Optimal Option Portfolio Strategies*

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Abstract

Options should play an important role in asset allocation. They allow for kernel spanning and provide access to additional (priced) risk factors such as stochastic volatility and negative jumps. Unfortunately, traditional methods of asset allocation (e.g. mean-variance optimization) are not adequate for options because the distribution of returns is non-normal and the short sample of option returns available makes it difficult to estimate the distribution. We propose a method to optimize option portfolios that solves these limitations. An *out-of-sample* exercise is performed and we show that, even when transaction costs are incorporated, our portfolio strategy delivers an annualized *Sharpe ratio* of 0.59 between January 1996 and September 2008.

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1 Introduction

Although options are well known to help span states of nature [Ross (1976)] and to provide exposure to (priced) risk factors like stochastic volatility and jumps,[1] they are seldom used in investment portfolios.[2] Part of the problem is that existing portfolio optimization methods, like the Markowitz mean-variance model, are ill suited to handle options. There are three main problems in option portfolio optimization. First, the distribution of option returns shows significant departures from normality and therefore cannot be described by means and variances alone. Second, the short history of returns available to the researcher severely limits the estimation of their complex distribution. For example, we only have data for S&P 500 options since 1996 which is not enough to estimate the moments of their return distribution with sufficient precision. Third, substantial transactions costs are prevalent in this market. For example, on average, at-the-money (ATM) options have a 5% relative bid-ask spread while out-of-the-money (OTM) options present relative bid-ask spreads of 10%.

We offer a simple portfolio optimization method that solves simultaneously these problems. Instead of a mean-variance objective, we maximize an expected utility function, such as power utility, which accounts for all the moments of the portfolio return distribution and, in particular, penalizes negative skewness and high kurtosis. We deal with the small sample of option returns by relying on data for the underlying asset instead. In our application, we use returns of the S&P 500 index since 1950 to simulate returns of the underlying asset going forward[3] and, from the definition of option payoffs, generate simulated option returns. Plugging the simulated option returns into the utility function and averaging across simulations gives us an approximation of the expected utility which can

[1] See Bates (1996), Bakshi, Cao, and Chen (1997), Andersen, Benzoni, and Lund (2002), and Liu and Pan (2003), among others.

[2] Mutual funds use of derivatives is limited [Koski and Pontiff (1999), Deli and Varma (2002), Almazan, Brown, Carlson, and Chapman (2004)]. Mutual funds generally face legal constraints in terms of short-selling, borrowing and derivatives usage. This does not happen with hedge funds. Most hedge funds use derivatives, but are just a small part of their holdings [Chen (2010), Aragon and Martin (2007)].

[3] We consider different alternatives for simulating the returns based on parametric distributions fitted to the data or simple bootstrap methods. We can also model a time-varying distribution of returns by simulating their distribution conditional on state variables such as realized volatility.

be maximized to obtain optimal portfolio weights. Note that only current option prices are needed in our procedure, since the payoff is determined by the simulations of the underlying asset.

We apply our model to the portfolio allocation between a risk-free asset and four options on the S&P 500 index with one month to maturity.[4] We define each option by choosing the most liquid contract in a predefined bucket around the specific moneyness. We consider an ATM call, an ATM put, a 5% OTM call and a 5% OTM put option.[5] These are liquid options and can be combined to generate a variety of final payoffs. To incorporate transaction costs, we follow Eraker (2007) and Plyakha and Vilkov (2008). For each option, we define two securities: a "long" option initiated at the ask quote and a "short" option initiated at the bid quote. We form a constrained optimization problem for these eight options and we enter "short" securities with a negative sign into the optimization algorithm. By not allowing short-selling, only one of two "options" is ever bought.

We study the performance of our optimal option portfolio strategies, which we denote by OOPS, in an out-of-sample (OOS) exercise. We find the optimal option portfolios one month before option maturity and examine the return that they would have had at maturity. The resulting time series of returns could have been obtained by an investor following our method in real time. We can then compute measures of performance such as Sharpe ratios or alphas to assess the interest of the method. It should be stressed that for the each OOS observation, only one month of option observations is needed. For the entire period of 153 monthly observations, 99% of the sample is OOS. This *per se* is significantly different from previous studies. OOPS have large Sharpe ratios in our sample period between January 1996 and September 2008. The best strategy yields a Sharpe ratio of 0.59. This compares well with the Sharpe ratio of the market in the same period of 0.20, or even in the full sample since 1950 of 0.40. In addition, several strategies present positive skewness and low excess kurtosis. We find that our strategies load significantly on all four options, and that the optimal weights vary over time. Finally, our optimal strategies are

[4]The S&P 500 index options (ticker SPX) are the most liquid equity options. Open interest for SPX was around 13 million in August 2009 and average daily volume of 708 thousand in 2008 (*www.cboe.com*).

[5]We do not include the S&P 500 in the asset universe since it is spanned by the options.

almost delta-neutral albeit with significant elasticity.

There is a related literature that investigates the returns of simple option trading strategies. Coval and Shumway (2001) show that short positions in crash-protected, delta-neutral straddles present Sharpe ratios around one and Saretto and Santa-Clara (2009) find similar values in an extended sample. Driessen and Maenhout (2006) confirm these results for short-term options on US and UK markets. Coval and Shumway (2001) and Bondarenko (2003) also find that selling naked puts offers high returns even after taking into account their considerable risk. However, these papers do not discuss how to optimally combine options into a portfolio. Interestingly, we find that our portfolios depart significantly from exploiting these simple strategies. For instance, there are extended periods in which the optimal portfolios are net long put options.

There are five papers closest that also address the optimal portfolio allocation with options. Liu and Pan (2003) model stochastic volatility and jump processes and derive the optimal portfolio policy for a CRRA investor between one stock, a 5% OTM put option, and cash. Although they get an analytic solution for the optimal option allocation, they need to specify a particular parametric model and estimate its parameters. They try different parameter sets and obtain ambiguous conclusions in terms of put weights. Also, they only require one option to complete the market since they either consider a pure jump risk or pure volatility risk setting. In our case, we can use any model (parametric or not) for the distribution of return of the underlying asset. Our paper is empirical in nature and no restrictions are imposed on the number of options that could be used in the optimization problem.

Eraker (2007) uses a standard mean-variance framework with a parametric model of stochastic volatility and jumps to choose between three risky assets: ATM straddles, OTM puts, and OTM calls. He provides a closed-form solution for weights and obtains an OOS annualized Sharpe ratio around 1.6. As in Liu and Pan (2003), he requires a long period to estimate the parametric model and the estimates are sensitive to the period under consideration. Our approach is more flexible in term of the distribution of returns and the number of options in the portfolio. Driessen and Maenhout (2007) analyze the importance of derivatives in portfolio allocation by using GMM to maximize the expected utility of returns for a portfolio of a stock, an option strategy, and cash. They use either an OTM put, an ATM straddle, or corresponding crash-neutral strategies. They conclude that positive put holdings that would implement portfolio insurance are never optimal given historical option prices. In contrast, we find that optimal weights are time-varying, and change signs during our sample period. Jones (2006) studies optimal portfolios to exploit the apparent put mispricing. He uses a general nonlinear latent factor model and maximizes a constrained mean-variance objective. He circumvents the short history of data by using option daily returns. The model is quite heavy with 57 parameters to estimate even when only factor is considered. This limits the practical usefulness of his approach. Constantinides, Jackwerth, and Savov (2009) study portfolios made up of either calls or puts with a targeted moneyness and they leverage-adjust their returns using options' elasticity. Although they find high Sharpe ratios, mostly for put strategies, these strategies yield negative skewness and high kurtosis.

The rest of this paper is organized as follows. Section 2 explains the methodology. Section 3 describes the data used. Section 4 presents results and findings. Robustness checks are performed in Section 5. Finally, we present some concluding remarks.

2 Portfolio Allocation

2.1 Methodology

We first define some terminology. Let time be represented by the subscript t and simulations indexed by n. Our portfolio allocation is implemented for one risk-free asset and a series of call and put options with one period to maturity. We assume that there are C call options indexed by c where $c = c_1, ..., c_C$, and P put options indexed by p where $p = p_1, ..., p_P$.[6] At time t, the value of the underlying asset is denoted by S_t and each option i has an exercise price of $K_{t,i}$. The risk-free interest rate from time t to $t+1$, known

[6] We include only options that are not redundant from put-call parity.

at time t, is denoted by rf_t. For each date t, weights are obtained through the maximization of the investor's expected utility of the end-of-period wealth, which is a linear function of simulated portfolio returns. The latter are derived from option returns which in turn depend on the underlying asset returns. The following steps describe in detail the algorithm used. Appendix A shows a simple illustration.

1. We simulate N underlying asset log-returns r_{t+1}^n where $n = 1, ..., N$. Several different possible simulation schemes can be used. Our simulation is performed under the empirical density not risk adjusted measure. We explain the simulation scheme in detail in Section 2.2.

2. The log-returns from step 1 are used to simulate next period's underlying asset value, given its current value

$$S_{t+1|t}^n = S_t \exp(r_{t+1}^n) \tag{1}$$

where $n = 1, ..., N$, and $S_{t+1|t}^n$ denotes the simulated underlying asset value in period $t + 1$ conditional on information up to time t.

