Stochastic energy market equilibrium modeling with multiple agents[1]

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Abstract

We present a simple approach to transform a deterministic numerical equilibrium model - where several agents simultaneously make decisions - into a stochastic equilibrium model. This approach is used to build a large stochastic numerical equilibrium model of the Western European energy markets where investment decisions must be taken before the uncertainty is revealed. We use the stochastic model to analyze the impact of economic uncertainty on the Western European energy markets; it is demonstrated that the equilibrium under uncertainty differs significantly from the deterministic outcome.

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1 Introduction

Agents in the European energy market have experienced considerable uncertainty: during the last financial crisis demand for energy dropped significantly, and in the future international climate treaties may trigger radical changes in the energy markets. Such abundant uncertainties can have huge consequences on investment in the energy industry. At the same time, if there is reluctance to invest in some technologies, for example, due to expectations about high fossil fuel prices or high taxes on greenhouse gas emissions, the market may look more promising for other technologies, like renewables. Thus to fully analyze the impact of uncertainty we need to take into account the *interdependence* of different technologies, energy goods and agents – this calls for a multi-dimensional *equilibrium* model that captures the essential characteristics of the energy industry.

It is, however, not trivial to solve, or even formulate, a model where many heterogeneous decision makers face uncertainty. Thus it is not surprising that most analyses assume full certainty, or, if uncertainty is analyzed, rely on simulations instead of examining the behavior of agents optimizing under uncertainty. Notable exceptions include stochastic equilibrium models for gas transportation network (Bjørndal et al. 2007), the electricity market (Ehrenmann and Smeers 2008, Gabriel and Fuller 2010), hydrothermal generation (Cabero et al. 2010), and the world-wide gas market (Egging 2010; 2013).

In this paper we present a framework for *stochastic equilibrium modeling*. Our main contribution is to offer an efficient way to transform a deterministic equilibrium model, where several agents simultaneously make decisions, into a stochastic equilibrium model. We illustrate that this transformation strategy works for a large-scale numerical deterministic multi-market equilibrium model of the Western European energy markets – LIBEMOD, see Aune et al. (2008).

We use the same software package (GAMS/PATH) to formulate and solve both the deterministic version of LIBEMOD and the stochastic LIBEMOD model. Thus, no programming of a stochastic solution algorithm is required. Hence, our second contribution to the literature is to demonstrate that is possible to formulate and solve a stochastic version of a large-scale equilibrium model without programming specific stochastic solution algorithms. This is in contrast to the papers cited above; Egging (2013), for example, focuses on implementation of a Benders Decomposition type of algorithm for large-scale stochastic multi-period mixed complementarity problems.

Third, we use the stochastic version of LIBEMOD to analyze the impact of economic uncertainty on the European energy markets. Here our contribution is to analyze the equilibrium when different types of heterogeneous agents simultaneously make decisions under uncertainty and all prices are endogenous.

In spirit, our approach to modeling uncertainty is similar to the discussion of uncertainty in Debreu's (1959, chapter 7) classic 'Theory of Value', where uncertainty is represented by a discrete event tree. In our terminology, each branch of Debreu's event tree is called a scenario. Hence, in our model uncertainty is represented by a set of scenarios. Each scenario is one possible future realization of the uncertainty, and each scenario is assigned a probability.

In order to properly handle uncertainty we build on the basic ideas of stochastic programming, see, for example, Kall and Wallace (1994). Here a crucial distinction is made between decisions made before the uncertainty is revealed, and decisions made afterwards. A key insight is that it is not valid to solve the model scenario by scenario and try to extract an overall picture from these solutions. There is, however, a large literature that does not make the distinction between decisions taken prior to and after the uncertainty is revealed; these studies go under many names, such as what-if analysis, sensitivity analysis, scenario analysis and also Monte Carlo simulations. Whereas this alternative approach is widely used, it does not have a sound theoretical basis. In particular, scenarios are not assigned probabilities but are rather seen as some of the possible future states; see, Higle and Wallace (2003) for an example showing what can go wrong. In contrast, in stochastic programming each scenario is assigned a probability and the sum of probabilities equals one.

Because one should make a clear distinction between decisions taken under uncertainty and decisions taken after the uncertainty has been revealed, the models of uncertainty presented in this paper have two periods. In period 1, some agents make decisions under uncertainty, typically to determine their future capacities through investments. In the beginning of period 2, the uncertainty is revealed and all agents learn the true state of the economy, that is, which scenario that has materialized. Then all agents make decisions; producers determine how much to produce (given the predetermined capacities), arbitrators determine how much to trade, and consumers determine how much to consume.

For each realization of the uncertainty, that is, for each scenario, the model determines supply of, and demand for, all goods from all agents and the corresponding vector of prices that clears all markets. In fact, the stochastic equilibrium model determines simultaneously all quantities (investment, production, trade and consumption) and all market clearing prices for *all* scenarios. The determination of quantities and prices are based on the assumption that all agents have rational expectations, that is, when investment decisions are taken in the first period agents take into account the probability distribution over the scenarios and the equilibrium prices of all scenarios.

In Section 2 we set up a simple equilibrium model with investment, production, trade and consumption. We solve the model both when there is no uncertainty and when investment decisions are taken under uncertainty. In particular, we show how the idea in Wets (1989) and Rockafellar and Wets (1991) to find the solution of a stochastic problem for a single agent can be used to solve a stochastic equilibrium model where several agents make decisions simultaneously. This gives us a guide on how to transform a deterministic equilibrium model into a stochastic equilibrium model. For a given specification of scenarios, the stochastic equilibrium model finds the optimal solution under uncertainty.

Note that although the formulation of the stochastic model is in the line with the basic ideas of stochastic programming, we do not solve the model by using stochastic programming; instead we build on an idea in Rockafellar and Wets (1991) on how to formulate the problem. This is an efficient strategy to transform a pre-existing deterministic model (LIBEMOD), where several agents simultaneously make decisions, to a stochastic equilibrium model (the stochastic version of LIBEMOD).

In Section 3 we present the deterministic version of the large-scale non-linear numerical equilibrium model LIBEMOD. We then use the guide from Section 2 to transform the deterministic LIBEMOD model into a stochastic equilibrium model where agents face uncertain prices when they make investment decisions. We generate the uncertainty by letting GDP growth rates and supply of coal and oil (from countries outside Western Europe) vary among the scenarios. The resulting equilibrium prices therefore differ across scenarios. Note that at the time of the investment decisions, *all* prices in *all* scenarios are uncertain.

We formulate the stochastic equilibrium model in the GAMS modelling environment (Brooke et al., 1998) and solve it with the PATH complementarity solver (Ferris and Munson, 1998). For our case, it is not necessary to use specialized solution approaches such as the

Bender's decomposition or the progressive hedging algorithm of Rockafellar and Wets (1991) as standard software is sufficiently powerful.

