The Human Development Index in an endogenous growth model

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Abstract: Van Zon and Muysken (2001) model both the education and the health sector based on the Lucas-Uzawa model of human capital accumulation. This endogenous growth model is extended here to explicitly consider education next to health and consumption in the CIES utility function. The model is solved for its long run equilibrium in which physical and human capital, consumption and output grow at constant and equal rates. The health level of the population is assumed to be constant. However, as the current situation in developing countries is far from being in a long run equilibrium state, special attention is given to the analysis of the dynamic properties of the system. The development of a country can be interpreted as the transition towards its long run equilibrium.

The long run equilibrium of the model can be solved numerically for a variety of parameter specifications. Central results are growth rate, labor shares in production, education and health sectors, propensity to consume or savings rate, return to capital and the health level, which is assumed to be constant in the long run equilibrium. The growth rate is positively related with the share of the population working in the education sector, savings rate and the health level. Labor in final output production and health sectors are substitutes. A higher preference of the present over the future has a positive influence on the final output production and consumption and a negative influence on education and health. Higher productivity in either one of the health and education sector increases the labor share in the respective sector. Still, the largest impact on the labor shares is given by the choice of weights of the corresponding variables in the utility function. As the weights for education and health increase from values close to zero to one third (implying an equal preference for consumption, education and health), so do the labor shares. Preliminary calculations of the transitional dynamics show that the economy gradually approaches its steady state for low starting values of physical and human capital or the health level.

Keywords: Lucas-Uzawa model, Human Development Index, Sub-Saharan Africa

1 Introduction

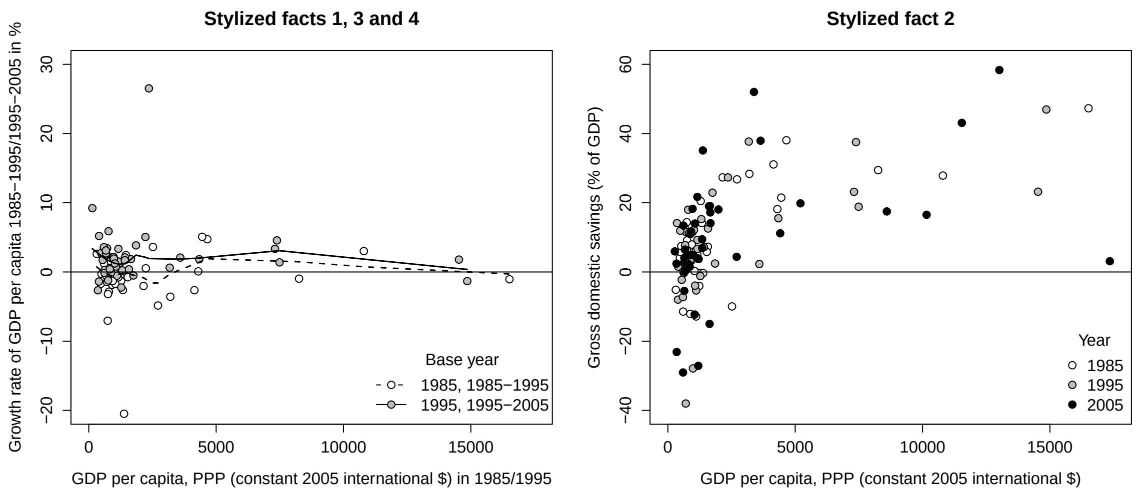
Sustainable development is a long-run concept; changes in investment and technology that support sustainable development are long-run processes. A standard way of analyzing a country's long-run development is the use of endogenous growth models. Their general conceptual framework seems to be particularly suited for studying sustainability issues. Most of the currently existing endogenous growth models for sustainable development were developed within and for industrial countries that are well endowed with physical capital, knowledge and technology. Most developing countries, especially in Africa, however, have low capital endowments, low rates of capital accumulation and low technological capabilities.

In 1988 Lucas showed that the neoclassical growth models of Solow (1956) or Denison (1961) are not models of economic development. They can well be used to model GDP growth rates for the US economy, but fail to explain possible differences between countries' income growth. This is due to the fact that the growth rate, which only depends on technological change and the labor share, is insensitive to changes in the remaining model parameters (Lucas, 1988, p. 15), which capture most of the country differences.

Still, nowadays only few endogenous growth approaches explaining the development of countries, especially for developing countries, exist. One approach is the unified growth theory developed by Oded Galor. This theory combines an endogenous growth model with elements from evolutionary economics, thus explicitly modeling the transition possibilities from one phase of development into the next phase. Several extensions of the basic model exist, covering various aspects of development, such as inequality, fertility or the gender gap, see for example Galor and Weil (1996); Galor and Moav (2000, 2004). More recently developed theoretical growth models for developing countries are the Cass-Koopmanns model as for example in Lecocq and Shalizi (2007) and the models by Pierre-Richard Agénor and Nihil Bayraktar (Agénor *et al.*, 2004, 2005) for analyzing progress toward the Millennium Development Goals (MDGs).

When using theoretical growth models to analyze growth dynamics in developing countries, however, it is most important to explicitly consider that these countries are far from being in a long-run equilibrium state. According to Steger (2000, p. 12) one possibility "to explain persistent underdevelopment, defined narrowly, within the framework of growth theory consists of interpreting real world growth dynamics as representing transition processes to dynamic equilibria". In other words, when using theoretical growth models, it is not the long-run equilibrium which represents development, but rather the transition of the economy from any state toward the equilibrium[1]. Steger (2000) identified four stylized facts of economic growth in devel-

[1] A short note on notation: the terms 'long-run equilibrium', 'dynamic equilibrium', 'balanced growth path' and 'steady state' are used almost interchangeably in the literature. Steger (2000, p. 12/13) explains the differences between these terms in detail, but in short, these refer to a state of the economy "in which the various quantities grow at constant (perhaps zero) rates" (Barro and Sala-i Martin, 2004, p.33).



Source: based on Data of WDI (2011)

Figure 1: Stylized facts of economic growth in developing countries: Sub-Saharan Africa

oping countries, that are replicated in the transition process toward the steady state in his models on subsistence consumption, productive consumption and endogenizing control variables in transitional dynamics The stylized facts are (Steger, 2000, p. 4):

1. *A considerable diversity in the growth rates of per capita income;*

2. *a positive correlation between the saving rate and the level of per capita income; and*

3. *a positive correlation between the growth rate and the level of per capita income, i.e. β-divergence.*

4. *More generally, many authors report β-divergence for the lower range of per capita income and β-convergence for the upper range of per capita income, i.e. a hump-shaped pattern of growth.*

Figure 1 shows data of Sub-Saharan African countries corresponding to these stylized facts. The diversity of growth rates (stylized fact 1) becomes immediately apparent when looking at the vertical spread of the dots in the left graph. This graph plots the ten-year average growth rates of Sub-Saharan African countries for the periods 1985 to 1995 and 1995 to 2005 on the vertical axis, and the corresponding GDP per capita in the base years (1985 and 1995) on the horizontal axis. Stylized fact 3, β-divergence, cannot be confirmed from this plot, however β-convergence (stylized fact 4) for the upper range of per capita incomes can well be the case as indicated by the two lines (solid and dotted), that decrease for higher values of per capita income. These two lines are simple linear regression lines for the two periods, with the 10-year average growth rate being the dependent and GDP per capita being the independent variable. The graph on the right plots the savings rate in 1985, 1995 and

2005 against the corresponding per capita GDP. Stylized fact 2 predicts a positive correlation between these two indicators, which can also be observed here.

GDP, GDP growth or the per capita versions of both are not the only important issues for developing countries. Also, as stated above, they have rather low physical capital endowments (though this starts to change, especially because of foreign direct investments from Asian emerging economies in African countries). However, they have large young populations, that are expected to grow in the future. This means that they have huge potential amounts of human capital. For this potential human capital to become productive human capital, the population needs to be well educated and healthy, both of which are major development concerns addressed by many development programs for Africa, captured, for instance, in the MDGs or in the Human Development Reports (HDR) by the United Nations Development Programme (UNDP).

A growth model for developing countries should therefore put special emphasis on these non-monetary issues of development. Van Zon and Muysken (2001), for example, explicitly model both the education and the health sector based on the Lucas-Uzawa model of human capital accumulation. In this chapter we take the van Zon and Muysken (2001) model and extend it to explicitly consider education next to health and consumption in the utility function.

The next section shortly introduces the Lucas-Uzawa model of human capital accumulation. In section 2 a development growth model is developed. The main feature of this model is that utility does not only depend consumption but also on health and education. Both the long-run equilibrium and the transition toward the long-run equilibrium of the model are analyzed. Section 4 concludes and introduces some ideas for future research.

2 The Lucas model, education and health

The van Zon and Muysken (2001) version of the Lucas model includes both consumption and health in the utility function. The new Human Development Index (HDI) as introduced in UNDP (2010) is a geometric average of three components: standard-of-living, health and education. Here, we will reformulate the utility function in terms of the HDI and therefore extend it to also include education. The section starts with a short presentation of the original Lucas model from 1988 as well as the addition of a health sector by van Zon and Muysken (2001).

By introducing education as schooling time into the model, which influences the level of human capital, Lucas (1988) allows for permanent differences in income across countries, but not for differences in growth rates. The optimization problem in the Lucas model is to maximize intertemporal utility W subject to the physical and human capital accumulation constraints, $\dot{K}$ and $\dot{h}$. The control variables in this problem are consumption c and the fraction of the population participating in the education sector z. Further variables are the size of the population L, capital stock K, a technology parameter A, the discount rate ρ, a productivity parameter in the human capital sector δ, human capital endowment per person h, and $(1-z)$ the fraction of the population employed in final output production Y:

$$\begin{aligned} W &= \max_{c,z} \int_0^\infty e^{-\rho t} u(c(t))L(t)dt \\ \dot{K} &= A\left[\left[1-z(t)\right]h(t)L(t)\right]^\alpha K(t)^{1-\alpha} - c(t)L(t) \\ \dot{h} &= \delta z(t)h(t). \end{aligned} \tag{1}$$

Van Zon and Muysken (2001) extend the basic Lucas model by introducing a health sector. Health aspects enter the model by van Zon and Muysken (2001) in three different ways (p.175): "First, a fall in the average health level of the population may be expected to cause a fall of the amount of effective labor services that the population can supply. Second, the generation of health takes scarce resources that have alternative uses (like the production of output or human capital), while third, a good health may be expected to influence utility directly." To model this, they first define life expectancy $T = \mu g$, with μ being a constant and g being the average health level of the population. They further assume that people are only working until the age of $R < T$, and that at each point t in time a cohort of n persons is born with health level $g(t)$ and education level $h(t)$, that leave the population by sudden death at the age of T.