3. Based on known strike prices for call options $K_{t,c}$ and put options $K_{t,p}$ and one-period simulated underlying asset values $S_{t+1|t}$ in equation (1), we simulate option payoffs at their maturity $t + 1$

$$C_{t+1|t,c}^n = \max(S_{t+1|t}^n - K_{t,c}, 0) \text{ and } P_{t+1|t,p}^n = \max(K_{t,p} - S_{t+1|t}^n, 0) \tag{2}$$

where $n = 1, ..., N$. Using these simulated payoffs in equation (2) and current option prices, returns are then computed by

$$r_{t+1|t,c}^n = \frac{C_{t+1|t,c}^n}{C_{t,c}} - 1 \text{ and } r_{t+1|t,p}^n = \frac{P_{t+1|t,p}^n}{P_{t,p}} - 1 \tag{3}$$

where $n = 1, ..., N$. To compute these returns, current options prices $C_{t,c}, c = c_1, ..., c_C$ and $P_{t,p}, p = p_1, ..., p_P$ at month t are used. Notice that only one-period ahead payoffs are simulated; the denominator of the return is the currently observed option price.

4. We construct simulated portfolio returns in the usual way

$$rp_{t+1|t}^n = rf_t + \sum_{c=c_1}^{c_C} \omega_{t,c}(r_{t+1|t,c}^n - rf_t) + \sum_{p=p_1}^{p_P} \omega_{t,p}(r_{t+1|t,p}^n - rf_t) \tag{4}$$

where $n = 1, ..., N$. Each simulated portfolio return is a weighted average of the asset returns and only the risk-free rate is not simulated.

5. We choose weights by maximizing expected utility over simulated portfolio returns

$$Max_{\omega}E[U(W_t(1+rp_{t+1|t}))] \approx Max_{\omega}\frac{1}{N}\sum_{n=1}^{N}U(W_t(1+rp^n_{t+1|t})). \quad (5)$$

The output is given by $\omega_{t,c}, c = 1, ..., C$ and $\omega_{t,p}, p = 1, ..., P$. We provide further details in Section 2.3.

6. One-period OOS performance is evaluated with realized option returns.

First, we determine the option realized payoffs

$$C_{t+1,c} = \max(S_{t+1} - K_{t,c}, 0) \text{ and } P_{t+1,p} = \max(K_{t,p} - S_{t+1}, 0) \quad (6)$$

Second, we find the corresponding returns

$$r_{t+1,c} = \frac{C_{t+1,c}}{C_{t,c}} - 1 \text{ and } r_{t+1,p} = \frac{P_{t+1,p}}{P_{t,p}} - 1 \quad (7)$$

Third, we determine the one-period OOS portfolio return

$$rp_{t+1} = rf_t + \sum_{c=c_1}^{c_C}\omega_{t,c}(r_{t+1,c} - rf_t) + \sum_{p=p_1}^{p_P}\omega_{t,p}(r_{t+1,p} - rf_t) \quad (8)$$

using the weights determined in step 5 of this algorithm.

2.2 Return simulation

The first step of the algorithm is to simulate one-period log-returns of the underlying asset. There are many possible approaches to do this. See Jackwerth (1999) for a survey of the literature. Aït-Sahalia and Lo (1998) and Jackwerth and Rubinstein (1996) present two examples of potential routes that we could follow to recover a density function in continuous or discrete time setting, respectively. We follow two approaches, unconditional and conditional simulation. Either is implemented in two ways, historical bootstrap and parametric simulation based on historical estimation of the parameters of the density. In all cases, in each month we use an information set corresponding to an expanding window of data of the underlying asset up to time t so that the results are out of sample.

The unconditional approach goes as follows. In the first place, following Efron and Tibshirani (1993), we bootstrap raw returns from the historical empirical distribution of the underlying distribution up to date t. Hence, we resample directly from historically observed returns. Implicitly, this corresponds to drawing returns from their empirical distribution (histogram). We denote this approach as *empirical.* Alternatively, we simulate returns using a parametric distribution f estimated from past data. We use two types of distribution. The first distribution, which is the most standard in the literature, is a Normal distribution with density $f(r|\hat{\mu}_t, \hat{\sigma}_t)$ where $\hat{\mu}_t$ is the sample mean and $\hat{\sigma}_t$ is the sample standard deviation. It is known that the normal distribution does not fit return data well, in particular, it does not capture the frequency of extreme events.[7] To extend our analysis to other types of distributions, we consider a family of distributions which is commonly designated by the Generalized Extreme Value distribution (GEV). The GEV distribution is defined by the density $f(r|\hat{\lambda}_t, \hat{\sigma}_t, \hat{\mu}_t)$ with shape parameter $\hat{\lambda}_t$, scale parameter $\hat{\sigma}_t$, and location parameter $\hat{\mu}_t$.[8] It provides a flexible framework that generalize several distributions. Notice that all estimated parameters are time-varying, since we use an expanding window up to time t to estimate them.[9]

So far, we have not taken into account that returns may be dependent. Both the bootstrap and the parametric density approaches assume that returns are i.i.d. However, it is well known that volatility clusters in time. To capture this issue, we use standardized

[7]Jackwerth and Rubinstein (1996) show that using a normal distribution to model returns, the probability of a stock market crash like the ones that we have witnessed in the past is 10^{-160}.

[8]The GEV density function is defined in the following way:

$$f(r|\lambda,\sigma,\mu) = \begin{cases} \frac{1}{\sigma}\left(1+\lambda\frac{r-\mu}{\sigma}\right)^{-1-1/\lambda} exp\left(-\left(1+\lambda\frac{r-\mu}{\sigma}\right)^{-1/\lambda}\right) & \text{for } \lambda \neq 0 \\ \frac{1}{\sigma} exp(-\frac{r-\mu}{\sigma}) exp(-exp(-\frac{r-\mu}{\sigma})) & \text{for } \lambda = 0 \end{cases}$$

The distributions depend crucially on the sign of the parameter λ. A positive sign denotes the Fréchet class which include well known fat-tailed distributions such as the Pareto, Cauchy, Student-t and mixture distributions. The zero parameter denotes the Gumbel class and includes the normal, exponential, gamma and lognormal distributions. A negative sign denotes the Weibull class which includes the uniform and beta distributions.

[9]Parameters are estimated by maximum likelihood using the built-in functions of MATLAB *normfit* and *gevfit*.

log-returns, which we denote by x. We select realized volatility as an estimate of volatility $rv_t = \sqrt{\sum_{d=1}^{D_t} r_{t,d}^2}$ where D_t is the number of days in month t and $r_{t,d}$ are the daily returns of day d in month t. Standardized log-returns are the ratio of log-return by the previous month's realized volatility $x_t = \frac{r_t}{rv_{t-1}}$. This is close in spirit to the filtered historical simulation of Barone-Adesi, Giannopolos, and Vosper (1999) in which volatility is estimated by a parametric model such as a GARCH(1,1).[10]

Table 1 presents summary statistics for raw and standardized returns for the period between 1950 and 2008 and two subperiods, before and after 1996. Both processes present a smaller mean in the second subperiod. While standardizing returns also reduce volatility in the second subperiod, the opposite happens to raw returns. Both processes present negative values for skewness around -0.50, but lower, in absolute terms, for standardized returns. Raw returns present an excess kurtosis in the first subperiod around 3 which reduces drastically in the latter period. Standardization almost eliminates kurtosis. This is due to less frequent extreme standardized returns, e.g., the Black Monday extreme negative return is now much smaller. The standardized return is now only 0.36 standard deviation units away from the mean while -6.13 standard deviation units away from the mean for raw returns. For the period between 1950 and 1995, the one and twelve-month autoregressive coefficients of raw and standardized returns are small and in the order of 0.03, whereas the autoregressive coefficient of the squared processes is of the order of 0.10 in absolute value. Performing Ljung-Box autocorrelation test for the residuals of raw and standardized returns, the residuals do not present autocorrelation. Using an ARCH test for the previous one and twelve months, the i.i.d. hypothesis is rejected for raw returns mainly in the period between 1950 and 1995. The i.i.d. hypothesis is not rejected for the case of standardized returns for any reasonable significance level.

The conditional approach uses standardized returns rather than raw returns and a slight modification of the algorithm in Section 2.1 is needed. The adjustment occurs in the

[10]Properties of standardized returns are presented, for instance, in Andersen, Bollerslev, Diebold, and Ebens (2001) for stock and Andersen, Bollerslev, Diebold, and Labys (2003) for exchange rates. They show that standardized returns are close to i.i.d. normal.

first and second steps of the algorithm.

1'. Simulate standardized returns

$$x_{t+1}^n = \frac{r_{t+1}^n}{rv_t} \tag{9}$$

To obtain x_{t+1}^n we consider the same two ways as in the unconditional simulation, bootstrapping and parametric simulation.

2'. Bootstrapped standardized returns are now scaled by the current standardized return

$$S_{t+1|t}^n = S_t \exp(x_{t+1}^n . rv_t) \tag{10}$$

where x_{t+1}^n are the simulated standardized returns from step 1' and rv_t is the realized volatility of the time period between $t-1$ and t, which is not simulated.

In either case, unconditional or conditional, current option prices are used in step 3. This intrinsically takes into account the recurrent change in expectations of the underlying asset conditional distribution (e.g., OTM put options become more expensive if investors think that the probability of a crash increased). Our concept of conditional variable is the scaling with lagged volatility.

