Estimation of the elasticity of substitution of production factors in CEE economies

Lukáš Rečka*

Abstract

This paper contributes to the literature on CES production function. We provide estimates of substitution elasticities for CEE countries. We apply non-linear estimation techniques to estimates substitution elasticities direct from the CES function. We find that in case of the industry as a whole, the (EL)K nesting structure fits the data best.

1. Introduction

In modern applied economics and especially in the field of environmental and climate policy, Computable General Equilibrium (CGE) models have become one of the leading tools to evaluate policy measures and alternative scenarios (Goulder, 2002, Böhringen, 2003). CGE, econometric, input-output or linear programming models use different types of nested production function with Constant Elasticity of Scale (CES) to describe the production of an economy (Kemfert, 1998). Substitution elasticities are key parameters in these models since measure the ease or difficulty of substituting between inputs in production. Therefore elasticities play an important role in any economic assessment of any policy (Paterson, 2012). Jacoby, Reilly, McFarland, & Paltsev (2006) perform a sensitivity analysis of structural parameters of their MIT-EPPA model and find that substitution elasticities between energy and value added (the capital-labour composite) are the main drivers of model results. But in spite of the importance of elasticities within the framework of applied quantitative simulations, the current situation of elasticities is rather unsatisfying and although the lack of adequate estimates of elasticities has been acknowledged surprisingly long time ago, the problem is still actual (Koesler & Schymura, 2012). In CGE models, "e*stimates for these elasticity values are often taken from the literature and are open to criticism for being outdated, poorly estimated or as often is the case, arbitrary*" (Paterson, 2012, p. 5). The criticism of estimates of substitution elasticities has several dimensions. First, the CES function in CGE model should have the same structure as the CES function used for elasticity estimation, since the elasticity of substitution differs across different kind of nesting structures of CES function. Second, As Van der Werf (2008) points out, many models use different values for the substitution elasticities, even when they use the same nesting structure. Furthermore, many models production function without any empirical foundation, only refer to other paper – also without empirical validation – for the nesting structures and elasticities chosen. So "*technology in generally modelled in a way that the modeler suits best, or to best answer the question under scrutiny*" (Van der Werf, 2008, p. 2966).

* Charles University Environment Center

Last but not least, the parameters of the CES production function could be regionally specific and as a result, the transferability of the estimates of substitution elasticities is limited. Most of the empirical studies focused on estimation of substitution elasticities in CES production function are made in well developed countries (e.g. Balistreri, McDaniel, & Wong (2003); Kemfert (1998); van der Werf(2008)) but to our knowledge, there is no study focused primarily on CEE countries so far. As a consequence, the lack of proper substitution elasticities estimates specific for CEE countries is higher than in other regions.

In this paper we seek to fill this gab and provide consistent estimates of substitution elasticities for industry sectors in CEE countries. We offer an empirical analysis of two dimensions by estimation three-input CES production function for all possible nesting structures. Accordingly, we report the accompanying substitution elasticities for each nesting structures and conclude which nesting structure fits the data best.

The remainder of the paper is organized as follows. We first provide a review of most recent studies focused on estimation of substitution elasticities. Then we specify our model in section 3. In section 4 we describe our data and present our estimation results. We explicitly test several non-linear estimation methods and their robustness and we compare our results with other estimates of substitution elasticities. The last section concludes.

2. Literature review

The need to estimate production function with nested structure is discussed since Berndt and Wood (1979). They argue that "*for production models involving more than two inputs there is no a priori reason to hypothesize either E-K substitutability or complementarity*". (Berndt & Wood, 1979). In 2-level nested ((KE)(LM)) production function, the authors distinguish the gross substitution effect and the expansion effect. They explain that the engineering studies supporting the *E-K* substitutability capture only the gross substitution effect and therefore these studies might not be in contradiction with econometric studies supporting the *E-K* complementarity, because the expansion effect could outweigh the gross substitution effect.

