Borrow short to lend long
A DSGE model of maturity transformation*

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Abstract

The present paper introduces maturity transformation, a core business of commercial banks, into the standard New Keynesian model. We assume that entrepreneurs rely on external finance with extended duration, as the capital stock which they purchase from borrowed funds requires time to become productive. Accordingly, the banking sector issues multi-period loans to entrepreneurs, financing these loans by short-term household deposits. The resulting maturity mismatch between banks’ assets and liabilities attenuates the impulse responses of output and employment to shocks emerging from the financial sector, whereas the transmission of a fiscal policy shock appears to be rather insensitive to the duration of loans.

Keywords: Banking; Confidence shock; Financial frictions; Maturity transformation

JEL Classification: C61; E32; E43; E51

*This paper is work in progress. Please do not quote without prior notification to the author!
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1 Introduction

The primal function of banks is to absorb differences between the characteristics of financial obligations issued by debtors (firms) and financial investments desired by creditors (households). Focusing on credit duration as one of these traits, this paper combines a Dynamic Stochastic General Equilibrium (DSGE) model with a banking sector that collects one-period deposits from private households and provides multi-period loans. It contributes thus to the existing macroeconomic literature, where banks can perfectly match the maturities of their assets and liabilities.

We set off from an otherwise standard New Keynesian DSGE model, where entrepreneurs rely on loans to purchase the productive capital stock from capital goods producers. The optimization problem of entrepreneurs is then modified by the assumption that capital becomes productive with a delay. As a consequence, a demand for credit with extended duration arises. Banks borrow short-term and lend long-term.

Macroeconomic research on the role of financial frictions in general equilibrium was initiated by Carlstrom and Fuerst (1997) as well as Bernanke et al. (1999). Their pioneering contributions have shown that financial frictions have a potentially amplifying effect on the propagation of shocks in standard Real Business Cycle models.

Since the beginning of the lingering financial and economic crisis, the number of DSGE models with explicit financial intermediation is virtually exploding. The spectrum covers micro-foundations of multiple interest rates by Goodfriend and McCallum (2007), the modeling of monopolistic competition among private banks e. g. by Gerali et al. (2009), and larger-scale models including financial frictions like the contributions of Christiano et al. (2009) or Cúrdia and Woodford (2009).

The vast majority of these share a crucial simplifying assumption that is hard to reconcile with the actual purpose of financial intermediaries. Even models which take into account financial contracts with differing duration (e. g. Christiano et al. (2009)) allow banks to perfectly match maturities between their assets and liabilities.

In general, however, the financial contracts issued by borrowers differ from the financial investment opportunities desired by creditors. As mentioned by Gurley and Shaw (1960) and emphasized by Fama (1980), the main task of commercial banks is to transform the characteristics of financial contracts to satisfy both lenders’ and depositors’ needs.
Figure 1: Average overall maturity of assets and liabilities in the balance sheet of Euro Area private financial institutions

Figure 1 documents a significant maturity mismatch in the balance sheet data of private financial institutions. For the Euro Area, the observed average duration of bank assets is about four times the duration of their liabilities. After 1999, this relationship has been relatively stable. When fulfilling their function as financial intermediaries, banks borrow short in order to finance longer-term lending.

Unsurprisingly, maturity transformation has been thoroughly investigated within partial-equilibrium models, e.g. by Flannery (1994), Rajan and Bird (2003) and Van den Heuvel (2006), as well as empirically, e.g. by Flannery and James (1984a), Flannery and James (1984b), and Akella and Greenbaum (1992).\(^1\)

Very recently, the topic has been approached within a general equilibrium framework. Andreasen et al. (2010) derive the need for maturity transformation from an infrequent adjustment of entrepreneurs’ productive capital stock as introduced by Calvo (1983). When incorporating a banking sector that matches the duration of credit contracts to the time period a borrower firm does not readjust its capital stock, the authors find real effects on business cycle dynamics.

\(^1\)For a more comprehensive discussion of related contributions see Freixas and Rochet (2008).
The present analysis draws on a simple reference model with financial intermediation, yet without a maturity mismatch. Entrepreneurs are unable to accumulate net worth and must therefore borrow each period in order to acquire productive capital. Banks receive one-period deposits from private households and issue one-period loans to entrepreneurs. In this scenario, the maturities of banks’ assets and liabilities are perfectly matched. The model is then augmented by a version of the time-to-build constraint introduced by Kydland and Prescott (1982)\textsuperscript{2}. The physical capital stock which is paid up front by entrepreneurs becomes productive with a delay of one or more periods. For this reason, the duration of loans entrepreneurs rely on for financing the capital stock must be extended accordingly. The representative bank still collects short-term deposits from households and holds reserves. Depending on credit duration, however, its balance sheet now contains two or more open lines of credit on the asset side. Entrepreneurial loans which do not mature until a latter period must be rolled over passively.

We subject the model to an unexpected increase in government consumption expenditure, a rise in the reserve ratio of commercial banks, and an unwarranted drop in the confidence of the representative household that financial intermediaries will service their deposit obligations. The latter two shocks are emerging directly from the banking sector. All three are of particular interest in situations of economic or financial distress. In our framework, the business cycle implications of a maturity mismatch between banks’ assets and liabilities vary with the exogenous disturbance considered. On the one hand, the impulse responses of output and employment to a fiscal policy shock are virtually insensitive to introducing maturity transformation. The opposing effects on consumption and investment apparently even out, especially in the very short run. In contrast, the percentage deviations from steady state in response to an unexpected rise in banking sector reserves and a confidence crisis between households and financial intermediaries are increasingly mitigated by a longer duration of loans.

The rest of the paper is structured as follows. Subsection 2.1 presents the timing and agents of our reference model, while 2.2 introduces the need for maturity transformation. In sections 3 and 4, we calibrate parameter values and derive the long-run implications of a maturity mismatch. The sensitivity with respect to credit duration of the impulse

\textsuperscript{2}Several less common formulations of time to build, some of them more closely related to the version used here, are discussed in Rouwenhorst (1991) and Edge (2007).
responses to exogenous disturbances is central to the dynamic analysis of our model. Section 6 concludes.

2 The Model Economy

The model is set up in infinite discrete time $t = 1, 2, \ldots$. The economic environment contains a representative private household, a representative capital goods producer, a representative entrepreneur, a limited number of retailers with monopolistic power, a homogeneous banking sector, and a combined fiscal and monetary authority.

2.1 The Reference Model

At the end of period $t$, entrepreneurs buy productive capital $k_t$ from capital goods producers at the going nominal price $Q_t$. Owning zero net worth, entrepreneurs rely on bank loans to finance the acquisition of the physical capital stock. In the following period, they determine their demand for labor and produce a wholesale output good. After production in period $t + 1$, the depreciated capital stock is sold to capital producers at the current price of capital. Note that entrepreneurs may thus make capital gains or losses – apart from the productivity of capital as an input – due to changes in the price of capital. The proceeds from selling output as well as the depreciated capital stock are used to reimburse creditor banks and to pay the wage bill.

The market for wholesale goods is perfectly competitive. Retailers purchase the output from entrepreneurs at a price equal to the marginal cost of production and differentiate it at zero costs. It is then sold in a monopolistically competitive market, where retailers set prices optimally subject to a downward-sloping demand curve for their heterogeneous final output. Final goods can be invested as well as consumed.

Capital goods producers combine the depreciated capital stock with investment in terms of final output bought from retailers. At the end of period $t + 1$, the newly produced capital stock is again supplied to entrepreneurs in a perfectly competitive market.

The representative private household enters period $t$ with nominal amounts of bank deposits and government bonds. Both assets pay nominal interest between $t - 1$ and $t$. Households also supply homogeneous labor to firms. The corresponding nominal wage is determined by a competitive labor market equilibrium.
The household sector receives profits from retailers, entrepreneurs, and private banks, if they accrue. Total funds – either carried over from period \( t - 1 \) or recently acquired – are spent on consumption of final goods, increase financial wealth in the form of deposit or government bonds, or pay a lump-sum tax to the authorities.

The representative financial intermediary is an interest-rate taker in the market for loans and deposits, respectively. It turns household deposits with a maturity of one period into loans of identical duration and holds liquidity reserves in a central bank account.

### 2.1.1 The Household Sector

Households live infinitely long and enter period \( t \) with nominal government bonds \( B_{t-1} \), bank deposits \( D_{t-1} \), and the respective interest payments. The representative household consumes a Dixit-Stiglitz (1977) aggregate

\[
c_t = \left( \int_0^1 (c_{j,t})^{\eta_p - 1} \frac{\eta_p}{\eta_p - 1} dj \right)^{\frac{\eta_p}{\eta_p - 1}}
\]

of heterogeneous final goods\(^3\), where \( \eta_p \) describes the corresponding elasticity of substitution, and supplies \( h_t \) units of labor. Households experience positive utility from \( c_t \) and disutility from working.

