Determinants of Precautionary Savings: Elasticity of Intertemporal Substitution vs. Risk Aversion

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Abstract

The purpose of this paper is to understand the effects of the elasticity of intertemporal substitution (EIS), the percentage change in intertemporal consumption in response to a given percentage change in the intertemporal price, and risk aversion on savings separately and determine which coefficient is more important factor for precautionary savings. This paper is an extension of Weil (1993)' paper where the determinants of precautionary savings can be studied analytically by assuming exponential risk utility function in Epstein-Zin (1989) preferences. In my model, the exponential risk utility function is not assumed in order to look at more general model. Thus, the problem is not analytically solvable anymore. Instead, the problem is solved numerically in order to determine which coefficient is more important factor for precautionary savings. The result is saving increases as EIS increases. Similarly, saving increases as the coefficient of risk aversion increases. More importantly, it is observed that EIS is a more important factor for precautionary savings than risk aversion because saving is more sensitive to changes in EIS than changes in risk aversion.

JEL classification: C61, C63, E21, E27

Keywords: Precautionary Savings, Epstein-Zin Utility, Risk Aversion, Household Dynamic Decision Problem

1 Introduction

The main question that this paper tries to answer is whether the elasticity of intertemporal substitution (EIS), the percentage change in intertemporal consumption in response to a given percentage change in the intertemporal price, or risk aversion is more important determinant of precautionary savings. This is an important question since a significant fraction of the capital accumulation that occurs in the United States is due to precautionary savings according to Zeldes (1989a), Skinner (1988) and Caballero (1990). Thus, knowing the important determinant of precautionary savings will be helpful to understand the capital accumulation mechanism in the U.S.

Zeldes (1989a) calculates the optimal amount of precautionary savings under uncertain income environment for the agents who have constant relative risk aversion utility. He finds that agents optimally choose to save more in an uncertain environment than they would have done in a certain environment when there is no borrowing or lending constraints. He uses numerical methods to closely approximate the optimal saving. In Deaton (1991)'s paper, the agents are restricted in their ability to borrow to finance consumption. However, nothing prevents these agents from saving and accumulating assets in order to smooth their consumption in bad states. In this environment, he shows that the behavior of saving and asset accumulation is quite sensitive to what agents believe about the stochastic process generating their income.

Aiyagari (1994) modifies the standard growth model of Brock and Mirman (1972) to include a role for uninsured idiosyncratic risk and borrowing constraints. In his model, there are a large number of agents who receive idiosyncratic labor endowment shocks that are uninsured. He analyzes its qualitative and quantitative implications for the contribution of precautionary saving to aggregate saving, importance of asset trading, and income and wealth distributions. He shows that aggregate saving is larger under idiosyncratic risk than certainty. Therefore, he demonstrates that two household with identical preferences over present and future consumption will under certainty save the same, but this does not necessarily imply that these two households will save the same in uncertain environments. In a recent work, Guvenen (2006) shows that aggregate investment is mostly determined by wealthy people who have high EIS and aggregate consumption is mostly determined by non-wealthy people who have low EIS. In his model, there are two different types of agents who differ in elasticity of intertemporal substitution and limited participation in the stock market. Limited participation is only used to create substantial wealth inequality similar to the data. Thus, difference in the elasticities is an important factor for determining savings.

My paper is an extension of Weil (1993)' paper where the determinants of precautionary savings can be studied analytically by assuming exponential risk utility function in Epstein-Zin (1989) preferences. This assumption makes the problem analytically solvable. Weil (1993) shows that savings increase in each of these cases:

- when persistence of income shocks increases
- when the coefficient of risk aversion increases
- when EIS increases

However, Weil does not rank the importance of these determinants in saving decisions.

The purpose of this paper is to understand the effects of the elasticity of intertemporal substitution (EIS) and risk aversion on savings separately and determine which coefficient is more important factor for precautionary savings. The numerical calculations are performed for the more general form of the Epstein-Zin utility function in order to calculate savings for different EIS and risk aversion, RA, coefficients to see which one is the more important determinant of precautionary savings. In this paper, I first look at the savings for different values of EIS by keeping the risk aversion coefficient constant. Then, savings are calculated by changing the risk aversion coefficients and keeping EIS constant. As a result, I obtained graph of savings for different EIS and risk aversion coefficients.

According to Chatterjee, Giuliano and Turnovsky (2004), most of the existing literature assumes that the preferences of the representative agent are represented by a constant elasticity utility function. While this specification of preferences is convenient, it is also restrictive in that two key parameters, the elasticity of intertemporal substitution and the coefficient of relative risk aversion, become directly linked to one another and cannot vary independently. This is a significant limitation and one that can lead to seriously misleading impressions of the effects that each parameter plays in determining the precautionary savings.

Arrow (1965) and Pratt (1964) introduced the concept of the coefficient of relative risk aversion and it is well defined in the absence of any intertemporal dimension. Hall (1978, 1988) and Mankiw, Rotemberg, and Summers (1985) established the concept of the elasticity of intertemporal substitution and it is well defined in the absence of risk. The standard constant elasticity utility function has the property that both parameters EIS and RA are constant, though it imposes the restriction EIS*RA = 1 with the widely employed logarithmic utility function corresponding to EIS=RA=1. Thus it is important to realize that in imposing this constraint the constant elasticity utility function is also invoking these separability assumptions according to Giuliano and Turnovsky (2003).

