.

The rest of the paper is structured as follows. In Section 2 we complement the evidence of a positive link between the depreciation rate and output growth shown in Figure 2 by estimating dynamic panel data growth models. In Section 3 we develop an endogenous AK growth model with endogenous capacity utilization that is consistent with the positive association between (1) short-term persistence and output growth and (2) the depreciation rate and output growth observed in the data. We also derive explicit measures of persistence on the basis of the parameters of the model and compare them with those obtained in a similar model but with exogenous depreciation. Section 4 concludes.

2 International Evidence on Depreciation and Output Growth Rates

2.1 Methodology

Having already presented some preliminary cross-country evidence regarding the positive association between the depreciation rate and economic growth, we now proceed to further investigate this issue for a sample of 101 countries and a subset comprising the OECD countries over the period 1970-2008 by exploiting both the cross-country and time dimensions of the data. We estimate a conventional dynamic panel data specification that regresses real per capita output growth on its main growth determinants according to standard growth theory:

$$y_{i,t} - y_{i,t-1} = \gamma y_{i,t-1} + \eta\prime X_{i,t} + \alpha_i + \delta_t + \varepsilon_{i,t} \qquad (1)$$

where y is the logarithm of real per capita GDP, α_i is a set of unobserved country-specific effects (to account for time-invariant country-specific structural characteristics) and δ_t is a set of time-specific effects (to account for common shocks affecting all countries in a given year). X represents a set of explanatory variables that includes the population growth rate (to capture the growth reduction in per capita terms due to increases in population), the secondary school enrollment rate (as a measure of human capital accumulation), trade size (computed as the ratio of exports plus imports to GDP), the agricultural share of GDP (to account for the negative effects of natural resource abundance due to induced rent-seeking and the Dutch disease),[6] and a variable accounting for the accumulation of physical capital. Unlike most previous studies, the share of gross domestic fixed capital formation over GDP is split into the share of net investment over GDP plus the depreciation rate.[7] The depreciation of fixed capital – measured as a proportion of GDP– represents the consumption of fixed capital as given by the replacement value of capital used up in the production process.[8] Finally, lagged

[6] See Gylfason and Zoega (2001b) for more details on these mechanisms.

[7] To the best of our knowledge, the only exception is Gylfason and Zoega (2001a) who estimate a similar specification to ours but using cross-sectional data averaged over the period 1965-1998 for a sample of 85 countries rather than panel data.

[8] The depreciation rates are World Bank staff estimates using data from the United Nations Statistics Division's National Accounts Statistics. According to the System of National Accounts (see United Nations, 2009), the formal definition of consumption of fixed capital is the decline, during the course of the accounting period, in the current value of the stock of fixed assets owned and used by a producer as a result of physical deterioration, normal obsolescence or normal accidental damage. It is important to stress the fact that the value of the assets not only declines because of physical deterioration but also due to the decrease in the demand for their services as a result of technological progress and the appearance of new substitutes for them (i.e. obsolescence). However, this variable only considers normal, expected rates of obsolescence, not unexpected ones. As with total output or intermediate

output ($y_{i,t-1}$) accounts for convergence dynamics of per capita output. The depreciation rate along with the other variables were retrieved from the *World Development Indicators* of the World Bank (2010).

Traditionally, this specification has been estimated via the Least Squares Dummy Variables (LSDV) estimator. This estimator, however, is unable to correct for the simultaneity bias due to the endogeneity of many of the regressors as well as for the omitted variable bias and the bias caused by the correlation of the lagged dependent variable and α_i. To deal with these shortcomings, Arellano and Bond (1991) propose to use the Generalized Method of Moments (GMM) instrumental variables dynamic panel data estimator called *difference estimator*. They difference equation (1):

$$(y_{i,t} - y_{i,t-1}) - (y_{i,t-1} - y_{i,t-2}) = \gamma(y_{i,t-1} - y_{i,t-2}) + \eta\prime(X_{i,t} - X_{i,t-1}) + +(\delta_t - \delta_{t-1}) + (\varepsilon_{i,t} - \varepsilon_{i,t-1}) \quad (2)$$

While differencing eliminates the Nickel (1981) bias caused by the correlation between lagged output and country-specific effects, it introduces a new bias caused by the correlation of the new error term ($\varepsilon_{i,t} - \varepsilon_{i,t-1}$) with the lagged dependent variable ($y_{i,t-1} - y_{i,t-2}$). The *difference estimator* uses previous realizations of the regressors to instrument for their current values in the first-differenced specification. Under the assumption of no serial correlation in the error term and the weak exogeneity of the regressors, Arellano and Bond (1991) propose to employ the following conditions:

$$E\left[y_{i,t-s}(\varepsilon_{i,t} - \varepsilon_{i,t-1})\right] = 0 \quad \text{for } s \geq 3; t = 4, ..., T, \quad (3)$$

$$E\left[X_{i,t-s}(\varepsilon_{i,t} - \varepsilon_{i,t-1})\right] = 0 \quad \text{for } s \geq 2; t = 3, ..., T, \quad (4)$$

Arellano and Bover (1995) and Blundell and Bond (1998) show that in the case of persistent regressors, lagged levels of the variables are weak instruments for the

consumption, consumption of fixed capital is calculated using current prices or rentals of fixed assets instead of employing historic costs as in business accounting. See more details in United Nations (2009) and OECD (2009).

first-differenced regressors. This leads to a fall in precision as well as to biased coefficients. To overcome these shortcomings, Arellano and Bover (1995) and Blundell and Bond (1998) suggest the use of the *system estimator* that utilizes instruments in levels and first-differences to improve efficiency.[9] The instruments for the first-differenced specification are the same as above. The instruments for the regression in levels are the lagged differences of the variables. In order to avoid using redundant instruments in first-differences that could lead to *overfitting bias*, we only employ the following additional moment conditions for the regression in levels:

$$E\left[(y_{i,t-s} - y_{i,t-s-1})(\alpha_i + \varepsilon_{i,t})\right] = 0 \quad \text{for } s = 1; \tag{5}$$

$$E\left[(X_{i,t-s} - X_{i,t-s-1})(\alpha_i + \varepsilon_{i,t})\right] = 0 \quad \text{for } s = 1; \tag{6}$$

A further concern in the use of the *system GMM estimator* is the downward bias associated with the standard errors of the estimates, particularly when the cross-sectional dimension is relatively small, which in turn may lead to spuriously significant regressors. To overcome this difficulty, we use the one-step estimator since the asymptotic standard errors for the two-step estimator are biased downwards. As a result, the asymptotic inference from the one-step standard errors are more reliable. In addition, we apply the small-size correction factors proposed by Windmeijer (2005).

The consistency of the *system estimator* depends on the validity of the instruments and the absence of serial correlation of second-order in the first-differenced error term. Therefore, we test these assumptions using the Sargan test for over-identifying restrictions and the test for second-order autocorrelation proposed by Arellano and Bond (1991). Failing to reject the null hypotheses of overall validity of the instruments and absence of second-order serial correlation in the first-differenced error for the respective tests would give support to an endogenous growth model with endogenous depreciation.

[9] More recently, Hauk and Wacziarg (2009) have shown that the *system estimator* appears to outperform the *difference estimator* in terms of the biases emerging from the presence of measurement error in growth regressions.

2.2 Empirical Findings

Before presenting the estimates obtained with the *system estimator* that corrects for endogeneity bias, the Nickel bias and omitted variable bias, Table 1 reports the results from the application of the LSDV estimator. The basic specification only includes as stochastic regressors the depreciation rate and net fixed capital formation over GDP, while the augmented specification further adds lagged output and four other additional variables such as population growth, the trade share of GDP, the secondary school enrollment rate and the agricultural share of GDP. We present the results for the OECD sample as well as for the full sample comprising 101 countries.[10]

[Insert Table 1 about here]

Interestingly, model (1) shows that there is a highly statistically significant coefficients on the depreciation rate and the net investment rate for the basic specification in the OECD sample. The regression coefficient on the depreciation rate capturing replacement investment is about ten times greater than the coefficient associated with net investment. Model (2) also yields a statistically significant coefficient on replacement investment which is substantially greater (about sevenfold) than the coefficient on net investment. This augmented specification also renders, as expected, a statistically significant negative coefficient on lagged output and population growth, while the coefficient on the trade share, secondary schooling and the agricultural share are statistically insignificant. The Wald statistics support the significance of country and time fixed effects.