The representation of the health sector is based on the Romer model from 1990 with the number of specializations in medicine, Ω, depending on the level of human capital h and a scaling factor π, $\Omega = \pi h$, and the supply of labor is measured in efficiency units, $hgnR$, where g is the level of health, n is the number of persons born in one year and R is the number of years the persons spend working. One crucial assumption of the model are decreasing returns to the provision of health services, reflected by $0 < \beta \leq 1$. The essential result of the analysis of the health sector itself is that "a higher share of employment in the health sector will result in a higher equilibrium health level, g^*, while human capital formation as such increases the speed of adjustment toward that equilibrium level". The health production model implies that the increase in health of one generation to the next is given by a function that brings all features of health production together:

$$\frac{dg}{dt} = \Psi \left(\frac{R}{\mu}\right)^\beta \pi^{(1-\beta)} \upsilon^\beta h, \tag{2}$$

where υ is the share of labor employed in the health sector, π is a scaling parameter of the effect of the level of education h and Ψ is a productivity parameter. By taking into account that with increasing medical knowledge and technology the perception of the health level is decreasing, van Zon and Muysken (2001) also include a term for the loss of labor time due to this effect in the health generation function (the second term in the square brackets):

$$\frac{dg}{dt} = \left[\Psi \left(\frac{R}{\mu}\right)^\beta \pi^{(1-\beta)} \upsilon^\beta - \zeta \pi g\right] h. \tag{3}$$

The model essentially becomes the following maximization problem, where the health status g is directly considered in the utility function:

$$\begin{aligned}
W &= \max_{c,z,v} \int_0^\infty e^{-\rho t} \frac{L}{1-\theta} \left[g^\gamma \left(\frac{C}{L} \right)^{(1-\gamma)} \right]^{1-\theta} dt \\
Y &= A \left[(1-z-v) hgnR \right]^\alpha K^{1-\alpha} \\
\dot{K} &= Y - C \\
\dot{h} &= \delta zgh \\
\dot{g} &= \left[\Psi \left(\frac{R}{\mu} \right)^\beta \pi^{(1-\beta)} v^\beta - \zeta \pi g \right] h.
\end{aligned} \tag{4}$$

Utility, represented through a CIES function with γ being the relative contribution of health to intertemporal utility and $1/\theta$ being the intertemporal elasticity of substitution, positively depends on the state of health g and consumption c. Output Y is represented by a Cobb-Douglas production function, where the term in the squared brackets is the fraction of the effective labor force available for final output production, a fraction z is used in human capital production and a fraction v in the health sector. The first term on the RHS of the $\dot{g}$ function represents the increase in the level of health from one cohort to the next due to increased human capital and health service accumulation. The second term represents the negative effect of technical change on health, e.g. stress or a lower perceived level of health due to more cure possibilities.

The model is not analytically solvable. The authors therefore present a graphical solution from which trade-offs between health and human capital accumulation as well as between health and consumption can be seen. The first trade-off is due to the labor force constraint. If more health services are needed, less labor is available for human capital and final output production. This lowers final output production directly and indirectly and hence also economic growth through a lower rate of human capital accumulation. Since health, too, (and not just consumption) increases utility, a higher level of health can offset the utility loss of lower consumption possibilities.

3 Modeling the HDI

The 'new' Human Development Indexis a geometric average of a standard-of-living, an education and a health indicator. This geometric average nicely resembles a utility function that can be used in an endogenous growth model.

Standard-of-living in the HDI is represented by Gross National Income (GNI) per capita. But as Stiglitz *et al.* (2009) emphasize, this is a measure of production rather than of the amount of money people are actually able to spend (disposable income) and so afford a certain standard of living, which is better reflected by consumption per capita. Therefore, the standard-of.living dimension of the HDI is here represented by consumption per capita $c = C/L$ in the utility function, where $L = nT$ is the size of the population, with n being the number of persons per cohort and T being life expectancy. To reflect the population increase due to increasing fertility and decreasing infant mortality rates, n is allowed to change over time. $T = \mu g$ depends on the general health level g of the population and a constant scaling parameter μ. The health index of the HDI is represented by life expectancy at birth.

Total human capital in the model is represented by $P = hL$, with h being the average level of human capital per capita. The average level of human capital directly enters the utility function. In addition, also the share of people employed in the education sector z is included in the utility function. Including both of these measures directly reflects the composite education indicator in the HDI, which is a geometric average (with equal weights) of the expected years of schooling (a quality parameter, here represented by h) and the average years of schooling (a quantity parameter, here represented by z).

The HDI weighs each of the components - standard of living (SoL), health ($Health$), and education (Edu) - by one third[2]:

$$\begin{aligned} HDI &= (SoL)^{1/3}(Health)^{1/3}(Edu)^{1/3} \\ &= c^{1/3}T^{1/3}(z^{1/2}h^{1/2})^{1/3} \\ &= c^{1/3}(\mu g)^{1/3}(z^{1/2}h^{1/2})^{1/3}, \end{aligned}$$

where μ (which is a scaling parameter) and z, the share of the working population in human capital production, are constant in the steady state. It should be noted, however, that, these weights of one third are decided upon by politicians and researchers who developed the HDI. Hence, these weights do not necessarily reflect utility preferences of the population, so that for the resulting constant intertemporal elasticity of substitution (CIES) utility function we use more general preference parameters γ_i[3]:

$$max_{c,z,v}U = \int_0^\infty e^{-\rho t}\frac{L}{1-\theta}\left[c^{\gamma_c}(\mu g)^{\gamma_g}h^{\gamma_h}z^{\gamma_z}\right]^{1-\theta}dt.$$

Output Y is produced using physical capital and healthy human capital, which is employed until retirement age R[4]:

$$Y = A\left[(1-z-v)hgnR\right]^{\alpha}K^{1-\alpha}, \tag{5}$$

with A being a technology parameter and v the share of the working population in the health sector. Naturally, $v + z << 1$. $r = (1-\alpha)Y/K$ is the return to capital and $q = cL/Y$ is the propensity to consume. Physical capital accumulation is given by

$$\dot{K} = \frac{dK}{dt} = Y - C = Y - Lc. \tag{6}$$

[2]Equation (2) in the Statistical Annex of UNDP (2010).

[3]with $\gamma_h = \epsilon_h\gamma_{hz}$, $\gamma_z = \epsilon_z\gamma_{hz}$, $\epsilon_h + \epsilon_z = 1$ and $\gamma_c + \gamma_g + \gamma_{hz} = 1$

[4]A constant value of the retirement age R, which therefore is independent of the health level, is one of the assumptions of van Zon and Muysken (2001). In the context of developing countries having an active labor force smaller than total population can be explained by the high share of labor in the informal sector.

Note, that we disregard the depreciation of physical capital for now. Human capital production is similarly modeled as in Lucas (1988) and in van Zon and Muysken (2001), but starts from the total amount of human capital in the population P. Total human capital is accumulated at rate $\dot{P}$ which depends on z, the share of the population working in the education/research sector[5], the total amount of human capital in the society P, the health level of the population g and a constant δ:

$$\dot{P} = \delta z g P, \tag{7}$$

so that the growth rate of the level of human capital per capita $h = P/L$ is

$$\begin{aligned} \hat{h} &= \left(\frac{\hat{P}}{L}\right) = \hat{P} - \hat{L} = \hat{P} - \hat{g} - \hat{n} \\ \Rightarrow \dot{h} &= h\frac{\dot{P}}{P} - h\frac{\dot{g}}{g} - h\frac{\dot{n}}{n} \\ &= \delta z g h - h\frac{\dot{g}}{g} - h\frac{\dot{n}}{n}, \end{aligned} \tag{8}$$

where we have used $L = nT = n\mu g$. The health production function used here is similar to van Zon and Muysken (2001), but does not depend on the level of human capital. It is possible to disregard the effect of the level of human capital in the health sector because of Baumol's Disease (Baumol, 1993) in this sector.[6] The main essence of this theory is that labor productivity increases in this sector are very limited, if present at all. The health level increases at rate:

$$\begin{aligned} \dot{g} = \frac{dg}{dt} &= \Psi\left(\frac{R}{\mu}\right)^{\beta} \sigma^{(1-\beta)} v^{\beta} - \zeta\sigma g \\ &= \zeta\sigma\left(\phi v^{\beta} - g\right), \end{aligned} \tag{9}$$

with $\phi = \frac{\Psi}{\zeta}\left(\frac{R}{\mu\sigma}\right)^{\beta}$. The current-value Hamiltonian following the maximization of intertemporal utility U, with control variables c, v and z and state variables g, h and K, is given by

$$\begin{aligned} H &= u + \lambda_K \dot{K} + \lambda_h \dot{h} + \lambda_g \dot{g} \qquad (10) \\ &= \frac{L}{1-\theta}\left[c^{\gamma_c}(\mu g)^{\gamma_g} h^{\gamma_h} z^{\gamma_z}\right]^{1-\theta} + \lambda_K\left[A\left[(1-z-v)hgnR\right]^{\alpha} K^{1-\alpha} - Lc\right] \\ &\quad + \lambda_h\left[\delta z g h - \frac{\zeta\sigma h\left(\phi v^{\beta} - g\right)}{g} - h\frac{\dot{n}}{n}\right] + \lambda_g\left[\zeta\sigma\left(\phi v^{\beta} - g\right)\right]. \end{aligned}$$

[5]This does not only include teachers, but also everyone else who is involved in human capital production ranging from primary and high school students to doctoral candidates and researchers in the R&D sector. Note that in the following text the description of this is often abbreviated to 'labor share in education' or the like.

[6]Additionally, including h in $\dot{g}$ results in undesirable out-of-steady state behaviour, which would give problems for the transitional dynamics presented later in this chapter.

For each of the co-state variables, the corresponding growth equations can be calculated either from the first order conditions of the control variables (Equations (30) - (32) on p. 25 in the Appendix), by solving these for the respective co-state variable λ_K, λ_h and λ_g, and then taking the derivative with respect to time, and from the first order conditions with respect to the state variables, that is from Equations (36) - (38). Using these results in combination with Equations (34) and (35), the co-state variables and their growth rates, which are displayed in Appendix A.2, can be eliminated from the system.