Although there are empirical studies about the value of the elasticity of intertemporal substitution, the results are different from each other. Hall (1988) and Campbell and Mankiw (1989) estimate EIS 0.1 based on macro data. Epstein and Zin (1991) provide estimates spanning the range 0.05 to 1, with clusters around 0.25 and 0.7. Attanasio and Weber (1993, 1995) find that their estimate of EIS is 0.3 using aggregate data and is 0.8 using cohort data. They propose that the aggregation implicit in the macro data may cause a significant downward bias in the estimate of EIS. Beaudry and van Wincoop (1995) estimate EIS near 1. More recent estimates by Ogaki and Reinhart (1999) suggest values of around 0.4. Moreover, Atkeson and Ogaki (1996) and Ogaki and Atkeson (1997) find evidence to suggest that the EIS increases with household wealth. As a result of these findings, the variation of EIS from 0.04 to 0.99 is used in the numerical calculations.

Similar to the elasticity of intertemporal substitution, the value the coefficient of risk aversion shows a discrepancy in the literature. Epstein and Zin (1991) conclude that their estimate of RA is near 1. In contrast, Kandel and Stambaugh (1991) take RA as 30 and Obstfeld (1994a) takes RA as 18. More recent study by Constantinides, Donaldson, and Mehra (2002) present that empirical evidence suggests that RA is most plausibly around 5. According to these findings, the variation of RA from 1.01 to 25 is used in the numerical calculations.

Zeldes (1989a), Deaton (1991) and Aiyagari (1994) use expected value of a discounted sum of time-additive utilities in the model, thus the motion of risk aversion and EIS is confused. As a result, it is not possible to look at the effects of EIS and risk aversion separately. According to Giuliano and Turnovsky (2003), this is important for two reasons. First, conceptually, EIS and RA impinge on the economy in quite independent, and in often conflicting ways. They therefore need to be decoupled if the true effects of each are to be determined. Risk aversion impinges on the equilibrium through the portfolio allocation process, and thus through the equilibrium risk that the economy is willing to sustain. It also determines the discounting of risk in deriving the certainty equivalent level of income. The intertemporal elasticity of substitution then determines the allocation of this certainty equivalent income between current consumption and future consumption. Second, the biases introduced by imposing the compatibility condition EIS*RA=1 for the constant elasticity utility function can be quite large, even for relatively weak violations of this relationship. According to Chatterjee, Giuliano and Turnovsky (2004), while one certainly cannot rule out using the constant elasticity utility function, as a practical matter, their results suggest that it should be employed with caution, recognizing that if the condition for its valid use is not met, very different implications may be drawn.

This paper follows Weil (1993) by using an Epstein-Zin utility function that permits risk attitudes to be disentangled from the degree of intertemporal substitutability. This facilitates the study of the effects of EIS and risk aversion separately. It is shown saving increases as EIS increases. Similarly, saving increases as the coefficient of risk aversion increases. More importantly, it is observed that EIS is a more important factor for precautionary savings than risk aversion because saving is more responsive to changes in EIS than changes in risk aversion. For example, starting from the benchmark preference parameters RA= 5 and EIS = 0.2, the constant elasticity utility function implies that doubling RA to 10 (and thus simultaneously halving EIS to 0.1 so that EIS*RA=1) would reduce the savings to 0.9148 when the savings in benchmark case is normalized to 1. On the other hand, when the EIS is doubled to 0.4 and RA is halved to 2.5, the savings increases to 1.4074. In the unrestricted utility function, if RA increases two times, RA=10, and EIS stays the same, the savings become 1.3838 whereas if EIS increases twice, EIS=0.4, and RA stays the same, the savings become 1.9083. Thus, the change in savings is much less sensitive to the degree of risk aversion than to the intertemporal elasticity of substitution.

The paper is structured as follows. Section 2 describes the model, by explaining the preferences and the optimization problem faced by individuals in the economy. The numerical results are presented and discussed in Section 3. Section 4 concludes the paper by outlining some directions for future research. The appendix describes the numerical solution of the model.

2 Model

Our model is the standard problem of a representative agent who lives for many periods and chooses optimal current consumption and next period's bond holding in order to maximize the utility function. The source of uncertainty considered is in exogenous future income and there exist no markets in which agents can insure against this uncertainty. Although agents can save by holding bonds, they are not able to borrow, i.e. there is a borrowing constraint.

Preferences

Following Weil's (1993) terminology, a representative agent whose preferences over deterministic consumption stream exhibit a constant elasticity of intertemporal substitution:

$$W(c_t, c_{t+1}, c_{t+2}, \dots) = \left[(1-\beta)\sum_{s=0}^{l} \beta^s \, c_{t+s}^{\varphi}\right]^{\frac{1}{\varphi}} \tag{1}$$

where $\rho = \frac{1}{(1-\varphi)} > 0$ is the elasticity of intertemporal substitution, EIS, and $\beta \in (0,1)$ is the constant exogenous discount factor. These preferences can be represented recursively as:

$$W(c_t, c_{t+1}, c_{t+2}, \dots) = U[c_t, W(c_{t+1}, c_{t+2}, c_{t+3}, \dots)] \tag{2}$$

$$= \left[(1-\beta)c_t^{\varphi} + \beta\,\{W(c_{t+1}, c_{t+2}, c_{t+3}, \dots)\}^{\varphi}\right]^{\frac{1}{\varphi}} \tag{3}$$

where U(.,.) is an aggregator function. Behavior towards risk is summarized by a constant coefficient of risk aversion, denoted by the parameter $\alpha > 1$.