The household chooses \( c_t, h_t, D_t, \) and \( B_t \) in order to maximize the expected discounted lifetime utility

\[
U_t = E_t \sum_{\nu=0}^{\infty} \beta^\nu u(c_{t+\nu}, c_{t+\nu-1}, h_{t+\nu}),
\]

where the instantaneous utility function for period \( t \) is given by

\[
u(c_t, c_{t-1}, h_t) = \frac{1}{1 - \sigma_c} \left( \frac{c_t}{c_{t-1}} \right)^{-\sigma_c} - \phi h_t \frac{h_t^{1+\sigma_h}}{1+\sigma_h}, \quad (1)
\]

For \( \varphi > 0 \), the household displays habit formation in consumption. \( \sigma_c \) denotes the household’s relative risk aversion and \( 1/\sigma_h \) its elasticity of intertemporal labor substitution.

Utility maximization is bounded by the budget constraint

\[
(1 + R_{t-1}) B_{t-1} + (1 + R_{t-1}^d) D_{t-1} + W_t h_t + \Pi_t \geq P_t c_t + B_t + D_t + T_t. \quad (2)
\]

In (2), \( R \) describes the nominal interest rate paid on government bonds, while \( R^d \) is the

\(^3\)Demand for consumption good \( j \) equals \( c_{j,t} = \left( \frac{P_{j,t}}{P_t} \right)^{-\eta_p} c_t \), with price index \( P_t = \left( \int_0^1 P_t(i)^{1-\mu} di \right)^{\frac{1}{1-\mu}} \).
nominal interest rate on deposits. Note that both financial investments are risk-free, here, i. e. neither the government nor private banks fail to service their debt.

$W_t$ is the nominal labor wage rate, while $\Pi_t$ represents net profits transferred to the private household in period $t$ by entrepreneurs, retailers, and banks. $T_t$ is a lump-sum tax payment to the authorities which is defined in subsection 2.1.6.

### 2.1.2 The Capital Goods Sector

After production in period $t$, capital producers purchase the stock of depreciated physical capital from entrepreneurs, combine it with a Dixit-Stiglitz aggregate investment good

$$i_t = \left( \int_0^1 (i_{j,t})^{\eta_p - 1} \eta_p^- d\eta_p \right)^{\eta_p - 1},$$

and turn it thus into new productive capital.

Since the marginal rate of transformation from a unit of used into a unit of new capital is 1, the prices of newly produced and previously installed physical capital must be the same. Accordingly, capital goods producers do not realize any capital gains or losses due to changes in the nominal price of capital goods $Q_t$.

The market for physical capital is perfectly competitive. Producers face convex investment adjustment costs $S(\phi, i_{t}/i_{t-1})$, where $S = S' = 0$ and $S'' = \phi_i > 0$ is a constant parameter in steady state. By choosing the optimal level of investment activity $i_t$, the representative capital goods producer maximizes the expected present value of all future discounted profits, expressed in terms of household utility:

$$\max_{i_t} E_t \left[ \sum_{\nu=0}^{\infty} \beta^\nu \tilde{\lambda}_{t+\nu} \Pi_{t+\nu}^k \right],$$

where the period $t$ nominal profit$^4$ is given by

$$\Pi_t^k = Q_t \left[ (1 - \delta)k_{t-1} + (1 - S(\phi_i, i_t/i_{t-1}))i_t \right] - Pt i_t - Q_t (1 - \delta)k_{t-1}. \quad (3)$$

The equation of motion of the aggregate physical capital stock in period $t$ has the standard notation

$$k_t = (1 - \delta)k_{t-1} + (1 - S(\phi_i, i_t/i_{t-1}))i_t. \quad (4)$$

In both equations (3) and (4), we assume quadratic investment adjustment costs of the following form:

$$S(\phi, i_t/i_{t-1}) = \frac{\phi_i}{2} \left( \frac{i_t}{i_{t-1}} - 1 \right)^2. \quad (5)$$

$^4$Note that this expression will be zero regardless of the particular state the model economy is in.
2.1.3 Entrepreneurs

A large number of identical and risk-neutral entrepreneurs with infinite expected horizon manages firms which produce a homogeneous wholesale good. They are unable to accumulate net worth which could be used to (partly) finance the expenditure for productive capital.\(^5\) Hence, the representative entrepreneur must borrow the entire cost of capital in period \(t\), namely a nominal amount

\[
L_t = Q_t k_t. \tag{6}
\]

The optimal demand for productive capital \(k_t\) by entrepreneurs depends on the expected marginal return and the expected marginal cost of capital over the duration of ownership. The expected marginal real return to entrepreneurs per unit of capital acquired in \(t\) is

\[
E_t \left( 1 + R^k_t \right) \equiv E_t \left[ \frac{r^k_{t+1} + (1-\delta)q_{t+1}}{q_t} \right]. \tag{7}
\]

It includes the return from production, i.e. the marginal productivity of \(k_t\), \(r^k_{t+1}\), as well as capital gains or losses from fluctuations in the real price of capital \(q_t = \frac{Q_t}{P_t}\).

From (6), the expected marginal cost of capital corresponds to the expected cost of external finance which equals the bank lending rate \(R^l_t\). The nominal interest payment at maturity of the loan expected by entrepreneurs as of period \(t\) is

\[
E_t \left( 1 + F_t \right) \equiv E_t \left( 1 + R^l_t \right). \tag{8}
\]

\((1 + F_t)\) is obviously predetermined, as \(R^l_t\) is set in the same instant that a loan is issued. Maximization of expected profits by entrepreneurs requires that the expected marginal return in \(t + 1\) equals the expected marginal cost of capital. Inserting (7) and (8) into the optimality condition in nominal terms,

\[
E_t \left[ \left( 1 + R^k_t \right) \frac{P_{t+1}}{P_t} \right] = E_t \left( 1 + F_t \right),
\]

yields

\[
E_t \left[ \frac{r^k_{t+1} + (1-\delta)q_{t+1}}{q_t} \right] = E_t \left[ \frac{1 + R^l_t}{\pi_{t+1}} \right]. \tag{9}
\]

\(^5\)We abstract from entrepreneurial net worth and focus on the effects of maturity transformation. The fact that entrepreneurs do not contribute own funds to investment projects implies an infinitely large leverage ratio. It would be straightforward to complement the model with a pro-cyclical external finance premium in the sense of Bernanke et al. (1999). For the sake of brevity, we abstain from reproducing the well-known financial accelerator.
The capital stock $k_t$ becomes productive in period $t + 1$. At this point, entrepreneurs decide how much labor to hire and produce the wholesale output good according to the Cobb-Douglas production function

$$y_t = k_t^\alpha h_t^{1-\alpha}, \quad (10)$$

where total factor productivity is assumed to be constant and equal to one.

### 2.1.4 The Retail Sector

Retailers purchase entrepreneurs’ wholesale output in a perfectly competitive market. Costless differentiation allows them to provide a heterogeneous final good which they sell to households and capital producers under monopolistic competition. Following Calvo (1983), each period a constant fraction $(1 - \xi_p)$ of retailers is allowed to optimize prices.\(^6\)

Retailer $j$, who receives a signal in period $t$, will thus reset $\tilde{P}_{j,t}$ to maximize the expected present value of discounted future profits

$$E_t\left[ \sum_{\nu=0}^{\infty} (\beta \xi_p)^\nu \tilde{\lambda}_{t+\nu} \Pi_{j,t+\nu} \right], \quad \text{with} \quad \Pi_{j,t+\nu} = (P_{j,t} - MC_t) y_{j,t}, \quad (11)$$

subject to the downward-sloping demand curve,

$$y_{j,t+\nu} = \left( \frac{P_{j,t+\nu}}{P_{t+\nu}} \right)^{-\eta_p} y_{t+\nu}, \quad (12)$$

for his final output good $y_{j,t}$. The latter can be consumed ($c_{j,t}$) or invested ($i_{j,t}$).

$P_{j,t+\nu}, \nu > 0$, is the price of retailer $j$, who sets $P_{j,t} = \tilde{P}_{j,t}$ in period $t$ and does not get an opportunity to re-optimize within $t+1, \ldots, t+\nu$. In these periods, the price of $j$ adjusts automatically according to $P_{j,t} = \tilde{\pi}_{p,t} P_{j,t-1}$ with an inflation index $\tilde{\pi}_{p,t} = \pi_{t-1}^{\nu_p} \pi^{1-\nu_p}$.