The final system of growth equations for all growth rates in variables g, v, z, r, q, $\hat{h}$, $\hat{q}$, $\hat{L}$, $\hat{n}$, $\hat{A}$ and the model parameters is

$$
\begin{aligned}
\hat{K} &= \frac{r(1-q)}{1-\alpha} && (11)\\
\hat{g} &= \zeta\sigma\frac{v^\beta\phi - g}{g} && (12)\\
\hat{h} &= gz\delta - \hat{g} - \hat{n} && (13)\\
\hat{c} &= \frac{1}{1-\gamma_c(1-\theta)}\left[(1-\theta)\left(\hat{g}\gamma_g + \hat{h}\gamma_h + \hat{z}\gamma_z\right) + r - \rho\right] && (14)\\
\hat{z} &= f(g, v, z, q, \hat{g}, \hat{h}, \hat{Y}, \hat{q}) && (15)\\
\hat{v} &= f(g, v, z, q, \hat{g}, \hat{h}, \hat{Y}, \hat{q}, \hat{c}, \hat{n}) && (16)\\
\hat{Y} &= \hat{A} + \hat{K} + \frac{-\alpha}{(1-z-v)}\left[v\hat{v} + z\hat{z} - gz\delta(1-z-v)\right] && (17)\\
\hat{r} &= \hat{Y} - \hat{K} && (18)\\
\hat{q} &= \hat{c} + \hat{L} - \hat{Y} && (19)\\
\hat{L} &= \hat{g} + \hat{n}. && (20)
\end{aligned}
$$

The proportional growth rates of z, i.e. $\hat{z} = Gz$, and v, i.e. $\hat{v} = Gv$, are non-linear rather long expressions and therefore not displayed here, but available upon request from the author.

3.1 Long-run balanced growth path

The long-run balanced growth path is defined here as the state of the system where the labor shares of physical capital production and in the education and health sectors are constant, i.e. $(1-z-v)$, z and v are constant. Furthermore, consumption c, physical and human capital, K and h respectively, grow at constant proportional rates. Following the reasoning of van Zon and Muysken (2001), we assume that the health level is constant along the long-run balanced growth path, so that $\hat{g} = \dot{g}/g = 0$, and hence $g^\star = \phi v^\beta$, implying that

$$\hat{h} = \dot{h}/h = \delta z g^\star = \delta z \phi v^\beta. \quad (21)$$

Life expectancy T as well as total population L are constant under the additional assumption that $\hat{n} = 0$, implying that is there is no exogenous fertility increase. Both the return to capital r and the savings rate $s = 1 - q$ are assumed to be constant in the long-run state of the economy. However, development in Sub-Saharan Africa seems to be far removed from such a long-run balanced growth path. Therefore, the long-run balanced growth analysis shown here only serves as a basis for the analysis of the system's transitional dynamics in Section 3.2.

The system of growth equations that is solved for the growth rates of c, h, K, Y, and the variables r and g consists of expressions (11), (14), (15), (17), (18) and (19). This gives

$$\kappa = \frac{\dot{c}}{c} = \frac{\dot{h}}{h} = \frac{\dot{K}}{K} = \frac{\dot{Y}}{Y} = \frac{\rho(q-1)}{E} \tag{22}$$

$$r = \frac{\rho(\alpha-1)}{E} \tag{23}$$

$$g^{\star} = \frac{1}{\delta z} \frac{(q-\alpha)\beta(z\alpha\gamma_c + q(-1+v+z)\gamma_z)\zeta\sigma\rho}{v(q-\alpha)\alpha\gamma_c\rho + [(v+v\beta+\beta)\gamma_c\alpha + \beta q(1-z-v)(\gamma_z-\gamma_g)]\,\zeta\sigma E} \tag{24}$$

with $E = \alpha - (1-\theta)(\gamma_c+\gamma_h) + q\left((1-\theta)(\gamma_c+\gamma_h) - 1\right)$.

Here, consumption per capita, average human capital, physical capital and final output production all grow at the same rate, κ. The final system of equations (*growthsys*) to solve for z, v, κ and $q = 1 - s$ are equations (13), (16), (22) and (24). The parameters in this system are the rate of time preference ρ, the intertemporal elasticity of substitution $1/\theta$, the relative contributions of the HDI components to intertemporal utility γ_i, productivity parameter δ of the education sector, a parameter $0 < \beta \leq 1$ reflecting decreasing returns in health generation, and a set of positive constants ϕ, ζ and σ that influence productivity in the health sector. The system of equations (13), (16), (22) and (24) is solved numerically[7] for different combinations of parameter values. To test the robustness of the system, the parameters are changed one-by-one for a sensitivity analysis. The parameter value ranges are displayed in columns 'minimum value' and 'maximum value' of Table 1. The initial set of parameter values is displayed in column 4 of the table. The initial values are taken from the literature, see for example the overview in Table 3.2 in Steger (2000). The transversality conditions as displayed in Appendix A.3 hold for this initial set of parameter values.

The relative contribution of consumption, education and health to intertemporal utility is most likely not equal. Most endogenous growth model assume that only consumption is part of the utility function, however - as also argued in van Zon and Muysken (2001) - health, too, has a significant influence on perceived well-being as well. Education does contribute to general utility, although for African and other developing countries education is relatively less important, considering that survival first of all depends one's health status and on the consumption of food and fresh water and basic living supplies such as clothing and housing. In this context Steger (2000) introduces a subsistence consumption utility function, assuming that there exists a minimum level of consumption below which nobody can survive. This specification could also be included in the present model, but since all components enter the utility function in a multiplicative way, while the health level is explicitly considered,

[7] Using *Mathematica*'s FindRoot routine.

consumption without health is not possible. The relative contributions of consumption, education and health have been set to $\gamma_c = 0.6$, $\gamma_z + \gamma_h = 0.05 + 0.05 = 0.1$ and $\gamma_g = 0.3$ respectively. Table 3 displays results for other values of the relative contributions that will be described in more detail later.

Figures 2 to 4 display results for the balanced growth rate κ (in all graphs on the vertical axis), the share of human capital in the education sector z and in the health sector v (in the graphs to the left), the share of human capital in the production sector $(1 - z - v)$ and the long-run savings rate $s = 1 - q$ in the graphs in the middle, and the long-run health level $g^\star$ and rate of return to capital r on the right. Low values of the changing parameter are displayed in dark (red) and higher values in lighter color (yellow) and vice versa for changes in δ and σ. The black dots indicate the solution corresponding to the initial choice of parameter values. Note that for some graphs the distance between two consecutive dots are are changing. This reflects a non-linear dependence of the variables on the corresponding parameter. In addition, sometimes a non-linear relation between the variables displayed in one graph becomes apparent. This is for example shown in the graphs of κ, $g^\star$ and r of a changing ρ or θ in Figure 2 (on the right).

The solution of the long run equilibrium based on the initial choice of parameter values results in a labor distribution across final production, education and health of 60%, 30% and 10%, respectively. The corresponding growth rate of physical and human capital, consumption and production, κ, is 0.02, the savings rate 10% (note that the axis label for 10% is 0.1), the rate of return to capital 8.5% and the general health level 0.69. This is a very abstract number, but it could be interpreted as the share of the population being healthy. However, $g^\star$ changes proportionally with parameter ϕ, so that the choice of ϕ directly influences the value of $g^\star$. Recall that life expectency is $T = \mu g$. Now for $\mu = 100$, life expectency in the long run equilibrium would be 69 years, which is approximately the global average life expectancy in 2010.

Both ρ and θ have a direct impact on the intertemporal utility, α on final output production, δ on human capital production and through the latter channel also on final output and intertemporal utility. β, ϕ, ζ and σ enter the system through the health sector and will therefore also have an influence on human and physical capital accumulation as well as intertemporal utility.

Changing ρ

According to Grossmann *et al.* (2010), a typical value for the rate of time preference ρ is 0.02. Steger (2000) reports values between 0.02 and 0.1 for ρ. The initial value chosen here is 0.05 and for the sensitivity analysis ρ increases from 0.01 to 0.07 in steps of 0.001. This increase in ρ, that is a rise in the preference for present utility over future utility, results in a decreasing long-run savings rate s (by -14pp[8]) as consumption today is valued more than consumption in the future, and a decreasing long-run growth rate κ (by -2pp). The share of human capital in the education sector