In Section 5 we compare the stochastic equilibrium with the deterministic equilibrium by using the expected values of the stochastic variables in the deterministic model. Thus we run the deterministic model with the expected GDP growth rates and the expected supply of coal and oil (from countries outside Western Europe). Our results indicate that uncertainty has a considerable impact on optimal investments. First, investment in electricity transmission is considerably higher when there is economic uncertainty than in the case of no uncertainty. Second, the composition of investment in electricity technologies differs significantly between the cases of economic uncertainty and no uncertainty. For example, optimal investment in wind power is much higher under economic uncertainty than in the case without uncertainty.

We also compare the stochastic equilibrium with Monte Carlo simulations. Usually, Monte Carlo simulations mean that the modeler draws a value from a probability distribution, runs the *deterministic* model with this realization, and then repeats the procedure many times. In our case, we have a discrete joint distribution over GDP growth rates and supply of coal and oil (from countries outside Western Europe), and the realization of a draw from this distribution corresponds to one scenario. Because the number of scenarios is “only” 10 (see Section 4), we do not repeatedly draw from the distribution but run the deterministic LIBEMOD model 10 times, that is, once for each of the 10 scenarios. To distinguish this procedure from the standard one, we refer to it as a Complete Monte Carlo Simulation (*CMCS*).

Using the probabilities of the scenarios, we calculate the average of the *CMCS* for each variable and compare these with the corresponding variables in the deterministic and the stochastic equilibrium. Typically, we find that the *CMCS* average is closer to the stochastic equilibrium than the deterministic solution. However, in some cases, and in particular for single countries and single technologies, the *CMCS* produces numbers that are far from those obtained with stochastic equilibrium modelling.

The rest of the paper is structured as follows. In Section 2 we present a simple analytical model that has the same basic multi-agent structure as LIBEMOD. We use this model to illustrate stochastic equilibrium modeling and compare this equilibrium to the one under no uncertainty, and also to Monte Carlo simulations (*CMCS* averages). Section 3 provides a description of the numerical model LIBEMOD. In Section 4 we describe the scenarios and in Section 5 we compare the equilibrium of the stochastic version of LIBEMOD with the equilibrium of the deterministic version of LIBEMOD and also with Complete Monte Carlo Simulations based on the deterministic version of LIBEMOD. Finally, Section 6 concludes.

2 A guide to transform deterministic models to stochastic models

The main purpose of this section is to show how the general idea in Rockafellar and Wets (1991) to find the solution of a stochastic problem for a *single* agent can be used to transform a *deterministic equilibrium* model with *multiple* agents into a stochastic equilibrium model with multiple agents. To this end we first set up a simple deterministic equilibrium model for a two-region electricity market. In each region there is production and consumption of electricity, and consumption in a region depends on one (utility) parameter. While the model is simple enough to be solved analytically, the basic formulation is similar to the numerical LIBEMOD model described in Section 3, or more generally to a CGE model.

The simplicity of the model makes the impact of uncertainty more transparent. We show that in the case of no uncertainty, there will be no investment in transmission between the two regions, and hence no trade in electricity. We demonstrate that this result does *not* depend on the values of the demand parameters. In contrast, with stochastic demand parameters there will be investment in transmission in the stochastic equilibrium. This capacity will be utilized if the realizations of the two stochastic demand parameters differ, which means that one region has higher demand for electricity than the other.

2.1 The deterministic model

We consider electricity production and consumption in two regions. In each region *i*, *i=1,2*, there is a representative producer *i*. Initially a producer has no production capacity, but he can invest in capacity at a constant unit cost *c*. There is no cost of operating the capacity, so production will equal capacity (K_i). There is also a transmission company which may invest in a transmission line between the two regions. Initially, there is no transmission capacity, but the transmission company can invest in capacity (K_T) at a constant unit cost (c_T).

The model has two periods (but we neglect discounting between the periods). In period 1 the agents may invest in capacity. In the beginning of period 2, the new capacities are available, and there is production and consumption like in any standard deterministic model. We assume that the electricity producer in market *i* can sell electricity in this market only, whereas the transmission company can buy electricity in one market and sell this electricity in the other market.

In period 1 the electricity producer knows that in the next period the price of electricity will be p_i. The electricity producer in region i will therefore maximize $(p_i - c)K_i$. The Kuhn-Tucker first-order complementarity condition of this problem is

$$p_i \leq c \perp K_i \geq 0$$

where the complementarity operator $\perp$ indicates that one or both of the weak inequalities must hold as strict equalities. In this case $p_i = c$ if it is optimal to invest in production capacity $(K_i > 0)$, whereas $p_i < c$ if it is not optimal to invest in production capacity $(K_i = 0)$.

In period 1 the transmission company determines its investment in transmission capacity. Let z_1 be electricity bought in market 1 by the transmission company. This quantity is exported to market 2 and then sold in market 2 by the transmission company. Correspondingly, let z_2 be electricity bought in market 2 by the transmission company and then exported to market 1. Profits of the transmission company are then $(p_1 - p_2)z_2 + (p_2 - p_1)z_1 - c_T K_T$. Of course, exports cannot exceed the transmission capacity, and hence in period 2 the transmission company faces the following two restrictions:

$$\begin{aligned} z_1 &\leq K_T \perp \gamma_1 \geq 0 \\ z_2 &\leq K_T \perp \gamma_2 \geq 0 \end{aligned}$$

where γ_i is the shadow price associated with the constraint on the amount of imports to market i. Maximizing profits with respect to transmission capacity and export quantities, the first-order conditions are:

$$\begin{aligned} \gamma_1 + \gamma_2 &\leq c_T \perp K_T \geq 0 \\ p_2 - p_1 &\leq \gamma_1 \perp z_1 \geq 0 \\ p_1 - p_2 &\leq \gamma_2 \perp z_2 \geq 0. \end{aligned}$$

In period 2, the electricity producers will use their entire production capacity (as long as demand is positive) because there are no costs of production.

In each region there is a representative consumer. His consumption of electricity (x_i) gives him gross utility $2\theta_i\sqrt{x_i}$, where θ_i is the utility parameter of the representative consumer in market *i*. This parameter may depend on a number of factors, for example, the temperature. Henceforth, a high parameter value is associated with cold weather, and therefore a high utility of electricity for heating. The consumer in region *i* maximizes his net utility $2\theta_i\sqrt{x_i} - p_i x_i$. The first-order condition for the consumer is:

$$x_i = \left(\frac{\theta_i}{p_i}\right)^2 .$$

Finally, the market clearing conditions are:

$$\begin{aligned} K_1 + z_2 - z_1 &= x_1 \\ K_2 + z_1 - z_2 &= x_2 . \end{aligned}$$

Hence, in each region domestic production of electricity (K_i) plus net imports of electricity $(z_j - z_i, j \neq i)$ is equal to consumption of electricity in this region (x_i).