$$\widehat{W} = (EW'^{1-\propto})^{\frac{1}{1-\alpha}} \tag{4}$$

Equation 4 defines the utility certainty equivalent of a lottery yielding a random utility level W' is $\widehat{W}$ for the representative agent where E is expectation operator. $\widehat{W}(\widetilde{c_{t+1}}, \widetilde{c_{t+2}}, \widetilde{c_{t+3}}, \dots)$ represents the certainty equivalent, conditional on time t information, of time t+1 utility. It is assumed that preferences over random consumption lotteries have the recursive representation with the aggregator function. Therefore, current utility becomes the aggregate of current consumption and the certainty equivalent of future utility as seen in Equation 5.

$$W(c_t, \widetilde{c_{t+1}}, \widetilde{c_{t+2}}, \dots) = U[c_t, \widehat{W}(\widetilde{c_{t+1}}, \widetilde{c_{t+2}}, \widetilde{c_{t+3}}, \dots)] \tag{5}$$

This utility function has both a constant elasticity of intertemporal substitution, $\rho = \frac{1}{(1-\varphi)}$, and a constant coefficient of risk aversion, α. This utility function distinguishes EIS and RA explicitly. This facilitates the study of the effects of EIS and risk aversion separately.

Utility Function

This utility function is used to calculate the determinants of precautionary savings:

$$U_t = [(1-\beta)(C_t)^{\varphi} + \beta(E_t(U_{t+1})^{1-\alpha})^{\frac{\varphi}{1-\alpha}}]^{\frac{1}{\varphi}}$$

where β is time discount factor, C_t is consumption today, $\rho = \frac{1}{(1-\varphi)}$ is the EIS and α is the coefficient of risk aversion. This type of utility preference allows us to disentangle the EIS and the risk aversion and examine their effects independently. Also, being third derivative of utility function is positive, $U''' > 0$, introduces prudence into the decisions of the consumer.

Weil (1993) assumes the exponential risk utility function in Epstein-Zin (1989) preferences and so the determinants of precautionary savings can be studied analytically. In other words, this assumption makes the problem analytically solvable.

However, in my model, the exponential risk utility function is not assumed in order to look at more general model. Thus, the problem is not analytically solvable anymore. Instead, the problem is solved numerically for the model that is more general than the model of Weil (1993).

Budget Set

When y denotes today's income, b denotes today's bond holding, C denotes today's consumption, b' tomorrow's bond holding and R denotes the interest rate, the budget constraint of the representative agent for each period becomes as seen in Equation 6 below.

$$C + b' \leq Rb + y \tag{6}$$

Household Dynamic Decision Problem

The agent solves her problem recursively in a given state. The optimal solution to this problem is characterized most simply in terms of a value function, V(y,b). The agent knows today's income, y, and bond holding, b, and chooses today's consumption, c, and tomorrow's bond holding, b', in order to maximize the utility function as a dynamic programming problem:

$$V(y,b) = \max_{C,b'} [(1-\beta)(C)^{\varphi} + \beta(E(V(y',b') \mid y)^{\frac{\varphi}{1-\alpha}}]^{\frac{1}{\varphi}}$$

s.t

$$C + b' \leq Rb + y \tag{7}$$

$$y' = \Gamma(y) \tag{8}$$

$$b' \geq 0 \tag{9}$$

$$C \geq 0 \tag{10}$$

where E denotes the mathematical expectation operator conditional on information available today. As said earlier, Equation 7 is the budget constraint. Equation 8 is the law of motion for income and it is a Markov Process getting two different income values, income low and income high, in the numerical calculations. Equation 9 shows the borrowing constraint and shows that asset holding or saving cannot be negative. Equation 10 shows that consumption cannot be negative. The time discount factor, β is chosen smaller than 1/R in order to prevent agents to save infinitely which is proved in Aiyagari (1994) that if β is larger than 1/R, agents save infinitely. Furthermore, the coefficient of risk aversion, α, is greater than 1 and the coefficient of EIS, $\rho = \frac{1}{(1-\varphi)}$, is between 0.04 and 0.99.

3 Results

In the model, the law of motion for income is a Markov Process in which agents can get only two different amounts of exogenous income, income low and income high. There is an assignment of the probability of getting the same income that defines the persistence of income shocks. As discussed in the introduction section, the EIS varies from 0.04 to 0.99 and the risk aversion (RA) varies from 1.01 to 25 as according to the estimates of these coefficients in the literature.

The model is simulated for 1000 periods in order to make the bond holdings converge to a stochastic steady state. Then, the agent's savings are summed from period 300 to 1000 and divided by 701. As a result, the findings are the average savings of the agent. The numerical solution of the model is explained explicitly in the Appendix.

For the time discount factor, β = 0.955 and the probability of getting the same income, persistence of income shocks, is 0.7, the savings are shown in Table 1 below:

Table 1: Savings when persistence is 0.7

Probability=0.7			EIS			
	0.05	0.1	0.2	0.4	0.8	0.99
Risk Aversion						
20	0.8795	1.1607	1.6916	2.7664	5.8416	9.7270
10	0.4863	0.9148	1.3838	2.3787	5.2578	9.0545
5	0.4057	0.8258	1.0000	1.9083	4.6570	8.2933
2.5	0.2903	0.4699	0.6960	1.4074	4.0172	7.4701
1.25	0.2279	0.3220	0.5079	1.0249	3.3490	6.5828
1.01	0.2010	0.2839	0.4476	0.8789	3.0111	6.2032

The benchmark preference parameters are RA= 5, EIS = 0.2 and the probability of getting the same income is 0.7. The savings in benchmark case is normalized to 1 and the savings for various parameters are proportions to the savings of benchmark case. For instance, if RA is doubled to 10 by implying the constant elasticity utility function (thus simultaneously halving EIS to 0.1 so that EIS*RA=1), the savings reduces to 0.9148. On the other hand, when the EIS is doubled to 0.4 and RA is halved to 2.5, the savings increases to 1.4074. In the unrestricted utility function, if RA increases two times, RA=10, and EIS stays the same, the savings become 1.3838 whereas if EIS increases twice, EIS=0.4, and RA stays the same, the savings become 1.9083. Thus, the change in savings is much less sensitive to the degree of risk aversion than to the intertemporal elasticity of substitution.