### 2.1.5 The Banking Sector

Throughout the paper, the financial sector consists of an infinite number of identical private banks which operate in two perfectly competitive markets for deposits and loans. In every period $t$, banks collect one-period deposits $D_t$ from households and convert them into entrepreneurial loans $L_t$ of the same maturity. While the provision of financial contracts is entirely costless, banks hold idle liquidity reserves $rr_t D_t$ on their deposits.\(^6\) Accordingly, the average duration until a given retailer may reset his price is $\frac{1}{\xi_p}$.\(^7\) The subscript $j$ can be dropped here, since all retailers that receive a signal in a given period solve the same profit maximization problem and will thus set the same optimal price.
The period $t$ balance sheet of the representative bank clarifies the perfect matching of maturities in the reference model:

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Investment Loans</td>
<td>$L_t$</td>
</tr>
<tr>
<td>Reserves</td>
<td>$rr_tD_t$</td>
</tr>
<tr>
<td>Deposits</td>
<td>$D_t$</td>
</tr>
</tbody>
</table>

The representative bank faces the following payment constraint in period $t$:

$$(1 + R_{t-1}^d)D_t + \Pi_t^b = rr_{t-1}R_{t-1} + (1 + R_{t-1}^d)L_{t-1} + D_t - rr_tD_t - L_t.$$  \hspace{1cm} (13)

According to (13), banks must finance the repayment of household deposits including interest and any dividend payments from either central bank reserves or settled loans. Making use of the balance sheet identity,

$$L_t + rr_tD_t \equiv D_t,$$  \hspace{1cm} (14)

for period $t$ and $t - 1$, we can solve for the bank’s instantaneous profit function

$$\Pi_t^b = R_{t-1}^dL_{t-1} - R_{t-1}^dD_{t-1}.$$  \hspace{1cm} (15)

The representative bank chooses $D_t$ and $L_t$ in order to maximize

$$E_t \left[ \sum_{\nu=0}^{\infty} \beta^\nu \lambda_{t+\nu} \Pi_{t+\nu}^b \right]$$

subject to the balance sheet identity (14).

As mentioned above, the liquidity reserves kept in a central bank account do not yield interest and lie idle.\(^8\) For this reason, a private bank has no incentive to hold any reserves beyond the prescribed amount.

We assume that the reserve ratio $rr_t$ follows a mean-reverting autoregressive process $rr_t = (rr_{t-1})^{\rho_{rr}} (\bar{rr})^{1-\rho_{rr}} \exp \left\{ \epsilon_t^rr \right\}$, where $\bar{rr}$ is the minimum reserve requirement defined by the monetary authority and $\epsilon_t^rr$ is a stochastic white noise process. An upward deviation from $\bar{rr}$ can thus be interpreted as an exogenous tightening of banks’ expectations concerning prospective liquidity requirements.

\(^8\)Considering the negligible interest payments on central bank deposits (e. g. the deposit facility of the ECB is at a level of 0.25% p. a. since April 2009), this seems to be a rather uncritical simplification.
2.1.6 Fiscal and Monetary Policy

The central bank follows a standard Taylor rule to determine the nominal policy rate, $R_t$, which also represents the interest rate on government bonds. By setting

$$ R_t = R_{t-1}^\rho \left[ R \left( \frac{\pi_t}{\pi_{\text{target}}} \right)^\phi_{\pi} \left( \frac{y_t}{y} \right)^\phi_{y} \right]^{1-\rho}, $$

monetary policy reacts to deviations of the gross rate of inflation $\pi_t$ from the targeted inflation level and to deviations of real output $y_t$ from its long-run equilibrium value. In steady state, $R_t = R_{t-1} = R$ implies $\pi = \pi_{\text{target}}$ and $y_t = y_{t-1} = y$.

While there is no currency in our model economy, the monetary authority still realizes a seignorage profit. Private banks must deposit reserves in a central bank account without receiving any interest payments on these funds. If seignorage profits are transferred to the treasury, the government budget constraint in period $t$ is

$$ T_t - G_t + B_t + rr_tD_t \geq (1 + R_{t-1}) B_{t-1} + rr_{t-1} D_{t-1}. $$

Ruling out Ponzi schemes for fiscal policy, government expenditure may not deviate systematically and persistently from tax income. We assume that $T_t$ is a constant fraction $\bar{g}$ of current GDP, whereas $G_t$ follows a stochastic AR(1) process which is stationary around the same share of $Y_t$. The primary budget deficit, defined herein as the difference between government income and government expenditure, will then reduce to the exogenous part of $G_t$, i.e. $T_t - G_t$ in (17) follows a mean-zero stochastic process. Hence, seignorage profits are used to service the interest payments on government bonds. This assumption allows us to endogenously pin down an equilibrium amount of $B_t$.

2.2 Maturity Transformation

In our model, the need for loans with longer maturity is motivated from the production side. More precisely, the capital stock requires time to build before it can be employed in the production process.

As in the reference model, entrepreneurs purchase productive capital $k_t$ at the end of period $t$, financing its acquisition by taking out a bank loan. However, now the purchased capital stock becomes productive with a delay of one or more periods and entrepreneurs

\footnote{Note that neither the reserve ratio $rr_t$ nor $D_t$ are choice variables to the authorities. An exogenous rise in government expenditure implies thus a contemporaneous increase in public debt $B_t$.}
decide about their demand for labor in a later period. More importantly, they require bank loans with extended maturity in order to be able to hold the capital stock until it finally becomes productive.

We thus explicitly assume a time-to-build setting, where the entire investment project – corresponding to the acquisition and ownership of the capital stock – must be financed up front, while the purchased capital can be employed in the production process only at the end of the project period.

As a consequence, the banking sector is no longer able to perfectly match maturities. Banks continue to collect one-period deposits, regardless of the length of the productivity delay, but convert these deposits into multi-period loans in order to satisfy entrepreneurs’ financial requirements. A maturity mismatch arises between the assets and the liabilities of commercial banks.

After production, entrepreneurs sell the depreciated capital stock to capital goods producers – again this implies the possibility of capital gains or losses. They moreover sell their wholesale output to retailers in a perfectly competitive market. The proceeds are used to pay the firm’s workers and creditors.

Private households, retailers, and the authorities remain unaffected with regard to the reference model. We thus revise only those sectors, where formal changes occur.

2.2.1 Capital Goods Producers

In this setting, new capital goods become productive with a delay of one or more periods. Entrepreneurs must therefore install the capital stock for a duration of $p = 2, 3, ...$ periods in order to employ it in the production process. The equation of motion of the aggregate physical capital stock in period $t$ varies thus slightly from the standard notation. Capital producers combine $k_{t-p}$ with $i_t$ to provide

$$k_t = (1 - \delta)k_{t-p} + \left(1 - \frac{\phi_t}{2} \left(\frac{i_t}{i_{t-1}} - 1\right)\right)^2 i_t.$$  

(18)

2.2.2 Entrepreneurs

The time required to incorporate the physical capital stock into the production process affects the planning horizon of entrepreneurs. Both the repayment of the period $t$ bank loan and the income from selling output and the depreciated productive capital accrue
in \( t + p \), now. We assume that loans are zero bonds, i.e. principal and interest payments for all \( p \) periods are settled at maturity with no intermediate coupons.

Profit maximization requires that the expected marginal return from holding capital until \( t + p \) equals expected marginal costs. Accordingly, entrepreneurs’ optimality condition (9) becomes

\[
E_t \left[ \frac{r_{t+p}^k + (1 - \delta)q_{t+p}}{q_t} \right] = E_t \left[ \frac{(1 + R_t^l)^p}{\pi_{t+1} \pi_{t+2} \cdots \pi_{t+p}} \right],
\]

while the physical capital stock enters the production function with a lag of \( p \) periods:

\[
y_t = A_t k_t^\alpha h_t^{1-\alpha}. \tag{20}
\]

### 2.2.3 The Banking Sector

The liabilities side of the representative bank’s balance sheet remains as in the reference model. Banks still collect deposits with a maturity of one quarter in each period, whereas they now issue loans with a duration of \( p \) periods. Accordingly, the asset side of the period \( t \) balance sheet contains \( p \) lines of unmatured credit with different residual terms as well as the bank’s liquid reserves.

The balance sheet identity requires that the amount of nominal household deposits \( D_t \) attracted in period \( t \) suffices to refinance the sum of compounded loans issued in previous periods, hold the required central bank reserves, and issue new loans \( L_t \).