Kemfert (1998) estimates the substitution elasticity of a nested CES production function for the entire German industry and individual industrial sectors. She estimates the elasticity for tree nested structures[1] and concludes that a nested CES function with a nest of capital and energy ((KE)L) is the most useful for the entire German industry, but for several industrial sectors a nest of capital and labour ((KL)E) might be closer to the reality.

To avoid issues related to linearization of estimation techniques by Kmenta approximation (Kmenta, 1967), Van der Werf (2008) and Okagawa and Ban (2008) use a cost function based approach, where they take advantage of the cost function associated with a specific production function and derive a linear system of equation from the corresponding optimal input demand. The disadvantage of this approach is that it requires exhaustive price data, which is in most cases rather difficult to obtain, especially for sector specific estimates. Van der Werf (2008) estimates a 2-level nested production function, using the industrial level data from 12 OECD countries in for 1978-1996. He finds that the nesting structure having capital and labour in the same node fits reality most closely. Similarly,

[1] ((KE)L), ((KL)E) and ((EL)K)

Okagawa and Ban (2008) estimate a nested CES function using another OECD dataset. Their data set is more refined compared to that used in Van der Werf (2008), where the data are disaggregated into 7 sectors; the Okagawa and Ban (2008) data set disaggregated into 19 sectors. Both study use cost function based approach to avoid problems

Henningsen & Henningsen (2011) re-estimate the elasticity of substation for the German industry provided by Kemfert (1998) using a non-linear least squares estimation method. Henningsen & Henningsen (2011) use the same data as Kemfert (1998), they apply several estimation approaches that yield robust results significantly different from those obtained by Kemfert (1998). Henningsen & Henningsen (2011) clearly show that the linear approaches using Kmenta approximation are not proper approaches for CES function estimation. They developed new R package **micEconCES** for economic analysis and economic modelling with a CES function (Henningsen & Henningsen, 2013). Koesler & Schymura (2012) use the World-Input-Output Database (WIOD) to estimate substitution elasticities for a three nested CES ((KL)E)M production function on the basis of non-linear least squares estimation method developed by Henningsen & Henningsen (2011). They estimate the substitution elasticity for 35 sectors pooled across all 40 countries over a period of 12 years (1995-2006) included in the Word-Input-Output Database[2] (WIOD). Koesler & Schymura (2012) confirm that non-linear estimation techniques perform significantly better than standard linear estimations using Kmenta approximation.

Su, Zhou, Ichi, Ren, & Mu (2012) estimate the China's elasticity of substitution of two-level CES KLE production function by three possible combination of nesting: (KL)E, (KE)L and (EL)K.

3. Model specification

As Henningsen & Henningsen (2011) and Koesler & Schymura (2012) show that micEconCES package provides a robust tool to substitution elasticity estimation, we use also it in this paper. The model specification follows Henningsen & Henningsen (2011):

The three- input nested CES function has following general form:

$$y = \gamma \left[\delta\left(\delta_1 x_1^{-\rho_1} + (1-\delta_1)x_2^{-\rho_1}\right)^{\rho/\rho_1} + (1-\delta)x_3^{-\rho}\right]^{-\nu/\rho}, \tag{1}$$

where y is the output quantity, x_1, x_2 and x_3 are input quantities , $\gamma \in (0, \infty)$ an efficiency parameter, δ and $\delta_1 \in (0,1)$ set the optimal distribution of inputs, ρ and $\rho_1 \in (-1,0) \cup (0,\infty)$ determine the (constant) elasticity of substitution, and $\nu \in (0,\infty)$ is equal to the elasticity of scale.

This specification of the CES function includes three special cases: for $\rho \to 0$, σ approaches 1 and the CES turns to the Cobb-Douglas form; for $\rho \to \infty$, σ approaches 0 and the CES turns to the Leontief production function; and for $\rho \to -1$, σ approaches infinity and the CES turns to a linear function if ν is equal to 1.[3]

For instance, x_1, x_2 and x_3 could be capital, labour and energy [(KL)E], respectively. We estimate all three nesting structure of the 2-level 3-input CES production function [(KL)E, (KE)L and (LE)K].