The three dimensional graph of savings according to different parameters of the elasticity of intertemporal substitution and risk averse is depicted in Figure 1. The figure demonstrates that, as similar to the results in the Weil(1993)'s paper , saving increases when the parameter of EIS increases by keeping risk aversion constant because an increase in the elasticity of intertemporal substitution increases the propensity to consume out of wealth and out of current income. Also, saving increases when the parameter of risk aversion increases by keeping EIS constant as expected since the more risk averse the agent is, the stronger his precautionary saving motive. More prominently, I observe that EIS is more important in precautionary saving decision than risk aversion since saving is more responsive to changes in EIS than changes in risk aversion as portrayed in Figure 2 and Figure 3.

Figure 1: 3-D graph of savings when persistence is 0.7

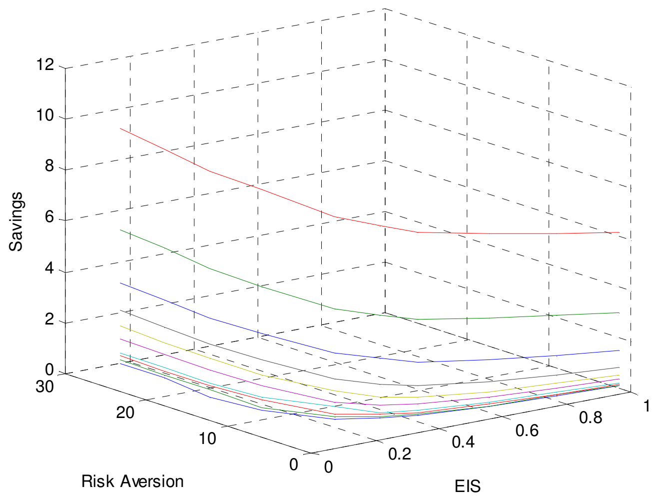


Figure 2: Savings when keeping EIS constant and when persistence is 0.7

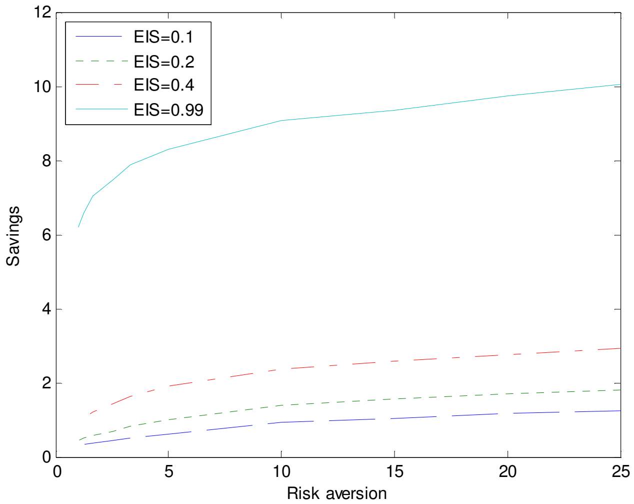
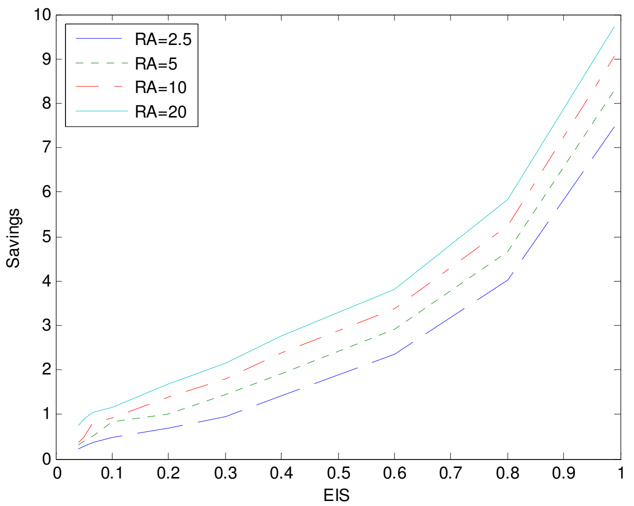


Figure 3: Savings when keeping RA constant and when persistence is 0.7



Also one can ask whether saving increases as if the persistence of income shocks increases as shown in the Weil(1993)'s paper. This is examined by increasing the probability of getting the same income. If the persistence of income shocks is increased to 0.8, savings are tabulated in Table 2.