The results for the full sample also support the statistical significance of the positive coefficient on replacement investment and net investment for both the basic and augmented specifications. In this case, the coefficient on depreciation appears to be between four and five times greater than that on net investment.

[10]Table A1 in Appendix A reports the identity of the countries in the sample and some descriptive statistics of the relevant variables.

This basic finding appears to contradict the standard predictions from traditional neoclassical or AK-type endogenous growth models with exogenous depreciation. For exogenous growth models the depreciation rate would only negatively affect transitional growth, while for AK models with exogenous depreciation it would exert a negative effect on long-run growth. As will be shown in Section 3, a positive link between the depreciation rate and long-term output growth appears more in line with the predictions of the AK model with endogenous depreciation.

Table 2 presents the results from the *system GMM estimator*. We find that the statistical significance, sign and size of the coefficients on replacement investment and net investment are very similar to those obtained with the LSDV estimator. Again, the size of the coefficient on the depreciation rate more than quadruples the size of the coefficient on the net investment rate. Interestingly, we reject the null hypotheses of overall validity of instruments and absence of second-order serial correlation with the Sargan and AR(2) tests for the basic specification with only depreciation and net investment. In contrast, when we add lagged per capita output and the rest of explanatory variables we fail to reject both null hypotheses, thereby supporting the validity of the augmented specification.

[Insert Table 2 about here]

Overall, the results in this section show that both the depreciation rate and per capita output growth appear to be positively correlated. The statistical and economic significance of the coefficients on the depreciation rate appear to be quite robust to changes in the sample of countries as well as in the set of explanatory variables. Interestingly, the size of the coefficient on replacement investment is much higher than that on net investment. Hence, these results indicate that traditional exogenous and endogenous growth theory (of the AK type) for which the depreciation rate affects negatively either transitional growth or long-run growth, may be neglecting some important features of the dynamics of growth. A natural candidate would be to consider an endogenous depreciation rate that directly depends on capacity utilization, which in turn varies procyclically with the state of the cycle. This feature is introduced in the AK model with endogenous capacity utilization presented in the next section.

3 The Setup of the Model

The framework is a simplified, stochastic version of Chatterjee's (2005) endogenous growth model: an AK-type growth model augmented by endogenous capital utilization. We consider this endogenous growth model because of its simplicity.

Consider a closed economy without a public sector. The economy is populated by a continuum of identical, infinitely lived agents which derive utility from the consumption of a final good and discount future utility at a rate $\beta \in (0,1)$. Preferences are given by $\sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}-1}{1-\sigma}$, where C_t denotes consumption. We assume that the labor supply is inelastic and we normalize it to unity.

The technology of the consumption good is described by the aggregate production function $Y_t = AZ_tU_tK_t$, where A is a scale parameter, U_tK_t is the flow of capital services derived from the available capital stock, Y_t denotes the corresponding flow of output, and Z_t is a temporary exogenous shock that captures the state of the technology. As suggested by Taubman and Wilkinson (1970) and Calvo (1975), and following Chatterjee (2005), we define the rate of capital utilization U_t as the intensity (measured in hours per week) with which the available capital stock is used. In this way, firms are provided by an extra margin to vary output, namely the intensive margin. The productivity shock Z_t is assumed to follow the autoregressive process: $\hat{z}_{t+1} = \rho\hat{z}_t + \varepsilon_{t+1}, \;\; 0 < \rho < 1$, where lower case letters represent logarithms, a circumflex on top of the variable denotes deviations from its steady state value and ε_t is a white noise.

In a closed economy without public sector all output is devoted to consumption or gross investment. Hence, the resource constraint of the economy is $C_t + I_t = Y_t$. The capital stock evolves according to $K_{t+1} = K_t(1-\delta_t) + I_t$. Following Greenwood, Hercowitz and Huffman (1988), we also assume that the rate of depreciation of the capital stock is a convex, constant elasticity function of its rate of utilization: $\delta(U_t) =$

$\frac{1}{\phi}{U_t}^{\phi}$, where $\phi > 1$ and $0 \leq \delta(U_t) \leq 1$. Note that, in contrast to the usual assumption in the growth literature, the marginal depreciation cost of capital utilization $\delta'(U_t)$ is variable. The parameter ϕ measures the elasticity of the depreciation rate with respect to the rate of capital utilization.[11] As is already known, due to the sensitivity of the depreciation rate of capital to the choice of capital utilization, it may not be optimal to fully utilize the capital. Obviously, this model collapses to the AK model considered by Fatás (2000) when full capital utilization, and therefore a constant depreciation rate, is assumed.

3.1 Solving the Model

In the absence of distortions, the allocations arising from a decentralized competitive economy coincide with those resulting from a centralized economy with a social planner. The dynamic program problem faced by the central planner is:

$$V(K_t, Z_t) = \max_{C_t, U_t} \left\{ \frac{C_t^{1-\sigma} - 1}{1-\sigma} + \beta E_t V(AZ_tU_tK_t - C_t + (1 - \frac{1}{\phi}{U_t}^{\phi})K_t, Z_{t+1}) \right\},$$

given K_t and Z_t and where E_t is the expectations operator conditional on the information available up to period t. The objective function is concave and the constraints are convex. Hence, the following set of FOC's characterizes the interior optimum:

$$AZ_t = {U_t}^{\phi-1}, \tag{7}$$

$$\begin{aligned} C_t^{-\sigma} &= \beta E_t \left\{ C_{t+1}^{-\sigma} \left[1 - \delta(U_{t+1}) + AZ_{t+1}U_{t+1}\right] \right\}, \\ &= \beta E_t \left\{ C_{t+1}^{-\sigma} \left[1 + (\phi - 1)\ \delta(U_{t+1})\right] \right\}, \end{aligned} \tag{8}$$

$$K_{t+1} = K_t(1 - \delta_t) + Y_t - C_t, \tag{9}$$

$$\lim_{t\to\infty} E_t \left\{ \beta^t C_t^{-\sigma} K_{t+1} \right\} = 0.$$

The interpretation of these optimality conditions is standard. Equation (7) determines the optimal choice of the capital utilization rate: the left hand-side represents the

[11] A plausible range for parameter ϕ seems to be $[1.4, 2)$. See Dalgaard (2003) and Chatterjee (2005) for a survey on this evidence.

marginal benefit of capital utilization and the right hand-side the marginal cost of capital utilization. Hence, in this setting, it is optimal to utilize capital less than fully, i.e. $U_t \in (0,1)$.[12]

In the long-run equilibrium, output, consumption and capital will grow at a common rate ($\tilde{G}$) and therefore there are no steady state levels for these variables. Let $\frac{Y_t}{K_t}$ and $\frac{C_t}{K_t}$ be the stationary variables for which we obtain the following steady state equilibrium: $\tilde{U} = (A)^{\frac{1}{\phi-1}}$, $\frac{Y_t}{K_t} = A\tilde{U}$, $\tilde{\delta} = \frac{1}{\phi}\frac{Y_t}{K_t}$, $\tilde{G} = \left\{\beta\left[1+(\phi-1)\,\tilde{\delta}\right]\right\}^{\frac{1}{\sigma}}$, and $\frac{C_t}{K_t} = (\phi-1)\,\tilde{\delta} + 1 - \tilde{G}$.

Let S_t be the proportion of income that is not consumed: $C_t = (1-S_t)Y_t$. In steady state the saving rate is given by $\tilde{S} = \frac{\tilde{G}-(1-\tilde{\delta})}{A}$. Hence, the long-run solution to this model is characterized by a constant saving rate, a constant but not full capital utilization rate, and a balanced growth path with output, consumption and capital growing at the same rate. These constant levels depend on the marginal product of capital services, A, and the elasticity of the depreciation rate with respect to the capital utilization rate, ϕ. Assume now that A is a country-specific technological parameter. Countries with a higher A will then have both a greater $\tilde{U}$ and therefore a greater $\widetilde{\delta}$. From the FOC (8) it is easy to verify that countries with a higher A will also have a greater long-run gross growth rate ($\widetilde{G}$).