[2] http://www.wiod.org/database/index.htm
[3] All three special cases take place for the nested function in case of ρ_1 also.

The substitution elasticity σ is determined as follows:

$$\sigma = 1/(1+\rho)$$

According to Henningsen & Henningsen (2011) we estimate the Hicks-McFadden (direct) elasticity of substitution between the inputs in the nest ($\sigma_{1,2} = 1/(1+\rho_1)$ and Allen-Uzawa (partial) elasticity of substitution between the nest and the third input ($\sigma_{12,3} = (1/(1+\rho)$).

4. Data and Estimation results

Although Koesler & Schymura (2012) argue in favour using the WIOD database as a consistent source of data for substitution elasticity estimation, we use only a part of the WIOD database. Our reason is that the WIOD database contains energy data only in form of *gross energy use* that includes energy supply and energy consumption and as a result there is a double counting of energy use. Therefore we use *Gross output* (Y), *Number of employees* (L), *Gross capital stock* (K) and *Gross value added* (VA) from the Socio-Economic Accounts of WIOD database and the industry *Final energy consumption* (E) we take from Eurostat Energy Balances.[4] All prices (Y, K, M) are converted to Euro 2005. Unfortunately, the Eurostat energy data does not fit to the the WIOD data ideally. Therefore we aggregated some sectors to get 13 industry sectors over a period of 15 years (1995-2009). After dropping observation with missing or zero values we obtain 1599 observation. Tables 1-3 describe summary statistics of the data and list countries and sectors included in our analysis.

Table 1 Summary statistics

Variable	Unit	Obs.	Mean	Std. Dev.	Min	Max
time	year	1599	2001.765	4.162649	1995	2009
Y	mil. €	1599	4185.948	6230.755	16.6	50113.6
K	mil. €	1599	64285.82	224873.9	2	2299303
E	ths. toe	1599	485.0669	1018.436	1	8199
L	thousands	1599	89.49781	130.9458	1	825

Table 2 Countries in the sample

Country	Freq.	Percent
CZE	167	10.44
EST	195	12.2
HUN	167	10.44
LTU	194	12.13
LVA	176	11.01
POL	169	10.57
ROU	191	11.94
SVK	170	10.63

[4] http://epp.eurostat.ec.europa.eu/portal/page/portal/energy/data/database

SVN	170	10.63

Table 3 Sectors analyzed

Sector	NACE rev.1	Freq.	Percent
Agriculture, Forestry & Fishing	AtB	124	7.75
Mining	C	128	8.01
Food&Tobaco	15t16	129	8.07
Textile&Lether	17t19	129	8.07
Wood	20	129	8.07
Paper&Publishing	21t22	129	8.07
Coke&Chemical industry	23t24	129	8.07
Non-metal Minerals	26	129	8.07
Metals	27-29	129	8.07
Transport equipment	34t35	124	7.75
Construction	F	126	7.88
Water transport	61	80	5
Air transport	62	114	7.13

We estimated substitution elasticities for all tree nesting structure first for the whole industry across the CEE countries and then for each sector separately. As Henningsen & Henningsen (2011) and Koesler & Schymura (2012) show that the estimates of CES function parameters can differ across different estimation methods, we apply a set of different optimisation algorithms for estimation of the whole industry, namely the Levenberg-Marquardt algorithm (LM) (Marquardt , 1963), PORT routines (Gay , 1990), the Differential Evolution algorithm (DE) (Storn and Price , 1997), Nelder-Mead routines (NM) (Nelder and Mead , 1965), the Simulated Annealing algorithm (SANN) (Kirkpatrick et al. , 1987; Cerny , 1985), the so called BFGS algorithm (Broyden , 1970; Fletcher , 1970; Goldfarb , 1970; Shanno , 1970) and restricted BFGS (L-BFGS-B). We employ also two-dimensional grid search (Grid) for ρ_1 and ρ using all gradient-based algorithms (Newton, BFGS, LM, PORT) and with these algorithms with starting values equals to the estimates from the corresponding grid search (Grid-Start). A detailed overview of all the estimations for the whole industry we run is given in Tables 8-10 in the Appendix. In the results we can see values out of economically meaningful region in some cases. In these cases we should consider only the values from PORTs and L-BFGS-B method, because these methods are constructed in the way to be consistent with economic theory.