2.3 Maximizing expected utility

In the fourth step, the investor maximizes the conditional expected utility of next period's wealth

$$\max_{\omega_{i,t} \in \mathbb{R}} \quad E[u(W_{t+1})]$$

subject to the usual budget constraint $W_{t+1} = W_t\,(1 + rp_{t+1})$. Maximizing expected utility takes into account different return distributions. If returns are normal, then rational investors only care about the mean and variance of portfolio returns. In practice, this is unlikely to hold, especially for option returns. Investors care about tail risk (extreme events), so mean and variance do not provide enough information for adequately perform asset allocation choice.

We use the power utility function (see Brandt (1999)). This utility function presents constant relative risk aversion (CRRA) and is given by

$$u(W) = \begin{cases} \frac{1}{1-\gamma} W^{1-\gamma}, & \text{if } \gamma \neq 1 \\ \ln(W), & \text{if } \gamma = 1 \end{cases}$$

where γ is the coefficient of relative risk aversion.[11] This utility function is attractive for two reasons. First, because of the homotheticity property, portfolio weights are independent of the initial level of wealth. So maximizing $E[u(W_{t+1})]$ is the same as maximizing $E[u\,(1 + rp_t)]$. Second, investors care about all moments of the distribution, in particular, skewness and kurtosis. Brandt, Goyal, Santa-Clara, and Stroud (2005) offer approximations to the optimal portfolio choice. We set the constant relative risk aversion parameter γ equal to 10 in order to make a conservative asset allocation choice. To the extent that the investor has lower risk aversion than this value, this works as a shrinkage mechanism and smooth of a portfolio weights. Rosenberg and Engle (2002) estimate for S&P 500 index option data over the sample period between 1991 and 1996 an empirical risk aversion of 7.36. Finally, notice that we could have used any other utility function in applying our methodology.[12]

2.4 Transaction Costs

There is a large body of literature that documents that transaction costs in the options market are quite large and are in part responsible for some pricing anomalies, such as violations of the put-call parity relation.[13] Hence, it is essential to include these frictions in our optimization problem. We only discuss the impact of transaction costs measured by the bid-ask spread. Other types of costs like brokerage fees and market price impact events may be substantial but are ignored here.

Figure 1 shows that between 1996 and 2008 options present substantial bid-ask

[11]For arguments lower than 0.001, we use a first-order approximation of this utility function to avoid extreme negative values, as is standard in the literature.

[12]Several more sophisticated models and implications for disappointment aversion are discussed by Driessen and Maenhout (2007).

[13]See, for instance, Phillips and Smith (1980), Baesel, Shows, and Thorp (1983), and Saretto and Santa-Clara (2009).

spreads. From Table 2 the bid-ask spread is on average \$1.20 for ATM options and \$0.60 for OTM options. Dividing this by mid prices, we measure relative bid-ask spreads for ATM option of around 5%, but for OTM options this increases to 10% on average. This relative bid-ask spread changes over time and for OTM options can attain levels up to 30%.[14]

We propose to incorporate transaction costs by decomposing each option into two "securities": a "bid option" and an "ask option". These mean that we initiate long positions at the ask quote and short positions at the bid quote, and the latter enters the optimization with a minus sign. This approach was applied by Eraker (2007) and Plyakha and Vilkov (2008). Then we run the previous algorithm as a constrained optimization problem by imposing no short selling. This means that in each month only one of the securities, either the bid or ask option, is ever bought. Note that the larger the bid-ask spread, the less likely will be an allocation to the security, since expected returns will be smaller.

3 Data

3.1 Securities

We analyze the optimal portfolio allocation from January 1996 to September 2008. This period selection is due to our availability of option data. We also use data that goes back to February 1950 for the simulation process. More specifically, we consider returns of the S&P 500 index, which extends from February 1950 through September 2008. Figure 2 presents the monthly time-series of S&P 500 and VIX indices[15] in the main period. This period encompasses a variety of market conditions as can be seen from the cycles in the index and from the evolution of volatility. Some of the events occurred in this period were the 1997 Asian crisis, the 1998 Russian financial crisis, the 1998 collapse of LTCM,

[14]Dennis and Mayhew (2009) shows that the effective spread is about 2/3 of the quoted spread, which is given in OptionMetrics. So quoted spread may overestimate the costs which seems like a conservative assumption in terms of OOPS performance.

[15]VIX is calculated and disseminated by CBOE. The objective is to estimate the implied volatility of short-term ATM options on the S&P 500 index over the next month. The formula uses a kernel-smoothed estimator that takes as inputs the current market prices for several call and put options over a range of moneyness ratio and maturities.

the 2001 Nasdaq peak, the 9-11 attack, the 2002 business corruption scandals (Enron and Worldcom), the Gulf War II, and the 2008 subprime mortgage crisis. The empirical analysis relies on monthly holding-period returns since microstructure effects tend to distort higher-frequency returns. For our empirical analysis, we use S&P 500 index closing prices extracted from Bloomberg. Based on this, we construct a time-series of monthly log-returns.

We use data from the OptionMetrics Ivy DB database for European options on the S&P 500 index.[16] The underlying asset is the index level multiplied by 100. These options are traded on the CBOE, and contracts expire on the third Friday of each month. The options are settled in cash, which amounts to the difference between the settlement value and the strike price of the option multiplied by 100, on the business day following expiration. This dataset includes daily highest closing bid and the lowest ask prices, volume, and open interest for the period between January 1996 and September 2008. In order to eliminate unreliable data, we apply a series of filters. First, we eliminate all observations for which the bid is less than $0.125 or greater than the ask price. Second, observations with no volume are also eliminated to mitigate the impact of non-trading. Finally, we exclude all observations that violate usual arbitrage bounds.

For the purpose of this study, we assume the risk-free interest rate to be represented by the 1-month US LIBOR rate. This series is extracted from Bloomberg for the period between January 1996 and September 2008.

3.2 Construction of option returns

Our asset allocation uses a risk-free asset and a set of risky "securities" . We define four options with different levels of moneyness: an ATM call, an ATM put, a 5% OTM call, and a 5% OTM put. This small number of securities keeps our model simple, but nevertheless generates flexible payoffs as a function of underlying asset price. OTM options are important for kernel spanning (Burachi and Jackwerth (2001) and Vanden (2004)) and a deep OTM put option is much more sensitive to negative jump risks. We do not allow

[16]From the CBOE report, these options trade under the ticker SPX and the average daily volume in 2008 was 707,688 contracts.

the investor to choose from all available contracts simultaneously, since our investor may then exploit small in-sample differences between highly correlated option returns, leading to extreme portfolio weights (see, e.g., Jorion (2000) for a discussion of this issue). These are also amongst the most liquid options.[17]

We choose 1-month maturity options. As reported by Burachi and Jackwerth (2001), most of the trading activity in S&P 500 index options is concentrated in the nearest contracts of less than 30 days to expiration. By choosing 1-month maturity we also prevent microstructure problems. This target maturity is also appealing since longer maturity option contracts may stop trading if the evolution of the underlying asset moves in such a way that options become very deeply ITM or OTM. Yet another advantage is that holding the options to maturity only incurs in transaction costs at the inception of the trade.

We then construct time-series of option returns. To that end, mid prices calculated as the midpoint of bid and ask prices are initially considered. We first find all available option contracts with exactly one-month to maturity.[18] We then define buckets for option moneyness (OM), measured by the ratio of the underlying price by the strike price subtracted by one, $S/K-1$. We set a range of moneyness between -2% and 2% for ATM options and a bound 1.5% away from 5% for OTM options. Basically, we fix target moneyness buckets conditional on 1-month maturity options.

Following this, at each month and for each bucket we are still left with several potential securities. However, we only want one option contract in each month. So we choose the option with lowest relative bid-ask spread, defined as the ratio between the bid-ask spread and the mid price. When more than one contract has the same ratio, we choose the one with largest open interest.[19] Finally, we construct the synthetic 1-month

[17] Using volume as a proxy for liquidity.

[18] There are other alternatives that we do not follow. In particular, Burachi and Jackwerth (2001), Coval and Shumway (2001), and Driessen and Maenhout (2006) select the options at the first day of each month and compute returns until the next month first day.

[19] We do not need further criteria, since this already defines a unique contract at each month for each option type and bucket.

hold-to-expiration option returns

$$r_{t,t+1} = \frac{\text{Payoff}_{t+1}}{\text{Price}_t} - 1$$

where Payoff_{t+1} is the payoff of the option at maturity calculated with the close price of the underlying asset at the day before settlement, and Price_t is the option price observed at the beginning of the period. We obtain a time-series of 153 observations for each option. Figure 3 presents the kernel density estimates for each option security. This reveals that the option return distribution departs significantly from the normal distribution, with considerable negative tail risk for any of the options considered.

Summary statistics of option sample grouped by moneyness (OM) for each time-series are presented in Table 2. ATM call and put options present an average moneyness of 0.35% and −0.24%, respectively, whereas OTM call and put options present an average moneyness of −4.03% and 4.45%, respectively. These numbers show how close each contract is to the mean value of each bucket. The volume for each contract is around 4,000 and there is an open interest close to 20,000. The mean implied volatility varies between 15% and 22% with moneyness, which confirms the known smile effect. Panel B of figure 1 presents the evolution of implied volatility of each option between 1996 and 2008. Implied volatility is low between 2003 and mid 2007's. We can recognize five pronounced peaks in these time-series corresponding to the year 1998 and the periods 2000-2002 and 2007-2008.