We rewrite the equilibrium dynamics of the model in terms of the saving rate, S_t. Combining conditions (7) and (8) and taking into account from the resource constraint (9) that $\frac{K_{t+1}}{K_t} = 1 - \frac{1}{\phi}\left(AZ_t\right)^{\frac{\phi}{\phi-1}}\left(1-\phi S_t\right)$, the following expression is obtained:

$$\left[\frac{1-\frac{1}{\phi}(AZ_t)^{\frac{\phi}{\phi-1}}(1-\phi S_t)}{(1-S_t)(AZ_t)^{\frac{\phi}{\phi-1}}}\right]^{\sigma} = \beta E_t\left\{\frac{1+\frac{1}{\phi}(AZ_{t+1})^{\frac{\phi}{\phi-1}}(\phi-1)}{\left[(1-S_{t+1})(AZ_{t+1})^{\frac{\phi}{\phi-1}}\right]^{\sigma}}\right\}. \qquad (10)$$

Since there is no closed-form solution to the equilibrium, we approximate it by linearizing both equations around the steady state values. From (10) we obtain the following

[12] This is in contrast to the existing growth literature which assumes a constant depreciation rate, implying a zero marginal cost of capital utilization and hence being optimal to fully utilize capital. As shown by Chatterjee (2005), there exists an optimal $U_t \in (0,1)$, under the mild condition $A < 1$.

first order stochastic difference equation:

$$a_1\hat{s}_t + a_2\hat{z}_t = a_3E_t\left(\hat{s}_{t+1}\right) + a_4E_t\left(\hat{z}_{t+1}\right),$$

where all a_i are functions of the parameters of the model and $\hat{x}_t$ denotes the deviation of variable X_t from its steady state value in logarithms.[13] Since $\hat{z}_t = \rho\hat{z}_{t-1} + \varepsilon_t$, the solution is given by:

$$\hat{s}_t = a\hat{z}_t, \quad \text{where} \quad a = \frac{\rho a_4 - a_2}{a_1 - \rho a_3}. \tag{11}$$

By linearizing the resource constraint around the steady state and by substituting the solution given by (11), we obtain the following expression for the deviations of capital growth from its steady state value $\tilde{G}$:

$$\widehat{\Delta k_{t+1}} = \theta\hat{z}_t, \quad \text{where} \quad \theta = A^{\frac{\phi}{\phi-1}}\frac{a\tilde{S} + \tilde{S}^{\frac{\phi}{\phi-1}} - \frac{1}{\phi-1}}{\tilde{G}},$$

where θ captures the contemporaneous impact of shocks on physical capital accumulation. Therefore, shocks have an effect on capital accumulation and growth varies procyclically. Plugging this expression into the production function and taking into account from (7) that $\hat{u}_t = \frac{1}{\phi-1}\hat{z}_t$, the deviations of output growth from its steady state value are given by the following moving average representation:

$$\widehat{\Delta y_t} = A(L)\varepsilon_t = \frac{\left[\frac{\phi}{\phi-1} - \left(\frac{\phi}{\phi-1} - \theta\right)L\right]}{1 - \rho L}\varepsilon_t, \tag{12}$$

where $A(L)$ is an infinite polynomial in the lag operator.

3.2 Persistence Results

This endogenous growth model has some important properties for growth and fluctuations. The model generates integrated time series, even though the underlying shocks are stationary. After the effects of these shocks vanish, output does not return to its trend level. That is, temporary shocks have permanent effects on output since they generate endogenous responses in the amount of resources allocated to growth. As a

[13] Appendix B provides the mapping between these parameters and the deep parameters of the model.

result, growth dynamics is an important component of the propagation mechanism in which the stochastic properties of the trend are endogenous. As argued by Fatás (2000), in this setting, output persistence is not simply equal to the persistence of disturbances since shocks endogenously generate changes in the capital accumulation rate that result in persistent responses of output.[14]

The permanent impact of a shock on the level of output equals the infinite sum of the moving average coefficients, which is $A(1)$.[15] For simplicity and to facilitate comparison with the results obtained by Fatás (2000), we also restrict our attention to $\sigma = 1$ in which case the utility function is logarithmic.[16] In the model under study the measure of persistence $A(1)$ is given by:

$$A(1) = \frac{\theta}{1-\rho}, \quad \text{where} \quad \theta = \frac{A^{\frac{\phi}{\phi-1}}}{1+\left(1-\frac{1}{\phi}\right)A^{\frac{\phi}{\phi-1}}} = \frac{\phi}{\phi-1}(1-\frac{\beta}{\tilde{G}}), \tag{13}$$

which is increasing in the parameter θ that represents the contemporaneous impact that shocks exert on the accumulation of physical capital. Hence, the greater the growth rate, the greater the effects of shocks on the output level through a higher response of capital accumulation to the shock.

Cochrane (1988) suggests another measure of persistence: the weighted sum of autocorrelations $V = \lim_{J\to\infty}\left[1+2\sum_{j=1}^{J}(1-\frac{j}{J+1})\psi_j\right]$, where ψ_j is the jth autocorrelation of the growth rate of output. In the model under study, V is given by:

$$V = \frac{\left(1-\rho^2\right)\theta^2}{(1-\rho)^2\left[\left(\frac{\phi}{\phi-1}\right)^2 - 2\rho(\frac{\phi}{\phi-1})\left(\frac{\phi}{\phi-1}-\theta\right)+\left(\frac{\phi}{\phi-1}-\theta\right)^2\right]}, \tag{14}$$

[14] A standard RBC model would predict no correlation between persistence and growth as growth would be treated as exogenous.

[15] In Appendix C we provide the derivation of the expressions linking this persistence measure and the variance ratio (V) with the θ parameter.

[16] When the standard AK model is considered, there is no closed-form solution to the equilibrium even for $\sigma = 1$. However, when the endogenous depreciation AK model is considered and $\sigma = 1$, there exists a closed-form solution to the equilibrium. Obviously, by using this closed-form solution we would obtain the same equation (12), with no need to calculate a. This is proven in Appendix B.

which is also increasing in θ.[17]

Therefore, all other things being equal, countries with a higher marginal product of capital services (A) will have a higher steady-state depreciation rate ($\widetilde{\delta}$), a higher long-run growth rate ($\tilde{G}$) and a greater contemporaneous response of capital growth to a given shock (θ).[18] Regardless of the measure considered, persistence is increasing in θ, and this yields a positive correlation between persistence and long-term growth rates. As previously indicated, this result is what Fatás (2000) observes for the G7 countries and accords with our findings for the larger sample of countries studied earlier. Further, this result is also consistent with the observed differences in cross-country utilization rates, the positive correlation between capital utilization and per worker income reported by Mayshar and Halevy (1997), and the positive correlation between depreciation and long-term average output growth shown above. Note that in an *exogenous* growth model the depreciation rate affects the output level, but not its growth rate in the long-run. Therefore, a standard growth model predicts no correlation between these variables. However, the endogeneity of growth by itself is not sufficient to generate this observed positive correlation, since in a standard AK-type endogenous growth model with an *exogenous* depreciation rate these variables are negatively related.

To sum up, this model shows that the degree of persistence is an increasing function of the depreciation rate (which in turn depends positively on the degree of capital utilization) as well as of long-term growth rates. The larger the depreciation rate is, the larger both the growth rate and the permanent impact of a shock on the level of output. The empirical analysis of the link between depreciation rates and output growth shown earlier is consistent with these implications of the model.

[17] Note that, $\lim_{\phi\to\infty}\theta = \frac{A}{A+1-\delta}$ and hence, $A(1)$ converges to the corresponding value for the standard AK model considered by Fatás (2000). Note also that $\lim_{\phi\to\infty} V = \frac{\theta^2\left(1-\rho^2\right)}{(1-\rho)^2\left[\theta^2+2(1-\rho)(1-\theta)\right]}$.

[18] The same result is obtained when the country-specific parameter is ϕ instead of A.

3.3 The Dynamic Response of Output

In order to directly compare the dynamic response of output in an AK model with and without endogenous depreciation, we analyze how this response is affected by changes in ϕ, which measures the elasticity of the depreciation rate with respect to the rate of capital utilization. As noted above, the endogenous depreciation model collapses to the AK model considered by Fatás (2000) when full capital utilization and therefore a constant depreciation rate is assumed. In fact, in the limit $A(1)$ and V converge to the corresponding values for the standard AK model with exogenous depreciation. Therefore, we can study the role that the endogeneity of the depreciation rate may play in explaining the response of output to a unit shock by varying a single parameter, ϕ.