The deterministic market equilibrium

The market equilibrium in this case is obvious. For prices approaching zero, demand is infinite. Hence, there will be production of electricity, which requires investment in production capacity in period 1; $K_i > 0$. With an interior solution for production capacity, we have $p_i = c$. Therefore, prices are equal between the two markets, and it will not be profitable to invest in transmission capacity to export electricity between the two regions $(K_T = 0)$. Hence, there will be no trade. Technically, $p_1 = p_2$ and $\gamma_1 = \gamma_2 = 0$. Thus the equilibrium is characterized as follows:

$$K_i = x_i = \left(\frac{\theta_i}{c}\right)^2$$
$$p_i = c$$
$$K_T = 0.$$

Note that no matter the value of $(\theta_1, \theta_2),$ the optimal solution is always $K_T = 0$. While this may seem like a very robust result, as demonstrated below $K_T = 0$ is not the equilibrium in the stochastic model.[2]

2.2 Modelling uncertainty

We now transform the deterministic model to a stochastic model by letting the demand parameters be random. Suppose that there are two possible values of $\theta_i \in \{\theta_L, \theta_H\}$ for each market. This makes four possible combinations: $(\theta_1, \theta_2) \in \{(\theta_L, \theta_L), (\theta_L, \theta_H), (\theta_H, \theta_L), (\theta_H, \theta_H)\}$. We denote each of the four outcomes as a scenario *s*, $s \in \{1,2,3,4\} = S$. The probability that scenario *s* materializes is q_s where $\sum_{s=1}^{4} q_s = 1$.

With uncertainty we need to specify the information available to the decision maker at the time of making the decision. We will assume that investment decisions are taken under uncertainty in period 1. In the beginning of period 2, agents learn the true scenario and trade, consumption and production decisions are taken. Hence, these decisions are determined after the uncertainty has been resolved.

Let us now consider the maximization problem of electricity producer *i*. The straightforward formulation would be to maximize

$$\sum_{s=1}^{4} q_s (p_{is} - c) K_i = \left(Ep_i - c\right) K_i$$

where Ep_i is the expected value of the price. This would give the first-order condition

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[2] For other examples for which the deterministic equilibrium differs qualitatively from the stochastic one, see Wallace (2000).

$$Ep_i \le c \perp K_i \ge 0.$$

While this would of course work, we want to find a strategy that makes the changes as small as possible when we move from a (pre-existing) deterministic model to a stochastic model. We want this strategy to be able to handle a large set of models, for example, multi-period models with learning, see section 2.4. To this end we employ a model formulation from Rockafellar and Wets (1991). To explain this approach, suppose we simultaneously solve the deterministic model for each of the four scenarios. This would simply amount to specify the first-order conditions four times, once for each scenario. We could do this by adding an index *s* for the scenarios to each variable. Thus the first-order condition for the electricity producers would be

$$p_{is} \le c \perp K_{is} \ge 0.$$

This condition has to be satisfied for each electricity producer *i* and each scenario *s*. Such a simultaneous solution would only require a scenario index on the variables. However, this will *not* be the solution to the stochastic problem: with no link between the scenarios, the production capacity would be $K_{is} = \left(\theta_{is} / c\right)^2$, and hence a producer would have a different capital stock for each scenario. But this does not make sense: because capital has to be chosen *before* the scenario is revealed, the capital stock must be the *same* in all scenarios. Therefore, $K_{i1} \ne K_{i2}$ cannot be a solution when capital is chosen *before* the firm knows the scenario. We therefore have to impose the condition that $K_{is} = K_{is'}$ for all $s, s' \in S$. Below this restriction is specified as $K_{is} = K_i$ for $s = 1, 2, 3, 4$ and it is referred to as the *implementability constraint*.

The discussion above implies that under uncertainty the investor cannot maximize profit for each individual scenario separately. With uncertainty, the aim of the electricity producer is to find the production capacity in each scenario (K_{is}) that solves the following problem:

$$\max \sum_{s=1}^{4} q_s (p_{is} - c) K_{is} \quad \text{subject to}$$
$$K_{is} = K_i \ \text{ for all } s.$$

The first-order conditions are

$$q_s p_{is} = q_s c + \omega_{is}^{'}, \;\; s = 1,2,3,4.$$
$$\sum \omega_{is}^{'} = 0, \;\; s = 1,2,3,4.$$

where ω' is the shadow price of the implementability constraint. Now, define $\omega_{is} = \frac{\omega_{is}^{'}}{q_s}$ - the probability adjusted shadow prices. The first-order conditions can then be rewritten as

$$p_{is} = c + \omega_{is}, \;\; s = 1,2,3,4.$$
$$E\omega_i = \sum_{s=1}^{4} q_s \omega_{is} = 0.$$

Compared to the first-order condition in the deterministic case ($p_i = c$), we have only added the (probability adjusted) shadow price of the implementability constraint (ω_{is}) and indexed all variables by *s*. In addition, we have a condition for the (probability adjusted) shadow price; its expected value should be zero. The number of equations has increased from 2 in the deterministic case (one for each region) to $2s+2$ in the stochastic case.

The first-order condition for investment in transmission capacity in the deterministic case is changed in the same way as the condition for investment in electricity production capacity; the first-order condition in the deterministic model is extended by an additive term ε_s, which is the (probability adjusted) shadow price of the implementability constraint $K_{Ts} = K_T$, $s = 1,2,3,4$, and all variables are indexed by s. In addition, the expected value of the shadow price ε is zero:

$$\gamma_{1s} + \gamma_{2s} \leq c_T + \varepsilon_s \perp K_{Ts} \geq 0$$
$$E\varepsilon = 0.$$

Actual transmission (trade) and consumption is decided in period 2, that is, *after* the scenario is known. Thus, to characterize these decisions no implementability constraint is needed; the conditions are therefore similar to the ones for the deterministic case, except that all variables are indexed by *s*. The first-order conditions for trade are thus

$$p_{2s} - p_{1s} \leq \gamma_{1s} \perp z_{1s} \geq 0$$
$$p_{1s} - p_{2s} \leq \gamma_{2s} \perp z_{2s} \geq 0,$$

whereas the first-order condition for the consumers is

$$x_{is} = \left(\frac{\theta_{is}}{p_{is}} \right)^2 .$$

The number of equations has increased from 2 in the deterministic case (one for each region) to $2s$ in the stochastic case.

Finally, market clearing requires

$$K_{1s} + z_{2s} - z_{1s} = x_{1s}$$
$$K_{2s} + z_{1s} - z_{2s} = x_{2s}.$$

Again, the only difference to the deterministic case is that all variables have been indexed by *s*.