If we compare our elasticities estimates with estimates in Kemfert (1998) and Henningsen & Henningsen (2011), we can see that for (KE)L nesting structure our estimates are closer to Kemfert (1998) but for (KL)E and (EL)K they are very close to Henningsen & Henningsen (2011)

Table 4 Comparison of our estimates of elasticities with Kemfert (1998) (K) and Henningsen & Henningsen (2011) (H)

		K	H	Own	K	H	Own	K	H	Own
Nesting structure			(KE)L			(KL)E			(EL)K	
Elasticities	σ_1	0.653	0.0667	0.5962	0.822	0.158-0.161	0.0849	0.422	Inf.	Inf.
	σ	0.846	Inf.	0.5658	0.458	Inf.	Inf.	0.146	0.118139	0.1296

From the results for the whole industry we can see that our results are robust with regard to the choice of the employed optimisation algorithm, for estimation elasticities of each sector we apply only BFGS, L-BFGS-B, PORT and BFGS, PORT, LM with grid search and with starting values from the corresponding grid search. Below, we present only the estimates from the method with the best fit according to the value of R^2 and Residual sum of squares (RSS). Tables 5-7 show the results for each sector in the corresponding nesting structure.

Table 5 Sectors: (KE) L

Sector	gamma	lambda	delta1	delta	rho1	rho	c	RSS	R2	Method
Agri.,F.&F.	43.6663	0.0248	-0.427	0.28	-0.1	-1	1	428621168	0.9384	LM
Mining	12.7189	0.05121	0.6378	0.07	0.323	-1	1	9899369	0.9883	PORT
Food&Tob.	42.014	0.02894	0.0551	0.052	-0.035	-1	1	132304822	0.9864	PORT
Textile&Let.	8.15566	0.02437	0.4251	0.566	0.066	0.545	1	38413057	0.9272	PORT
Wood	2.94E-13	0.01547	0.2277	0.782	0.09	0.263	1	17623792	0.9169	LM
Paper&Publ.	13.9144	0.014	0.9999	0.725	14.63	0.701	1	14140925	0.9808	LM-GR-Start
Coke& Chem.	2.6155	0.0602	0.5669	1	-1	6.4	1	180801938	0.945	PORT-GR
Minerals	3.93163	0.03936	1.35e-16	0.99	70.37	1.914	1	10423227	0.9793	LM-GR-Start
Metals	3.27847	0.04974	0.7176	1	-1	12.83	0	353583385	0.955	PORT
Transport eq.	10.7452	0.08237	0.148	1	-0.049	15.27	0	406295319	0.8857	PORT
Construction	7.93381	0.03842	1	0.986	10.17	2.569	0	662408165	0.9484	
Water tran.	28.2841	0.03437	1	0.048	10.9	-1	0	680502	0.8287	PORT
Air transport	11.5989	0.01678	0.0361	0.146	-0.065	-1	1	611943	0.9365	PORT

Table 6 Sectors: (KL)E

Sector	gamma	lambda	delta1	delta	rho1	rho	c	RSS	R2	Method
Agri.,F.&F.	60.4296	0.00914	-0.31	0.935	0.3	-1	1	484656501	0.93	LM
Mining	12.655	0.0507	0.175	0.639	0.1153	-1	1	10185053	0.988	PORT
Food&Tob.	40.2892	0.02914	0.016	0.941	-3E-04	-1	1	132680697	0.986	PORT