[8]pp = percentage points

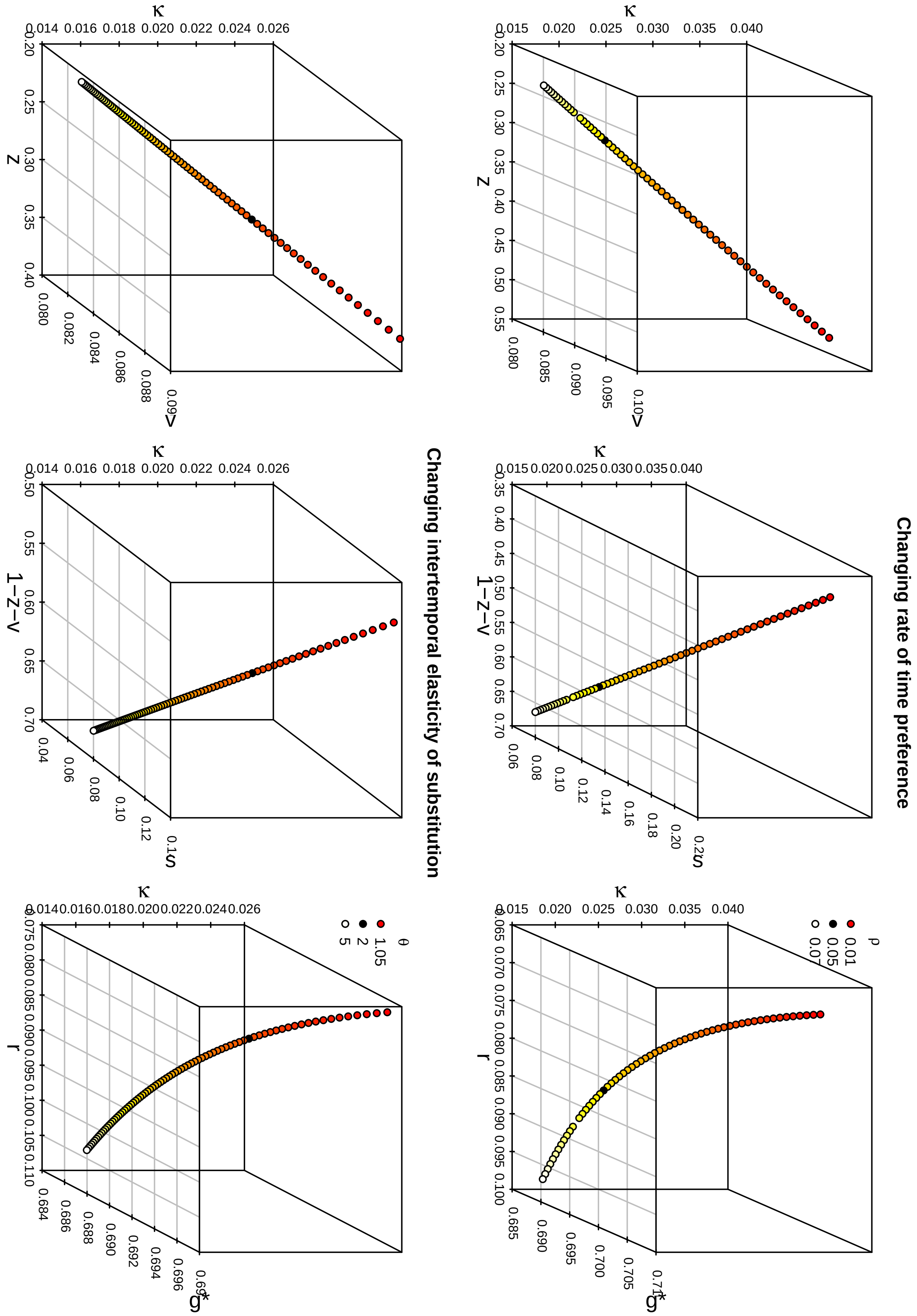


Figure 2: Changing ρ and θ

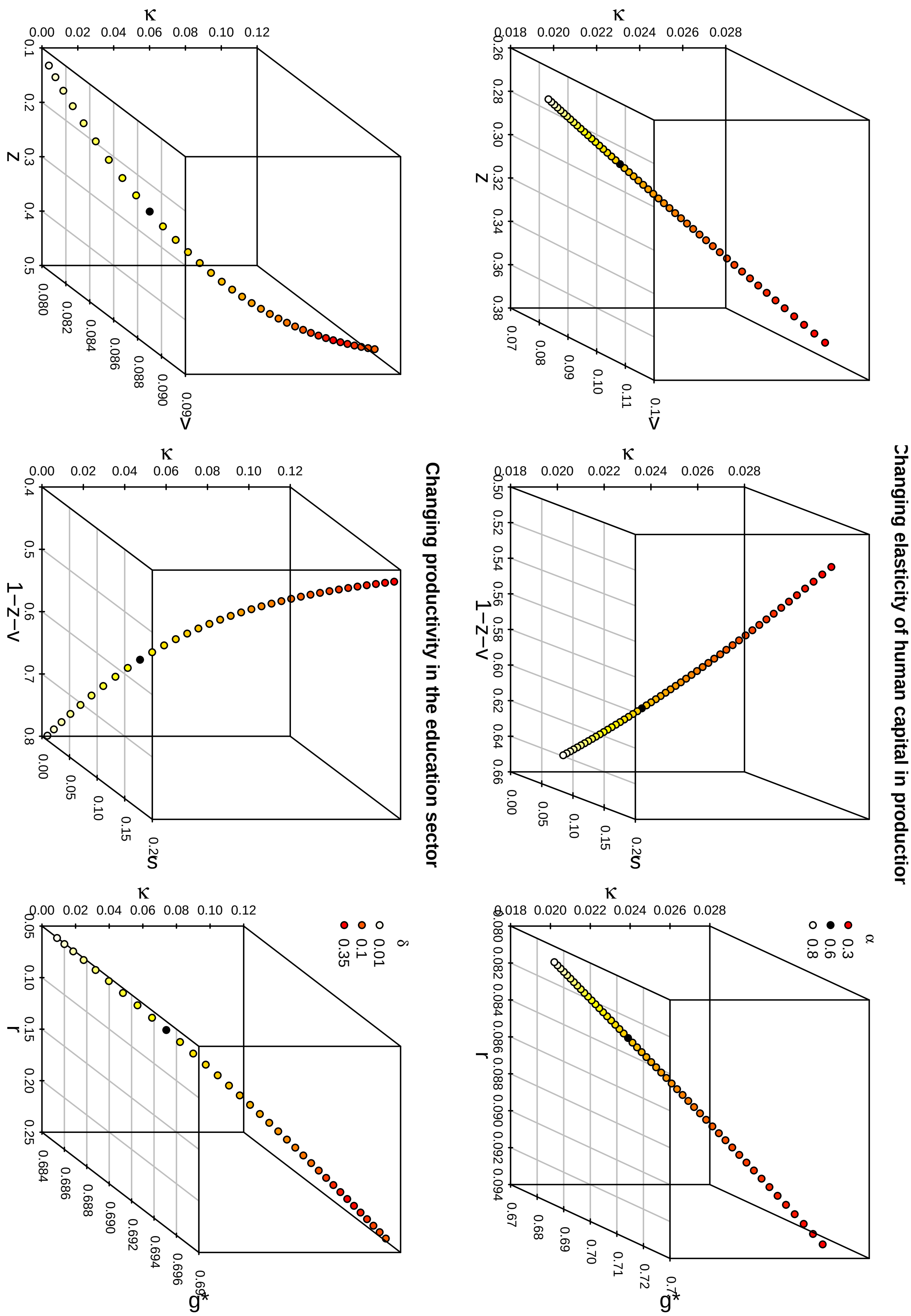


Figure 3: Changing α and δ or ϕ

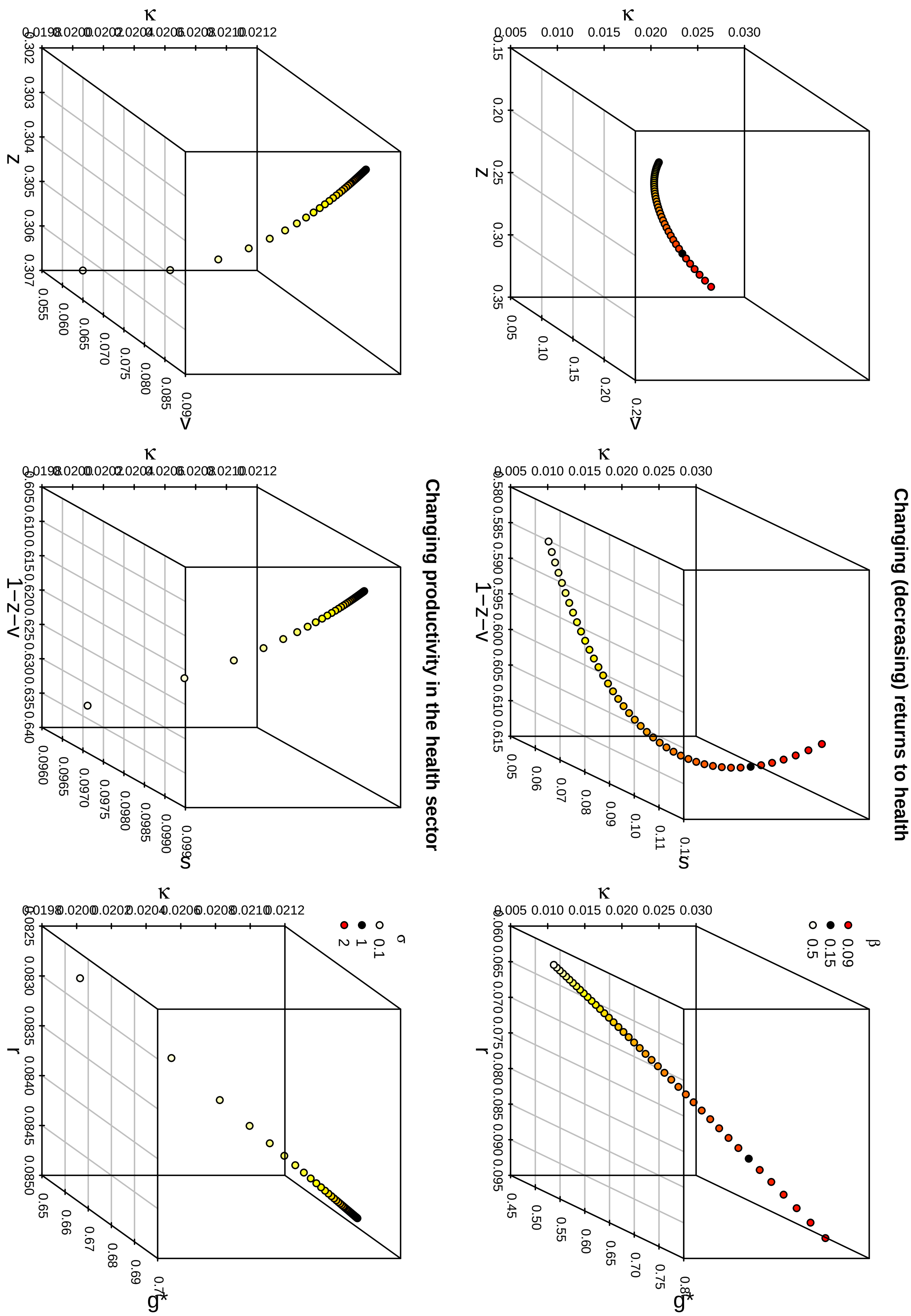


Figure 4: Changing β and σ or ζ

Table 1: Set of parameter values

Parameter	Minimum value	Initial value	Maximum value	Step size
ρ	0.01	0.05	0.07	0.001
θ	1.05	2.00	5.00	0.050
α	0.30	0.60	0.80	0.010
δ	0.01	0.10	0.35	0.010
β	0.09	0.15	0.50	0.010
ϕ		1.00		
σ	0.10	1.00	2.00	0.100
ζ		1.00		

z is cut in half from more than 50% to 25%, while the share in the production sector increases from 39% to 67%. The share of human capital in the health sector, v, only changes from 10% to 8%. $g^\star$ is slightly reduced, while r increases by 3pp.

Changing θ

Guvenen (2006) reviews the discussion on the intertemporal elasticity of substitution, $1/\theta$. Some economists, such as Hall (1988), believe that it is close to zero. Others, however, claim that it is closer to one, with a $\theta = 2$ being high according to Lucas (1990). Ortigueira and Santos (1996) choose $\theta = 1.5$. Here, the initial value is $\theta = 2$, resulting in an intertemporal elasticity of substitution of 0.5. For the sensitivity analysis θ is increased from 1.05 to 5.0, resulting in a range of 0.95 to 0.2 of the intertemporal elasticity of substitution (IES). This decrease in the IES results in a significantly decreasing share of human capital in human capital production z, and a corresponding substantial increase in the share of human capital $(1 - z - v)$ in final output production Y. An increasing value of θ furthermore implies a decreasing value of the balanced growth rate κ, the long-run savings rate s and the steady state health level $g^\star$, as shown in the lower three graphs in Figure 2.