The stochastic solution

As noted above, the deterministic solution implies $p_i = c$ for any value of (θ_1, θ_2). For the stochastic solution, this is no longer the case as $p_{is} = c + \omega_{is}$ and the shadow price of the implementability constraint will generally be non-zero. Thus prices may deviate from unit cost. The intuitive reason is that an electricity producer has to decide on a capacity *before* demand is known. Thus the same amount of electricity is supplied when demand is low as when demand is high, and consequently the price will be low when demand is low and high when demand is high.

With uncertainty, the realizations of θ_1 and θ_2 may differ and therefore prices may differ between regions. This provides a market for transmission. It is easy to choose parameters such that the optimal stochastic solution is in fact a positive investment in transmission capacity. Thus, what seemed like a robust result in the deterministic model is not true in the stochastic model.

Note that the stochastic equilibrium determines production, consumption, trade flows and prices in *all* scenarios simultaneously (in addition to investments – these do not differ between scenarios). For each decision variable determined under uncertainty (here investments), we introduce one (probability adjusted) shadow price, and a corresponding equation stating that the expected value of this shadow price is zero. For variables that are determined *after* the uncertainty has been revealed, the first-order conditions from the deterministic case are not changed but all these variables are indexed by the scenario because in general their values depend on the scenario. Hence, if there are l relations in the deterministic model, k of these are related to investment decisions, and there are s scenarios, the number of relations in the stochastic model is $ls+k.$ If then the number of scenarios is doubled, the number of relations increases by $ls.$ Thus, whereas it is easy to transform a deterministic model to a stochastic one, the challenge may be to solve the model due to computational challenges. This suggests that the number of scenarios should not be "too high".

2.3 Monte Carlo simulations

Monte Carlo simulation is a method for numerical integration. If we want to compute an expectation $Ef(\theta)$ where θ is a random variable, then Monte Carlo simulation may come handy. We simply draw a number of realizations of the stochastic variable $\theta,$ compute $f(\theta)$ and take the average. This crude method yields a valid estimate of $Ef(\theta)$ and it can be much improved upon, see, for example, Judd (1998, chapter 8).

For a numerical equilibrium model, Monte Carlo may seem as a viable option to assess the impact of uncertainty. We could draw a number of realizations of the stochastic variables and use the average as the expected outcome under uncertainty. Tempting as it is, there is a fundamental problem with this approach.

A key characteristic of equilibrium models is maximizing agents. Let $\pi(x,\theta)$ be the objective function of an agent where x is the vector of decision variables of this agent. Because the parameter θ is uncertain, the agent will maximize under uncertainty, that is, his choice follows from $\operatorname{argmax}_x E\pi(x,\theta).$ Assume instead that we rely on Monte Carlo simulation. Then we would use the following two-stage procedure: First, for each realization of the stochastic variable θ we find the choice of the agent from $f(\theta) = \text{argmax}_x \pi(x,\theta)$. Next, we compute the average of the choices and use this as the prediction of the choice made by the agent. Because $Ef(\theta) = E[\arg\max_x \pi(x,\theta)] \neq \arg\max_x E[\pi(x,\theta)]$, the Monte Carlo method does not produce a valid estimate of the behavior of the agent.

We now illustrate this general discussion with our model. The following system of equations corresponds to a Monte Carlo simulation of the original model:

$$p_{is} = c$$
$$K_{is} = x_{is} = \left(\frac{\theta_{is}}{p_{is}}\right)^2$$
$$\text{K}_T = 0.$$

Here we obtain one value (solution) for each endogenous variable p_{is}, x_{is}, K_{is} for each scenario *s*, that is, for each realization of θ_{is}. In particular, the electricity capacities K_{is} will differ between the scenarios. Monte Carlo simulations thus simply ignore the fact that producers in the economy do not know which scenario that will materialize when they decide on investment. Put differently: Under Monte Carlo simulations the solution is found under the false assumption that producers consider the future as certain – which scenario that for sure will materialize differs between the simulations.

Comparing the Monte Carlo approach and the assumption that agents take the uncertainty into account when making decisions, we note some major differences. In the Monte Carlo simulations, $p_{is} = c$ in all scenarios. Thus there is no variation in the price, but electricity production and electricity capacity will be different in each scenario. Moreover, there is no investment in transmission capacity. In contrast, with optimizing agents under uncertainty electricity capacities, which are determined *before* the producer knows which scenario that will materialize, and production, which is equal to capacity, do not differ between the scenarios. Moreover, under uncertainty there is investment in transmission capacity, and the prices - $p_{is} = \theta_{is} / \sqrt{x_{is}}$ - differ between the scenarios because the parameters θ_{is} differ between the scenarios. To sum up: variables that differ between scenarios under Monte Carlo simulations do not differ between scenarios under uncertainty, and vice versa.

More fundamentally, according to economic theory uncertainty change the *behavior* of agents (compared with the case of no uncertainty). This is captured by the stochastic model but not by Monte Carlo simulations. For each Monte Carlo simulation, a realization of the stochastic variables, that is, one set of parameter values, is drawn from a probability distribution and then one finds the equilibrium in the resulting *deterministic* model. By simulating *n* times one finds *n* equilibria, all obtained from the same deterministic model with different parameter values. Needless to say, the realizations (parameter values) will in general differ between each of the *n* runs, but agents neglect uncertainty simply because the model is deterministic.

2.4 Discussion

We argued above that by adding a capital variable to each scenario and then keeping implementability as separate constraints, we minimized the changes in transforming a deterministic model to a stochastic model. This may not be entirely true for a model with no learning (like the one above). Consider the model except for the transmission part. The first-order equations we derived above were

$$\begin{aligned} p_s\sqrt{x_s} &= \theta_s \text{ for all } s \in S \\ p_s &= c + \omega_s \text{ for all } s \in S \\ x_s &= K_s \text{ for all } s \in S \\ K_s &= K \text{ for all } s \in S \\ E\omega &= 0. \end{aligned}$$

Alternatively, we can use only one capital variable. Then implementability is automatically satisfied, and hence we drop the implementability constraint. The equations then become

$$\begin{aligned} p_s\sqrt{x_s} &= \theta_s \text{ for all } s \in S \\ Ep_s &= c \\ x_s &= K \text{ all } s \in S. \end{aligned}$$

The first-order condition in the deterministic scenario $(p_s = c)$ is now replaced by $Ep_s = c$, that is, an expectation is introduced, but we no longer need $E\omega = 0$.

The first approach has some advantages. First, in our experience it is less work to write the code for the first approach (using the existing numerical deterministic model as the starting point); all variables in the deterministic model are replaced by variables indexed by scenario and a single shadow price is included in each investment FOC, avoiding the need for introducing expected values of all variables entering the FOCs. The solution is also found easier/faster.