Textile&Let.	8.11529	0.02308	0.375	0.605	0.3025	-1	1	37498130	0.929	PORT
Wood	9.47E-14	0.01604	0.452	0.445	0.1907	-0.143	1	17590957	0.917	LM
Paper&Publ.	7.58316	0.0143	1	0.791	13.2	-0.4	0	12488986	0.983	LM-GR-Start
Coke& Chem.	8.5012	0.0576	0.899	0.963	0.9704	-1	1	197758264	0.94	PORT-GR
Minerals	4.8249	0.0398	1	0.912	13.6	-1	1	10436633	0.979	LM-GR-Start
Metals	3.70208	0.05348	1	0.808	9.7609	-1	1	353321835	0.955	PORT
Transport eq.	14.3181	0.06113	0.459	0.475	0.2324	-1	1	441575564	0.876	PORT
Construction	7.44317	0.03719	0.999	1	4.1051	3.3418	1	668176501	0.948	
Water tran.	3.82566	0.03669	1	0.623	6.8701	-0.082	0	691704	0.826	PORT
Air transport	5.82663	0.01805	2E-04	0.448	-1	-0.384	1	634143	0.934	PORT

Table 7 Sectors: (EL) K

Sector	gamma	lambda	delta1	delta	rho1	rho	c	RSS	R2	Method
Agri.,F.&F.	8.04737	0.008014	0.1012	1.46E-08	-1	12	1	559855189	0.919	LM
Mining	11.7894	0.050614	0.2503	0.857455	-1	0.1149	1	9840388	0.988	PORT
Food&Tob.	40.6803	0.029109	0.0568	0.987803	-1	-0.016	1	132558463	0.986	PORT
Textile&Let.	7.6794	0.015606	0.1709	0.684572	-1	0.3002	1	38964088	0.926	PORT
Wood	2.48E-13	0.015561	0.6983	0.822587	0.179	0.1075	1	17644614	0.917	LM
Paper&Publ.	8.2616	0.01374	0.1123	8.94E-11	-0.4	68.366	0	11875420	0.984	LM-GR-Start
Coke& Chem.	5.8929	0.0583	0.9999	0.5346	4.523	0.9138	1	199392254	0.939	PORT-GR
Minerals	5.74215	0.038596	0.8127	1.23E-11	0.83	101.59	1	10076235	0.98	LM-GR-Start
Metals	6.35499	0.0492	0.0117	0.007724	-1	3.0128	1	395801119	0.95	PORT
Transport eq.	16.0955	0.059437	1	0.877025	45.32	0.0145	0	442402075	0.876	PORT
Construction	7.45371	0.037174	0.0048	0.000969	3.368	4.0551	1	668168488	0.948	
Water tran.	5.8836	0.03188	0.2556	0.039642	-1	1.7621	1	711141	0.821	PORT
Air transport	10.5988	0.016341	0.1538	0.968643	-1	-0.029	1	612408	0.936	PORT

5. Conclusion

This paper contributes to the literature on CES production function estimation strongly linked with CGE and other models using elasticities of substitution as parameters. We apply non-linear estimation techniques to estimates substitution elasticities direct from the CES function. We focus on CEE countries, because as we know no other estimate for CEE countries has been done before. We find that in case of the industry as a whole, the (EL)K nesting structure fits the data best. But it is no surprise that the sector specific estimates fit the data much more than in case of estimates of the industry as a whole. Therefore it seems to be desirable to estimate sector specific elasticities of substitution. Another finding is that the non-linear estimation methods tend to be consistent rather within a sector than within a nesting structure.