Table 3 reports summary statistics of returns of the various securities present in this study. The S&P 500 index experiences an average monthly return of 0.3% over the sample period corresponding to an annualized Sharpe ratio of 0.20.[20] The main performance measure in this study is the Sharpe ratio.[21] We also compute the certainty equivalent of an investor with risk aversion of 10 and, in addition, we also incorporate descriptive statistics

[20] This is not in line with usual stated Sharpe ratios for the US market in the order of 0.50. The main reason for this low Sharpe ratio is the period in question which is not a very long sample period.

[21] The main problem of a Sharpe ratio is that it only takes into account the first two moments, mean and standard deviation. Broadie, Chernov, and Johannes (2009) show that although Sharpe ratio is not the best measure to evaluate performance in an option framework, other alternative measures as Leland's alpha or the Manipulation Proof Performance Metric face the same problems. See Bernardo and Ledoit (2000) and Ingersoll, Spiegel, Goetzmann, and Welch (2007) for problems with Sharpe ratio.

of the return distribution, in particular, skewness and excess kurtosis. The S&P 500 index returns also present negative skewness and excess kurtosis. Options present large negative average monthly returns which range from -3.1% to -51.6%. This suggests that writing options may work out as a good strategy since annualized Sharpe ratios range from 0.10 to 1.02 for this period.[22] However, this has negative tail risk which may be too onerous in some months. The returns to writing options have a maximum of 100% but the minimum ranges from -459% to -2,349% depending on the option. This leads to large negative skewness and excess kurtosis between 4.82 and 35.70. The last row of this table shows a strategy that allocates the same weight to each option. DeMiguel, Garlappi, and Uppal (2009) argue that a naive $1/N$ uniform rule is generally good. Using this rule for our four risky assets, we obtain smoother skewness and kurtosis, but the Sharpe ratio is close to zero.

The construction of "bid" and "ask" securities is straightforward. For the chosen contract, we use the *bid* quote and the *ask* quote, respectively. The descriptive statistics of these contracts are very similar to the ones using *mid* prices and are not presented for space reasons.

4 Results

Table 4 presents summary statistics of OOS returns for the OOPS between January 1996 and September 2008. The first point is that all strategies display annualized Sharpe ratios greater than the market. Based on the unconditional approach, the OOPS strategies present negative skewness (close to -4) and substantial excess kurtosis (around 30). Notice the extreme negative returns which may achieve -75% per month. The best strategy is the one that uses a Normal distribution with an annualized Sharpe ratio of 0.27, although the Sharpe ratios are not truly very different across simulation methods.[23] Certainty equivalent values are ridiculously negative, due to the extreme returns.

[22]Coval and Shumway (2001) and Eraker (2007) show that writing put options earns Sharpe ratios of this magnitude.

[23]These strategies are i.i.d. as it can be shown using an ARCH test or a Ljung-Box test. Results not reported for space reasons. Hence, there is no need to correct annualized Sharpe ratios for the potential autocorrelation in strategy returns.

On the other hand, conditional strategies present returns with positive skewness (around 1) and smoother excess kurtosis (around 9). This intrinsically defines a narrower distribution which limits the downside risk. Annualized Sharpe ratios are now of the order of 0.50 and certainty equivalent of 5%. Another point to stress is the fact that conditional strategies always present lower standard deviation than unconditional strategies. These strategies present negative returns for only 40% of the months. We next analyze in more detail the OOS returns of the conditional OOPS using the GEV distribution. The negative extreme returns, which are relatively small, happen in September 2007 (-8.7%) and September 2001 (-6.9%), two periods in which two events were totally unexpected by the options market and the stock market as well. On the other hand, positive extreme returns happen in February 1996 (16.3%), July 1996 (11.6%), May 1997 (13.3%), November 1999 (13.1%) and October 2008 (14.1%).

Table 5 reports average weights of each option in each strategy (panel A) and the proportion of months with positive weights (panel B). This latter measure allows us to confirm if the security is, on average, "long" or "short" since the mean may be affected by outlier values. We complement the analysis with a picture with the evolution of weights. Figure 4 represents one example of what happens in terms of weights of the four risky assets for the parametric simulation using a GEV distribution for conditional OOPS. The main conclusion is that all these four assets have significant weights. Figure 4 shows that these four option weights offset each other and in some periods of time this offsetting is not always the same. For instance, in November 1998 we write -10.20% of OTM put option and balance it with an ATM put option weight of 10.96%. In September 1999, call options are more relevant. We write an ATM call with weight of -9.95% and hold a long position of 5.04% in an OTM call. In September 2001, we write OTM options, -4.74% for call and -18.16% for put, and we hold a long position of ATM options, 10.87% for call and 22.46% for put. These weights are quite different over time. The OOPS is quite different from the simple short put strategies described in the literature.

The first conclusion from Table 5 is that the sign of the position in each option is not really related to the way we choose to simulate returns. On average, we hold long positions of ATM puts and OTM calls and short positions of OTM puts. The holdings of ATM call options change more. The second point is that simulating the conditional distribution of returns leads to an allocation with smaller weights. The mean of the maximum and the minimum weights of all securities are smaller for conditional OOPS and the mean of the sum of absolute value of weights of these eight securities is clearly lower than for unconditional OOPS. This makes our portfolio less leveraged.

We can check that generically ATM options balance OTM options and this is even more true when extreme weights are set. Correlation figures confirm these results. The strongest correlated pairs are the ATM and OTM calls and the ATM and OTM puts in the order of -0.70. For the other pairs the correlation is lower than 0.5 in absolute value. Moreover, put options seem to play a more important role in the allocation than call options. This can be checked in terms of individual assets and comparing the sum of absolute weights of calls and puts. The latter is three times more for unconditional OOPS and twice as much for conditional OOPS. Moreover, peaks are most pronounced in put options weights. The most pronounced weight was due to 9/11. This seems quite intuitive since this event was not expected by the market in any way. This was really something that was not priced in the options market, and therefore the reason why this affected allocation so much. Following this event, the period surrounding LTCM bailout is also very volatile in terms of weights. September 2008 also holds larger weights but still not as much as the previous two.

Our results are not in line with Liu and Pan (2003) since they tend to buy OTM put options. Nevertheless, we point out that they do not have the choice between different levels of moneyness and for conditional OOPS we also have a positive net position in put options which agrees with their result. Our results confirm partially Driessen and Maenhout (2007) since we short OTM put option, but we never write straddles.

The bottom right picture in Figure 4 presents the evolution of the risk-free security weight. The mean weight is 98.2% and 81% of the months is less than 100%. Hence, no borrowing is the most standard case. The maximum weight is 107.1% in June 1996 and the minimum is achieved in August 1998 and February 2007 around 92.6%.

To measure the sensitivity of this portfolio to potential changes in the underlying asset we first analyze the delta. Delta is the Black-Scholes delta. Panel A of Table 6 presents summary statistics of the delta for each of the three strategies using unconditional and conditional OOPS. The main conclusion is that the portfolio delta is very close to zero, ranging from -0.06 to 0.02 with a mean around 0. Most months, the portfolio delta is negative. Conditional OOPS present narrower delta magnitudes implying less risk. A better measure of risk is the elasticity of the portfolio. The elasticity of an option is the product of the delta by the ratio between underlying asset value by the option value. This measure has the advantage that it takes into account the option leverage. Panel B of Table 6 presents summary statistics for elasticity. The mean value is around -10, hence an increase of 1% in the underlying asset, will impact the portfolio by -10%, on average. This means that we typically hold a large net short position. However, our strategies are clearly targeted to prevent extreme bad outcomes. A negative return for the OOPS strategies against a positive return for the market only happens in 28% of the months and with an implied monthly average return of only -1.54%. Notice that in only 50% of the months we have an elasticity greater than 8 in absolute value. If we contrast individual option elasticity[24] to the elasticity of OOPS, we can conclude that the optimal portfolio have relatively much smaller elasticity. The main difference of strategies simulated from unconditional and conditional OOPS is that the latter present less extreme negative elasticity. The elasticity evolution in the period between January 1996 and September 2008 is presented in Figure 5. These series are quite volatile and there are three main periods below -20, such as the second half of the years 1998, 1999 and 2007. In particular, in August and September this series attains the lowest value around -45.

[24] Elasticity for individual options is presented in Table 2.

5 Robustness Checks

In this section we perform some robustness diagnostics, but only for conditional OOPS, since the previous section has shown how well these strategies behave. The first is the impact of choosing fewer assets in each OOPS. Several authors use only a restricted set of options to develop optimal strategies.[25] Table 7 presents the most relevant statistics for understanding their performance. Each row presents a different portfolio choice. Each number represents a different option choice considered in the strategy. The digits 1, 2, 3, and 4 denote the ATM call, the ATM put, the 5% OTM call, and the 5% OTM put options, respectively. When more than one digit defines the number, that strategy involves the combination of options coded as before. The strategy with four option securities yields across the different simulation methods simultaneously the best Sharpe ratio (around 0.50) and certainty equivalent values (around 5%) with low kurtosis and positive skewness.

Now we analyze portfolios formed by only one option. There is only one strategy that yields a positive Sharpe ratio (the OTM call option), but then presents a negative annualized certainty equivalent due to the excess kurtosis. On the other hand, strategies with positive annualized certainty equivalent, present negative Sharpe ratios although holding lower excess kurtosis. A strategy with only a 5% OTM put option yields both negative Sharpe ratio and certainty equivalent value. Next, we analyze portfolios formed with two options. They all present positive Sharpe ratio, consistently around 0.40. There is a strategy that combines a ATM call option with a 5% OTM put option that achieves a Sharpe ratio of 0.52 for the empirical simulation, but then falls to 0.27 when using GEV simulation. The same happens to the certainty equivalent.