Table 2: Savings when persistence is 0.8

Probability=0.7			EIS			
	0.05	0.1	0.2	0.4	0.8	0.99
Risk Aversion						
20	1.1238	1.5943	2.3077	3.6695	7.6213	12.5308
10	0.6277	1.1794	1.8532	3.0868	6.7874	11.6185
5	0.5128	0.8944	1.2281	2.3364	5.7818	10.4551
2.5	0.3268	0.5706	0.7397	1.5477	5.8636	9.0654
1.25	0.2195	0.3768	0.5013	1.0132	3.5628	7.4995
1.01	0.1911	0.2767	0.4350	0.8577	3.0529	6.7754

For the parameters RA=5 and EIS=0.2, the savings is 1.2281. It means there is about 22.8 % increase if the persistence increases from 0.7 to 0.8 since the savings in the benchmark case is normalized to 1 and in the benchmark case preference parameters are RA= 5, EIS = 0.2 and the probability of getting the same income is 0.7. In the constant elasticity utility function, if RA is multiplied by 4 and RA becomes 20 (thus simultaneously halving EIS to 0.05 so that EIS*RA=1), the savings reduces to 1.1238 from 1.2281. The percentage reduction is 8.5 %. On the other hand, when the EIS is multiplied by 4 to make EIS=0.8 and RA becomes to 1.25, the savings increases to 3.5628 and the percentage raise is 190.1 %. In the unrestricted utility function, if RA increases four times, RA=20, and EIS stays the same, the savings become 2.3077 whereas if EIS increases four times, EIS=0.8, and RA stays the same, the savings become

5.7818. As seen from percentages, it is clear that saving is much more responsive to changes in EIS than to changes in risk aversion.

The persistence of income shocks is a determinant of the strength of precautionary savings motive. The more persistent the income process, the more responsive current consumption to fluctuations in current income. Therefore, the more persistence in income shocks leads to a stronger precautionary savings motive as seen in Table 2.

The three dimensional graph of savings according to different parameters of the elasticity of intertemporal substitution and risk aversion when the persistence of income shocks is 0.8 is depicted in Figure 4 below. Also, the savings when keeping EIS constant and when keeping RA constant portrayed in Figure 5 and Figure 6 respectively.

Figure 3: 3-D graph of savings when persistence is 0.8

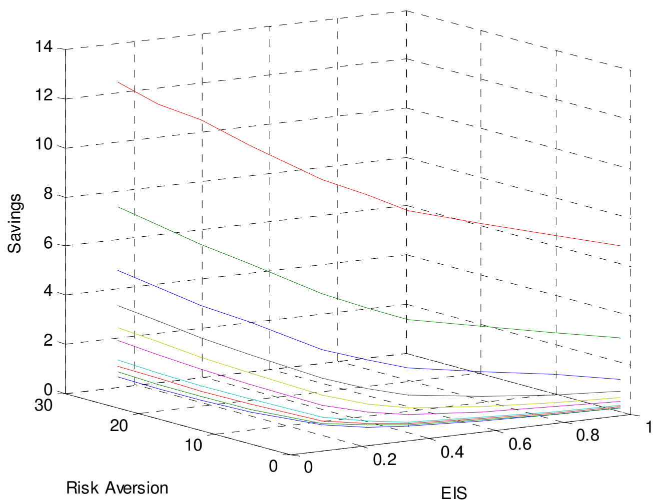


Figure 5: Savings when keeping EIS constant and when persistence is 0.8

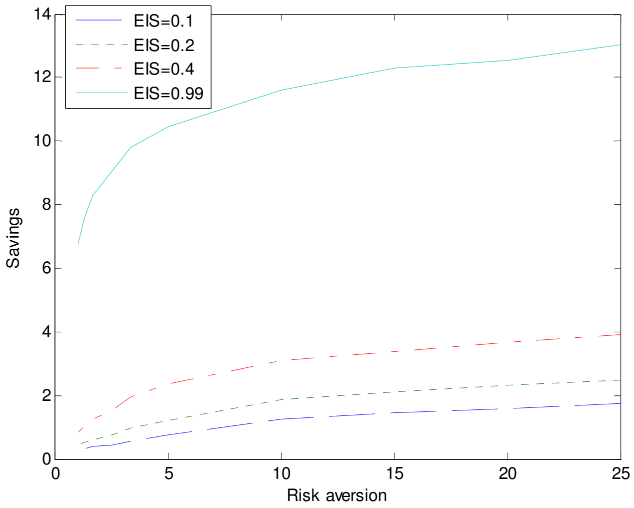
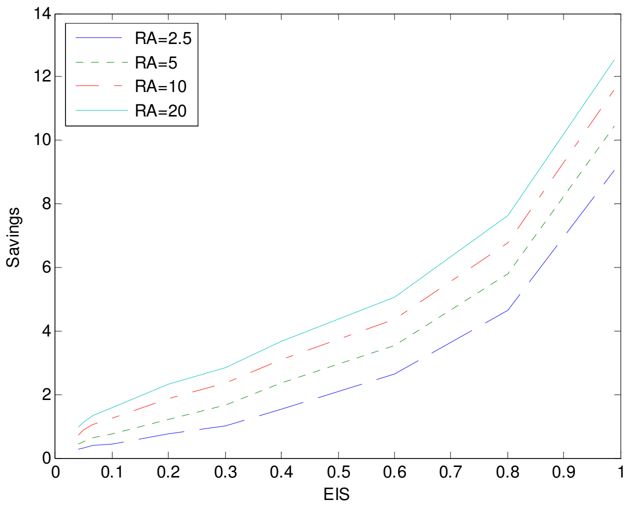


Figure 6: Savings when keeping RA constant and when persistence is 0.8



It is explained that savings increase when persistence of income shocks increases in Weil(1993)'s paper. It is shown in the figures below that the savings when probability is 0.8 is larger than the savings when probability is 0.7. It is also observed the same result that EIS is a more significant determinant of savings than risk aversion for each probability.