In order to illustrate the response of output to a shock, we must assign values to the following technology parameters: A, ϕ and ρ. We assign these values based on micro-evidence and long-run properties of the economy. The existing empirical studies that estimate the elasticity parameter ϕ suggest that an empirically plausible range for this parameter value is [1.4, 2). We will take $\phi = 1.7$ as the benchmark value.[19] Epstein and Denny (1980) estimate a depreciation rate about 13 percent per annum on average for US manufacturing over the period 1947-1971.[20] Given ϕ and $\tilde{\delta}$, the value for A is derived from the long-run solution of the FOC (7) which establishes that $A = \tilde{U}^{\phi-1}$ and, therefore, $A\tilde{U} = \tilde{U}^{\phi} = \phi\tilde{\delta}$. This yields $A = 0.537$.

We can derive the MA representation (or impulse response function) for the level

[19] Greenwood, Hercowitz and Huffman (1988) and Finn (1995) estimate ϕ to be approximately 1.4 for US manufacturing, while Burnside and Eichenbaum (1996) obtain a value for ϕ of 1.56. Dalgaard (2003) finds the corresponding estimate for Denmark to equal 1.7. Finally, Basu and Kimball (1997) obtain a point estimate for ϕ equal to 2.

[20] Interestingly, this value roughly coincides with the depreciation rate obtained for the US economy with data from the *World Development Indicators* averaged over the period 1970-2008, which equals 12.86.

of log GNP by rewriting equation (12):

$$\begin{aligned} y_t &= y_{t-1} + ln\tilde{G} + A(L)\varepsilon_t \\ &= y_0 + t\ ln\tilde{G} + B(L)\varepsilon_t \end{aligned}$$

where $B_i = \sum_{j=0}^{i} A_j$. Therefore, the limit of B_i is $A(1)$, which measures the response of y_{t+i} to a shock at time t for a large i.[21]

Figure 3 illustrates the impulse response functions for the endogenous depreciation model with different values of ϕ. We calculate these responses by setting $\rho = 0.9$. This figure shows that as the elasticity of depreciation with respect to the rate of capital utilization ϕ increases, the permanent effect of a shock on output increases. In Appendix D we prove that $A(1)$ increases with ϕ.

[Insert Figure 3 about here]

As mentioned above, if the depreciation rate were treated as exogenous, $A(1)$ would converge to the corresponding value of the standard AK model in Fatás (2000), i.e. $\lim_{\phi\to\infty} A(1) = \frac{A}{A+1-\delta}\frac{1}{1-\rho}$ and, hence, this model would overstate the long-run impact of a shock on the level of output.[22] The logic behind this result is the following. Temporary shocks become persistent in both models as they have an effect on the amount of resources allocated to capital accumulation. However, when the depreciation rate is endogenous, temporary shocks affect capital accumulation through two channels: the marginal product of capital and the depreciation rate. In this setting, it is optimal to utilize capital less than fully and, therefore, shocks have a lower impact on the marginal

[21] In Appendix D we provide the derivation of B_i.

[22] We can compare these two models by controlling for the long-run depreciation rate. Let a bar on top of a variable denote its steady-state value for the standard AK model. When the standard AK model is considered, the long-run growth rate is given by $\bar{G} = \beta[A + 1 - \bar{\delta}]$. Therefore, controlling for the depreciation rate ($\tilde{\delta} = \bar{\delta}$), we obtain $\bar{G} - \tilde{G} = \beta A(1 - \tilde{U}) > 0$. Similarly, we obtain $\bar{\theta} - \tilde{\theta} = \frac{A}{A+1-\bar{\delta}} - \frac{A\tilde{U}}{1+A\tilde{U}-\tilde{\delta}} = \frac{A(1-\delta)}{[1+A\tilde{U}-\tilde{\delta}][A+1-\bar{\delta}]}(1-\tilde{U}) > 0$. Hence, the standard AK model overstates the long-run impact of a shock on the level of output.

product of capital than in a standard AK model, which leads to a lower effect on capital accumulation. Moreover, the fact that the depreciation rate is procyclical further smooths the impact on capital accumulation. As a result, shocks have a lower impact on growth, thereby rendering a lower persistence measure A(1) than in a standard AK model. Likewise, the persistence measure V that we obtain for the whole empirically plausible range of ϕ is also lower than in a standard AK model.

In addition, from Figure 3 we infer that there exists a $\breve{\phi}$ for which B_i is a constant equal to A(1). For $\phi < \breve{\phi}$, B_i is a decreasing and convex function while for $\phi > \breve{\phi}$, B_i becomes an increasing and concave function. Therefore, the higher the value of ϕ, the smaller the impact of the shock, although the permanent impact of the shock becomes larger.

Summing up, the dynamic response of output depends on the value of ϕ, which determines the degree of sensitivity of the depreciation rate of capital to the choice of capital utilization. As the elasticity of depreciation with respect to the rate of capital utilization increases, the permanent effect of a shock on output rises. As a result, the standard AK model overstates the permanent impact of a shock on the level of output.

4 Summary and Conclusions

Cross-country differences in output persistence have already been well documented by Campbell and Mankiw (1989) and Cogley (1990). Further, Fatás (2000) finds a strong positive correlation between the persistence of fluctuations and long-term average growth rates for a sample that includes the G7 countries and eight additional OECD countries. We confirm these findings for a larger sample of 101 countries with data extending over the period 1970-2008. The standard RBC models with exogenous productivity shocks cannot account for this evidence, while Fatás (2000) shows that the standard AK endogenous growth model is able to generate this positive correlation.

Moreover, empirical evidence documents large differences in capital utilization rates across countries and a positive correlation between capital utilization and per capita income as well as between depreciation and long-term average per capita income growth. Both standard exogenous and endogenous growth models, however, assume that capital services are a constant proportion of the underlying capital stock and treat depreciation as an exogenous parameter. We have extended the existing evidence on the relationship between output growth and depreciation rates through dynamic panel data estimations by using a sample of 101 countries over the period 1970-2008. The evidence is not consistent with the predictions of standard exogenous growth and AK-type endogenous growth models, for which the depreciation rate affects negatively either transitional growth or long-run growth, respectively.

We have then attempted to reconcile empirical evidence and the predictions on persistence, long-term growth and capital utilization rates by allowing the depreciation rate to be sensitive to the rate of capital utilization in an otherwise standard AK model. We find that, in this setting, a full utilization rate of capital is not optimal, depreciation is endogenously determined, and the implications of the model are consistent with the observed cross-country evidence showing that: (1) the degree of persistence is an increasing function of output growth, and (2) output growth is positively associated with the depreciation rate.

In addition, we have shown that the standard AK growth model (compared to the AK model with endogenous capacity utilization) overstates the long-run impact of a shock on the level of output. Despite the fact that temporary shocks become persistent in both models (since they have an impact on the amount of resources allocated to capital accumulation), it turns out that when the depreciation rate is endogenous, temporary shocks affect capital accumulation through two channels: the marginal product of capital and the depreciation rate. Since it is optimal to utilize capital less than fully, shocks have a lower impact on the marginal product of capital than in a standard AK model, which leads to a lower effect on capital accumulation. Moreover, the procyclicality of the depreciation rate further smooths the impact on capital accumulation, which results in a lower persistence measure of short-term fluctuations than in a traditional AK model.

We conclude that the interaction between growth and variable capital utilization rates generates theoretical predictions that are closer to the empirical evidence. In addition, the analysis may also have the potential to generate fruitful insights for public policy studies and may help shed light on various aspects of business cycles research. Baxter and Farr (2005), for example, study the relevance of the capital utilization decision into an otherwise standard international busines cycle model in explaining several central issues in this area. They find that variable capital utilization by itself does not provide an internal propagation mechanism that improves the model's ability to explain the observed persistence in macro-aggregates. By allowing not only capital utilization but also growth to be endogenously determined, the models might help overcome these shortcomings. This is an aspect that merits future research.