Second, we get the comparison with the Monte Carlo solution almost for free; we just drop the two last equations. Finally, the first approach handles more complex models, for example, non-trivial information structures, more easily: Suppose the set of all scenarios is partitioned into two disjoint sets $S = S_1 \cup S_2$ where $S_1 \cap S_2 = \varnothing$, and assume that investors know, at the time of investment, which of these two subsets that will materialize, even if this was not known at some earlier stage Then the implementability constraint becomes

$$K_s = K^j \text{ for all } s \in S_j \text{ for } j = 1,2$$
$$\sum_{s \in S_j} q_s \omega_s = 0 \text{ for } j = 1,2.$$

The rest of the model is not changed. Thus for a given information structure, we can handle implementability with separate implementability constraints without altering the rest of the model.

In the alternative approach we would need to change the main equations to get:

$$p_s \sqrt{x_s} = \theta_s \text{ for all } s \in S$$
$$\sum_{s \in S_j} q_s p_s = c \text{ for } j = 1,2$$
$$x_s = K^j \text{ for all } s \in S_j \text{ and } j = 1,2.$$

Here the number of capital variables will depend on the number of information sets.

More complex information structures are relevant in dynamic models with learning. In that case investors may only know that $s \in S$ in the first period, but with new information the investor may learn that $s \in S_j$ for some *j* when the second investment is to be made. The simpler it is to administer such information structures, the easier it is to analyze the effect of learning.

3 LIBEMOD

Below we describe the numerical deterministic equilibrium model LIBEMOD. This model will be transformed to a stochastic equilibrium model using the guide from Section 2. In Section 5 we use the stochastic version of LIBEMOD to study decisions under uncertainty.

LIBEMOD allows for a detailed study of the energy markets in Western Europe, taking into account factors like fossil fuel extraction, inter-fuel competition, technological differences in electricity supply, transport of energy through gas pipelines/electricity lines and investment in the energy industry. The model determines all energy prices and all energy quantities invested, extracted, produced, traded and consumed in each sector in each Western European country. The model also determines all prices and quantities traded in world markets, as well as emissions of CO_2 by country and sector

The core of LIBEMOD is a set of competitive markets for seven energy goods: electricity, natural gas, oil, steam coal, coking coal, lignite and biomass. All energy goods are extracted, produced and consumed in each country in Western Europe. Natural gas and electricity are traded in Western European markets using gas pipelines and electricity transmission lines that connect pairs of model countries; this corresponds to the modelling, as well as the equilibrium conditions, of transmission in Section 2 except that for a number of pipes/electricity lines there are pre-existing capacities in LIBEMOD. There are competitive world markets for coking coal, steam coal and oil, but only domestic (competitive) markets for lignite and biomass. While fuels are traded in annual markets, there are seasonal (summer vs. winter) and time-of-day markets for electricity.

In each country in Western Europe (henceforth referred to as a model country) there is demand for all types of energy from three groups of end users; the household segment (including service and the public sector), the industry segment and the transport sector. In addition, there is intermediate demand for fuels from fuel-based electricity producers. Demand from each end-user group (in each model country) is derived from a nested multi-good multi-period constant elasticity of substitution (CES) utility function; this is a truly non-linear function, making LIBEMOD a non-linear model.[3]

Extraction of all fossil fuels, as well as production of biomass, is modelled by standard supply functions. Energy is traded between countries. In addition, there are domestic transport and distribution costs for energy; these differ across countries, energy carriers and user groups.[4] For all energy goods, there is a competitive equilibrium; this is the case i) for all goods traded in a model country, ii) for oil and coal traded in world markets, and iii) for transport services of natural gas and electricity between model countries. The price of each transport service consists of a unit cost and a non-negative capacity term; the latter ensures that demand for transport does not exceed the capacity of the gas pipe/electricity line. The capacities for international transport consist of two terms: pre-existing capacities (according to observed capacities in the data year of the model) and new capacities, that is, investments. The modelling of investment in LIBEMOD is similar to the modelling of investment in Section 2, that is, only profitable investments are undertaken.

We now turn to electricity supply, which is the most detailed model block in LIBEMOD. In each model country electricity can (with several exceptions) be produced by a number of technologies: steam coal power, lignite power, gas power, oil power, reservoir hydropower, pumped storage hydropower, nuclear power, waste power, biomass power and wind power.

Below we explain in detail electricity supplied from the combustion of fuels. The other electricity technologies are modelled similarly, but they are characterized by additional technology specific features; for example, for reservoir hydro, total availability of water in a season, that is, the amount of water at the end of the previous season plus water inflow in the present season, must equal total use of water, that is, water used to produce electricity plus water

[3] There are also other non-linear functions in LIBEMOD, for example, in extraction of fossil fuels.

[4] End-users also face different types of taxes, in particular value added taxes. The end-user price of an energy good is the sum of i) the producer price of this good, ii) costs of transport and distribution of this energy good (which differ between countries, end-user groups and energy goods), iii) end-user taxes (which also differ between countries, end-user groups and energy goods), and finally iv) the value of losses in transport and distribution.

saved for the next season. Moreover, water filling at the end of the season cannot exceed the reservoir capacity. For wind power, sites differ wrt. the number of hours its blows, and hence the energy capacity differs across plants using the same technology. The main differences from the model in Section 2 are that in LIBEMOD i) there are a number of different types of operating costs, ii) there are several electricity technologies, iii) the efficiency of technologies varies, and iv) the composition of capacities in electricity production technologies differs between countries.

In each model country there are five pre-existing fuel technologies: gas power, steam coal power, lignite power, bio power and oil power, as well as four new technologies using the same fuels (except lignite). In general, for each old technology and each model country, efficiency varies across electricity plants. However, instead of specifying heterogeneous plants for each old technology (in each model countries), we model the supply of electricity from each old technology (in each model countries) as if there were one single plant with decreasing efficiencies; this implies increasing marginal costs. For each type of a new fuel-based technology, we assume, however, that all plants have the same efficiency (in all model countries).

There are five types of costs involved in electricity supplied from combustion of fuels. First, there are non-fuel monetary costs directly related to production of electricity, formulated as a constant unit operating cost c^O. When y_t^E is the production of power in period *t*, the monetary cost in each period is $c^O y_t^E$, which must be summed over all periods to get the total annual operating cost. Second, there are fuel costs, with a fuel input price of P^{XF} and an annual input quantity of x^{DF}.

Because the capital cost of the installed power capacity K^P is sunk, it should not affect behaviour, and it will therefore be disregarded in our model. On the other hand, there will be costs related to the maintenance of capacity. In addition to choosing an electricity output level, the producer is assumed to choose the level of power capacity that is maintained, K^{PM}, thus incurring a unit maintenance cost c^M per power unit (GW). Fourth, if the producer chooses to produce more electricity in one period than in the previous period in the same season, he will incur start-up or ramping up costs. In LIBEMOD these costs are partly expressed as an extra fuel requirement (and therefore included in the fuel costs above), but also as a monetary cost c^S per unit of started power capacity (K_t^{PS}) in each period.