Since investment loans to entrepreneurs are zero bonds, by definition, callable assets are the only source of income to banks. Before maturity, banks do not receive interest payments on credit contracts, while they must pay interest on deposits each period.\(^{10}\)

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>Investment Loans</td>
<td>( L_t )</td>
</tr>
<tr>
<td>( (1 + R_{t-1}^l) L_{t-1} )</td>
<td>Household Deposits</td>
</tr>
<tr>
<td>( (1 + R_{t-2}^l)^2 L_{t-2} )</td>
<td>( D_t )</td>
</tr>
<tr>
<td>\vdots</td>
<td></td>
</tr>
<tr>
<td>( (1 + R_{t-p+1}^l)^p L_{t-p+1} )</td>
<td></td>
</tr>
<tr>
<td>Reserves</td>
<td>( RR_t D_t )</td>
</tr>
</tbody>
</table>

\(^{10}\)Banks must passively roll over all entrepreneurial loans which have not yet matured in the period under consideration, i.e. the duration of which is longer than the time passed since the loan was issued. These open assets in the balance sheet necessarily affect the equilibrium amount of new loans \( L_t \).
In period $t$, the representative bank collects principal and interest $(1 + R_{t-p}^l) L_{t-p}$ from entrepreneurs and combines the proceeds with its current liquidity reserves $rr_{t-1} D_{t-1}$ to service household deposits including interest $(1 + R_{t-1}^d) D_{t-1}$. In case a surplus remains after clearing the balance sheet, a dividend $\Pi_{t}^b$ is transferred to private households which own the bank. Accordingly, the representative bank faces the payment constraint

$$(1 + R_{t-1}^d) D_{t-1} + \Pi_{t}^b = rr_{t-1} D_{t-1} + (1 + R_{t-p}^l) L_{t-p} + D_t - rr_t D_t - L_t.$$

(21)

Substituting into (21) from the period $t$ balance sheet identity,

$L_t + (1 + R_{t-1}^d) L_{t-1} + (1 + R_{t-2}^d) L_{t-2} + \ldots + (1 + R_{t-p+1}^d) L_{t-p+1} = (1 - rr_t) D_t,$

(22)

and from the same identity shifted one period backwards, we can solve for the bank’s instantaneous balance sheet profit

$$\Pi_{t}^b = R_{t-1}^d L_{t-1} + R_{t-2}^d (1 + R_{t-2}^d) L_{t-2} + \ldots + R_{t-p+1}^d (1 + R_{t-p+1}^d) L_{t-p+1} - R_{t-1}^d D_{t-1}.$$

(23)

As in the reference model, the representative bank chooses the optimal volume of deposits and credit in order to maximize the expected present value of all future profits, i.e.

$$\max_{L_t, D_t} E_t \left[ \sum_{\nu=1}^{\infty} \beta^\nu \lambda_t^{\nu} \Pi_{t+\nu}^b \right],$$

subject to the balance sheet identity (22).

3 Calibration

Most of our model parameters are very common (see e.g. King and Rebelo (1999)) and calibration is hence straightforward. We choose standard values for the capital income share $\alpha = 0.35$, the quarterly depreciation rate of productive capital $\delta = 0.025$, and the household discount factor $\beta = 0.995$. The relative risk aversion $\sigma_c$ and the inverse Frisch elasticity of labor supply $\sigma_h$ are set equal to 1 and 1/3, respectively.

A persistence parameter $\varphi = 0.65$ corresponds to a significant level of habit formation in private household consumption. Moreover, we set the investment adjustment cost coefficient $\phi_i$ to 2.5 and the retailers’ mark up on real marginal costs to 20%, i.e. $\eta_p = 6$.

All three conform to the parameter values estimated by Christiano et al. (2005).
The monetary authority targets a quarterly inflation rate of 0.5% and demands a minimum reserve ratio on bank deposits of $r_{rr} = 2\%$.

As Taylor-rule coefficients, we choose $\rho = 0.5$ for the central bank’s interest rate smoothing, $\varphi_{\pi} = 1.5$ for its response to inflation deviations, and $\varphi_{y} = 0.2$ for the response to output deviations. The steady-state government share in GDP equals 20%.

On average, retailers are allowed to adjust the price of the final good every 4 quarters, i.e. $\xi_p = 0.75$. The weight of $\pi_{t-1}$ in the inflation index, $\iota_p$, is 0.5. In the resulting hybrid Phillips curve, both previous period and expected future inflation determine the dynamics of the current inflation rate.

The stochastic processes for government expenditure, households’ confidence in the banking sector, and liquidity reserves shocks display autocorrelations of $\rho_g = 0.8$, $\rho_\theta = 0.8$, and $\rho_{rr} = 0.75$. The calibration is summarized in table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.35</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.995</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.025</td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>1</td>
</tr>
<tr>
<td>$\sigma_h$</td>
<td>1/3</td>
</tr>
<tr>
<td>$\phi_h$</td>
<td>1.48</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>0.65</td>
</tr>
<tr>
<td>$\pi_{\text{target}}$</td>
<td>0.005</td>
</tr>
<tr>
<td>$\tau_T$</td>
<td>0.02</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.5</td>
</tr>
<tr>
<td>$\varphi_{\pi}$</td>
<td>1.5</td>
</tr>
<tr>
<td>$\varphi_y$</td>
<td>0.2</td>
</tr>
<tr>
<td>$\phi_i$</td>
<td>2.5</td>
</tr>
<tr>
<td>$\eta_p$</td>
<td>6</td>
</tr>
<tr>
<td>$\xi_p$</td>
<td>0.75</td>
</tr>
<tr>
<td>$\iota_p$</td>
<td>0.5</td>
</tr>
<tr>
<td>$\bar{g}$</td>
<td>0.2</td>
</tr>
<tr>
<td>$\rho_g$</td>
<td>0.8</td>
</tr>
<tr>
<td>$\rho_{rr}$</td>
<td>0.75</td>
</tr>
<tr>
<td>$\rho_\theta$</td>
<td>0.8</td>
</tr>
</tbody>
</table>

4 Stationary Equilibrium

To evaluate the model’s steady state, we drop any time indices from first order and equilibrium conditions, and ignore the three exogenous disturbances. With a few predetermined specifications, the model has a steady-state solution in closed form.

The steady-state interest rates on government bonds, i.e. the monetary policy rate, and deposits are determined by the corresponding FOCs of the representative household. In the absence of shocks, these interest rates solve identical equilibrium conditions. $R$ and $R^d$ must thus be identical.

\[\text{These values represent the European Central Bank's officially propagated idea of price stability and a minimum reserve system.}\]

\[\text{Note that } \phi_h \text{ is calibrated to ensure that steady-state employment in each model equals } 1/3 \text{ of the representative household's time endowment. The corresponding parameter values for credit durations of two, four, and eight quarters are } 1.39, 1.29, \text{ and } 1.20, \text{ respectively.}\]
Since banks are bound to hold a fraction of deposits as idle reserves, the interest rate on loans, $R^l$, exceeds $R^d$. However, the model fails to determine an endogenous steady-state reserve ratio. We therefore impose $rr = r\gamma$, as banks have no incentive to hold reserves beyond the minimum required by the monetary authority. The numerical characteristics of the calibrated model’s stationary equilibrium are summarized in table 2.

Table 2: Steady-State Characteristics (Annualized)

<table>
<thead>
<tr>
<th>model setup</th>
<th>$r^k$</th>
<th>$k/y$</th>
<th>$c/y$</th>
<th>$i/y$</th>
<th>$g/y$</th>
<th>$\Pi'/y$</th>
<th>$R = R^d$</th>
<th>$R^d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>reference</td>
<td>0.121</td>
<td>2.412</td>
<td>0.559</td>
<td>0.241</td>
<td>0.200</td>
<td>0.167</td>
<td>0.040</td>
<td>0.041</td>
</tr>
<tr>
<td>duration=2</td>
<td>0.142</td>
<td>2.055</td>
<td>0.595</td>
<td>0.206</td>
<td>0.200</td>
<td>0.167</td>
<td>0.040</td>
<td>0.041</td>
</tr>
<tr>
<td>duration=4</td>
<td>0.184</td>
<td>1.582</td>
<td>0.642</td>
<td>0.158</td>
<td>0.200</td>
<td>0.167</td>
<td>0.040</td>
<td>0.041</td>
</tr>
<tr>
<td>duration=8</td>
<td>0.270</td>
<td>1.079</td>
<td>0.692</td>
<td>0.108</td>
<td>0.200</td>
<td>0.167</td>
<td>0.040</td>
<td>0.041</td>
</tr>
</tbody>
</table>

A delay in the usability of capital in production reduces its value to entrepreneurs which are now forced to borrow external funds for several periods and to pay interest on these. Accordingly, entrepreneurs demand a higher marginal product of capital $r^k$ – annualized productivity increases from 12.1 to around 27% – and chose to hold an optimal physical capital stock in steady state that decreases with extended credit duration. The annual capital-to-output ratio falls from 2.4 to 1.08 with respect to the reference model, if the acquisition of capital is financed by eight-period loans.\(^{13}\)

Even though the rate of depreciation is the same throughout all model setups, a higher capital-to-output ratio in the reference setting implies a higher share of investment expenditure in GDP in order to maintain the capital stock. In turn, a smaller fraction of output – around 56% compared to up to 70% in the extended model – remains for private consumption. Note that, regardless of maturity characteristics, no adjustment costs arise in steady state. The economy’s output is either consumed by the representative household and the government or invested into new productive capital. Corporate banks, entrepreneurs, and capital producers make zero profit in stationary equilibrium. Accordingly, the only source of dividend income to private households are retailers’ profits which are determined by the steady-state mark up over marginal costs $\eta_p$. The output shares of profits must therefore be invariable in $p$.