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Appendix A: Descriptive Statistics

[Insert Table A1 about here]

Appendix B: Parameters in the Linearization

Let $\hat{x}_t$ denote the deviation of variable X_t from its steady state value in logarithms. From the log-linearization of equation (10), we obtain the following first order stochastic difference equation:

$$a_1\hat{s}_t + a_2\hat{z}_t = a_3E_t\left(\hat{s}_{t+1}\right) + a_4E_t\left(\hat{z}_{t+1}\right), \tag{A.1}$$

where

$$a_1 = \left[\frac{\tilde{G}}{(1-\tilde{S})A^{\frac{\phi}{\phi-1}}}\right]^{\sigma}\sigma\tilde{S}\left[\frac{A^{\frac{\phi}{\phi-1}}}{\tilde{G}} + \frac{1}{1-\tilde{S}}\right],$$

$$a_2 = \left[\frac{\tilde{G}}{(1-\tilde{S})A^{\frac{\phi}{\phi-1}}}\right]^{\sigma}\sigma\frac{1}{\phi-1}\left[\frac{A^{\frac{\phi}{\phi-1}}(\phi\tilde{S}-1)}{\tilde{G}} - \phi\right],$$

$$a_3 = \beta\frac{\tilde{S}}{1-\tilde{S}}\left\{\left[\frac{1}{(1-\tilde{S})A^{\frac{\phi}{\phi-1}}}\right]^{\sigma}\sigma + \frac{\phi}{\phi-1}A^{\frac{\phi}{\phi-1}(1-\sigma)}\frac{1}{1-\tilde{S}}\sigma\right\},$$

$$a_4 = \beta\left\{A^{\frac{\phi}{\phi-1}(1-\sigma)}\frac{1}{1-\tilde{S}}(1-\sigma) - \left[\frac{1}{(1-\tilde{S})A^{\frac{\phi}{\phi-1}}}\right]^{\sigma}\sigma\frac{\phi}{\phi-1}\right\}.$$

Since $\hat{z}_t = \rho\hat{z}_{t-1} + \varepsilon_t$, the solution is given by:

$$\hat{s}_t = a\hat{z}_t, \tag{A.2}$$

where $a = \frac{\rho a_4 - a_2}{a_1 - \rho a_3}$. Note that, when $\sigma = 1$, $a_1 = \frac{\tilde{S}}{1-\tilde{S}}(\frac{\phi-1}{\phi} + A^{\frac{\phi}{1-\phi}})$, $a_2 = -\frac{\phi}{\phi-1}A^{\frac{\phi}{1-\phi}}$, $a_3 = \beta a_1$, $a_4 = \beta a_2$, and therefore $a = \frac{1-\tilde{S}}{\tilde{S}}\frac{\phi}{\phi-1}\frac{A^{\frac{\phi}{1-\phi}}}{\frac{\phi-1}{\phi}+A^{\frac{\phi}{1-\phi}}}$. By plugging this value into the expression for θ we obtain $\theta = \frac{A^{\frac{\phi}{\phi-1}}}{1+\frac{\phi-1}{\phi}A^{\frac{\phi}{\phi-1}}}$ as reported in equation (13).

As mentioned earlier, when $\sigma = 1$, there is a closed-form solution to the equilibrium. It is easy to show that the solution is given by:

$$\frac{C_t}{K_t} = (1-\beta)[1 + \frac{\phi - 1}{\phi}(AZ_t)^{\frac{\phi}{\phi-1}}],$$

$$\frac{K_{t+1}}{K_t} = \beta[1 + \frac{\phi - 1}{\phi}(AZ_t)^{\frac{\phi}{\phi-1}}].$$

Obviously, by using this closed-form solution we can get the same θ, with no need to calculate a. By log-linearizing the last equation around the steady state, we obtain:

$$\tilde{G}(1 + \widehat{\Delta k_{t+1}}) = \beta + \beta\frac{\phi - 1}{\phi}A^{\frac{\phi}{\phi-1}}(1 + \frac{\phi}{\phi - 1}\hat{z}_t),$$

and dropping the constants we get,

$$\widehat{\Delta k_{t+1}} = \beta\frac{A^{\frac{\phi}{\phi-1}}}{\tilde{G}}\hat{z}_t = \theta\hat{z}_t,$$

where $\theta = \frac{A^{\frac{\phi}{\phi-1}}}{1+\frac{\phi-1}{\phi}A^{\frac{\phi}{\phi-1}}}$ as shown earlier.

Appendix C: Persistence and the Parameter θ

The persistence measure $A(1)$ can be calculated from equation (12). This equation can be rewritten as:

$$\begin{aligned}
\widehat{\Delta y_t} &= \left[\frac{\phi}{\phi - 1} - \left(\frac{\phi}{\phi - 1} - \theta\right)L\right]\left[1 + \rho L + \rho^2 L^2 + ...\right]\varepsilon_t \\
&= \left[\frac{\phi}{\phi - 1} + d_1 L + \rho d_1 L^2 + \rho^2 d_1 L^3 + ...\right]\varepsilon_t,
\end{aligned}$$

where $d_1 = [\frac{\phi}{\phi-1}\rho - (\frac{\phi}{\phi-1} - \theta)]$. Adding up these coefficients, we obtain:

$$P^J = \frac{\phi}{\phi - 1} + \sum_{j=1}^{J}\rho^{j-1}d_1. \qquad (A.4)$$

Therefore, the infinite sum of all coefficients will give us the following measure for the permanent impact of a given shock on the level of output

$$A(1) = \lim_{J\to\infty} P^J = \frac{\phi}{\phi - 1} + \frac{d_1}{1 - \rho} = \frac{\theta}{1 - \rho}.$$

Cochrane (1988) suggests another measure of persistence:

$$V = \lim_{J\to\infty} \left[1 + 2\sum_{j=1}^{J} (1 - \frac{j}{J+1}) \psi_j \right], \qquad (A.5)$$

where ψ_j is the jth autocorrelation of the growth rate of output.

The persistence measure V can be calculated by rewriting equation (12) as an ARMA(1,1) process:

$$(1-\rho L)\widehat{\Delta y_t} = \left[\frac{\phi}{\phi-1} - \left(\frac{\phi}{\phi-1} - \theta \right) L \right] \varepsilon_t = b_1 \varepsilon_t - b_2 \varepsilon_{t-1}$$

with $b_1 = \frac{\phi}{\phi-1}$ and $b_2 = \frac{\phi}{\phi-1} - \theta$.

The autocorrelation coefficients of this ARMA(1,1) process are the following: $\psi_1 = \frac{\rho b_1^2 - \rho^2 b_1 b_2 + \rho b_2^2 - b_1 b_2}{b_1^2 - 2\rho b_1 b_2 + b_2^2)}$ and $\psi_\tau = \rho \psi_{\tau-1}$ for $\tau > 1$. Given this structure for the autocorrelation coefficients we obtain $V = \lim_{J\to\infty} \left[1 + 2\psi_1 \sum_{j=1}^{J} (1 - \frac{j}{J+1}) \rho^{j-1} \right] = 1 + 2\frac{\psi_1}{1-\rho}$.

By substituting the corresponding coefficients b_1 and b_2 into ψ_1:

$$V = \frac{\left(1-\rho^2\right)\theta^2}{(1-\rho)^2 \left[\left(\frac{\phi}{\phi-1}\right)^2 - 2\rho(\frac{\phi}{\phi-1}) \left(\frac{\phi}{\phi-1} - \theta\right) + \left(\frac{\phi}{\phi-1} - \theta\right)^2 \right]} \qquad (A.6)$$

which is increasing in θ as long as $\theta < 2\frac{\phi}{\phi-1}$.

Appendix D: The Moving Average Representation for the Level of Log GNP

We can derive the MA representation (or impulse response function) for the level of log GNP by rewriting equation (12):

$$\begin{aligned} y_t &= y_{t-1} + ln\tilde{G} + A(L)\varepsilon_t \\ &= y_{t-1} + ln\tilde{G} + \left[\frac{\phi}{\phi-1} + d_1 L + \rho d_1 L^2 + \rho^2 d_1 L^3 + ... \right] \varepsilon_t, \end{aligned}$$

where $d_1 = [\frac{\phi}{\phi-1}\rho - (\frac{\phi}{\phi-1} - \theta)]$. By iterating, we obtain:

$$\begin{aligned} y_t &= y_0 + t\ ln\tilde{G} + B(L)\varepsilon_t \\ &= y_0 + t\ ln\tilde{G} + \left[B_0 + B_1 L + B_2 L^2 + B_3 L^3 + ... \right] \varepsilon_t, \end{aligned}$$

with $B_0 = \frac{\phi}{\phi - 1}$, $B_1 = B_0 + d_1$, $B_2 = B_0 + d_1 + \rho d_1$, and hence $B_i = B_0 + \sum_{j=1}^{i} \rho^{j-1} d_1$ which is equal to P^J in $(A4)$. Therefore, the limit of B_i is $A(1)$, which measures the response of y_{t+i} to a shock at time t for a large i (since $\frac{\partial y_{t+i}}{\partial \varepsilon_t} = B_i$).