For investments in *new* power capacity, K^{inv}, there are annualised capital costs c^{inv} related to investments; this corresponds fully to the modelling of investment in Section 2.

The short-run variable cost equation is (when indices for country and technology are supressed):

$$C^{P} = \sum_{t \in T} c^{o} y_{t}^{E} + P^{XF} x^{DF} + c^{M} K^{PM} + \sum_{t \in T} c^{S} K_{t}^{PS} \tag{1}$$

where T is the set of time periods.

The revenue of the power producers come potentially from two sources. First, there is revenue from regular sale of electricity produced in each time period; $P_{t}^{YE} y_{t}^{E}$. Second, each agent can also sell maintained capacity that is used as reserve power capacity K_{t}^{PR} for which he receives a price P_{t}^{KPR} from the system operator. The profit of each power producer is then the two revenue sources less the short run variable costs and any costs of new investments:

$$\Pi^{E} = \sum_{t \in T} P_{t}^{YE} y_{t}^{E} + \sum_{t \in T} P_{t}^{KPR} K_{t}^{PR} - C^{P} - c^{inv} K^{inv} \tag{2}$$

The producer maximises profits given several constraints. Below, the restrictions on the optimisation problem are given in solution form, where the Kuhn-Tucker multiplier – complementary to each constraint – is also indicated. The first constraint requires that maintained power capacity K^{PM} should be less than or equal to total installed power capacity K^{P}:

$$K^{PM} \leq K^{P} \perp \lambda^{E} \geq 0, \tag{3}$$

where λ^{E} is the shadow price of installed power capacity.

Second, in each period maintained capacity can be allocated either to production of electricity or to reserve power. Since production is measured in energy units (TWh) while maintained and reserve capacity is measured in power units (GW), this can best be expressed by a constraint that production should be bounded by the energy equivalent of maintained power capacity net of reserve power capacity, i.e., the number of hours available for electricity production in each period, ψ_{t}, multiplied by net power capacity $K^{PM} - K_{t}^{PR}$ in that period:

$$y_t^E \leq \psi_t \left(K^{PM} - K_t^{PR} \right) \perp \mu_t \geq 0. \tag{4}$$

All power plants need some down-time for technical maintenance. Therefore, total annual production cannot exceed a share (ξ) of the maintained capacity:

$$\sum_{t \in T} y_t \leq \xi \sum_{t \in T} \psi_t K^{PM} \perp \eta \geq 0. \tag{5}$$

Notice that this is an annual constraint, so the producer may choose in which period(s) technical maintenance will take place.

Fourth, as mentioned above, start-up and ramping up costs are incurred if electricity production varies between periods in the same season. This cost depends on the additional capacity that is started at the beginning of each period, that is, on the difference between capacity use in one period and capacity use in the previous period in the same season. The start-up capacity (K_t^{PS}) must therefore satisfy the following requirement:

$$y_t^E / \psi_t - y_u^E / \psi_u \leq K_t^{PS} \perp \phi_t \geq 0, \tag{6}$$

where y_t^E / ψ_t is actual capacity used in period *t* and y_u^E / ψ_u is actual capacity used in the previous period *u=t-1* in the same season. Each produced quantity y_t^E is thus involved in two inequalities, one for period *t* and one for period *t+1*, which together imply two different non-negative start-up capacities. Note that the maximum value of $y_t^E / \psi_t - y_u^E / \psi_u$ is K^{PM}, and hence K_t^{PS} can never exceed K^{PM}.

We now turn to the fuel requirement, which consists of two parts. The first is related to the quantity of electricity produced by the direct input requirement function $x^E(y_t^E)$, which is the quantity of fuel needed to produce the given quantity of electricity and which captures the energy efficiency of the transformation process. In LIBEMOD the direct input requirement function is quadratic:

$$x^E(y_t^E) = \nu^0 y_t^E + \nu^1 \frac{\left(y_t^E\right)^2}{\psi_t} \tag{7}$$

where ν^0 and ν^1 are parameters (to be calibrated). The second part is the additional fuel required to start extra capacity, or ramp up an already started power plant, which is assumed proportionate to the start up capacity by a factor ν^S:

$$\sum_{t\in T}\left(x^E(y_t^E) + \nu^S K^{PS}\right) \le x^{DF} \perp \pi \ge 0. \tag{8}$$

For fuel power technologies, the Lagrangian of the optimisation problem is:

$$\begin{aligned} \mathcal{L}^E &= \sum_{t\in T} P_t^{YE} y_t^E + \sum_{t\in T} P_t^{KPR} K_t^{PR} - C^P - c^{inv} K^{inv} \\ &-\lambda^E \left\{K^{PM} - K^P\right\} - \sum_{t\in T}\mu_t \left\{y_t^E - \psi_t\left(K^{PM} - K_t^{PR}\right)\right\} \\ &-\eta\left\{\sum_{t\in T} y_t^E - \xi \sum_{t\in T}\psi_t K^{PM}\right\} - \sum_{t\in T}\phi_t\left\{ y_t^E/\psi_t - y_u^E/\psi_u - K_t^{PS}\right\} \\ &-\pi\left\{\sum_{t\in T}\left(x^E(y_t^E) + \nu^S K^{PS}\right) - x^{DF}\right\} \end{aligned} \tag{9}$$

where period *u* is the previous period in the same season as period *t*. In addition to production of electricity in each period y_t^E, each electricity producer chooses the amount of reserve power capacity to sell in each period K_t^{PR}, the quantity of fuel to buy x^{DF}, the capacity to maintain K^{PM}, the capacity to start up each period K_t^{PS}, and, for new technologies only, the level of investment K^{inv}.

After insertion of the cost equation (1) in the Lagrangian (9), the first-order condition with respect to produced electricity in each period is:

$$P_t^{YE} - c^O \le \mu_t + \eta + \frac{1}{\psi_t}\left(\phi_t - \phi_u\right) + \pi\nu_t \perp y_t^E \ge 0 \tag{10}$$

where *u* is the period *following* *t* in the same season, and $\nu_t = \partial x^E(y_t^E)/\partial y_t^E$ is the marginal inverse efficiency in period t. Hence, in each period positive electricity production $y_t^E > 0$

requires that the difference between the price of electricity P_t^{YE} and the marginal operating cost of production c^O should be equal to the sum of suitably weighted shadow prices. The first term in this sum is the shadow price of the period available energy capacity restriction (4), where $\mu_t > 0$ reflects that increased production in period *t* is not possible for a given maintained capacity K^{PM} net of reserve power K_t^{PS}. Outside of optimum, if the left hand side of (10) is greater than the right hand side and the restriction (3) is not binding, it may be possible to increase maintained capacity to facilitate increased electricity production. Once optimum is reached, and (10) holds, increasing maintained capacity is either not possible or not worthwhile.