References

Balistreri, E. J., McDaniel, C. a, & Wong, E. V. (2003). An estimation of US industry-level capital–labor substitution elasticities: support for Cobb–Douglas. *The North American Journal of Economics and Finance*, *14*(3), 343–356. doi:10.1016/S1062-9408(03)00024-X

Berndt, E. R., & Wood, D. O. (1979). Engineering and Econometric Interpretations of Energy-Capital Complementarity. *The American Economic Review*, *69*(3), 342–354. Retrieved from http://www.jstor.org/stable/1807369

BÖHRINGER, C., RUTHERFORD, T.F., and WIEGARD,W. (2003): Computable General Equilibrium Analysis: Opening a Black Box, in: ZEW Discussion Paper, No. 03-56.

BROYDEN, C.G. (1970): The Convergence of a Class of Double-rank Minimization Algorithms, in: Journal of the Institute of Mathematics and Its Applications, Vol. 6, pp. 76-90.

FLETCHER, R. (1970): A New Approach to Variable Metric Algorithms, in: Computer Journal, Vol. 13, pp. 317-322.

GOLDFARB, D.A. (1970): A Family of Variable Metric Updates Derived by Variational Means, in: Mathematics of Computation, Vol. 24, pp. 23-26.

Goulder, Lawrence H. (Lawrence Herbert) (2002). *Environmental policy making in economies with prior tax distortions*. Edward Elgar, Cheltenham

Henningsen, A., & Henningsen, G. (2011). Econometric Estimation of the “ Constant Elasticity of Substitution ” Function in R : Package micEconCES.

Henningsen, A., & Henningsen, G. (2013). Package “ micEconCES ” - Analysis with the Constant Elasticity of Scale (CES) function. Retrieved from http://www.micecon.org

Jacoby, H. D., Reilly, J. M., McFarland, J. R., & Paltsev, S. (2006). Technology and technical change in the MIT EPPA model. *Energy Economics*, *28*(5-6), 610–631. doi:10.1016/j.eneco.2006.05.014

Kemfert, C. (1998). Estimated substitution elasticities of a nested CES production function approach for Germany. *Energy Economics*, *20*(3), 249–264. doi:10.1016/S0140-9883(97)00014-5

Kmenta, J. (1967). On estimation of the CES production function. *International Economic Review*, *8*(2), 180–189. Retrieved from http://www.jstor.org/stable/10.2307/2525600

Koesler, S., & Schymura, M. (2012). Substitution Elasticities in a CES Production Framework. An Empirical Analysis on the Basis of Non-Linear Least Squares Estimations. *wiod.org*. Retrieved from http://www.wiod.org/conferences/groningen/Paper_Koesler_Schymura.pdf

Paterson, N. (2012). Elasticities of Substitution in Computable General Equilibrium Models.

SHANNO, D.F. (1970): Conditioning of Quasi-Newton Methods for Function Minimization, in: Mathematics of Computation, Vol. 24, pp. 647-656.

Su, X., Zhou, W., Ichi, K., Ren, H., & Mu, H. (2012). Capital stock-labor-energy substitution and production efficiency study for China. *Energy Economics*, *34*(4), 1208–1213. doi:10.1016/j.eneco.2011.11.002

Van der Werf, E. (2008). Production functions for climate policy modeling: An empirical analysis. *Energy Economics*, *30*(6), 2964–2979. Retrieved from http://econpapers.repec.org/article/eeeeneeco/v_3a30_3ay_3a2008_3ai_3a6_3ap_3a2964-2979.htm