Second, we analyze the effect of risk aversion on portfolio choice. Table 8 presents the most relevant statistics for understanding strategies performance. Each row presents a different risk aversion parameter, γ. There are two approaches that we follow. The first

[25]Liu and Pan (2003) use one stock, a 5% OTM put option, and cash, Eraker (2007) chooses between three risky assets, ATM straddles, OTM puts, and OTM calls. Driessen and Maenhout (2007) analyze the choice between stock, an option strategy and cash. They use either an OTM put, an ATM straddle or corresponding crash-neutral strategies. Jones (2006) only uses put options. Constantinides, Jackwerth, and Savov (2009) uses either call or put options.

is to understand the effect of changing the risk aversion parameter, γ, from 10 to 5, 3, or 2. From Panel A of this table, as γ decreases, the skewness becomes even more positive, but kurtosis also increases. This impacts negatively the overall performance as the Sharpe ratio is lower. The certainty equivalent is only negative when γ is 2. These effects are due to holding higher weights for the risky securities. Nonetheless, the overall results show the outstanding behavior of OOPS. The second approach involves the use of a mean-variance utility function $U = \mu_p - \frac{\gamma}{2}\sigma_p^2$. Panel B of the same table shows the results. Although the strategies involve positive skewness, they have substantial excess kurtosis and yield modest annualized Sharpe ratios. The problem associated with the strategies is that their mean delta is 0.20 and mean elasticity is 231. This is a high-leverage strategy with mean weights of 82% on OTM call option and -14% on OTM put option. This yields a strategy 60% of the months where all money is lost compensated by opportunistic high returns. This strategy could hardly be implemented. This shows the importance of using an objective function that penalizes skewness and kurtosis.

6 Conclusion

We offer a new method for portfolio optimization with options. Our approach relies on simulating option payoffs from a given distribution of the underlying asset and using those returns to maximize an expected utility function. This takes into account the complex distribution of option returns and the investor's preferences for high-order moments. Another advantage of our approach is that it does not rely on a long time series of option returns which in practice does not exist. Our approach also takes into account transaction costs in a simple manner.

We apply the method out of sample in the period 1996 to 2008 with impressive results. The method is straightforward to implement (it can be applied on a spreadsheet) and requires virtually no computing resources. We obtain high Sharpe ratios and other good performance measures that take into account the non-normality of the return distribution.

Appendix A

We fix two time periods, $t = 1$ and $t = 2$. In the former, we run our optimization problem and obtain the weights of each security for our asset allocation. In the latter, we perform our OOS exercise. We set the underlying asset value at period 1 equal to 1.00, $S_1 = 1$, and at period 2 equal to 0.98, $S_2 = 0.98$, only known at period 2, and the one-period risk-free interest rate equal to 10%, $r_f = 0.1$, the same for both periods. We assume two call options (ATM, c_1, and OTM, c_2) and two put options (ATM, p_1, and OTM, p_2). Their strike prices are given by $K_{1,c_1} = K_{1,p_1} = 1$, $K_{1,c_2} = 1.05$ and $K_{1,p_2} = 0.95$. The prices of the options at date $t = 1$ are given by $C_{1,c_1} = 0.04$, $C_{1,c_2} = 0.0008$, $P_{1,p_1} = 0.07$, and $P_{1,p_2} = 0.02$. We assume a power utility with $\gamma = 10$.

1. Simulate the underlying asset log-return			
$r_2^1 = 0.0500$	$r_2^2 = 0.0100$	$r_2^3 = -0.0400$	$r_2^4 = -0.1000$

2. Find the next period underlying asset value			
$S_{2\|1}^1 = 1.0513$	$S_{2\|1}^2 = 1.0101$	$S_{2\|1}^3 = 0.9608$	$S_{2\|1}^4 = 0.9048$

3.a. Determine simulated option payoffs at maturity			
$C_{2\|1,c_1}^1 = 0.0513$	$C_{2\|1,c_1}^2 = 0.0101$	$C_{2\|1,c_1}^3 = 0.0000$	$C_{2\|1,c_1}^4 = 0.0000$
$C_{2\|1,c_2}^1 = 0.0013$	$C_{2\|1,c_2}^2 = 0.0000$	$C_{2\|1,c_2}^3 = 0.0000$	$C_{2\|1,c_2}^4 = 0.0000$
$P_{2\|1,p_1}^1 = 0.0000$	$P_{2\|1,p_1}^2 = 0.0000$	$P_{2\|1,p_1}^3 = 0.0392$	$P_{2\|1,p_1}^4 = 0.0952$
$P_{2\|1,p_2}^1 = 0.0000$	$P_{2\|1,p_2}^2 = 0.0000$	$P_{2\|1,p_2}^3 = 0.0000$	$P_{2\|1,p_2}^4 = 0.0452$
3.b. And corresponding returns for each option			
$r_{2\|1,c_1}^1 = 0.2818$	$r_{2\|1,c_1}^2 = -0.7487$	$r_{2\|1,c_1}^3 = -1.0000$	$r_{2\|1,c_1}^4 = -1.0000$
$r_{2\|1,c_2}^1 = 0.5589$	$r_{2\|1,c_2}^2 = -1.0000$	$r_{2\|1,c_2}^3 = -1.0000$	$r_{2\|1,c_2}^4 = -1.0000$
$r_{2\|1,p_1}^1 = -1.0000$	$r_{2\|1,p_1}^2 = -1.0000$	$r_{2\|1,p_1}^3 = -0.4398$	$r_{2\|1,p_1}^4 = 0.3595$
$r_{2\|1,p_2}^1 = -1.0000$	$r_{2\|1,p_2}^2 = -1.0000$	$r_{2\|1,p_2}^3 = -1.0000$	$r_{2\|1,p_2}^4 = 1.2581$

4. Construct the simulated portfolio return			
$rp_{2\|1}^1 = 0.10+$	$rp_{2\|1}^2 = 0.10+$	$rp_{2\|1}^3 = 0.10+$	$rp_{2\|1}^4 = 0.10$
$+\omega_{2,c_1} \times 0.2818+$	$+\omega_{2,c_1} \times (-0.7487)+$	$+\omega_{2,c_1} \times (-1.0000)+$	$+\omega_{2,c_1} \times (-1.0000)$
$+\omega_{2,c_2} \times 0.5589+$	$+\omega_{2,c_2} \times (-1.0000)+$	$+\omega_{2,c_2} \times (-1.0000)+$	$+\omega_{2,c_2} \times (-1.0000)$
$+\omega_{2,p_1} \times (-1.0000)+$	$+\omega_{2,p_1} \times (-1.0000)+$	$+\omega_{2,p_1} \times (-0.4398)+$	$+\omega_{2,p_1} \times (0.3595)+$
$+\omega_{2,p_2} \times (-1.0000)$	$+\omega_{2,p_2} \times (-1.0000)$	$+\omega_{2,p_2} \times (-1.0000)$	$+\omega_{2,p_2} \times 1.2581$

5. Choose weights by maximizing expected utility over simulated returns $\frac{1}{4}\sum_{n=1}^{4}\frac{(1+rp_{2\|1}^n)^{-9}}{-9}$
$\omega_{2,c_1} = 0.02, \omega_{2,c_2} = -0.03, \omega_{2,p_1} = -0.01, \omega_{2,p_2} = -0.12$ and $E(U) = -0.0369$

6.a. Determine option actual payoffs...	6.b. and returns for each option
$C_{2,c_1} = 0.0000$	$r_{2,c_1} = -1.0000$
$C_{2,c_2} = 0.0000$	$r_{2,c_2} = -1.0000$
$P_{2,p_1} = 0.0200$	$r_{2,p_1} = -0.7143$
$P_{2,p_2} = 0.0000$	$r_{2,p_2} = -1.0000$
6.c. Determine one-period OOS portfolio return	
$rp_2 = 0.2511$	

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Figure 1

Bid-ask spread and implied volatility

This figure represents monthly observations of relative bid-ask spreads (the ratio of absolute bid-ask spread and mid price) and implied volatility (using Black-Scholes model) measures for the period between January 1996 and September 2008 for four options: an ATM call, an ATM put, a 5% OTM call, and a 5% OTM put options. Each option security was created as defined in Section 3.2. Results in percentage.

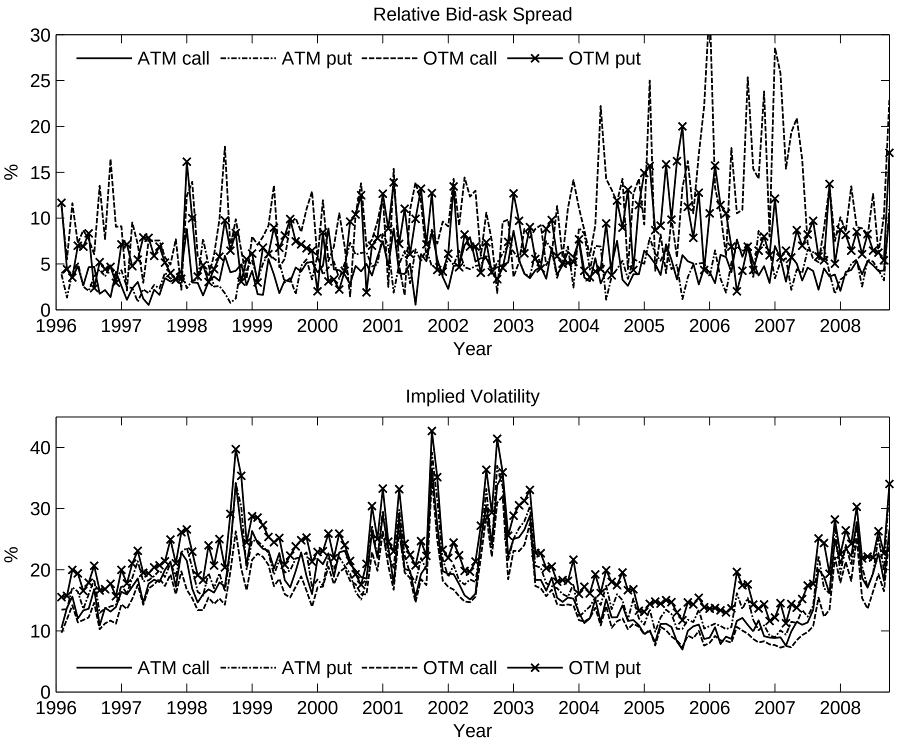

Figure 2
S&P 500 Index and VIX

This figure represents monthly observations of S&P 500 Index and VIX Index in the period between January 1996 and September 2008.