\(^{13}\)Note that the steady decrease in $k/y$ does not arise from the pure delay in capital productivity but from the cost of external finance which increases with the duration of bank loans.
5 Business Cycle Properties

To analyze the dynamic behavior, we log-linearize the four versions of our model around the corresponding steady state. Each linearized system of difference equations is then exposed to three stochastically occurring disturbances in order to extract the impulse-response functions of endogenous variables to these shocks.

Besides the reference model with one-period deposits and loans, we simulate the extended version with credit durations of two, four, and eight quarters, respectively. Stepping from \( p = 1 \) to \( p = 2 \) allows for a direct comparison of maturity matching with maturity transformation. \( p = 4 \) approximates quite well the largely stable empirical relation between average duration of banks’ assets and liabilities as illustrated in Figure 1. Finally, \( p = 8 \) represents an average duration of investment projects that has been observed in the data (see e. g. Mayer (1960) and Hall (1977)).

5.1 A Fiscal Policy Shock

Consider an unanticipated increase in the stochastic part of real government consumption expenditure by five percent of its steady-state value, i. e. one percent of stationary GDP. This extra spending instantaneously increases the authorities’ requirement for financial resources. Remember our assumption that both taxation and the reserve ratio of private banks follow an exogenous process. For this reason, either current government debt \( B_t \) or current household deposits \( D_t \) have to adjust, as both variables’ previous period values are predetermined states.

In fact, households reduce their deposit holdings in order to absorb the increased supply of government bonds. This drop in \( D_t \) initiates a credit crunch reflected in a lower real price of capital. Demand-driven output and inflation imply a hump-shaped rise in all three interest rates. Figure 2 displays the simulated impulse responses of selected real and nominal variables to this expansionary fiscal policy shock.

The public spending shock is propagated through monetary policy as well as the banking sector, resulting in a crowding out effect on both private consumption and investment.

\(^{14}\)The solutions of the dynamic stochastic models have been implemented in Dynare on Matlab.
\(^{15}\)Based on survey data, Mayer (1960) estimates the average time – weighted by project size – that passes between the decision to undertake an investment and the completion of it to 21 months, while Hall (1977) identifies a delay until productivity of about two years.
\(^{16}\)As the impulse responses of \( R^{d} \) are identical to those of \( R^{d} \), only the latter are depicted in Figure 2.
Figure 2: Selected impulse responses to a debt-financed fiscal policy expansion $\varepsilon^g_t$

Note: Horizontal axes display the time passed in quarters since the event of the exogenous disturbance. Vertical axes measure the resulting percentage deviations from the model’s deterministic steady state.

Moreover, a higher marginal utility of consumption induces households to work more, even in the face of a lower real wage rate. While employment increases and the productive capital stock takes time to adjust, economic activity rises temporarily.

Maturity transformation clearly influences the impulse responses of several considered variables, whereas the time path of others remains largely unaffected. In the reference case with perfect matching, banks can flexibly adjust both their assets and liabilities. Accordingly, loans and deposits decrease one-for-one, allowing for an immediate rise in public debt by 5.5 percent relative to steady state.

With extended duration of bank credit, however, a growing share of loans must be rolled
over passively, forcing banks to delay the adjustment of deposits. As less deposits are withdrawn, on impact, the increase in government bonds slows down.

On the other hand, the percentage drop in newly issued loans is more pronounced, if banks’ financial resources are bound. The resulting shortage of funds which are essential for acquiring productive capital implies qualitatively and quantitatively similar corrections in the real price of capital. Investment impulse responses follow a hump-shaped pattern due to positive adjustment costs.

To attract an amount of household deposits that is sufficient to refinance the outstanding lines of credit, banks must increase $R^d$ by a higher number of basis points than in the reference model. Yet, this interest rate spread is only marginal. It takes about four quarters, until a maturity mismatch visibly accelerates the steady-state convergence of consumption relative to the setting with one-period loans.

In this model, maturity transformation amplifies the responses of investment, bank loans, and the real price of capital to a budget deficit shock, whereas the percentage deviations from steady state of government bonds and bank deposits become less pronounced both on impact and during adjustment. The opposing effects of longer-run loans on consumption and investment – the two main components of GDP – appear to compensate each other over the business cycle. Accordingly, the impulse responses of employment and output are largely insensitive to introducing a maturity mismatch in the balance sheet of corporate banks.

5.2 A Liquidity Reserves Shock

The second stochastic disturbance emerges from the banking sector itself. Figure 3 displays the impulse responses to an exogenous increase in the reserve ratio of corporate banks from 0.02 to 0.04, i. e. a doubling in the overall reserve ratio. As mentioned, this kind of shock could be interpreted as a sudden fear of banks to suffer a shortage of liquidity or a severe decrease in prospective household deposits.

An exogenous increase in the reserve ratio has two direct effects on the financial sector. It tightens the balance sheet constraint of banks and lowers the amount of funds available

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17 Remember that the steady-state output share of consumption increases with the duration of loans.

18 Some readers might doubt the empirical relevance of the shock’s size. However, in times of financial distress, insecurity, and mistrust, excess reserves of the same size as or even larger magnitude than required reserves do not seem to represent an overly pessimistic scenario.
to make entrepreneurial loans. Assuming perfect competition among corporate banks, higher reserves moreover bring about an expansion of the spread between the two retail interest rates. $R^d$ clearly rises, on impact, whereas $R^d$ does not fall unambiguously – due to simultaneous anti-inflationary monetary policy.\textsuperscript{19}

In order to ensure that the balance sheet identity remains satisfied, the credit supply is reduced, while the amount of deposits increases mechanically to offset the rise in $rr_t$ by two percentage points. Households substitute bank deposits for government bonds.\textsuperscript{20}

\textsuperscript{19}The subcycles emerging in figure 3 and figure 4 below, in particular for $\pi$ and $R$ respectively $R^d$, are an artefact of time-to-build as demonstrated e. g. by Rouwenhorst (1991). Again, $R$ equals $R^d$.

\textsuperscript{20}The authorities’ budget constraint is largely unaffected by this substitution, as seignorage profits increase with the reserve ratio. This compensates the government for not being able to issue new bonds.

---

\textbf{Figure 3:} Impulse responses to an exogenous shift in banks’ reserve ratio through $\varepsilon^r_t$

Note: Horizontal axes display the time passed in quarters since the event of the exogenous disturbance. Vertical axes measure the resulting percentage deviations from the model’s deterministic steady state.
They furthermore increase their consumption expenditure, as relatively lower interest payments on deposits reduce the opportunity cost of consumption.

The corresponding drop in the marginal utility of consumption is not compensated by an adequate increase in the real wage rate. It is thus reflected by the hump-shaped downturn in employment. Accordingly, output declines in a qualitatively similar fashion.

With a larger share of bank assets tied up in reserves, demand for productive capital is considerably restrained. As a consequence, investment activity decreases and the real price of capital falls.

In this second simulation, maturity transformation again has a visible impact on the post-disturbance behavior of key variables. The impulse responses of household deposits and government bonds are virtually unaffected by the duration of credit. The observed effects appear to be transmitted exclusively through the variation in bank lending as well as through the retail interest rates channel.

As described above, the drop in newly issued loans is amplified, when undue loans bind an increasing part of banks’ assets. Moreover, the size of the interest rate differential arising after a reserves shock depends on the duration of credit. Any period’s market-clearing loan rate will remain effective for all debt obligations issued in this period until they mature. The optimal \( R_t \) must therefore incorporate expectations regarding the deposit rate for the subsequent \( p \) periods. The longer the planning horizon, the smaller will thus be the spread opening between \( R_t \) and \( R^d \).