Figure 7: Savings when EIS=0.2 for persistence 0.7 and 0.8

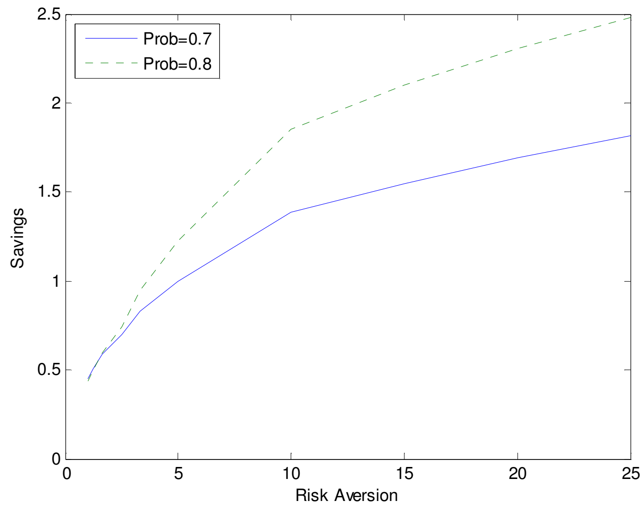
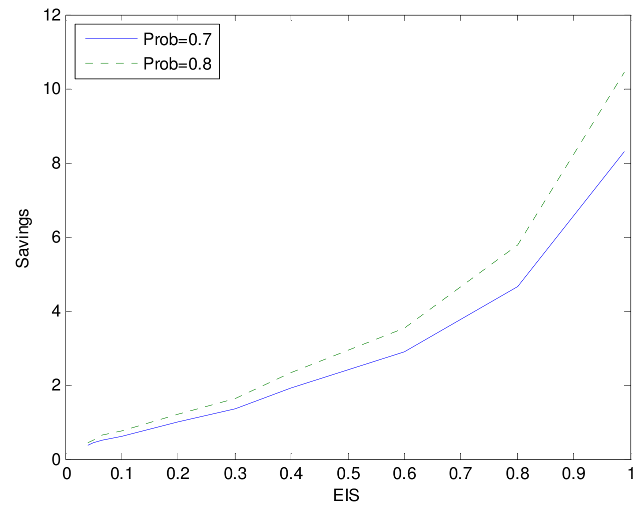


Figure 8: Savings when Risk Aversion=5 for persistence 0.7 and 0.8



The savings are calculated also when persistence of income shock is 0.5. When the persistence is 0.5, the savings for the benchmark parameters, EIS=5 and RA=1, is 0.55 and so there is 45 % decrease if the persistence decreases from 0.7 to 0.5. In the constant elasticity utility function, if RA is multiplied by 5 and RA becomes 25 (thus simultaneously halving EIS to 0.04 so that EIS*RA=1), the savings reduces to 0.41 from 0.55 and so the percentage reduction becomes 25 %. On the other hand, when the EIS is multiplied by 5 to make EIS=0.99 and RA becomes to 1.01, the savings increases to 4.26 and the percentage raise is 675 %. The similar results are obtained that EIS is more important determinant of precautionary savings as shown in the figures below.

Figure 9: Savings when keeping EIS constant and when persistence is 0.5

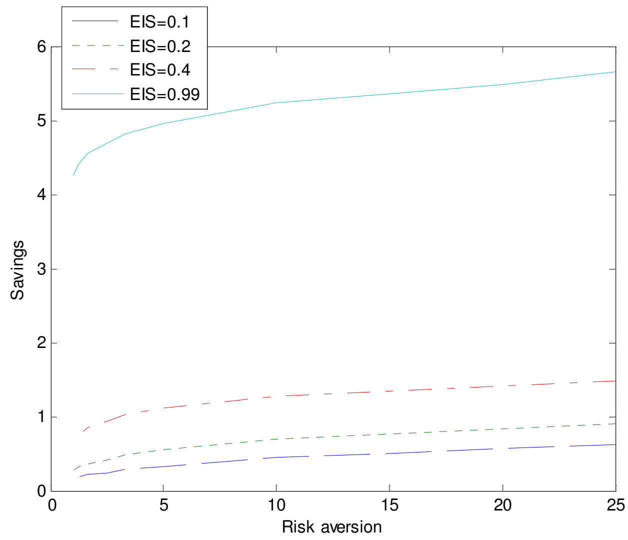
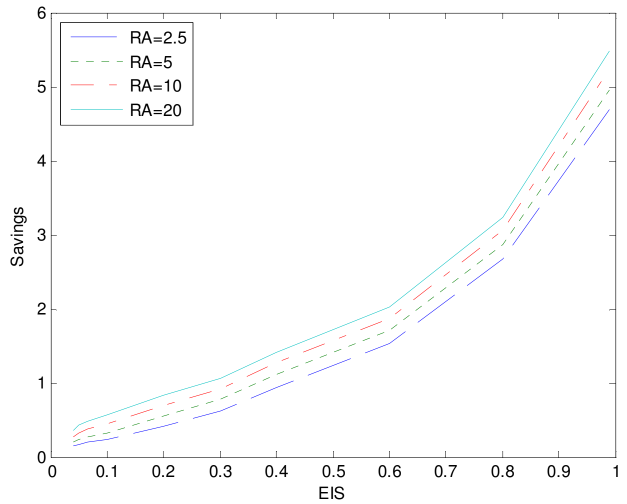


Figure 10: Savings when keeping RA constant and when persistence is 0.5



If the persistence decreases from 0.7 to 0.6, there is 31 % decrease in the savings for the benchmark parameters since the savings 0.69 in this case. In the constant elasticity utility function, if EIS is multiplied by 2 and EIS becomes 0.4 (thus simultaneously halving RA to 2.5 so that EIS*RA=1), the savings rise to 1.02 from 0.69 and so the percentage raise becomes 48 %. On the other hand, when the RA is multiplied by 2 to make RA= 10 and EIS becomes to 0.1, the savings shrinks to 0.62 and the percentage reduction is 10 %. In the unrestricted utility function, if EIS increases two times, EIS=0.4, and RA stays the same, the savings become 1.35 whereas if RA increases two times, RA=10, and EIS stays the same, the savings become 0.95. The increase is 96 % in the first case and 38 % in the second case. As seen from percentages, it is clear that saving is much more responsive to changes in EIS than to changes in risk aversion. The results are as portrayed in the Figure 11 and Figure 12 below.