Figure 3 shows the impulse response functions for the level of log GNP for different values of ϕ. In each case, B_i approaches $A(1)$ as i approaches ∞. This figure shows that as ϕ increases, the impact effect of the shock decreases, but the permanent effect of a shock on output increases. It can be easily proved that:

$$\frac{\partial B_0}{\partial \phi} = -\frac{1}{(\phi - 1)^2} < 0$$

$$\frac{\partial A(1)}{\partial \phi} = \frac{A^{\frac{\phi}{\phi-1}}(\frac{1}{\phi-1} - \frac{\phi}{(\phi-1)^2})lnA}{1 + \frac{A^{\frac{\phi}{\phi-1}}(\phi-1)}{\phi}} - \frac{A^{\frac{\phi}{\phi-1}}(-\frac{A^{\frac{\phi}{\phi-1}}(\phi-1))}{\phi^2} + \frac{A^{\frac{\phi}{\phi-1}}}{\phi} + \frac{A^{\frac{\phi}{\phi-1}}(\phi-1)(\frac{1}{\phi-1} - \frac{\phi}{(\phi-1)^2})lnA}{\phi})}{(1 + \frac{A^{\frac{\phi}{\phi-1}}(\phi-1)}{\phi})^2} > 0$$

since $A < 1$ and $\phi > 1$.

When $\lim_{\phi \to \infty} B_0 = 1$ and for a large i we have that $\lim_{\phi \to \infty} B_i = \frac{\bar{\theta}}{1-\rho}$, which is exactly the $A(1)$ value for the standard AK model considered by Fatás (2000). Thus, this model overstates the long-run impact of a shock on the level of output, since $\bar{\theta} > \tilde{\theta}$.

Moreover, there exists a $\breve{\phi}$ for which B_i is a constant equal to $A(1)$. This $\breve{\phi}$ is obtained by solving $B_0 = A(1)$, that is, $\frac{\phi}{\phi-1} = \frac{1}{1-\rho} \frac{A^{\frac{\phi}{\phi-1}}}{1+\left(1-\frac{1}{\phi}\right)A^{\frac{\phi}{\phi-1}}}$. Given A and ρ, we can easily obtain such $\breve{\phi}$. In our case, we obtain $\breve{\phi} = 1.807663$. For $\phi < \breve{\phi}$, B_i is a decreasing and convex function while for $\phi > \breve{\phi}$, B_i becomes an increasing and concave function. Therefore, the higher the value of ϕ, the lower the impact effect of the shock, although the permanent impact of the shock becomes larger.

FIGURES

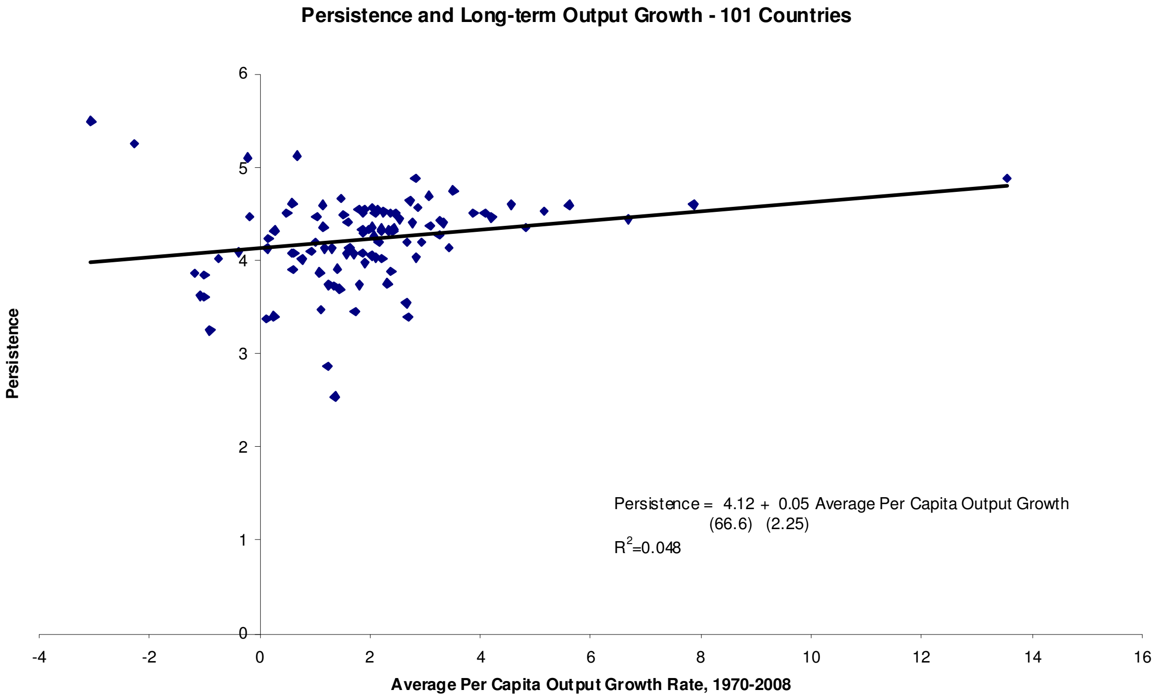


FIGURE 1: Short-term persistence and long-term output growth, 1970-2008

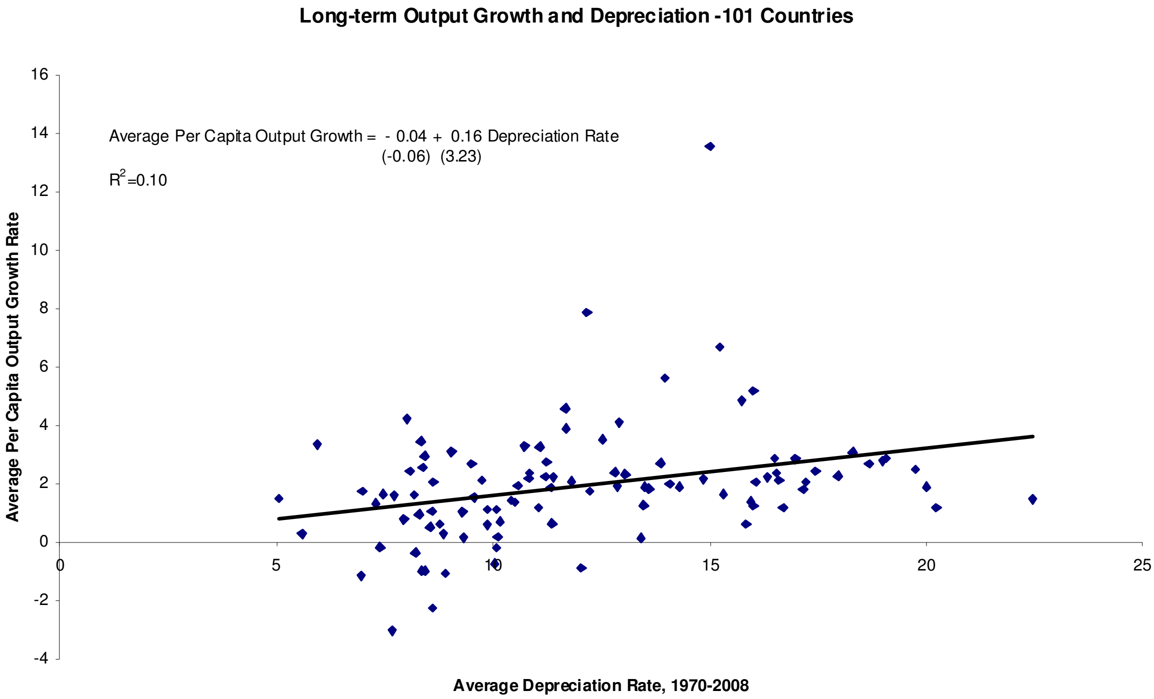
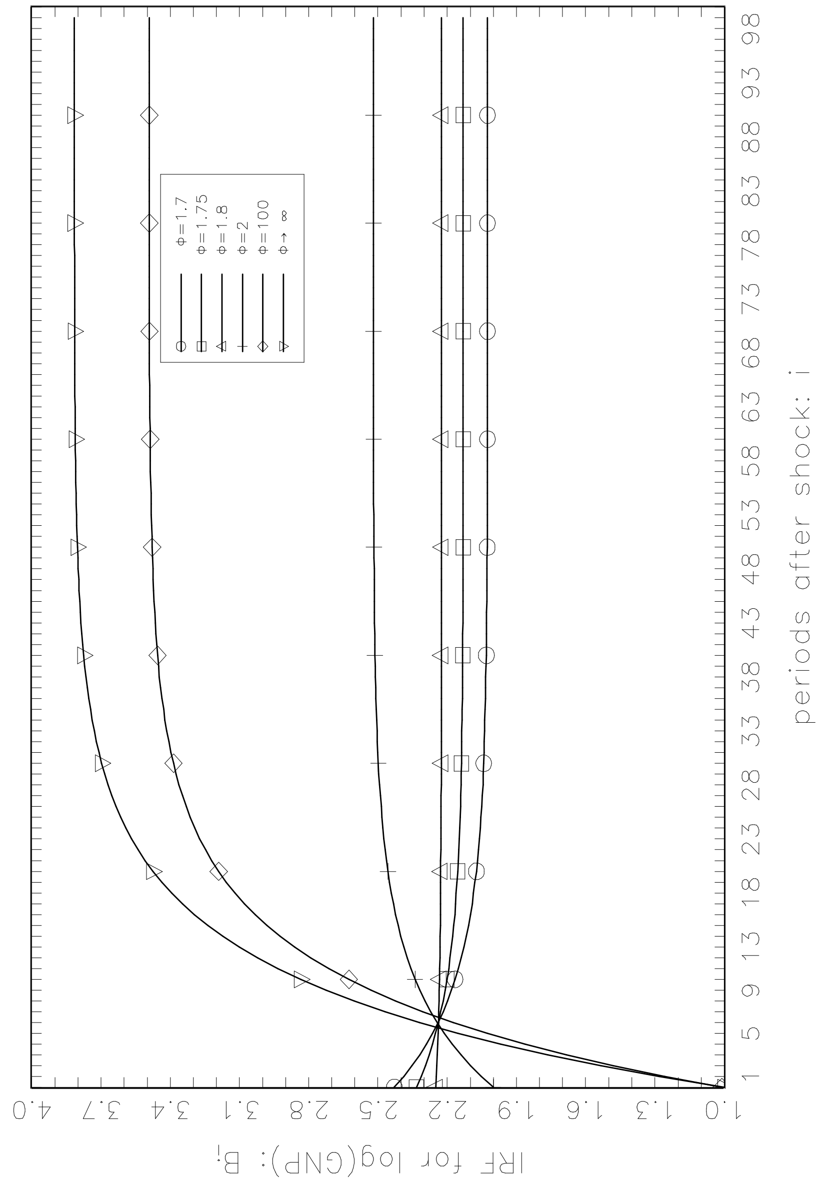