In economies/societies where the future is valued higher (as reflected by low values of ρ and θ), less labor is employed in final output production and more in the education sector. The propensity to consume is lower, i.e. the savings rate higher and the growth rate is higher as well, compared to economies/societies with a preference for present utility over future utility.

Changing α

The relative contribution of healthy human capital to production Y (measured by the partial output elasticity of healthy human capital) is α, while the relative contribution of physical capital K is $(1 - \alpha)$. In the top plots in Figure 3, α increases from 0.3 to 0.8 in steps of 0.01. This range covers the plausible parameter range rather well, as the usual value of α is assumed to be about 0.6, see for example Ortigueira and

Santos (1996) or Steger (2000). Increasing α from 0.3 to 0.8 implies decreasing shares of human capital in both the education and the health sector. A high value of α, reflected by the light dots at the lower right end of the curves in the top plots of Figure 3, implies that labor is relatively more important than capital in final output production. Lower values of α (moving up the curves from the light dots in the lower corners to the darker dots at the top) imply that labor becomes relatively less important, e.g. when moving from an agricultural based economy to a more capital based economy, the share of human capital (that is the share of labor) employed in the final output sector is lower, while the shares employed in education and health and the long-run health level increase. As the contribution of capital (1-α) grows, the savings rate is higher as well, as is r, the rate of return to capital.

Changing δ or ϕ

δ and ϕ are productivity parameters in the education and health sector, respectively. As δ and ϕ only enter *growthsys* as a product $\delta\phi$, the reaction of the system to changes in either one of these parameters is exactly the same. For this analysis we have normalized ϕ to 1. The reactions induced by changes in δ, as explained below, are also valid for proportional changes in ϕ. The only difference is the direct proportional impact of ϕ on the health level $g^\star$.

The higher δ, the higher the productivity in the education sector and the faster the increase in the level of human capital and hence the growth rate. With increasing δ a higher share (z) of the working population is employed in the education sector, at the expense of the share $(1 - z - v)$ of people employed in the production sector. For low values of δ, that is low productivity in the education sector reflected by the light (white-yellow) dots in the lower left/right in the plots in the bottom of Figure 3, the share of labor in final output production is high, the savings rate s and the rate of return to capital r are low as is the growth rate κ. As the productivity in the education sector increases, the share of labor in the other two sectors decreases, resulting in a higher long-run health level. The rate of return to capital and the savings rate increase as well.

Changing β

Decreasing returns in the health sector are reflected by $0 < \beta < 1$. As β increases, v^β decreases (for a constant v). The share of labor in the health sector v increases from 0.05 to 0.23 as β increases from 0.09 to 0.5. Still, as ϕ remains constant, the increase in v does not offset the increasingly decreasing returns in the health sector, so that $g^\star = \phi v^\beta$ decreases as shown in Figure 4. The share of labor in the other two sectors however decreases sharply, as do the savings rate and the rate of return to capital. What is not considered here is that ϕ also depends on β. The effect of a changing β can therefore be amplified or reduced by its effect on ϕ.

Changing σ or ζ

σ and ζ influence the variety of products, i.e. a variety of treatment methods or, more genera, the extent of medical knowlegde, and the general productivity in the health sector. Just as $\delta\phi$, $\sigma\zeta$ only enter *growthsys* as a product, hence inducing the same reactions by the system. In Figure 4 ζ is constant at a value equal to 1, while σ increases from 0.1 to 2. The results also apply to $\sigma = 1$ and a changing ζ. For low values of σ, the effect of increasing it by 0.1 is large, as reflected by the large gaps between two consecutive dots in Figure 4. For higher values of σ, the effect becomes much smaller. An increase in σ has a positive effect on κ, v, r and g. The savings rate, and the share of labor in the education and final output sectors on the other hand slightly decrease.

Relative valuation of consumption, education and health

As for all equilibrium solutions in endogenous growth models, the results displayed here are based on assumptions that are - if at all - only applicable to long-run balanced growth paths. The parameter values chosen for the empirical analysis are based on those found in the literature or they are simply normalized to one as in the case of ϕ, σ or ζ. As in van Zon and Muysken (2001), a positive correlation between the savings rate s and the balanced growth rate κ is apparent, confirming the necessary increase in the savings rate to keep the capital-output ratio at a constant level in order to sustain higher growth. This model does not confirm the negative correlation that van Zon and Muysken (2001) found between the share of human capital in education z and in health v as displayed in Table 1 on p. 180 of van Zon and Muysken (2001). This can be explained by the main difference of the model presented here to the van Zon and Muysken model, which is the inclusion of education in the utility function. This inclusion reflects a higher valuation of both the quality of human capital h and the quantity of human capital z, which makes z relatively more important in the model. We have a negative correlation between labor in the health sector v and labor in the final output production sector $(1 - z - v)$. When changing the relative preferences of the different components in the utility function, i.e. the γ's as displayed in Table 3, changes in γ_z have a higher impact on the overall system than changes (of the same order) in the other variables, as will be explained in the next paragraphs. The share of labor in the education sector is positively correlated with the growth rate and the savings rate, underlining the importance of a good education for sustainable long-run growth. The share of labor in the health sector is positively correlated with the long-run growth rate as long as the decreasing returns in the health sector are maintained.

Table 3 displays equilibrium results of this model for different specifications of the relative contribution of consumption, education and health in the utility function. The upper part of the table shows how the model outcomes change when increasing the relative contribution of education and health from values close to zero to values that are slightly higher than their initial values. The relative valuation of education

Table 2: Responses to parameter changes

	κ	z	v	$1-z-v$	s	r	$g^\star$
ρ	$-$	$-$	$-$	$+$	$-$	$+$	$-$
θ	$-$	$-$	$-$	$+$	$-$	$+$	$-$
α	$-$	$-$	$-$	$+$	$-$	$-$	$-$
δ or ϕ	$+$	$+$	$+$	$-$	$+$	$+$	$+$
β	$-$	$-$	$+$	$-$	$-$	$-$	$-$
σ or ζ	$+$	$-$	$+$	$-$	$+$	$+$	$+$

and health remains the same. At first the relative contribution of consumption to intertemporal utiliy is almost one, hence resembling a common assumption of endogenous growth models. The corresponding growth rate is very low, as are the shares of labor in both education and health sector. More than 90% of the population is working in final output production and the propensity to consume q is very high, resulting in a savings rate s of only 2.9%. The return to capital is relatively low, as is the health level $g^\star$. As education and health contribute more to intertemporal utility, the corresponding labor shares, the growth rate κ, the savings rate, the rate of return to capital and the health level increase significantly. Labour in final output production decreases with the relatively lower contribution of consumption to utility.

In the second part of the table the relative contribution of consumption to utility also decreases from values close to one to one third, while the relative contributions of education ($\gamma_z + \gamma_h$) and health (γ_g) are equal and increase from values close to zero to one third. For $\gamma_h = \gamma_z = 0.005$ and $\gamma_g = 0.01$ the growth rate κ is less than 0.3% and savings rate s and health level $g^\star$ are very low as well, with values of 2% and 0.49, respectively. 6% of the population are busy with human capital production (z) and less than one percent is employed in the health sector (v). As the relative contribution of education and health to utility rise, so do the corresponding labor shares. When $\gamma_h + \gamma_z = \gamma_g = \gamma_c = 1/3$ (the last row in the second part of Table 3) the growth rate amounts to almost 4%, and the savings rate, the rate of return to capital and the value of the health level are substantially increased as well. The share of labor in education and health have increased by a factor 10 or more, while the share of labor in final output production dropped from 93% to 35%. It should be noted, that an equal valuation of education, health and consumption, as suggested by the weighting scheme of the HDI, implies a labor share of more than 50% in the education sector, while the share of labor in the health sector remains unchanged (at about 8 to 9%) compared to the initial relative contributions $\gamma_c = 0.6$, $\gamma_h = \gamma_z = 0.05$ and $\gamma_g = 0.3$. A relatively higher contribution of education and a relatively lower contribution of consumption to utility result in a distribution of labor across sectors that cannot be observed in the real world[9]. However, if the valuation of 1/3, 1/3, 1/3 as suggested

[9]For example, the sum of labor in the German education (2.3 million) + R&D sector (0.2 million excluding researchers employed in private industrial enterprises or businesses) + the part of the population (above the age of 10) that is currently educated (11 million) is about 29% of the total population that is working or in education (36 million + 11 million). The number of persons employed

Table 3: Relative valuation of consumption, education and health

γ_c	γ_h	γ_z	γ_g	κ	z	v	1-z-v	s=(1-q)	r	$g^\star$
Increasing γ_h, γ_z and γ_g to their equilibrium levels										
0.960	0.005	0.005	0.030	0.0043	0.0794	0.0156	0.9049	0.0292	0.0584	0.5360
0.920	0.010	0.010	0.060	0.0072	0.1244	0.0260	0.8496	0.0451	0.0639	0.5784
0.840	0.020	0.020	0.120	0.0112	0.1806	0.0419	0.7775	0.0633	0.0709	0.6214
0.680	0.040	0.040	0.240	0.0178	0.2644	0.0709	0.6647	0.0882	0.0806	0.6723
0.600	0.050	0.050	0.300	0.0210	0.3030	0.0859	0.6111	0.0991	0.0846	0.6919
0.450	0.100	0.100	0.350	0.0306	0.4369	0.0935	0.4696	0.1257	0.0975	0.7008
Increasing γ_h, γ_z and γ_g: equal importance of education and health										
γ_c	γ_h	γ_z	γ_g	κ	z	v	1-z-v	s=(1-q)	r	$g^\star$
0.980	0.005	0.005	0.010	0.0029	0.0593	0.0084	0.9322	0.0208	0.0558	0.4887
0.960	0.010	0.010	0.020	0.0056	0.1048	0.0149	0.8803	0.0366	0.0610	0.5320
0.920	0.020	0.020	0.040	0.0089	0.1576	0.0223	0.8200	0.0530	0.0673	0.5654
0.840	0.040	0.040	0.080	0.0137	0.2295	0.0324	0.7381	0.0724	0.0758	0.5979
0.680	0.080	0.080	0.160	0.0217	0.3420	0.0481	0.6099	0.0984	0.0882	0.6343
0.333	0.167	0.167	0.333	0.0395	0.5766	0.0809	0.3425	0.1447	0.1093	0.6857
Decreasing γ_H, increasing γ_z: decreasing importance of education quality relative to education quantity										
γ_c	γ_h	γ_z	γ_g	κ	z	v	1-z-v	s=(1-q)	r	$g^\star$
0.333	0.328	0.005	0.333	0.0333	0.4972	0.0696	0.4332	0.1265	0.1054	0.6705
0.333	0.323	0.010	0.333	0.0336	0.5011	0.0701	0.4288	0.1273	0.1057	0.6712
0.333	0.313	0.020	0.333	0.0342	0.5083	0.0711	0.4206	0.1287	0.1063	0.6727
0.333	0.293	0.040	0.333	0.0352	0.5213	0.0729	0.4058	0.1313	0.1073	0.6752
0.333	0.253	0.080	0.333	0.0369	0.5428	0.0760	0.3812	0.1359	0.1085	0.6794
0.333	0.167	0.167	0.333	0.0395	0.5766	0.0809	0.3425	0.1447	0.1093	0.6857
0.333	0.080	0.253	0.333	0.0414	0.5998	0.0843	0.3158	0.1526	0.1085	0.6901
0.333	0.040	0.293	0.333	0.0421	0.6082	0.0856	0.3062	0.1561	0.1078	0.6916
0.333	0.020	0.313	0.333	0.0424	0.6119	0.0862	0.3020	0.1579	0.1073	0.6923
0.333	0.010	0.323	0.333	0.0425	0.6136	0.0865	0.2999	0.1587	0.1071	0.6927
0.333	0.005	0.328	0.333	0.0426	0.6144	0.0866	0.2990	0.1592	0.1070	0.6928

by the HDI was indeed correct, that is education and health were equally important as consumption, the current distribution of labor is far from optimal. In fact, more resources should be allocated to human capital production and health.