Appendix

Table 8 Industry (KE)L

gamma	lambda	delta1	delta	rho1	rho	c	RSS	R2	Method
10.2221	0.04153	0.9413	0.6945	0.6773	0.7673	1	1.2E+10	0.8063	Newton
10.2221	0.04153	0.9413	0.6945	0.6773	0.7673	1	1.2E+10	0.8063	BFGS
10.2221	0.04153	0.9413	0.6945	0.6773	0.7673	1	1.2E+10	0.8063	PORT
10.2216	0.04153	0.9412	0.6946	0.677	0.7674	1	1.2E+10	0.8063	LM
10.1333	0.0424	1	0.6099	4.032	0.5845	1	1.22E+10	0.803	NM
10.2224	0.04153	0.9413	0.6944	0.6775	0.7672	1	1.2E+10	0.8063	NM-LM
10.2221	0.04153	0.9413	0.6945	0.6773	0.7673	1	1.2E+10	0.8063	NM-PORT
12.2002	0.0366	0.9245	0.6146	0.6138	0.6637	NA	1.21E+10	0.8052	SANN
10.2224	0.04153	0.9413	0.6944	0.6774	0.7672	1	1.2E+10	0.8063	SANN-LM
10.2221	0.04153	0.9413	0.6945	0.6773	0.7673	1	1.2E+10	0.8063	SANN-PORT
10.0526	0.04156	0.9448	0.7107	0.7	0.8	1	1.2E+10	0.8063	Newton-GR
10.2221	**0.04153**	**0.9413**	**0.6945**	**0.6773**	**0.7673**	**1**	**1.2E+10**	**0.8063**	**Newton-GR-Strart**
10.0527	0.04156	0.9448	0.7107	0.7	0.8	1	1.2E+10	0.8063	PORT-GR
10.222	0.04153	0.9413	0.6945	0.6773	0.7673	1	1.2E+10	0.8063	PORT-GR-Start
10.0526	0.04156	0.9448	0.7107	0.7	0.8	1	1.2E+10	0.8063	LM-GR
10.2219	0.04153	0.9413	0.6945	0.6773	0.7673	1	1.2E+10	0.8063	LM-GR-Start

Table 9 Industry (KL)E

gamma	lambda	delta1	delta	rho1	rho	c	RSS	R2	Method
8.5322	0.0329	1	1	13.336	-22.78	0	10479903886	0.83107	Newton
7.3797	0.0396	0.9813	0.9999	2.4582	-9.496	1	10286894915	0.83418	BFGS

6.091	0.041	0.9999	0.8154	4.7621	-1	1	10394973875	0.83244	L-BFGS-B
6.1064	**0.041**	**1**	**0.8176**	**10.784**	**-1**	**1**	**10374984245**	**0.83276**	**PORT**
6.0367	0.0406	1	0.8699	7.8226	-1.934	1	10273499366	0.8344	LM
12.478	0.0364	0.5569	0.9915	0.6781	-2.931	1	10676805176	0.8279	NM
9.8129	0.04	0.7129	0.9833	0.899	-2.87	NA	10441935248	0.83168	SANN
6.1063	0.041	1	0.8176	10.8	-1	1	10374984257	0.83276	PORT-GR
6.1063	0.041	1	0.8176	10.8	-1	0	10374984259	0.83276	PORT-GR-Start

Table 10 Industry (EL)K

gamma	lambda	delta1	delta	rho1	rho	c	RSS	R2	Method
7.77347	0.03577	0.00046	1.34E-11	-3.105	16.1867	0	9403038520	0.8484	Newton
8.86172	0.04029	7.99E-10	0.166504	-8.578	1.22942	1	9024898159	0.8545	BFGS
6.79753	0.04139	0.039316	1.15E-05	-1	7.13274	1	10064286840	0.8378	L-BFGS-B
6.8806	**0.04023**	**0.040036**	**2.36E-05**	**-1**	**6.71295**	**1**	**10061703079**	**0.8378**	**PORT**
9.22607	0.03983	1.23E-76	0.207718	-71.37	1.08961	0	8964783066	0.8555	LM
10.0716	0.04298	3.22E-07	0.350598	-6.149	0.77528	1	9157846438	0.8524	NM
6.55736	0.04371	0.009003	0.029966	-1.776	2.13685	NA	9744463520	0.8429	SANN
6.87729	0.04023	0.04004	2.05E-05	-1	6.8	1	10061705508	0.8378	PORT-GR
6.87729	0.04023	0.04004	2.05E-05	-1	6.8	0	10061705508	0.8378	PORT-GR-Start