FIGURE 2: Depreciation and long-term output growth, 1970-2008

FIGURE 3: Impulse response function for the level of log(GNP).



TABLES

TABLE 1: LSDV Panel Growth Estimations

	(1) LSDV	(2) LSDV	(3) LSDV	(4) LSDV
	OECD		Full Sample	
Depreciation [a]	0.039***	0.057***	0.045***	0.042***
	(4.87)	(3.65)	(6.43)	(5.33)
NFCF [a]	0.004**	0.008***	0.011***	0.009***
	(2.20)	(2.94)	(6.10)	(3.92)
Population Growth		-0.009**		-0.009***
		(-2.40)		(-3.58)
Trade [a]		0.011		0.028***
		(0.61)		(3.02)
Secondary Schooling [a]		-0.008		-0.009
		(-0.83)		(-1.13)
Agriculture Share [a]		0.002		-0.002
		(0.49)		(-0.75)
Lag GDP p.c. [a]		-0.036***		-0.028***
		(-2.92)		(-3.59)
Specification Tests				
Wald (joint)	26.20	86.10	47.61	116.7
	[0.000]	[0.000]	[0.000]	[0.000]
Wald (Dummy)	792.5	9885	755.6	381.2
	[0.000]	[0.000]	[0.000]	[0.000]
Wald (time)	734.6	1086	489.6	1.290e+004
	[0.000]	[0.000]	[0.000]	[0.000]
AR(1) test	3.688	0.079	3.087	1.355
	[0.000]	[0.937]	[0.002]	[0.175]
AR(2) test	2.030	-0.453	-0.408	-1.226
	[0.042]	[0.650]	[0.683]	[0.220]
R^2	0.355	0.439	0.264	0.363
Observations	981	393	3347	1124

Notes: The dependent variable is the growth rate of real per capita output. [a] indicates that the variable is included in the regression in log-levels. *,** and *** indicate significance at 10%, 5% and 1%, respectively. Robust t-statistics are given in parenthesis. Figures in brackets for the specification tests represent probabilities of non-rejection of the null hypothesis. A Wald-type test is given under the name Wald (dummy) to test for the joint-significance of the deterministic components given by the constant and time effects. Wald(joint) tests for the joint-significance of both the long-run and the short-run coefficients. All Wald-type tests are distributed as a χ^2 with the number of degrees of freedom equal to the number of restrictions. The tests labelled by AR(1) and AR(2) test for the presence of first and second-order correlation in the residuals of the model.

Table 2: Dynamic Panel Growth Estimations

	(5) SYS-GMM	(6) SYS-GMM	(7) SYS-GMM	(8) SYS-GMM
	OECD		Full Sample	
Depreciation [a]	0.033***	0.061***	0.048***	0.041***
	(4.62)	(4.20)	(4.27)	(4.96)
NFCF [a]	0.003**	0.009***	0.011***	0.008***
	(2.05)	(3.15)	(3.82)	(3.03)
Population Growth		-0.009*		-0.007 **
		(-1.89)		(-2.29)
Trade [a]		0.004		0.037***
		(0.21)		(3.11)
Secondary Schooling [a]		-0.004		-0.015*
		(-0.32)		(-1.88)
Agriculture Share [a]		0.003		-0.001
		(0.78)		(-0.41)
Lag GDP p.c. [a]		-0.059***		-0.035***
		(-2.71)		(-2.62)
Specification Tests				
Wald (joint)	22.38	102.8	18.72	62.09
	[0.000]	[0.000]	[0.000]	[0.000]
Wald (dummy)	867.2	3.494e+004	858.5	395.2
	[0.000]	[0.000]	[0.000]	[0.000]
Wald (time)	757.3	1001.	405.9	3787
	[0.000]	[0.000]	[0.000]	[0.000]
AR(2) test	-2.628	0.2732	-2.413	-0.500
	[0.009]	[0.785]	[0.016]	[0.617]
Sargan test	1599	794.2	3241	1810
	[0.014]	[1.000]	[0.000]	[1.000]
Observations	981	393	3347	1123

Notes: The dependent variable is the growth rate of real per capita output. [a] indicates that the variable is included in the regression in log-levels. *,** and *** indicate significance at 10%, 5% and 1% , respectively. Robust t-statistics are given in parenthesis. Figures in brackets for the specification tests represent probabilities of non-rejection of the null hypothesis. A Wald-type test is given under the name Wald (dummy) to test for the joint-significance of the deterministic components given by the constant and time effects. Wald(joint) tests for the joint-significance of both the long-run and the short-run coefficients. All Wald-type tests are distributed as a χ^2 with the number of degrees of freedom equal to the number of restrictions. All the models are estimated with the *system estimator* of Arellano and Bover (1995). The Sargan statistic for over-identifying restrictions testing the validity of the instruments under the null hypothesis is distributed as a χ^2 with J-K degrees of freedom, where J is the number of instruments and K is the number of regressors. The AR(2) statistic tests for the presence of second-order correlation in the residuals of the model and is distributed as a standard normal distribution.