The lower part of Table 3 assumes an equal valuation of consumption, education and health as has just been discussed. To identify which of the two education variables, the quality-related level of human capital h or the quantity-related share of the working population busy with human capital production z, brings about the changes in the labor distribution, γ_h drops from (1/3-0.005) to 0.005, while γ_z increases from 0.005 to (1/3-0.005). The effects on all model outcomes, but the share of labor in final output production, are positive. This means that a higher valuation of education

in the German health sector is 4.5 million and, thus, about 9.6% of the total population that is working or in education. Employment data for 2010 published in the System of National Accounts, DESTATIS (2011) and data on education for 2008 is published in BMBF (2010). The corresponding shares in African developing countries are very likely to be significantly lower than that.

quantity[10] relative to education quality increases the number of people occupied with human capital production and health generation, and implies a higher growth, savings and return to capital rates as well as a higher value of the health level. This result corresponds to Millennium Development Goal 2 Target 2 A: Ensure that, by 2015, all children everywhere, boys and girls alike, will be able to complete a full course of primary schooling.

3.2 Transitional dynamics

Steger (2000) emphasizes the importance of interpreting the development of a country as its transition toward the steady state. Jonathan Temple (2003, p. 509) regarding the same issue (also cited in Trimborn *et al.* (2006)): "Ultimately, all that a long-run equilibrium of a model denotes is its final resting point, perhaps very distant in the future. We know very little about this destination, and should be paying more attention to the journey."

The analysis of transitional dynamics in this extended version of the Lucas model is based on an algorithm provided by Trimborn *et al.* (2008) and implemented in Mathematica by Manuel Bichsel (2011). For the calculations the system of ten growth equations (11) to (20) is reduced to a system in six variables, which are the state variables K, h and g and the control variables c, z and v. This dynamic system is then first transformed into its reduced form and then into its equivalent scale adjusted form, which describes the evolution over time of the growth rates relative to their long-run growth values.

The values for an equilibrium balanced growth path solution, from now on called BGP solution, were chosen given the results for the long run equilibrium analysis of the previous section, that is the initial parameter values displayed in Table 1. The values chosen for the remaining model variables are normalized to one, with the exception of the growth rate of technology $\hat{A}$ and the exogenous population increase $\hat{n}$, which are set to zero. Note that from *growthsys* (as defined on p. 9) we obtain $\kappa^\star$, $z^\star$, $v^\star$ and $q^\star$. From Equations (23) and (24) we have values for $r^\star$ and $g^\star$. Physical capital K is normalized to 1. Initial values for h and c can then be calculated by combining equations for the rate of return to capital $r = (1-\alpha)Y/K$ and the propensity to consume $q = cL/Y$.

This BGP solution has two stable (negative) eigenvalues and one equal to zero, or at least very close to zero, as is common for scale adjusted systems (Steger, 2011): λ_K=1.0465, λ_h=-1.01712, λ_g=0.175334, λ_c=-0.0920515, λ_z=0.0596788, λ_v=$3.02041 * 10^{-8}$.

To visualize the transition path, the initial values of the state variables have been set equal to fractions of their BGP solution. These fractions are 0.01, 0.1 and 0.5 for capital K, and 0.3, 0.5 and 0.7 for human capital h and health level g. As it

[10] Recall that this does not only include teachers, but also everyone else who is devoted to human capital production ranging from primary and high school students to doctoral candidates and researchers in the R&D sector.

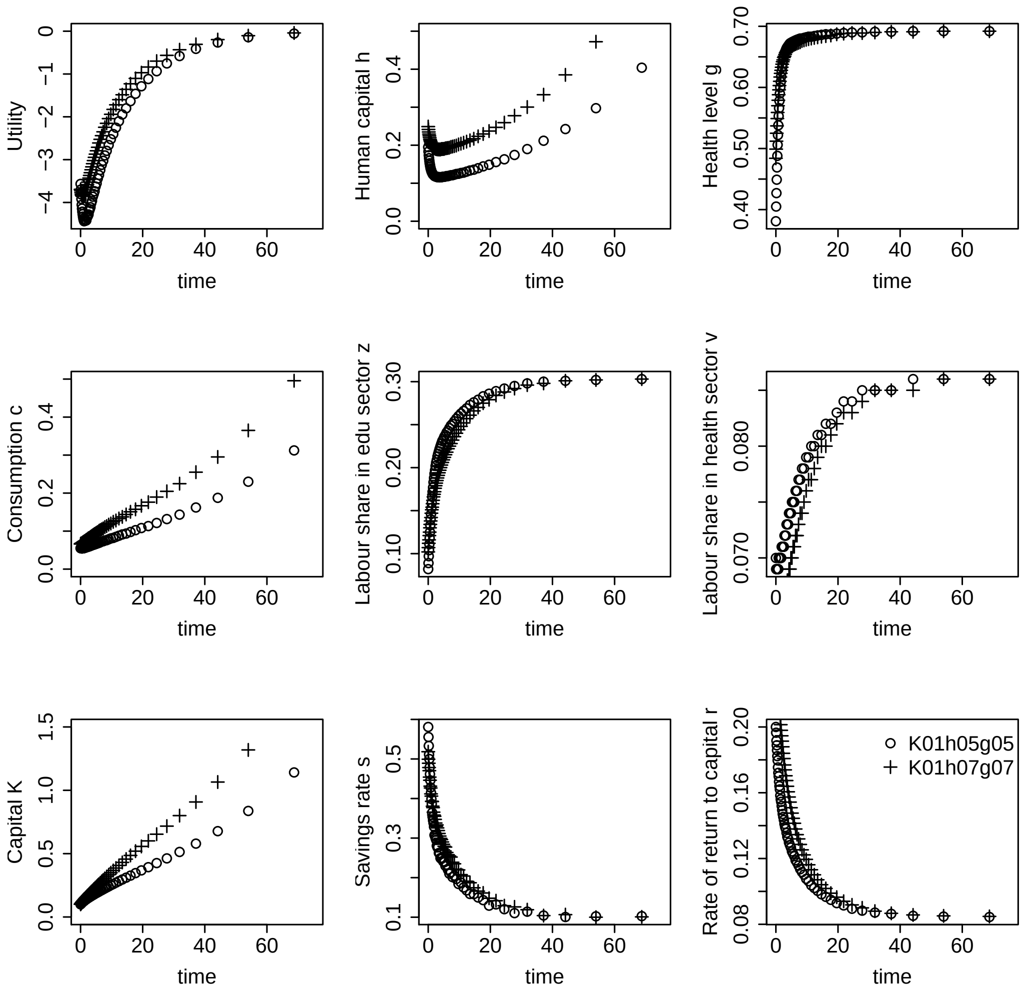


Figure 5: Shocking K, h and g

is very unlikely that only one of the variables is out of its long run equilibrium, a more thorough analysis is conducted for a joint shock on K, h and g and the corresponding transition paths are displayed in Figures 5 through 6. The analysis of the individual shocks, which serves the purpose of identifying the individual effects, is available from the author upon request. For all shocks it proved to be the case that those variables that have definite long run equilibrium values, such as g, z, v, s and r always converge to their respective values. For the remaining variables we have observed that their growth rates converge to the corresponding long run balanced growth rate κ. In addition, the transition paths of the present value of instantaneous utility $u = e^{-\rho t} L/(1-\theta) \left[c^{\gamma_c} h^{\gamma_h} z^{\gamma_z} g^{\gamma_g}\right]^{1-\theta}$ are displayed.

In case of developing countries, all physical and human capital as well as the health status of the population are far below something that could be a long run equilibrium. The discussion in Steger (2000) clearly shows that his stylized facts of economic growth in developing countries could be modeled as being part of the transition process toward the long-run equilibrium.

Figures 5 and 6 show the transition paths of an economy, which has very low initial levels of physical capital, human capital and health, toward the future balanced growth equilibrium. For the African developing countries the current state of the European countries could be seen as this future resting point of the economy. The 'shocks' chosen for this exercise are a value of capital K of 0.1 of the BGP solution value (which was normalized to one), a human capital endowment of 0.5 or 0.7, which is about the ratio of the share of literate people in African countries to literate people in Europe[11]. Life expectancy in Europe is about 80 years, while that in Sub-Saharan Africa only is about 50 years, resulting in a ratio of 0.6. Hence, for the initial health level the two fractions we have chosen values $g_0 = 0.5g^\star$ and $g_0 = 0.7g^\star$. Results are displayed for two cases: case 0.5, with fractions of the BGP solution values 0.1 for K and 0.5 for both h and g, and case 0.7, with 0.1 for K and 0.7 for both h and g. Figure 5 plots the transition paths for both cases 0.5 (K01h05g05, circles) and 0.7 (K01h07g07, plus-signs).

Using the algorithm of Trimborn *et al.* (2008) it is possible to rescale the transition path to reflect actual time. Figure 5 displays the first 75 years of the transition phase on the horizontal axis, as most of the adjustment takes place during this period. The health level (top right plot) strongly increases from the start, resulting in an initial decrease in the level of human capital (top center plot). The reason is the very low health level, which directly influences the rate of human capital accumulation. For values of g that reduce the overall input in the education sector δzg to values relatively lower than the rate of health increase, the level of human capital (that is the quality indicator of education) decreases even further. Only for a sufficient level of average health g combined with a higher share of the population involved in human capital accumulation z, an actual increase in average human capital h is possible, compare Equation 13.