While the percentage deviation from steady state of \( R_t \) becomes less pronounced with increasing duration of credit contracts, it applies for a longer period, in turn. It implies thus a stronger reduction in entrepreneurs’ demand for productive capital, in investment activity, and in the real price of capital.

Household consumption expenditure changes most dramatically in the reference model. Again, this is due to the fact that the bank deposit rate, i. e. the opportunity cost of consumption, declines faster when banks are able to match maturities.

The associated decrease in the marginal utility of consumption dominates the impulse responses of employment and output. The percentage deviation from steady state of both \( h \) and \( y \) is clearly attenuated by a growing maturity mismatch. For a credit duration of eight quarters, the output gap is narrowed by 69\% on impact and by 42.5\% after one year,
relative to the benchmark case. Regarding employment, 69% and 36%, respectively, of the percentage deviation from steady state are absorbed.

5.3 A Negative Confidence Shock

We finally consider an uncommon exogenous disturbance which might appear to conflict with the rational expectations hypothesis. When introducing the household sector in subsection 2.1.1, both government bonds and bank deposits were considered equivalently risk-free alternatives of financial investment.

Let us assume that this still holds, i.e. neither authorities nor corporate banks fail to settle any of their principal and interest liabilities towards households, ex post. Imagine that the representative household is, nevertheless, occasionally struck by a sudden while temporary loss of confidence in the banking sector, fearing that its bank deposits will not be repaid completely at maturity. In light of the recent financial crisis – especially the bank run on British Northern Rock in mid-September 2007 –, purely expectation-driven economic fluctuations are certainly worth analyzing.

Formally, we incorporate this kind of shock by means of a multiplicative stochastic disturbance \( e^{-\theta_t} \) in the household’s first order condition with respect to deposits (A.4).

This translates into an additive disturbance \( \theta_t \) in the corresponding loglinearized equation (B.4). At the same time, the household budget constraint remains unchanged, as the ex-ante loss of confidence in banks turns out to be unwarranted, ex post.

The impulse responses to an orthogonal increase in \( \theta_t \) – expressed in terms of percentage deviations from steady state – are illustrated in Figure 4.

The negative confidence shock is reflected in an instantaneous rise in the equilibrium interest rate on deposits. Nevertheless, households withdraw financial funds from their bank accounts, investing part of them into government bonds instead. At the same time, the marginal utility of consumption must increase over time, implying an immediate jump and a consecutive decrease in private consumption.\(^{21}\)

A truncated liabilities side of the balance sheet forces banks to boil down the asset side, as well. Due to the exogenously determined reserves ratio, this requires a considerable reduction in newly issued loans. As a consequence, the impulse responses of investment activity and the real price of capital are qualitatively very similar to the case of a positive

\(^{21}\)In fact, the shock lowers the representative household’s expected opportunity cost of consumption.
Figure 4: Selected impulse responses to a rupture of household confidence in banks $\varepsilon_t^\theta$

Note: Horizontal axes display the time passed in quarters since the event of the exogenous disturbance. Vertical axes measure the resulting percentage deviations from the model’s deterministic steady state.

liquidity reserves shock. Note that the impulse responses of output and employment are again determined by the time path of the marginal utility of consumption.

A negative shock to private investors’ confidence in the banking sector drives a wedge between the expected returns $- R$ and $R^d$ on the saving alternatives of a household, as the corresponding Euler equations are otherwise identical. The flight from deposits into government bonds and consumption will accordingly be the more pronounced, the larger is this interest rate spread.

In response to the contemporaneous development of inflation, the monetary policy rate increases, on impact, if and only if the duration of bank assets exceeds the duration of
bank liabilities. This increase in \( R \) retards the rise in private consumption expenditure. It corresponds thus to an anti-inflationary monetary contraction.

In this particular scenario, maturity transformation implies a higher effectiveness of automatic policy stabilizers. The percentage deviation from steady state of bank deposits, government bonds and consumption, respectively, is increasingly attenuated by a longer duration of credit contracts. For instance, the initial drop in deposits for \( p = 8 \) amounts to less than one fifth of the deviation in the reference model.

If a larger share of assets is tied up in previous period loans, the reduction in newly issued entrepreneurial loans is nevertheless somewhat more pronounced. Accordingly, the downturn in investment activity and the fall in the real price of capital are amplified with maturity transformation.

Remember that the steady-state share of investment in GDP is falling in the duration of credit contracts. As a consequence, the effect of maturity transformation on consumption dominates the impulse responses of output and employment. For both \( y \) and \( h \), the deviations from stationary equilibrium are mitigated by longer-term bank loans.

Our analysis of the impulse responses to a pure expectations shock highlights the real economic consequences even of ex-post unfounded concerns and uncertainty. Moreover, the model predicts a smoother overall adjustment back to the long-run steady state, when corporate banks engage in maturity transformation.

6 Conclusion

This paper combines a New Keynesian DSGE model with a corporate banking sector that collects one-period deposits from households in order to provide multi-period loans to entrepreneurs. It contributes thus to the existing macroeconomic literature, where banks perfectly match the maturities of their assets and liabilities. The need for maturity transformation arises from the assumption that the physical capital stock requires time to become productive.

Due to the cost of external finance, a delay in productivity reduces entrepreneurs’ return on capital and implicates a lower capital intensity of production in long-run equilibrium. Accordingly, the steady-state share of investment in output decreases, whereas that of consumption increases with the duration of credit contracts.
In the dynamic simulation exercise, we find that a demand shock – in our case a debt-financed increase in government expenditure – is amplified for some and attenuated for other variables, when introducing a maturity mismatch into the reference model. Note that the opposing influences of consumption and investment even out over the business cycle. As a consequence, the impulse responses of output and employment remain largely unaffected by the extent of maturity transformation.

On the contrary, the impulse responses to a positive liquidity reserves shock inherent to the banking sector are increasingly moderated by multi-period loans.

To a certain extent, our results for an unwarranted drop in household confidence in the banking sector match the finding of a credit maturity attenuator in response to a similar shock by Andreasen et al. (2010). However, maturity transformation only attenuates the business cycle for quarterly deposits and credit durations below one year in their model, while output and employment fluctuations in our framework appear to be unambiguously dampened with increasing $p$.\(^{22}\)

\(^{22}\)Note, moreover, that the adverse financial shock in our model arises from a different mechanism.
Appendix A. Short-Run Equilibrium Conditions

The stochastic short-run equilibrium is characterized by the following equations in the endogenous and exogenous variables of the respective model setup.

Appendix A.1 The Reference Model

The FOCs of the representative household’s utility maximization in real terms are

\[ \lambda_t c_t = \left( \frac{c_t}{c_{t-1}} \right)^{1-\sigma_c} - \beta E_t \left[ \varphi \left( \frac{c_{t+1}}{c_t} \right)^{1-\sigma_c} \right] \]  \hspace{1cm} (A.1)

\[ \lambda_t w_t = \phi_h h_t^{\theta_h} \]  \hspace{1cm} (A.2)

\[ \lambda_t = \beta E_t \left[ \frac{\lambda_{t+1}}{\pi_{t+1}} (1 + R_t) \right] \]  \hspace{1cm} (A.3)

\[ \lambda_t = \beta E_t \left[ \frac{\lambda_{t+1}}{\pi_{t+1}} (1 + R^d_t) \right] \exp \{ \theta_t \} \]  \hspace{1cm} (A.4)

\[ c_t + b_t + d_t = \frac{1 + R_{t-1}}{\pi_t} b_{t-1} + \frac{1 + R^d_{t-1}}{\pi_t} d_{t-1} + w_{t+1} \left( \frac{\Pi_t}{\Pi} \right), \]  \hspace{1cm} (A.5)

where (A.4) contains the confidence shock. In (A.5), \( \tau_t \) represents real lump-sum taxes. Capital goods producers optimally invest in accordance with

\[ \frac{1}{q_t} + \phi_i \left( \frac{i_t}{i_{t-1}} - 1 \right) \frac{i_t}{i_{t-1}} = 1 - \phi_t \left( \frac{i_t}{i_{t-1}} - 1 \right)^2 + \beta E_t \left[ \frac{\lambda_{t+1}}{\pi_t} \frac{q_{t+1}}{q_t} \left( \frac{i_{t+1}}{i_t} - 1 \right) \left( \frac{i_{t+1}}{i_t} - 1 \right)^2 \right] \]  \hspace{1cm} (A.6)

\[ k_t = (1 - \delta) k_{t-1} + \left[ 1 - \phi_t \left( \frac{i_t}{i_{t-1}} - 1 \right)^2 \right] i_t. \]  \hspace{1cm} (A.7)