TABLE A1: Descriptive Statistics

	Country Name	Code	Per Capita Output Growth		Depreciation Rate		Net Investment Rate	
			Mean	**Std Error**	**Mean**	**Std Error**	**Mean**	**Std Error**
1	Austria	AUT	2.42	1.75	17.45	3.10	6.60	3.17
2	Belgium	BEL	2.20	1.81	16.34	2.72	4.29	3.30
3	Denmark	DNK	1.80	1.88	17.16	2.73	3.41	3.27
4	Finland	FIN	2.67	2.86	18.68	3.27	4.61	4.88
5	France	FRA	1.98	1.61	14.08	2.44	6.75	3.13
6	Germany	DEU	2.04	1.57	16.06	2.69	5.54	3.47
7	Greece	GRC	2.38	3.29	12.81	2.26	9.18	4.00
8	Hungary	HUN	2.69	3.41	13.86	3.65	10.98	6.71
9	Iceland	ISL	2.84	3.38	16.51	3.79	7.33	4.13
10	Ireland	IRL	3.87	3.06	11.68	2.18	9.97	3.79
11	Italy	ITA	2.04	2.05	17.22	2.89	4.96	3.14
12	Luxembourg	LUX	3.07	3.24	18.30	4.88	2.64	4.82
13	Netherlands	NLD	2.09	1.55	16.60	2.68	5.24	3.40
14	Norway	NOR	2.77	1.64	19.00	2.98	5.50	4.29
15	Portugal	PRT	2.86	3.80	19.07	3.86	6.42	4.82
16	Spain	ESP	2.35	2.00	16.54	3.19	7.86	3.26
17	Sweden	SWE	1.88	1.93	13.50	2.24	5.76	2.97
18	Switzerland	CHE	1.15	2.12	20.23	3.90	4.95	3.87
19	Turkey	TUR	2.41	3.91	8.07	3.82	11.19	4.67
20	United Kingdom	GBR	2.13	1.89	14.85	2.86	3.29	2.72
21	Australia	AUS	1.87	1.65	20.00	4.07	5.82	3.35
22	China	CHN	7.86	3.88	12.16	3.42	19.71	5.33
23	Hong Kong	HKG	4.83	4.43	15.73	2.42	9.34	4.15
24	Indonesia	IDN	4.21	3.60	8.01	3.29	16.92	4.97
25	Japan	JPN	2.46	2.49	19.75	3.35	9.39	5.29
26	Korea Rep	KOR	5.61	3.42	13.96	2.72	16.35	4.20
27	Malaysia	MYS	4.08	3.59	12.91	2.46	15.34	6.94
28	New Zealand	NZL	1.16	2.39	16.70	3.06	5.95	3.49
29	Philippines	PHL	1.41	3.32	10.41	2.22	10.35	5.75
30	Singapore	SGP	5.16	3.96	16.01	3.40	18.53	7.76
31	Thailand	THA	4.56	3.85	11.67	2.42	16.84	6.03
32	Argentina	ARG	1.25	5.95	13.48	4.78	8.43	6.12
33	Bolivia	BOL	0.57	3.04	9.87	1.56	5.93	2.86
34	Brazil	BRA	2.31	3.99	13.05	3.26	6.43	3.73
35	Chile	CHL	2.83	5.00	16.98	4.57	3.02	5.99
36	Colombia	COL	2.07	2.30	11.81	2.40	5.94	2.14
37	Costa Rica	CRI	2.21	3.39	11.38	6.67	8.99	5.28
38	Dominican Rep.	DOM	3.43	4.19	8.33	3.40	11.34	4.93
39	Ecuador	ECU	1.81	3.59	13.60	3.43	6.36	4.02
40	El Salvador	SLV	1.03	4.15	8.58	3.52	7.29	4.07
41	Guatemala	GTM	1.17	2.40	11.05	1.50	4.47	2.45
42	Honduras	HND	1.31	3.06	7.29	1.79	14.83	4.54
43	Jamaica	JAM	0.68	4.77	10.17	2.51	12.27	5.18
44	Mexico	MEX	1.71	3.30	12.23	2.38	7.70	2.26
45	Nicaragua	NIC	-0.75	6.36	10.04	2.79	11.35	6.10
46	Panama	PAN	2.03	4.48	8.61	1.53	9.58	4.46

47	Paraguay	PRY	1.86	3.91	11.33	3.69	9.75	2.95
48	Peru	PER	1.11	5.47	9.87	3.37	10.89	4.92
49	Uruguay	URY	1.91	5.13	10.57	4.93	4.47	6.53
50	Venezuela	VEN	0.13	5.65	13.42	3.30	8.99	5.61
51	Algeria	DZA	1.35	5.16	10.50	3.17	19.65	6.23
52	Egypt	EGY	3.09	2.83	9.04	2.21	12.40	5.40
53	Iran Islamic Rep.	IRN	1.46	7.24	22.45	13.31	5.40	16.08
54	Israel	ISR	2.25	2.73	17.96	2.98	4.51	4.73
55	Jordan	JOR	2.65	6.76	9.50	3.08	17.12	7.94
56	Morocco	MAR	2.16	4.25	10.82	1.94	12.42	4.98
57	Saudi Arabia	SAU	1.23	7.94	16.01	10.56	3.54	12.94
58	Syrian Arab Republic	SYR	2.21	7.16	11.20	2.50	11.66	5.74
59	Tunisia	TUN	3.25	3.42	11.08	1.83	14.42	4.24
60	Canada	CAN	1.87	2.01	14.31	1.26	6.76	1.80
61	United States	USA	1.90	2.00	12.86	1.01	5.93	1.11
62	Bangladesh	BGD	1.52	4.03	9.55	2.78	10.40	3.76
63	India	IND	3.27	3.17	10.73	1.84	10.84	3.96
64	Nepal	NPL	1.48	2.64	5.04	2.46	13.83	3.80
65	Pakistan	PAK	2.37	2.40	10.84	2.02	5.71	2.58
66	Sri Lanka	LKA	3.32	2.07	5.94	1.65	16.54	4.72
67	Benin	BEN	0.50	3.02	8.54	2.26	7.46	4.01
68	Botswana	BWA	6.67	5.41	15.23	3.91	12.77	7.49
69	Burkina Faso	BFA	1.60	3.16	8.17	2.20	10.36	3.72
70	Burundi	BDI	0.27	5.41	5.58	1.68	5.44	5.60
71	Cameroon	CMR	1.09	5.92	10.07	3.39	10.33	7.39
72	Central African Republic	CAF	-0.99	4.02	8.36	1.93	1.52	3.09
73	Chad	TCD	0.94	9.27	8.29	3.01	5.53	12.94
74	Congo Dem Rep	ZAR	-3.05	5.23	7.67	2.08	3.64	5.26
75	Congo Rep.	COG	1.63	6.00	15.32	5.53	11.94	9.70
76	Cote d'Ivoire	CIV	-1.08	4.23	8.89	2.95	6.29	7.38
77	Equatorial Guinea	GNQ	13.54	19.16	15.01	8.31	32.11	28.46
78	Gabon	GAB	1.37	10.89	15.94	8.12	15.65	9.12
79	Gambia The	GMB	0.63	3.27	11.36	3.55	8.54	4.04
80	Ghana	GHA	0.59	4.49	8.76	1.72	7.15	8.43
81	Guinea-Bissau	GNB	-0.18	7.57	7.39	1.51	17.90	9.25
82	Kenya	KEN	1.01	4.34	9.29	2.00	9.52	3.10
83	Lesotho	LSO	2.94	6.58	8.41	2.95	29.20	16.32
84	Liberia	LBR	-2.27	20.49	8.60	3.00	2.46	6.04
85	Madagascar	MDG	-1.00	4.39	8.42	3.06	5.03	6.50
86	Malawi	MWI	0.77	5.53	7.94	2.18	9.94	5.41
87	Mali	MLI	1.63	5.18	7.45	2.77	11.82	4.77
88	Mauritania	MRT	0.13	4.37	9.32	1.77	11.92	15.83
89	Mauritius	MUS	3.51	3.47	12.53	2.50	11.42	3.45
90	Niger	NER	-1.17	6.04	6.96	2.33	5.42	5.17
91	Nigeria	NGA	1.58	6.31	7.71	4.72		
92	Rwanda	RWA	1.73	10.95	6.98	2.43	7.97	3.05
93	Senegal	SEN	0.16	3.67	10.11	3.28	8.68	4.57
94	Sierra Leone	SLE	0.26	7.09	8.85	3.01	1.85	4.13
95	South Africa	ZAF	0.60	2.45	15.84	3.43	5.07	5.18
96	Sudan	SDN	2.10	5.79	9.73	3.03	3.04	4.02
97	Swaziland	SWZ	2.72	4.07	11.24	3.03	9.82	8.92
98	Togo	TGO	-0.38	5.74	8.21	2.13	9.42	3.67

99	Uganda	UGA	2.54	3.22	8.37	2.78	5.62	5.89
100	Zambia	ZMB	-0.90	4.03	12.04	4.45	6.45	7.96
101	Zimbabwe	ZWE	-0.22	6.25	10.08	4.69	7.04	6.84

Notes: The real per capita GDP growth figures that appear in the Table are multiplied by 100. Net Investment Rate stands for real Net Fixed Capital Formation over real GDP. The depreciation of fixed capital measured as a proportion of GDP represents the consumption of fixed capital as given by the replacement value of capital used up in the production process. The source of these data is the *World Development Indicators* of the World Bank (2009).