Figure 11: Savings when keeping EIS constant and when persistence is 0.6

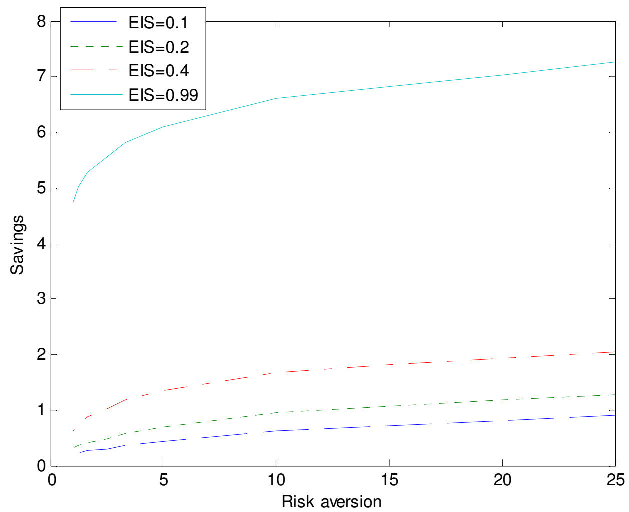
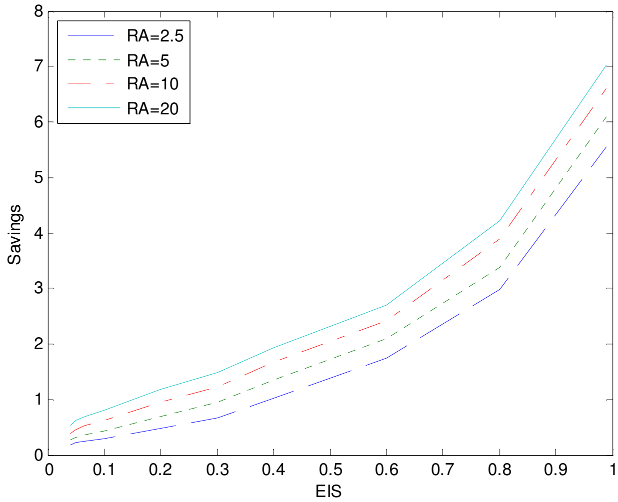


Figure 12: Savings when keeping RA constant and when persistence is 0.6



As mentioned earlier, the persistence of income shocks is a determinant of the strength of precautionary savings motive. The more persistence in income shocks leads to a stronger precautionary savings. It is shown in the Figure 13 and Figure 14 below that the savings when the persistence of income shocks is 0.6 is larger than the savings when persistence is 0.5. The ratio of the savings when persistence is 0.5 to the savings when persistence is 0.6 ranges from 0.70 to 0.89 by comparing the savings with the same parameter values for the coefficients of elasticity of intertemporal substitution and risk aversion. The range is wider when keeping the EIS=0.2 constant and changing the coefficient of risk aversion than when keeping the RA=5 constant and changing the coefficient of elasticity of intertemporal substitution as seen in Figure 13 and Figure 14 below.

Moreover, it is also observed the same result that EIS is a more crucial determinant of savings than risk aversion for each persistence of income shocks since saving is more sensitive to changes in EIS than in risk aversion.

Figure 13: Savings when EIS=0.2 for persistence 0.5 and 0.6

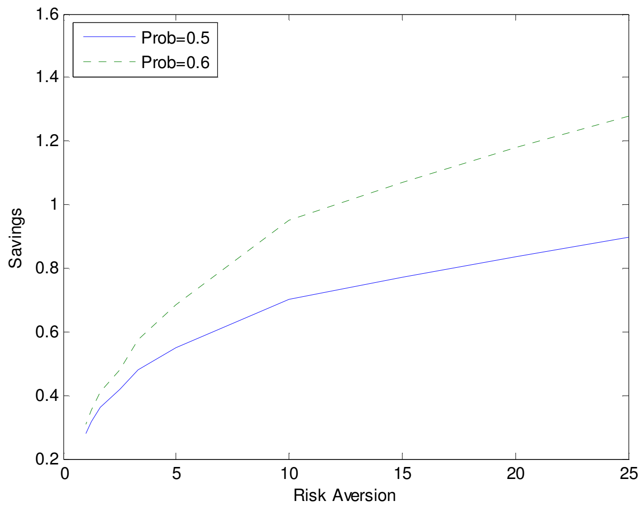
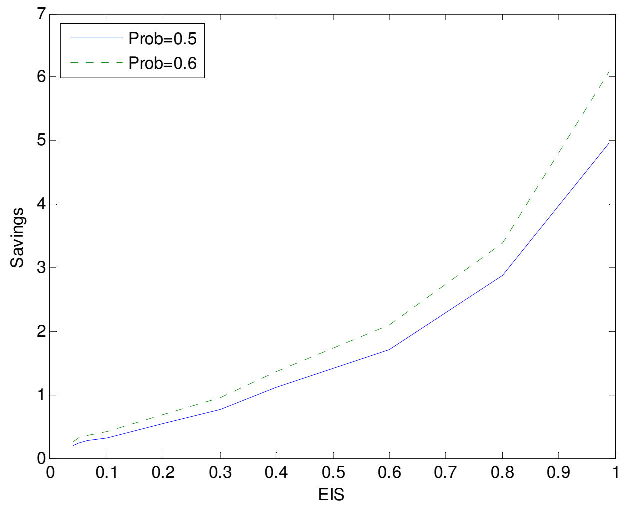