Both labor shares in education and health increase toward their steady state values, somewhat slower for the smaller shock, that is higher initial values (0.7) for h and g, marked by the plus-signs in Figure 5. The share of the population involved in human capital accumulation increases strongly. Recall that this result relates to MDG 2, which targets a 100% primary school enrollment rate, indicating the need for more education. Both the savings rate and the rate of return to capital are very high initally and then decrease toward their steady state values, again at a slower rate for the 0.7 case.

For low levels of capital, investment returns are usually high as the capital stock increases. For countries with high capital stocks, the return to capital is lower reflecting decreasing marginal returns to capital. High capital returns tend to induce high savings rates, which is also the case here. Still, when following Steger's line of argumentation (Steger, 2000), saving is not possible for consumption levels below sub-

[11]The median share of literate people in Sub-Saharan Africa over the past three decadesis 58%. Literacy in Europe is almost 100%, so that choosing 0.5=50%/100% and 0.7=70%/100% seems to be an appropriate approximation of the possible range.

sistence consumption. However, in this model the effect of the capital shock dominates the effect of education and health shocks, which induce negative saving rates.

Consumption c and capital K, displayed in the center and bottom plots of the left column, grow from the start. While capital accumulation is initially very high and then slows down after about the first five years, consumption only increases slowly at first. Instantaneous utility is derived from consumption, education quality h and quantity z and the health level. The initial decline in human capital per capita has a significant influence on instantaneous utility as can be seen from the top left plot in Figure 5. However, as soon as human capital accumulation increases again, utility follows. In contrast to the result of the sensitivity analysis in Table 3 in the previous section regarding the relative valuation of consumption, both education indicators and health (where h only had a limited impact on the long run growth rate), h does have a significant impact on utility, as can be seen here.

The two plots at the top of Figure 6 display the transition paths toward the long-run constant consumption-output-ratio q and the long run constant capital-output ratio $(1-\alpha)/r$. The different shades of gray indicate the time, the first years in light gray, then getting darker. For both cases 0.5 and 0.7 output is relatively higher than both consumption and capital. The long run growth trajectories, i.e. the straight lines, are reached from below, indicating that most effort is put into building up the capital stock. However, the immediate increase in health is even stronger, which reduces both human capital and physical capital endowments per capita as the population growth is too high[12].

This effect is displayed in the graph in the middle of Figure 6, which shows the movements of human capital (circles and plus signs) and per capita physical capital (dots and stars) relative to the average health level. Per capita endowments of both types of capital first decline as the health level increases, but then start increasing toward the long-run growth trajectory (which in this case is horizontal as we assumed a constant long-run health level). For human capital the turning point is very close to the long run health level indicating the importance of a good health for education and hence productive human capital.

The lower two graphs in Figure 6 display the evolution of the labor shares over the first 50 years. At that point in time the deviation from the long run equilibrium values is less than 1% for both initial values 0.5 and 0.7 of health and education. The labor shares are only 8.2% and 10.2%, respectively, for education at the start, and 7% and 6.1%, respectively for education. In the 0.5 case, the labor share in health is actually higher in the beginning than in the 0.7 case. This is because labor is immediatly allocated to the health sector in order to increase health if the initial health level is insufficient. This stresses the importance of a sufficiently high health level of the population for achieving growth in the other sectors.

Analyzing the entire transition path provides valuable insights into the dynamics of the modeled economy. Whether, and if so, how these dynamics change with the underlying parameters is an interesting question, which is left for future research.

[12] Recall that total population directly depends on the health status: $L = n\mu g$.

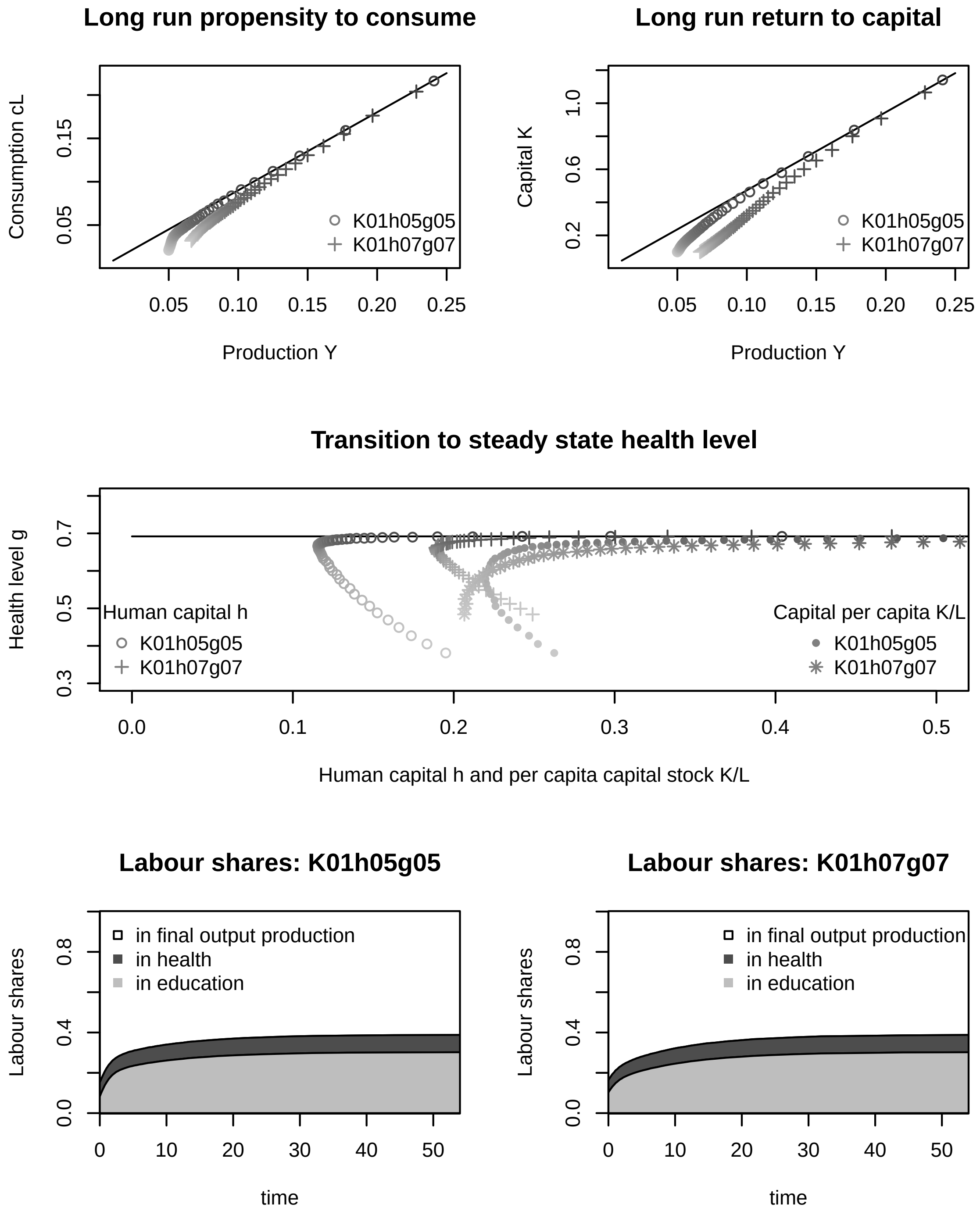


Figure 6: Shocking K, h and g: Transition paths

4 Model implications and future research

Endogenous growth models are a standard way of analyzing a country's long term development. This chapter has introduced a growth model that is in line with current discussions on the measurement of the welfare of nations. Both Stiglitz *et al.* (2009) and OECD (2011) emphasize the importance of considering non-monetary development issues such as education and health in addition to the standard economic measures such as GDP or its growth rate. The growth model developed here extends the model of van Zon and Muysken (2001), which is based on the Lucas-Uzawa model of human capital accumulation, by including both health and education next to consumption in the utility function. This explicit consideration results in a relatively high share of the population in the education sector as can be seen from the sensitivity analysis of the long run balanced growth situation with regard to various parameter combinations. During the transition process the importance of health and education is clearly visible. Without an adequate health level there is a constraint on the availability of productive labor. Only when the population is sufficiently healthy, human and physical capital endowments per person can increase. This shows that even though health and both education and economic growth are complementary in the long run, they are substitutes in the short run. This result also shows the importance of further empirical investigation, such as the analyses in Chapter 4 and 5, using longer time series to identify short run and long run complementary development processes and development policies.

The actual transition paths of the economy depend on the parameter constellation, which implies the actual production technologies, health provision and education systems, and the initial conditions. The analysis of the transitional dynamics in this chapter shows the working principles of this model, but leaves the investigation of development trajectories corresponding to other parameter combinations for future research. The immediate reactions of the system are very strong, that is the predicted immediate convergence process is very fast. This may however result from the underlying optimality assumption in this optimum growth model. Still, this shows that the model is not yet able to replicate all stylized facts of economic growth in developing countries that were introduced in the introduction to this chapter. There is no positive relation between the growth rate and the level of per capita income, i.e. there is no β-divergence for low levels of per capita income (stylized fact 3). As the economy approaches the long run balanced growth path (BGP), growth rates slow down, so that stylized fact 4, β-convergence for higher income levels, is modeled here. Stylized facts 1 and 2, a considerable diversity of growth rates and a positive correlation between savings rate and per capita income cannot directly be observed from the two transition paths analyzed here.

Future research should include an analysis of the model dynamics for different parameter specifications, as has been done for the steady state. A further idea is to also introduce the concept of subsistence consumption into the model as suggested by Steger (2000). In additionthe model could be extended to include the provision of energy as well.