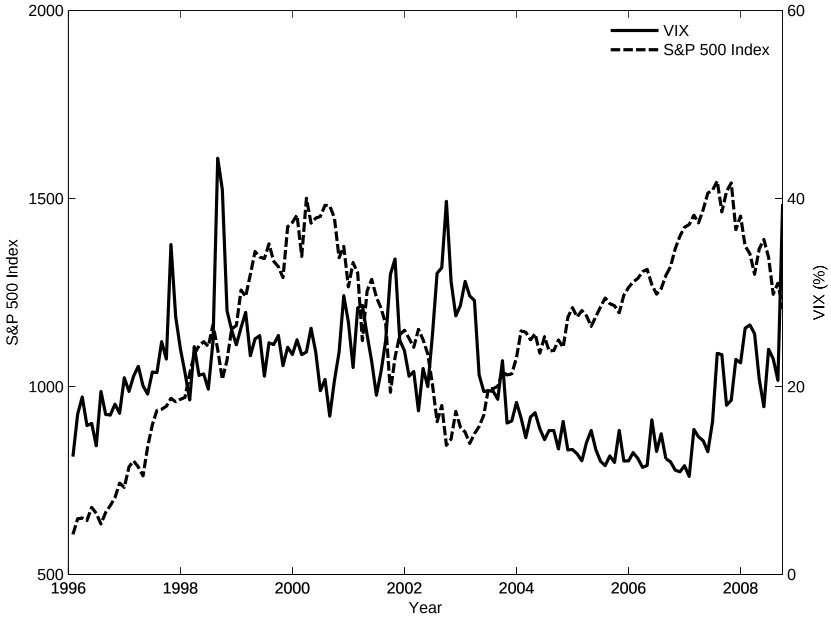

Figure 3
Densities of monthly option returns

This figure represents the densities of monthly raw returns of a long position in ATM and 5% OTM options over the S&P 500 index estimated by normal kernel smoothing for the period between January 1996 and September 2008.

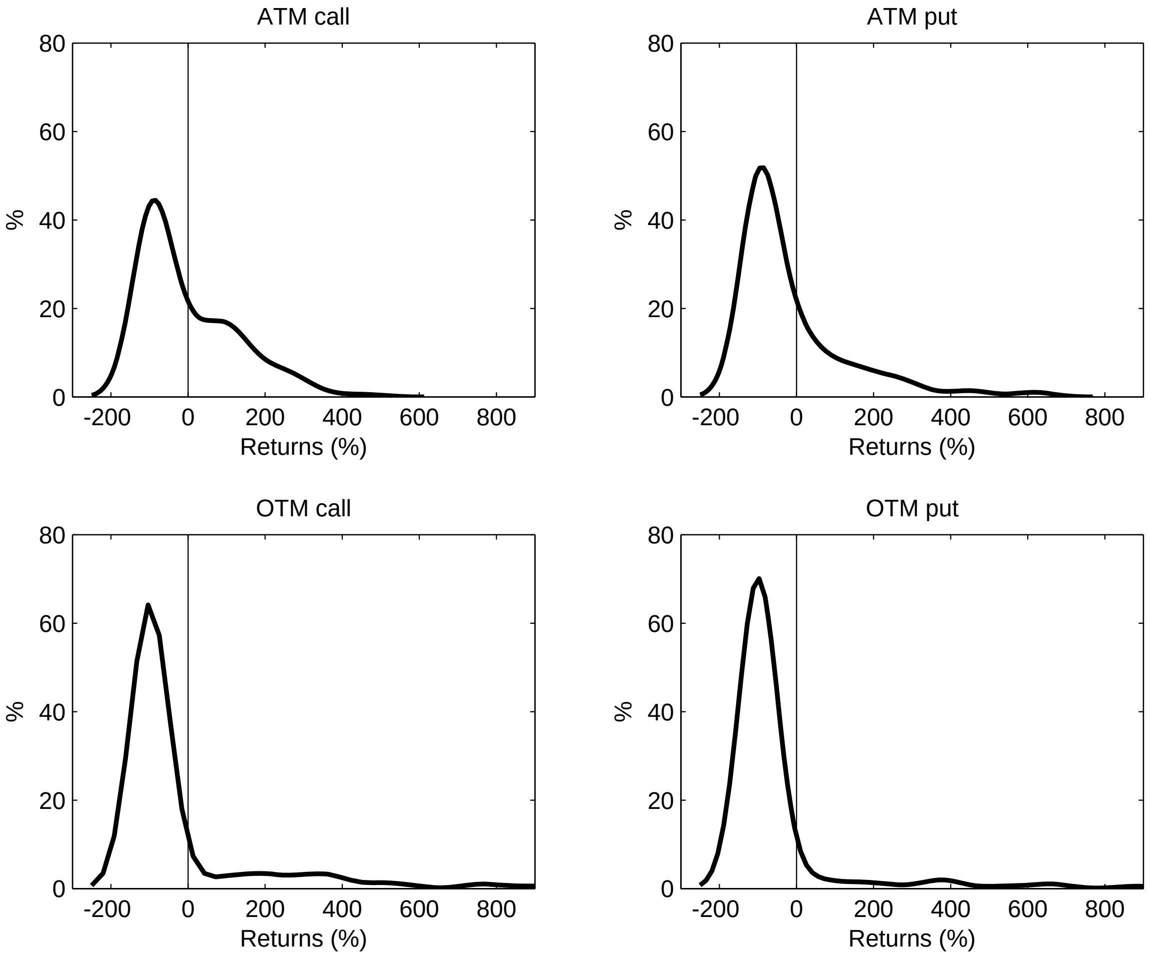

Figure 4

OOPS time-series weights

This figure represents the monthly weights of conditional OOPS constructed by simulation from a GEV distribution. Top left panel presents weights for ATM call and 5% OTM call options. Top right panel presents weights for ATM put and 5% OTM put options. Bottom left panel presents weigths for the difference between ATM options and OTM options. Bottom right panel presents the weight of the risk-free security. Period: January 1996 to September 2008.

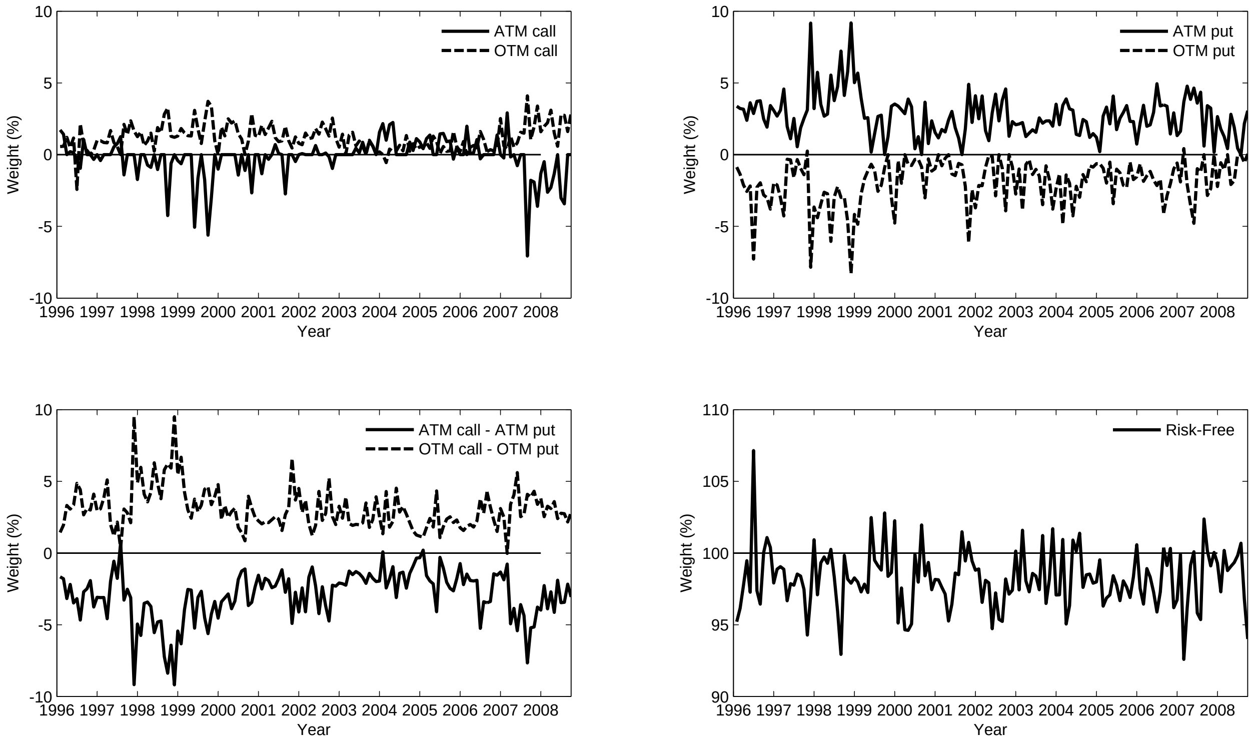

Figure 5
OOPS elasticity

This figure presents conditional OOPS portfolio elasticity constructed by simulation from a GEV distribution. Period: January 1996 to September 2008.