Figure 14: Savings when Risk Aversion=5 for persistence 0.5 and 0.6



4 Conclusion and Discussion

In this paper, I attempt to determine the important factors of precautionary saving. Saving under temporal risk aversion and intertemporal substitution usually exceeds the certainty-equivalent level of saving and this type of prudent behavior is called the precautionary motive for saving. Precautionary saving arises when consumers are risk averse and have elastic intertemporal preferences and so hedge against unanticipated future declines in income. The precautionary motive induces individuals to save in order to provide insurance against future periods in which their incomes are low or their needs are high according to Van der Ploeg (1993). I look at the effects of EIS and risk aversion to savings separately by using Epstein-Zin (1989) recursive utility function. I use Epstein-Zin (1989) utility since this utility permit risk attitudes to be disentangled from the degree of intertemporal substitutability and provides a motive for precautionary saving.

According to Chatterjee, Giuliano and Turnovsky (2004), most of the existing literature assumes that the preferences of the representative agent are represented by a constant elasticity utility function. While this specification of preferences is convenient, it is also restrictive in that two key parameters, the elasticity of intertemporal substitution and the coefficient of risk aversion, become directly linked to one another and cannot vary independently. This is a significant limitation and one that can lead to seriously misleading impressions of the effects that each parameter plays in determining the precautionary savings. With the diversity of empirical evidence suggesting that this constraint, EIS*RA=1, may or may not be met, it is important that studies of these two parameters impinges on the equilibrium in very distinct and in some respects conflicting ways. Therefore, the general conclusion to be drawn is that errors committed by using the constant elasticity utility function, even for small violations of the compatibility condition within the empirically plausible range of the parameter values, can be quite substantial. While one certainly cannot rule out using the constant elasticity utility function, as a practical matter, their results suggest that it should be employed with caution, recognizing that if the condition for its valid use is not met, very different implications may be drawn.

Hall (1988) points out that intertemporal substitution by consumers is a central element of most modern macroeconomic models. Weil (1993) shows that when the coefficient of elasticity of intertemporal substitution increases savings increase. Atkeson and Ogaki (1996) develop and estimate a model of preferences which formalizes the intuition that poor consumers have a lower intertemporal elasticity of substitution than do rich consumers because expenditure inelastic goods (necessary goods) are less substitutable over time than are expenditure-elastic goods. Guvenen (2006) shows that aggregate saving is mostly determined by wealthy people who have high EIS and aggregate consumption is mostly determined by non-wealthy people who have low EIS. Weil (1993) and Van der Ploeg (1993) show that when the coefficient of risk aversion increases savings increase. The saving increases as EIS increases and as the coefficient of risk aversion increases is observed in this paper. More importantly, it is examined that EIS is a more important factor for precautionary savings than risk aversion because saving is more responsive to changes in EIS than changes in risk aversion. This finding sheds new light on precautionary savings. Knowing that EIS is more significant contributor to the precautionary savings is important since a significant fraction of the capital accumulation that occurs in the United States is due to precautionary savings according to Zeldes (1989a).

The main limitation of the model of precautionary savings I have introduced in this paper is in future income process. The Markov process is used in the paper where the future income takes only two different values, high income and low income, for simplicity. Investigating other income processes would be a good improvement and future research for giving more representation of the precautionary savings motive. Yet, this model sheds new light on the determinant of precautionary savings in multi-period economics and determines the coefficient of elasticity of intertemporal substitution is a more important factor for precautionary savings than the coefficient of risk aversion because saving is more responsive to changes in the coefficient of elasticity of intertemporal substitution than changes in the coefficient of risk aversion.

Appendix: Numerical Solution

This section describes the numerical solution of the model. The state values of the agent are today's income and bond holding. Then, agent chooses the today's consumption and tomorrow's bond holding, none of them can be negative. Tomorrow's income is determined as a law of motion.

Step1: Initialization

- The interest rate, discount factor, coefficient vectors of EIS and risk aversion are determined. There are two different income values, income low and income high, and different probabilities ranging from 0.5 to 0.8 for the Markov process of income so uncertainty in income in the model comes from this process. EIS changes from 0.04 to 0.99 and Risk aversion changes from 1.01 to 25.The interest rate can be two different values, either 1.03 or 1.04. Thus, calculations are performed for these each different values of income, EIS, risk aversion, interest rate and probabilities.
- There are 100 grid points for the initial bond holdings. I execute value function iteration and determine tomorrow's bond holding for each case by initially assuming b'=b. I am able to use the linear interpolation to evaluate tomorrow's bond holding and the value function for off the grid points since the value function is linear in individual wealth in Epstein-Zin preferences .

Step 2: Household Dynamic Decision Problem

- I start with a household who has an initial income and zero bond at first period and the household decides for current consumption and bond holding of second period. I iterate the process unless the bond holding process converges to a stochastic steady state. I observe 1000 iterations are adequate for the convergence.

- For the income process, I generate pseudo random process for each probabilities of the Markov Process by using “randsrc” function in MATLAB. I generate two different pseudo random processes for two different income values according to probabilities and then produce the real income process that the agent faces in the iteration from those random processes.

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