A Appendix

A.1 First order conditions

The first order conditions (F.O.C.s) are calculated from the present value Hamiltonian, Equation (10), and can be subdivided into three groups: F.O.C.s with respect to the control variables c, z and v:

$$\frac{\partial H}{\partial c} = 0, \frac{\partial H}{\partial z} = 0, \frac{\partial H}{\partial v} = 0, \tag{25}$$

w.r.t. the state variables K, h and g

$$\dot{\lambda}_K = -\frac{\partial H}{\partial K} + \rho\lambda_K, \dot{\lambda}_h = -\frac{\partial H}{\partial h} + \rho\lambda_h, \dot{\lambda}_g = -\frac{\partial H}{\partial g} + \rho\lambda_g, , \tag{26}$$

and with respect to the co-state variables λ_K, λ_h and λ_g. The latter are already defined by the rate of physical and human capital accumulation $\dot{K}$, $\dot{h}$, and $\dot{g}$. Thus, we get the growth rates of capital accumulations

$$\begin{aligned}
\frac{\dot{K}}{K} &= (Y - Lc) = \frac{r}{1-\alpha}(1-q) = \frac{r}{1-\alpha}s && (27)\\
\frac{\dot{h}}{h} &= \delta z g - \frac{\zeta\sigma\left(\phi v^{\beta} - g\right)}{g} - \frac{\dot{n}}{n} && (28)\\
\frac{\dot{g}}{g} &= \frac{\zeta\sigma\left(\phi v^{\beta} - g\right)}{g} && (29)
\end{aligned}$$

The F.O.C.s with respect to the control variables are given by

$$\begin{aligned}
\frac{\partial H}{\partial c} &= L\gamma_c \frac{1}{c}\left[c^{\gamma_c}(\mu g)^{\gamma_g} h^{\gamma_h} z^{\gamma_z}\right]^{1-\theta} - \lambda_K L \ \hat{=}\ 0 && (30)\\
\frac{\partial H}{\partial z} &= L\gamma_z \frac{1}{z}\left[c^{\gamma_c}(\mu g)^{\gamma_g} h^{\gamma_h} z^{\gamma_z}\right]^{1-\theta} \\
&\quad + \lambda_K A\left[(1-z-v)hgnR\right]^{\alpha} K^{1-\alpha}\frac{-\alpha}{(1-z-v)} + \lambda_h \delta g h \ \hat{=}\ 0 && (31)\\
\frac{\partial H}{\partial v} &= \lambda_K A\left[(1-z-v)hgnR\right]^{\alpha} K^{1-\alpha}\frac{-\alpha}{(1-z-v)} \\
&\quad + \lambda_h h\frac{1}{g}\zeta\sigma\phi\beta v^{\beta-1} + \lambda_g\zeta\sigma\phi\beta v^{\beta-1} \ \hat{=}\ 0. && (32)
\end{aligned}$$

Equation (30) implies that

$$\left[c^{\gamma_c}(\mu g)^{\gamma_g} h^{\gamma_h} z^{\gamma_z}\right]^{1-\theta} = \lambda_K \frac{c}{\gamma_c}. \tag{33}$$

Using this together with the rates of capital accumulation and the propensity to consume, the F.O.C.s with respect to z and v reduce to

$$\lambda_h = -\frac{1}{gh\delta}\lambda_K Y\left[\frac{-\alpha}{(1-z-v)} + \frac{q\gamma_z}{\gamma_c z}\gamma_z\right] \tag{34}$$

and also

$$\lambda_g = \lambda_K \frac{\alpha v^{1-\beta} Y}{(1-v-z)\beta\zeta\sigma\phi} + \lambda_h \frac{h}{g} \tag{35}$$

The F.O.C.s with respect to the state variables K, h and g are

$$\begin{aligned}
\dot{\lambda}_K &= -\frac{\partial H}{\partial K} + \rho\lambda_K \\
&= -\lambda_K(1-\alpha)A\left[(1-z-v)hgnR\right]^{\alpha} K^{-\alpha} + \rho\lambda_K \\
&= -\lambda_K(1-\alpha)\frac{Y}{K} + \lambda_K\rho = \lambda_K(-r+\rho)
\end{aligned} \tag{36}$$

$$\begin{aligned}
\dot{\lambda}_h &= -\frac{\partial H}{\partial h} + \rho\lambda_h \\
&= -L\gamma_h \frac{1}{h}\left[c^{\gamma_c}(\mu g)^{\gamma_g} h^{\gamma_h} z^{\gamma_z}\right]^{1-\theta} \\
&\quad -\lambda_K \alpha h^{\alpha-1} A\left[(1-z-v)gnR\right]^{\alpha} K^{1-\alpha} \\
&\quad -\lambda_h\left[\delta z g - \frac{\zeta\sigma\left(\phi v^{\beta} - g\right)}{g} - \frac{\dot{n}}{n}\right] + \rho\lambda_h \\
&= -\lambda_K \frac{Y}{h}\left[\frac{q}{\gamma_c}\gamma_h - \alpha\right] + \lambda_h\left[\rho - \frac{\dot{h}}{h}\right]
\end{aligned} \tag{37}$$

$$\begin{aligned}
\dot{\lambda}_g &= -\frac{\partial H}{\partial g} + \rho\lambda_g \\
&= -L\gamma_g \frac{1}{g}\left[c^{\gamma_c}(\mu g)^{\gamma_g} h^{\gamma_h} z^{\gamma_z}\right]^{1-\theta} \\
&\quad -\lambda_K \alpha g^{\alpha-1} A\left[(1-z-v)hnR\right]^{\alpha} K^{1-\alpha} \\
&\quad -\lambda_h h\left[\delta z - \frac{\zeta\sigma\left(\phi v^{\beta}\right)}{-g^2}\right] + \lambda_g \frac{\zeta\sigma\left(\phi v^{\beta}\right)}{-g^2}\rho\lambda_h
\end{aligned} \tag{38}$$

A.2 Differential equations of co-state variables

To differential equations for the co-state variables are obtained by solving Equations (30) to (32) for λ_K, λ_h and λ_g, respectively, then differentiating with respect to time and dividing both sides by the corresponding co-state variable

$$\frac{\dot{\lambda}_K}{\lambda_K} = \left[\gamma_c(1-\theta)-1\right]\frac{\dot{c}}{c} + (1-\theta)\left[\beta\gamma_g\frac{\dot{v}}{v} + \gamma_h\frac{\dot{h}}{h} + \gamma_z\frac{\dot{z}}{z}\right] \tag{39}$$

$$\frac{\dot{\lambda}_h}{\lambda_h} = \frac{\dot{\lambda}_K}{\lambda_K} + \frac{\dot{Y}}{Y} - \frac{\dot{h}}{h} - \frac{\dot{g}}{g} + \frac{\dot{q}}{q} - \frac{\dot{z}}{z} + \frac{z(1-v)\frac{\dot{z}}{z} - v\frac{\dot{v}}{v} - \frac{\dot{q}}{q}(1-v-z)\alpha\gamma_c}{(1-z-v)\left(z\alpha\gamma_c - q(1-v-z)\gamma_z\right)} \tag{40}$$

$$\begin{aligned}
\frac{\dot{\lambda}_g}{\lambda_g} &= \frac{g^2 vz\alpha\left[\left(\frac{\dot{\lambda}_K}{\lambda_K} + \frac{\dot{Y}}{Y}\right)(1-z-v) + z\frac{\dot{z}}{z} + v\frac{\dot{v}}{v}\left(1-z-(1-z-v)\beta\right)\right]\gamma_c\delta}{(1-z-v)\left[g^2 vz\alpha\gamma_c\delta + v^{\beta}\beta\left(z\alpha\gamma_c - q(1-z-v)\gamma_z\right)\zeta\sigma\phi\right]} \\
&\quad + \frac{v^{\beta}\beta z\left[\left(\frac{\dot{\lambda}_K}{\lambda_K} + \frac{\dot{Y}}{Y} + 2\frac{\dot{g}}{g}\right)(1-z-v) + v\frac{\dot{v}}{v} + z\frac{\dot{z}}{z}\right]\zeta\sigma\phi}{(1-z-v)\left[g^2 vz\alpha\gamma_c\delta + v^{\beta}\beta\left(z\alpha\gamma_c - q(1-z-v)\gamma_z\right)\zeta\sigma\phi\right]}
\end{aligned} \tag{41}$$

$$+\frac{-v^{\beta}\beta q\left[\frac{\dot{q}}{q}-2\frac{\dot{g}}{g}+\frac{\dot{Y}}{Y}-\frac{\dot{z}}{z}+\frac{\dot{\lambda}_K}{\lambda_K}\right](1-z-v)^2\gamma_z\zeta\sigma\phi}{(1-z-v)\left[g^2vz\alpha\gamma_c\delta+v^{\beta}\beta\left(z\alpha\gamma_c-q(1-z-v)\gamma_z\right)\zeta\sigma\phi\right]}$$

A.3 Tansversality conditions in the long run equilibrium

The transversality conditions, the sum of the state variables' growth rates and their respective co-state variable's growth rate has to be negative, which implies that the following inequalities should hold:

$$\hat{K}+\hat{\lambda}_K-\rho = \frac{(1-q)(\gamma_c+\gamma_h)(1-\theta)\rho}{\alpha-(1-q)(\gamma_c+\gamma_h)(1-\theta)-q}-\rho<0 \tag{42}$$

$$\begin{aligned}\hat{h}+\hat{\lambda}_h-\rho &= \frac{qr\left(z\alpha\gamma_c^2\left(z\left(1+\gamma_c(\theta-1)\right)+\gamma_z(\theta-1)\right)+(v-1)\gamma_z(\theta-1)\right)\left(\rho+v^{\beta}z\alpha\delta\phi\right)}{q(1-z-v)\gamma_z-z\alpha\gamma_c}\\ &\quad \frac{qz(1-v-z)\alpha\gamma_c\gamma_z(\theta-1)\left(\rho+v^{\beta}z\alpha\delta\phi\right)\left(\gamma_c\rho-v^{\beta}z\gamma_h\delta\phi\right)}{q(1-z-v)\gamma_z-z\alpha\gamma_c}-\rho<0\end{aligned} \tag{43}$$

$$\begin{aligned}\hat{g}+\hat{\lambda}_g-\rho &= F\left[(1-v-z)\beta\gamma_z\zeta(\theta-1)\sigma(\gamma_c\rho-v^{\beta}z\gamma_h\delta\phi)\right]\\ &\quad +F\left[r\gamma_c\left((v-1)\beta\gamma_z\zeta\sigma(\theta-1)\right)\right]\\ &\quad +F\left[r\gamma_c\left(z\left(\beta\gamma_z\zeta\sigma(\theta-1)+\alpha\left(1+\gamma_c(\theta-1)\right)\left(\beta\zeta\sigma+v^{1+\beta}\delta\phi\right)\right)\right)\right]-\rho<0\end{aligned} \tag{44}$$

with

$$F=\frac{-qz\alpha\gamma_c(\beta\zeta\sigma+v1+\beta\delta\phi)(\rho+v\beta z\alpha\delta\phi)}{\beta\left[z\alpha\gamma_c-q(1-z-v)\gamma_z\right]\zeta\sigma+v^{1+\beta}z\alpha\gamma_c\delta\phi} \tag{45}$$

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