The sum of shadow prices also contains the shadow price of the annual energy capacity η, and the difference (measured per hour) between the shadow price of capacity used in this period and in the following period, where $\phi_t > 0$ reflects that production in period t cannot be increased for given K_t^{PS}. The final term πv_t reflects the value of fuel input needed to produce an extra unit of electricity.

Second, the first-order condition with respect to reserve power capacity sold in each period is:

$$P_t^{KPR} \leq \mu_t \psi_t \perp K_t^{PR} \geq 0 \tag{11}$$

so that for positive reserve power sales the reserve power price must equal the shadow value of increasing the power capacity available to produce electricity. The marginal unit of power capacity should be worth the same either it is sold as reserve power (P_t^{KPR}) or used to produce electricity ($\mu_t \psi_t$) expressed in value per power unit (MUSD/GW).

Third, the first-order condition with respect to fuel input demand is:

$$\pi \leq P^{XF} \perp x^{DF} \geq 0 \tag{12}$$

which trivially states that at positive input demand, the shadow price of the input is equal to its market price.

Fourth, the first-order condition with respect to maintained capacity is:

$$\sum_{t \in T} \psi_t \left\{ \mu_t + \eta \, \xi \right\} \leq c^M + \lambda^E \perp K^{PM} \geq 0, \tag{13}$$

that is, the cost of increasing maintained capacity marginally – the sum of the maintenance cost (c^M) and the shadow price of installed capacity (λ^E) – should be equal to the value of increased annual production following from this policy (or maintained capacity should be zero). Because increased maintained capacity raises both potential periodic electricity production and potential annual electricity production, in each period the value of increased production (per hour) is the sum of the shadow price of periodic energy capacity (μ_t) and the shadow price of the annual energy capacity adjusted by the maximum operating time ($\eta \, \xi$).

Fifth, the first-order condition with respect to the start-up capacity is:

$$\phi_t \leq c^S + \pi \nu^S \perp K_t^{PS} \geq 0, \tag{14}$$

that is, in each period the shadow price of start-up capacity ϕ_t, which reflects the benefit of increased start-up capacity through higher production, should be equal to the sum of the monetary start-up cost c^S and the cost of the extra fuel input $\pi \nu^S$, or alternatively, the start-up capacity should be zero.

Equations (10) and (14) imply that *if* a plant is producing in one period, costs will increase if the plant does not also produce in the previous period because the plant will incur a start-up cost. By the same token, if the marginal benefit of a start-up is positive in the period after the one we examine ($\phi_u > 0$), then this allows a greater benefit of a start-up in this period since if capacity is already used in this period, we can also use it in the next period without incurring additional start-up costs. Hence, the start-up component tends to smooth out production from a plant over the day. However, smooth production combined with high demand during the day and low demand at night will lead to increased price variation between day and night.

The final FOC with respect to a decision variable is for investment. Using the fact that for new technologies total capacity will be equal to investment, $K^P = K^{inv}$, the investment criteria can be written as

$$\lambda^{E} \leq c^{inv} \perp K^{inv} \geq 0. \tag{15}$$

Relation (15) implies that if investment is positive, the annualised investment cost must equal the shadow price of installed capacity, i.e. the increase in operating surplus resulting from one extra unit of capacity.

In addition to the FOCs with respect to the decision variables, that is, (10)-(15), the FOCs with respect to the multipliers recover the original optimisation restrictions (3)-(8).

In this paper we use a version of LIBEMOD that is calibrated on data for 2000. Several parameters are set individually to be in line with the data sources, see Aune *et al.* (2008). The remaining parameters, for example, the CES utility parameters and parameters characterizing the efficiency distribution of pre-existing power plants are determined in tailor-made calibration procedures. Technically, these parameters are solutions of simultaneous systems of relations that specify technical and economic requirements imposed on the calibration equilibrium.

As stated above, the model determines all quantities and market clearing prices in the energy industry in Western Europe, as well as all prices and quantities traded in the world markets, see Aune *et al.* (2008) for a detailed documentation of LIBEMOD. For some applications of the model, see Golombek *et al.* (2011) on the potential of CCS electricity technologies in Western Europe, Golombek *et al.* (2013a) for price and welfare effects of different schemes for allocating free emission quotas in the EU, and Golombek *et al.* (2013b) for an examination of liberalizing different segments of the European energy industry.

We have changed LIBEMOD from a deterministic model to a stochastic model following the methodology outlined in Section 2: (i) all endogenous variables, that is, quantities and prices, depend on the scenario, and (ii) for each variable that has to be determined *before* the agent knows which scenario that will materialize, here investments in the energy industry (capacities for power plants, international electricity transmission and international gas transmission), we impose the requirement that the agent has to choose the same value for all scenarios. The latter requirement leads to probability adjusted shadow prices; each of these enters one of the first-order conditions for variables that are determined before the uncertainty is resolved. In addition, the expected value of each probability adjusted shadow price is zero. Once the agents know which scenario that has materialized, all the remaining variables are determined in a standard way by identifying the vector of prices that clear all markets. In solving the model we find *all* the S price vectors simultaneously with the corresponding quantities in period 2 (production, trade and consumption) and the investment levels in period 1.

4 Scenarios

Because our stochastic model is too complex for analytical treatment, we need to discretize the distribution of all stochastic parameters. This is handled by introducing scenarios. Technically, scenarios are multi-dimensional vectors representing possible values of the underlying random variables. Scenarios can be regarded either as sample points from this multi-dimensional distribution or as the discrete support of a multi-dimensional distribution that approximates the original distribution. These two views lead to the same models and approaches. In the stochastic programming literature, scenarios are typically viewed as atoms of an approximating distribution, see, for example, King and Wallace (2012), chapter 4, for a discussion.

Specification of a discrete distribution with limited cardinality is frequently termed *scenario generation*. An obvious choice is sampling, but because the number of scenarios (atoms) we can handle is limited, sampling might leave us with a random problem, which might produce random results. In the limit, sampling will always produce a good discretization, but that is useless if the resulting model cannot be solved; see King and Wallace (2012).

The goal of scenario generation is therefore to generate a discrete distribution with as few atoms as possible for a given accuracy of the solution, or to maximize the accuracy of the solution for a given number of atoms. If the starting point is historical data and one believes they describe the future adequately, then the historical data are used as the (empirical) distribution to approximate (if one has no knowledge of the distributional form). This is the strategy in the present paper.

Below we use the stochastic LIBEMOD model to examine agents making investment decisions under uncertainty. We assume that agents make investment decisions in the year 2010 (period 1).[5] The new capacities are assumed to be available in 2030 (period 2), and once these capacities are available the uncertainty is revealed; all agents learn the true state of the economy, that is, which scenario that has materialized. Then agents make production, trade and consumption decisions under no uncertainty.