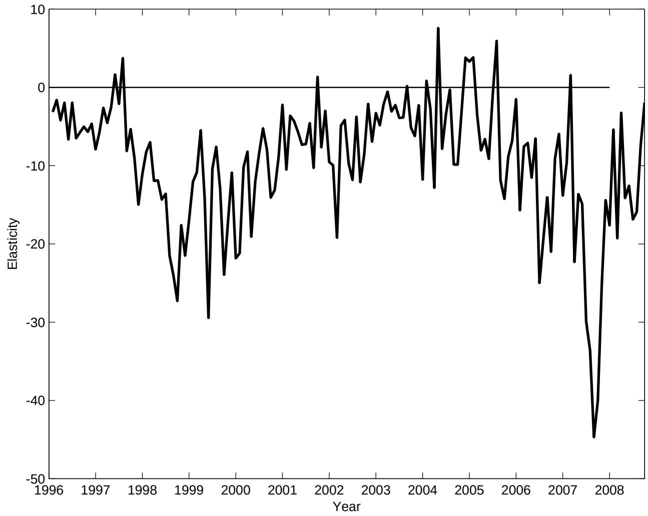

Table 1

S&P 500 Index Returns – Summary Statistics

This table reports summary statistics (number of observations, mean, standard deviation, minimum, 5% percentile, first quartile, median, third quartile, 95% percentile, maximum, skewness, excess kurtosis, one and twelve month autocorrelation for the residuals and square residuals) and tests (one and twelve month LjungBox and Arch tests; *p-values* of each test are presented in squared brackets) for raw returns and standardized returns for S&P 500 index. Standardized returns are raw returns divided by realized volatility of previous month. The results are presented for three periods: February 1950-December 1995, January 1996-September 2008, and February 1950-September 2008.

	Raw returns			Standardized returns		
	1950-1995	1996-2008	1950-2008	1950-1995	1996-2008	1950-2008
Obs	551	153	704	551	153	704
Mean	0.7%	0.3%	0.6%	22%	11%	19%
Std Dev	4.0%	4.3%	4.1%	129%	95%	122%
Min	-24.5%	-16.0%	-24.5%	-488%	-334%	-488%
$q_{0.05}$	-6.2%	-8.1%	-6.3%	-206%	-141%	-191%
$q_{0.25}$	-1.7%	-2.0%	-1.8%	-59%	-49%	-56%
$q_{0.50}$	0.9%	0.9%	0.9%	28%	21%	26%
$q_{0.75}$	3.4%	3.5%	3.4%	101%	75%	97%
$q_{0.95}$	6.7%	7.0%	6.8%	215%	149%	207%
Max	15.1%	8.9%	15.1%	354%	288%	354%
Skew	-0.56	-0.67	-0.59	-0.33	-0.46	-0.33
Exc Kurt	2.97	0.86	2.41	0.54	0.58	0.69
$\rho_1(z)$	0.03	0.01	0.02	0.03	0.01	0.03
$\rho_{12}(z)$	0.03	0.08	0.04	-0.01	0.04	0.00
$\rho_1(z^2)$	0.10	0.07	0.10	-0.11	-0.10	-0.09
$\rho_{12}(z^2)$	0.02	0.07	0.03	0.02	0.04	0.05
$Q_1(z)$	0.44	0.01	0.44	0.47	0.01	0.47
	[0.51]	[0.92]	[0.51]	[0.49]	[0.92]	[0.49]
$Q_{12}(z)$	11.78	8.86	11.78	9.81	10.27	9.81
	[0.46]	[0.71]	[0.46]	[0.63]	[0.59]	[0.63]
Arch(1)	9.60	0.62	9.60	3.96	1.54	3.96
	[0.00]	[0.43]	[0.00]	[0.05]	[0.21]	[0.05]
Arch(12)	22.07	13.16	22.07	12.09	4.79	12.09
	[0.04]	[0.36]	[0.04]	[0.44]	[0.96]	[0.44]

Table 2

Summary statistics of options

This table reports averages of option moneyness, bid-ask spread, relative bid-ask spread, volume, open interest, implied volatility, delta and elasticity for four options: ATM call, ATM put, 5% OTM call, and 5% OTM put options using mid prices. Option moneyness is defined as *S/K-1*. Bid-ask spread is the difference between ask price and bid price. Relative bid-ask spread is the ratio of the bid-ask spread by the mid price. Volume is the contract's volume at the day when there is one month to expiration. Open interest is the open interest prevalent at the day with one month to expiration. Implied volatility is the annualized volatility of the option with one-month to maturity using the Black-Scholes model. Delta is the Black-Scholes delta. Elasticity is the product of delta by the ratio of underlying asset value and option price. Period: January 1996-September 2008.

	ATM Call	ATM Put	OTM Call	OTM Put
Option moneyness	0.35%	-0.24%	-4.03%	4.45%
Bid-ask spread	1.24	1.18	0.58	0.70
Relative bid-ask spread	4.43%	4.68%	10.22%	7.48%
Volume	3,494	3,955	4,130	4,376
Open interest	16,213	18,413	18,676	29,819
Implied volatility	17.49%	18.53%	15.54%	21.51%
Delta	0.57	-0.48	0.20	-0.21
Elasticity	26.83	-25.75	49.29	-30.93

Table 3

Summary statistics of returns for individual securities

This table reports summary statistics (mean, standard deviation, minimum, maximum, skewness, excess kurtosis, annualized Sharpe ratio) for a buy-and-hold strategy in several return assets: S&P 500 index, 1-month US Libor, a ATM call option, a ATM put option, a 5% OTM call option and a 5% OTM put option. Option returns are based on contracts with one-month to maturity and each month a contract is selected such that it has the minimum bid-ask spread and, when draws are available, the largest open interest. Statistics are presented for monthly returns and computed over one-month period prior to the maturity date. The last row presents a strategy that allocates a weight of 1/4 to each one of the options previously selected. The period under consideration is between January 1996 and September 2008.

	Mean	Std Dev	Min	Max	Skew	Exc Kurt	Ann SR
S&P 500	0.3%	4.3%	-16.0%	8.9%	-0.68	0.86	0.20
1m US Libor	0.3%	0.1%	0.1%	0.6%	-0.49	1.74	-
ATM call	-3.1%	120.3%	-100%	459%	1.22	3.93	-0.10
ATM put	-19.8%	131.1%	-100%	595%	2.13	7.82	-0.53
OTM call	-10.3%	279.6%	-100%	2,349%	5.22	38.70	-0.13
OTM put	-51.6%	177.1%	-100%	1,139%	4.56	25.68	-1.02
1/N Rule	-21.2%	105.5%	-100%	625%	2.86	13.78	-0.01

Table 4

OOPS returns – Summary Statistics

This table reports summary statistics (mean, standard deviation, minimum, maximum, skewness, excess kurtosis, annualized Sharpe ratio, annualized Certainty Equivalent) for OOS one-month OOPS returns based on three different strategies for the period from January 1996 to September 2008. This is performed for unconditional and conditional OOPS. The three strategies differ in terms of simulation of the underlying asset returns, bootstrapping, simulation from a normal or GEV distribution using historical moments. Statistics are presented for monthly returns and computed over one-month period prior to the maturity date.

	Mean	Std Dev	Min	Max	Skew	Exc Kurt	Ann SR	Ann CE
Panel A: Market returns								
S&P 500	0.3%	4.3%	-16.0%	8.9%	-0.68	0.86	0.20	-5.9%
Panel B: Unconditional OOPS returns								
	Mean	Std Dev	Min	Max	Skew	Exc Kurt	Ann SR	Ann CE
Empirical	0.6%	9.1%	-39.0%	87.9%	4.9	59.5	0.21	-52.3%
Normal	1.1%	10.0%	-75.1%	21.1%	-4.1	27.9	0.27	-100.0%
GEV	1.1%	10.6%	-86.8%	21.6%	-4.8	36.3	0.25	-100.0%
Panel C: Conditional OOPS returns								
	Mean	Std Dev	Min	Max	Skew	Exc Kurt	Ann SR	Ann CE
Empirical	0.8%	3.0%	-7.4%	14.9%	2.1	7.7	0.51	5.1%
Normal	0.9%	3.8%	-18.8%	15.0%	-0.1	6.3	0.48	0.5%
GEV	1.0%	3.7%	-8.7%	16.3%	1.3	3.5	0.59	4.7%

Table 5

OOPS weights

This table presents mean time-series weights for three strategies (empirical, normal and GEV) as explained in Section 2 using unconditional and conditional OOPS in the period from January 1996 to September 2008. *Sum Abs* (*Sum Abs Calls*, *Sum Abs Puts*) denotes the sum of absolute weights of the four options (call options, put options).