Profit maximization of entrepreneurs – who are also the producers of wholesale output in this economy – leads to the following FOCs:

\[ E_t \left[ \frac{1 + R^d_t}{\pi_{t+1}} \right] = E_t \left[ \frac{r^k_{t+1} + (1 - \delta) q_{t+1}}{q_t} \right] \]  \hspace{1cm} (A.8)

\[ r^k_t = mc_t \alpha \frac{y_t}{k_{t-1}} \]  \hspace{1cm} (A.9)

\[ w_t = mc_t (1 - \alpha) \frac{y_t}{h_t} \]  \hspace{1cm} (A.10)

\[ y_t = k_{t-1} h_t^{1-\alpha} \]  \hspace{1cm} (A.11)

Whenever retailers receive a signal, they adjust their prices in order to satisfy

\[ \tilde{p}_t = \frac{\eta_p}{\eta_p - 1} \cdot \frac{E_t \sum_{\nu=0}^{\infty} (\beta \xi_p)^{\nu} \lambda_{t+v} y_{t+v} mc_{t+v} (X_{t,v})^{-\eta_p}}{E_t \sum_{\nu=0}^{\infty} (\beta \xi_p)^{\nu} \lambda_{t+v} y_{t+v} (X_{t,v})^{1-\eta_p}}, \]  \hspace{1cm} (A.12)
where
\[ X_{t+\nu} = \tilde{\pi}_{t+\nu} \tilde{\pi}_{t+2} \tilde{\pi}_{t+1} \] with inflation index \( \tilde{\pi}_t = \pi_{1-t+\nu} \pi_{1-t+\nu} \).

From the evolution of the aggregate price level
\[ P_t = \left[ (1 - \xi_p) P_t^{1-\eta_p} + \xi_p (\tilde{\pi}_t P_{t-1})^{1-\eta_p} \right] ^{\frac{1}{1-\eta_p}}, \]
we can solve for
\[ \tilde{\pi}_t = \left[ \frac{1 - \xi_p (\tilde{\pi}_t / \pi_t)^{1-\eta_p}}{1 - \xi_p} \right] ^{\frac{1}{1-\eta_p}}. \]

Perfect competition in the banking sector implies that both the balance sheet identity and the equilibrium interest-rate differential are determined by the common reserve ratio:
\[ l_t = (1 - rr_t) d_t \] (A.14)
\[ R_t^d = (1 - rr_t) R_t^d \] (A.15)

The central bank follows a Taylor-type monetary policy rule and the authorities use any seigniorage profits from corporate bank’s reserves to pay interest on public debt.
\[ R_t = R_{t-1}^d \left[ R \left( \frac{\pi_t}{\pi_{\text{target}}} \right) \psi_y \left( \frac{y_t}{y} \psi_y \right)^{1-\rho} \right] \] (A.16)
\[ g_t - \tau_t = rr_t d_t - \frac{rr_{t-1} d_{t-1}}{\pi_t} + b_t - \frac{(1 + R_{t-1}) b_{t-1}}{\pi_t} \] (A.17)

Remember that only retailers make nonzero profit in the steady state, while the expected return of entrepreneurs and banks is zero due to perfect competition. Ex post, however, the latter two might face profits or losses, as well, which are equally transferred to the representative household. The sum of real corporate transfers therefore contains the net of retail, entrepreneurial, and bank profits:
\[ \frac{\Pi}{P_t} = q_t (1 - \delta) k_{t-1} + (1 - (1 - \alpha) m c_t) y_t - \frac{1}{\pi_t} q_{t-1} k_{t-1} - \frac{R_{t-1}^d}{\pi_t} d_{t-1}. \] (A.18)

In linear form, the stochastic processes for the model’s three exogenous variables are
\[ \gamma_t = \rho_g \gamma_{t-1} + \varepsilon^\gamma_t, \quad \text{where} \quad \gamma_t = \tilde{\gamma}_t - \tilde{\tau}_t \] (A.19)
\[ rr_t = \rho_{rr} rr_{t-1} + (1 - \rho_{rr}) \varepsilon^{rr}_t \] (A.20)
\[ \theta_t = \rho_{\theta} \theta_{t-1} + \varepsilon^\theta_t, \] (A.21)

where all \( \varepsilon^i_t, i = g, rr, \theta \) are identically and independently distributed disturbances.
Appendix A.2  Maturity Transformation

Delaying productivity of the capital stock influences the timing in several of the above equilibrium conditions. While the household FOCs and the optimal investment decision of capital goods producers remain entirely unaffected, the law of motion of the aggregate capital stock changes to

\[ k_t = (1 - \delta) k_{t-p} + \left( 1 - \frac{\phi_t}{2} \left( \frac{i_t}{i_{t-1}} - 1 \right) \right) i_t. \]  

(A.6')

Furthermore, we must adjust the following optimality conditions of entrepreneurs:

\[ E_t \left[ \frac{1 + R^d_t}{\pi_{t+1} \pi_{t+2} \cdots \pi_{t+p}} \right] = E_t \left[ \frac{r^{k_t} + (1 - \delta) q_{t+p}}{q_t} \right] \]  

(A.8')

\[ r^{k_t} = mc_t \alpha y_t k_{t-p} \]  

(A.9')

\[ y_t = A_t k^\alpha_{t-p} h_t^{1-\alpha}. \]  

(A.11')

As illustrated in section 2.2.3, banks can no longer perfectly match the maturities of their assets and liabilities. Instead, they maximize profits subject to an extended balance sheet with the sum of unmatured loans on the asset side. The period t balance sheet constraint of the representative bank depends on the time index operator p as follows:

\[ l_t + \frac{1 + R^d_{t-1}}{\pi_t} l_{t-1} + \frac{(1 + R^d_{t-2})^2}{\pi_t \pi_{t-1}} l_{t-2} + \ldots + \frac{(1 + R^d_{t-p+1})^{p-1}}{\pi_t \cdots \pi_{t-p+1}} l_{t-p+1} = (1 - rr_t) d_t \]  

(A.13')

Accordingly, the optimal supply of deposits and credit by banks implies a relationship

\[ \left( R^d_t - \frac{R^d_t}{1 - rr_t} \right) + \beta \frac{\lambda_{t+2}}{\lambda_{t+1}} (1 + R^d_t) \left( R^d_t - \frac{R^d_{t+1}}{1 - rr_{t+1}} \right) + \beta^2 \frac{\lambda_{t+3}}{\lambda_{t+1}} (1 + R^d_t)^2 \left( R^d_t - \frac{R^d_{t+2}}{1 - rr_{t+2}} \right) \]

\[ + \ldots + \beta^{p-1} \frac{\lambda_{t+p}}{\lambda_{t+1}} (1 + R^d_t)^{p-1} \left( R^d_t - \frac{R^d_{t+p-1}}{1 - rr_{t+p-1}} \right) = 0 \]  

(A.14')

between the deposit and the loan interest rates, \( R^d_t \) and \( R^l_t \), in short-run equilibrium. Monetary policy and seignorage income remain unaffected by maturity transformation, while the net transfer of corporate profits to the representative household changes to:

\[ \Pi_P = q_t (1 - \delta) k_{t-p} + (1 - (1 - \alpha) mc_t) y_t + \frac{R^d_{t-1}}{\pi_t} l_{t-1} + \frac{R^d_{t-2}}{\pi_t \pi_{t-1}} l_{t-2} + \ldots + \frac{R^d_{t-p+1}}{\pi_t \cdots \pi_{t-p+2}} l_{t-p+1} - \frac{R^d_{t-p}}{\pi_t \cdots \pi_{t-p+1}} l_{t-p}. \]  

(A.17')
Appendix B. The Models in Loglinear Form

Below, \( \hat{z}_t \) represents the percentage deviation of variable \( z \) from its long-run equilibrium value in period \( t \). The denotations \( \hat{R}_t, \hat{R}^d_t, \) and \( \hat{R}^l_t \) have a somewhat different meaning. The interest rates on government bonds, deposits, and loans enter the loglinear system of equations in terms of absolute, i.e., percentage point deviations from steady state. Letters without a time subscript denote the respective variable’s stationary value.