	Unconditional OOPS			Conditional OOPS		
	Empirical	Normal	GEV	Empirical	Normal	GEV
	Panel A: Time-series weights mean					
ATM Call	-1.3%	-1.2%	-0.6%	-0.4%	-0.5%	-0.2%
Long	0.2%	0.2%	0.3%	0.1%	0.1%	0.3%
Short	1.5%	1.4%	0.9%	0.5%	0.7%	0.5%
ATM Put	-1.2%	2.3%	3.8%	0.8%	2.4%	2.7%
Long	0.3%	2.3%	3.8%	0.8%	2.4%	2.7%
Short	1.5%	0.1%	0.0%	0.0%	0.0%	0.0%
OTM Call	-0.1%	0.0%	0.2%	0.9%	1.1%	1.2%
Long	0.3%	0.4%	0.5%	0.9%	1.1%	1.2%
Short	0.4%	0.4%	0.4%	0.0%	0.0%	0.0%
OTM Put	-0.2%	-4.5%	-5.3%	-0.7%	-1.9%	-1.9%
Long	0.4%	0.2%	0.1%	0.0%	0.1%	0.0%
Short	0.5%	4.6%	5.4%	0.8%	1.9%	1.9%
Max	0.7%	2.5%	3.9%	1.3%	2.7%	2.9%
Min	-2.1%	-4.6%	-5.4%	-1.0%	-2.2%	-2.2%
Sum Abs	5.1%	9.6%	11.3%	3.2%	6.4%	6.6%
Sum Abs Calls	2.4%	2.4%	2.0%	1.6%	2.0%	2.0%
Sum Abs Puts	2.7%	7.2%	9.3%	1.6%	4.5%	4.6%
	Panel B: Proportion of positive weights					
ATM Call	20.9%	17.0%	22.9%	17.6%	19.6%	27.5%
Long	21.6%	17.0%	22.9%	17.6%	19.6%	27.5%
Short	61.4%	62.1%	50.3%	43.1%	49.0%	30.7%
ATM Put	30.1%	80.4%	96.1%	69.9%	94.1%	97.4%
Long	30.7%	79.7%	96.1%	69.9%	94.1%	97.4%
Short	58.2%	2.6%	0.7%	2.0%	0.7%	0.0%
OTM Call	41.2%	59.5%	67.3%	92.8%	95.4%	96.1%
Long	41.2%	59.5%	67.3%	92.8%	95.4%	96.1%
Short	37.3%	22.9%	19.6%	3.3%	2.0%	2.6%
OTM Put	4.6%	1.3%	1.3%	3.3%	1.3%	1.3%
Long	5.2%	1.3%	2.0%	3.3%	1.3%	1.3%
Short	77.1%	98.7%	100.0%	81.7%	92.8%	91.5%

Table 6

OOPS risk measures – Delta and Elasticity

This table presents summary statistics regarding delta and elasticity (time-series mean, standard deviation, minimum, 5% percentile, median, 95% percentile, maximum) of three strategies (empirical, normal and GEV) as explained in Section 2 using unconditional and conditional OOPS in the period from January 1996 to September 2008. Panel A presents results for delta and Panel B presents results for elasticity.

Panel A: Delta						
	Unconditional OOPS			Conditional OOPS		
	Empirical	Normal	GEV	Empirical	Normal	GEV
Mean	0.00	-0.01	-0.01	0.00	-0.01	-0.01
Std	0.01	0.01	0.01	0.01	0.01	0.01
Min	-0.04	-0.04	-0.06	-0.03	-0.03	-0.03
$q_{0.05}$	-0.01	-0.02	-0.02	-0.01	-0.02	-0.02
$q_{0.50}$	0.00	-0.01	-0.01	0.00	-0.01	-0.01
$q_{0.95}$	0.01	0.00	0.00	0.01	0.00	0.00
Max	0.02	0.01	0.01	0.01	0.01	0.01
Panel B: Elasticity						
	Unconditional OOPS			Conditional OOPS		
	Empirical	Normal	GEV	Empirical	Normal	GEV
Mean	-2.08	-9.27	-11.45	-3.03	-10.03	-9.46
Std	8.99	9.21	10.57	7.10	8.33	8.32
Min	-67.67	-51.92	-79.85	-36.71	-41.94	-44.69
$q_{0.05}$	-17.32	-24.54	-27.91	-17.49	-26.54	-24.85
$q_{0.50}$	-1.28	-7.55	-9.94	-2.42	-8.80	-7.93
$q_{0.95}$	7.66	1.91	2.34	5.68	1.07	1.52
Max	22.68	6.14	9.50	12.70	8.24	7.56

Table 7

Robustness checks - Different securities choice

This table presents OOS skewness, excess kurtosis, annualized Sharpe Ratio and annualized Certainty Equivalent for three strategies (empirical, normal and GEV) as explained in Section 2 using conditional OOPS in the period from January 1996 to September 2008. Each row represents a different option choice. 1 denotes ATM call option, 2 denotes ATM put option, 3 denotes 5% OTM call option, 4 denotes 5% OTM put option, and the remaining strategies are just simultaneous choice of more than one option according to this codification, e.g., 13 denotes ATM call and 5% OTM call options. This means that "bid" and "ask" securities are considered in each strategy.

	Conditional OOPS											
	Empirical				Normal				GEV			
Asset choice	Skew	Exc Kurt	Ann SR	Ann CE	Skew	Exc Kurt	Ann SR	Ann CE	Skew	Exc Kurt	Ann SR	Ann CE
1	1.75	4.95	-0.10	1.78%	1.76	4.87	-0.09	2.04%	1.67	4.17	-0.13	1.32%
2	2.91	11.36	-0.02	1.43%	2.98	12.12	-0.05	1.46%	2.63	8.75	-0.12	0.46%
3	6.45	52.78	0.07	-0.87%	6.31	50.60	0.07	-0.52%	6.32	51.35	0.04	-1.92%
4	3.77	15.80	-0.26	-2.65%	3.84	16.50	-0.28	-2.36%	3.64	13.89	-0.38	-4.29%
12	1.36	4.23	0.39	4.40%	1.53	5.06	0.41	4.33%	1.55	4.97	0.26	2.47%
13	4.23	28.95	0.07	-5.32%	4.93	37.13	0.08	-7.58%	5.20	39.46	0.03	-10.80%
14	1.67	6.13	0.52	4.84%	2.05	8.16	0.49	4.16%	1.99	7.40	0.27	0.40%
23	3.55	20.78	0.33	1.59%	4.42	32.16	0.30	0.41%	4.81	36.02	0.18	-2.71%
24	4.19	22.35	0.44	1.77%	4.02	20.49	0.43	0.34%	3.90	19.14	0.27	-5.10%
34	5.66	45.96	0.42	1.21%	5.60	45.42	0.42	0.32%	5.53	44.08	0.26	-5.26%
123	2.23	8.96	0.22	2.46%	1.29	5.94	0.13	-0.02%	2.05	7.15	0.11	0.23%
124	3.80	21.57	0.12	-2.15%	1.64	9.82	-0.09	-17.51%	2.85	12.48	-0.01	-9.63%
134	1.75	6.98	0.00	-4.74%	0.50	6.39	-0.18	-27.83%	2.60	11.73	-0.09	-10.41%
234	3.10	14.43	0.37	3.06%	2.84	14.22	0.34	0.78%	4.19	24.15	0.29	0.38%
1234	2.09	7.70	0.51	5.14%	-0.07	6.28	0.48	0.54%	1.32	3.46	0.59	4.72%

Table 8

Robustness checks - Different risk aversion levels

This table presents OOS skewness, kurtosis, annualized Sharpe Ratio and annualized Certainty Equivalent for three strategies (empirical, normal and GEV) as explained in Section 2 for conditional OOPS in the period from January 1996 to September 2008. Panel A uses CRRA utility function whereas Panel B uses mean variance utility. Each row represents a different risk aversion parameter.

	Conditional OOPS - CRRA utility											
	Empirical				Normal				GEV			
γ	Skew	Exc Kurt	Ann SR	Ann CE	Skew	Exc Kurt	Ann SR	Ann CE	Skew	Exc Kurt	Ann SR	Ann CE
2	2.86	13.28	0.35	-1.14%	1.98	8.38	0.43	-3.17%	2.20	8.75	0.38	-7.99%
3	2.44	10.45	0.40	3.13%	1.51	5.71	0.52	3.38%	1.76	5.98	0.47	0.44%
5	2.13	8.51	0.45	4.81%	1.21	4.22	0.61	5.96%	1.49	4.31	0.55	4.07%
10	2.09	7.70	0.51	5.14%	-0.07	6.28	0.48	0.54%	1.32	3.46	0.59	4.72%
	Conditional OOPS - Mean Variance utility											
	Empirical				Normal				GEV			
γ	Skew	Exc Kurt	Ann SR	Ann CE	Skew	Exc Kurt	Ann SR	Ann CE	Skew	Exc Kurt	Ann SR	Ann CE
2	4.99	35.38	0.26	-	4.99	35.31	0.24	-	5.02	35.46	0.15	-
3	5.02	35.79	0.27	-	5.02	35.70	0.22	-	5.06	35.88	0.15	-
5	5.06	36.33	0.30	-	5.06	36.11	0.20	-	5.11	36.37	0.13	-
10	6.20	51.67	0.32	-	5.16	37.20	0.24	-	5.15	36.95	0.17	-