Appendix B.1 The Reference Model

Note that both the order and meaning of the following equations is identical to those in Appendix A.1.

\[
\begin{align*}
\lambda c (\hat{\lambda}_t + \hat{c}_t) &= (e^{1-\varphi})^{1-\sigma_c} \cdot \{ (1 - \sigma_c) (\hat{c}_t - \varphi \hat{c}_{t-1}) - \beta \varphi E_t [(1 - \sigma_c) (\hat{c}_{t+1} - \varphi \hat{c}_t)] \} \quad (B.1) \\
\hat{\lambda}_t + \hat{w}_t &= \sigma_c \hat{h}_t \quad (B.2) \\
\hat{\lambda}_t &= \hat{\lambda}_{t+1} - \hat{\pi}_{t+1} + \frac{\hat{R}_t}{1 + R} \quad (B.3) \\
\hat{\lambda}_t &= \hat{\lambda}_{t+1} - \hat{\pi}_{t+1} + \frac{\hat{R}^d_t}{1 + R^d} + \hat{\theta}_t \quad (B.4) \\
c\hat{c}_t + T \hat{\pi}_t &= \frac{(1 + R) b}{\pi} \left( \hat{R}_t - \hat{\pi}_t - \hat{b}_t \right) + \frac{(1 + R^d) d}{\pi} \left( \hat{R}^d_t - \hat{\pi}_t - \hat{d}_t \right) \\
&\quad + \frac{\Pi \hat{R}^l_t}{\Pi P_t} - b \hat{b}_t - d \hat{d}_t \quad (B.5) \\
\hat{q}_t &= q \varphi_t (1 + \beta) \hat{t}_t - \beta q \varphi_t E_t \hat{t}_{t+1} - q \varphi_t \hat{t}_{t-1} \quad (B.6) \\
\hat{k}_t &= (1 - \delta) \hat{k}_{t-1} + \delta \hat{t}_t \quad (B.7) \\
\hat{l}_t &= \hat{q}_t + \hat{k}_t \quad (B.8) \\
\hat{R}^l_t &= E_t \left[ \frac{r^k}{q} \hat{r}_t^{k+1} + (1 - \delta) \hat{q}_{t+1} - \frac{r^k}{q} \hat{q}_t + \frac{1 + R^d}{\pi} \hat{\pi}_{t+1} \right] \quad (B.9) \\
\hat{r}_t^{k} &= \hat{m} \hat{c}_t + \hat{y}_t - \hat{k}_{t-1} \quad (B.10) \\
\hat{w}_t &= \hat{m} \hat{c}_t + \hat{y}_t - \hat{h}_t \quad (B.11) \\
\hat{y}_t &= \alpha \hat{k}_{t-1} + (1 - \alpha) \hat{h}_t \quad (B.12) \\
\hat{\pi}_t &= \frac{\beta \xi_p}{1 + \beta \xi_{p't}} E_t \hat{\pi}_{t+1} + \frac{\xi_p}{1 + \beta \xi_{p't} \xi_p} \hat{\pi}_{t-1} + \frac{(1 - \xi_p) (1 - \xi_{p'p})}{\xi_p (1 + \beta \xi_{p'p})} \hat{m} \hat{c}_t \quad (B.13) \\
\hat{\lambda}_t &= \frac{\xi_{p't}}{1 - rr} \hat{r}_{t-1} \quad (B.14) \\
\hat{R}^d_t &= (1 - rr) \hat{R}^l_t - \left( R^l r r \right) \hat{r}_{t-1} \quad (B.15) \\
\hat{R}_t &= \rho \hat{R}_{t-1} + (1 - \rho) (\varphi_x \hat{\pi}_t + \varphi_y \hat{y}_t) \quad (B.16)
\end{align*}
\]
\[ \eta (\hat{\eta} + \hat{d}_t) = \frac{\eta}{\pi} (\hat{\eta} - \hat{\pi}_t + \hat{d}_t - \hat{\pi}_t) \]  
\[ + \frac{b}{d} \hat{b}_t + \frac{(1 + R)b}{\pi \cdot d} \left( \hat{\eta} + \hat{\pi}_t - \hat{\pi}_t \right) + g \gamma \eta_t \quad (B.17) \]

\[ \frac{\hat{\eta}_t}{\hat{P}_t} = q (1 - \delta) k (\hat{q}_t + \hat{k}_t) + y [1 - (1 - \alpha) mc] \hat{y}_t - (1 - \alpha) mc \cdot y \cdot \hat{m}_c \]
\[ - \frac{q \cdot k}{\pi} (\hat{q}_t + \hat{k}_t - \hat{\pi}_t) - \frac{R^d}{\pi} d \left( \frac{\hat{r}_{t+1}}{R^d} + \hat{d}_{t-1} - \hat{\pi}_t \right) \quad (B.18) \]

**Appendix B.2 Maturity Transformation**

As in appendix A.2, we restrict the loglinear specification of the model with maturity transformation to those equations which actually differ from the reference model:

\[ \hat{k}_t = (1 - \delta) \hat{k}_{t-p} + \hat{\delta}_t \]  
\[ \hat{R}_t = \frac{\pi}{p(1 + R^d)^p - \eta^2} E_t \left[ \frac{\eta}{q} \hat{R}_{t+p-1} + (1 - \delta) \hat{q}_{t+p} - \frac{\eta}{q} \hat{q}_t \right] \]
\[ + \frac{1 + R^d}{\pi p} E_t \left( \hat{\pi}_{t+1} + \hat{\pi}_{t+2} + \ldots + \hat{\pi}_{t+p} \right) \quad (B.8') \]

\[ \hat{r}_t = \hat{m}_c + \hat{y}_t - \hat{k}_{t-p} \]  
\[ \hat{y}_t = \alpha \hat{k}_{t-p} + (1 - \alpha) \hat{\delta}_t \]
\[ r_t \hat{r}_t = (1 - r_t) \frac{d}{r_t} \hat{d}_t - \hat{\ell}_t - \frac{1 + R^d}{\pi} \left( \hat{R}_{t+1} - \hat{\pi}_t \right) - \frac{(1 + R^d)^2}{\pi^2} \left( \frac{2 \hat{R}_{t+p-2} + \hat{I}_{t-p} - \hat{\pi}_t - \hat{\pi}_{t-1}}{1 + R^d} \right) \]
\[ - \ldots - \frac{(1 + R^d)^{p-1}}{\pi^{p-1}} \left( (p-1) \hat{R}_{t-p+1} + \hat{I}_{t-p} - \hat{\pi}_1 - \hat{\pi}_{t-1} - \ldots - \hat{\pi}_{t-p+2} \right) \quad (B.13') \]

\[ \hat{R}_t = \left[ 1 + \frac{\beta}{\pi} (1 + R^d) + \beta^2 (1 + R^d)^2 + \ldots + \frac{\beta^{p-1}}{\pi^{p-1}} (1 + R^d)^{p-1} \right] \times \]
\[ \left[ \frac{\hat{R}_t}{1 - r_t} + \frac{R^d}{(1 - r_t)^2} \hat{r}_t + \frac{1 + R^d}{(1 - r_t)^3} \hat{r}_{t+1} + \ldots \right] \]
\[ + \frac{\beta^{p-1}}{\pi^{p-1}} (1 + R^d)^{p-1} \left( \frac{\hat{R}_{t+p-2}}{1 - r_t} + \frac{R^d}{(1 - r_t)^2} \hat{r}_{t+p-1} \right) \quad (B.14') \]

\[ \frac{\hat{\eta}_t}{\hat{P}_t} = q (1 - \delta) k (\hat{q}_t + \hat{k}_t) + y [1 - (1 - \alpha) mc] \hat{y}_t - mc \cdot y (1 - \alpha) \hat{m}_c \]
\[ + \frac{1 + R^d}{\pi} \left( \hat{R}_{t-1} + \hat{I}_{t-1} - \hat{\pi}_t \right) + \frac{R^d}{\pi^2} \left( \frac{R^d}{R^d} + \frac{R^d}{1 + R^d} \hat{I}_{t-1} - \hat{\pi}_t - \hat{\pi}_{t-1} \right) + \ldots \]
\[ + \frac{R^d}{\pi^{p-1}} \left( \frac{R^d}{R^d} + \frac{R^d}{1 + R^d} \hat{I}_{t-p+1} \hat{I}_{t-p+1} - \hat{\pi}_t - \hat{\pi}_{t-1} \ldots - \hat{\pi}_{t-p+2} \right) \]
\[ - \frac{(1 + R^d)^{p-1}}{\pi^p} \left( \frac{(p-1) \hat{R}_{t-p} + \hat{I}_{t-p} - \hat{\pi}_t - \hat{\pi}_{t-1} - \ldots - \hat{\pi}_{t-p+2} \right) \]
\[ - \frac{R^d}{\pi} \left( \frac{\hat{R}_{t+1}}{R^d} + \hat{d}_{t-1} - \hat{\pi}_t \right) \quad (B.17') \]
References