We assume that the sources of uncertainty are the crude oil price, the coking coal price, the steam coal price, and the GDP in each model country in 2030. Because all prices are endogenous in LIBEMOD, also oil and coal prices, uncertain oil and coal prices are simulated by shifts in the slope of the inverse supply functions for oil and coal in the non-model countries (countries outside Western Europe):

$$P_{js} = a_j + b_j \beta_{js} y_{js}. \tag{16}$$

Here, P_{js} is the (world-market) price of fuel *j*, $j =$ crude oil, coking coal and steam coal, in scenario *s*, y_{js} is supply of fuel *j* from the non-model countries in scenario *s*, and a_j and b_j are parameters. Further, β_{js} is a fuel- and scenario-specific shift parameter, which is estimated on data (see below). The higher the value of β_{js}, the higher is the slope in the inverse supply function, which, *cet. par.*, suggests a high price. In the calibration, we normalize the expected value of the shift parameter to 1: $\sum_s q_s \beta_{js} = 1$ where q_s is the probability of scenario *s*. Hence, in the deterministic case the slop is simply b_j and if $\beta_{js} > 1$ the slope in scenario *s* is steeper than the slope in the deterministic case.

We use a similar procedure for growth. Starting from the observed level of GDP for each model country *m* in 2010, we calculate GDP of country *m* in scenario *s* in 2030, henceforth referred to as γ_{ms}, by using country-specific growth rates for the period 2010 to 2030.

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[5] Because our data year is 2000, an alternative interpretation is that investment decisions are taken in 2000, and investors correctly foresee the *true* development between 2000 and 2010. This development is taken into account for the complete investment period 2000 to 2030.

Next, we construct measures related to aggregate GDP in 2030. In the deterministic case, aggregate GDP is measured by its expected value ($\sum_s q_s \sum_m \gamma_{ms}$). Under uncertainty, we use a parameter that measures aggregated GDP in scenario *s* relative to the expected value of aggregate GDP:

$$\bar{\gamma}_s = \frac{\sum_m \gamma_{ms}}{\sum_{s'} q_{s'} \sum_m \gamma_{ms'}}. \tag{17}$$

As seen from (17), the higher the value of $\bar{\gamma}_s$, the higher is aggregate GDP in scenario *s* relative to aggregate GDP in the deterministic case. If $\bar{\gamma}_s > 1$, then aggregate GDP in scenario *s* exceeds aggregate GDP in the deterministic case.

To sum up: each scenario is characterized by i) a specific shift in the inverse supply function of crude oil in the non-model countries, ii) a specific shift in the inverse supply function of coking coal in the non-model countries, iii) a specific shift in the inverse supply function of steam coal in the non-model countries, and iv) GDP growth rates for the period 2010-2030 for each model country. The growth rates determine aggregate GDP of the model countries in 2030.

We now describe how we have constructed the scenarios. First, we assume that investors use only *historic information* to assess which scenarios that may materialize and to estimate the probability of each scenario. Hence, the information set of investors does not include probability distributions of future events; we do not have data that provide a solid foundation for including probability distributions of future events as part of the information set of investors. Note that we assume that the information set does not vary among the investors, and we will also assume that the assigned scenario probabilities of the investors (q_s) are "the true one". Under this assumption, the stochastic equilibrium provides the social optimal investment levels.

To construct the scenarios we use data for the period 1970 to 2009 for the world prices of crude oil, coking coal and steam coal, and GDP growth rates for each model country. World fossil prices for recent years are from IEA (2002, 2011), while prices for earlier years have been collected from EIA (2011) for crude oil and from McNerney et al. (2011) for coal. GDP levels are from OECD (2011). Predicted world demand and supply are calculated using the LIBEMOD demand and supply systems and the prices and GDP levels of the data sources for each year, country and fuel. The shift parameters are then calibrated as the ratio of predicted demand to predicted supply for each fuel in each year. Using this shift parameter in the inverse supply functions (16) of the model would then have cleared all fuel markets in all data years.

We group the period 1970 to 2009 into four 10-year periods, henceforth termed 70, 80, 90 and 00. Define $k_t = (k_t^1, k_t^2, k_t^3, ..., k_t^{n+3})$, $t = 70, 80, 90, 00$. Here, k_t^1 shows the shift parameter (β) in the inverse supply function of crude oil for the non-model countries in the 10-year period *t*. Similarly, k_t^2 shows the shift parameter in the inverse supply function of coking coal for the non-model countries in the 10-year period *t*. Finally, k_t^i shows the growth rate for country *i*, $i = 4, 5, ..., n+3$, in period *t*.

A scenario is characterized by two k_t, which we symbolize by $k_{t'} \times k_{t''}$ where t' and t'' are two 10-year periods. This gives us 4^2 scenarios, and we assume that each of these scenarios have the same probability $p\ (=1/16)$. However, in our model the sequence of the two 10-year periods is of no importance. Hence, we consider $k_{t'} \times k_{t''}$ and $k_{t''} \times k_{t'}$ $(t' \neq t'')$ as *one* scenario, and the probability that it materializes is *2p*. This leaves us with 10 unique scenarios. Six of these are characterized by $k_{t'} \times k_{t''}$, $t' \neq t''$, that is, the change between 2010 and 2020, and the change between 2020 and 2030, are non-identical, and the probability of each of these six scenarios is 2*p*. The remaining four scenarios are characterized by $k_{t'} \times k_{t'}$, that is, the change between 2010 and 2020, and the change between 2020 and 2030, are identical. The probability of each of these four scenarios is *p*.

Table 1 provides information about the 10 scenarios, in particular the β_{js} and the $\bar{\gamma}_s$. In scenario 1 the development between 2010 and 2030 is characterized by two identical 10-year periods, namely the 70ies, see the second column in Table 1. The probability of this scenario is 0.0625, see the third column in Table 1. For crude oil, the shift parameter in the inverse supply function of the non-model countries, β_{js}, is 4.08. From (16) we know that a higher value of this parameter tends to shift the inverse supply function upwards, which will, *cet. par.*, raise the price of oil. However, the price of oil will also depend on world demand for oil, which depends (partly) on aggregate GDP in the model countries as well as the coal prices (because of cross-price effects in demand). If, *hypothetically*, aggregate GDP and coal prices did not differ between the scenarios, the price of oil would be higher in scenario 1 than in the deterministic case simply because 4.08 > 1 (the shift parameter is measured relative to the value in the deterministic case, see discussion above). Moreover, because scenario 1 has the highest value of the shift parameter for crude oil, the price of oil would be highest in this scenario.

[Table 1]

Table 1 also provides information about the shift parameters of coking coal and steam coal as well as the index $\overline{\gamma}_s$ for aggregate GDP for the model countries in 2030. For the GDP index, a value exceeding one indicates that GDP is higher than in the deterministic case. Hence, scenario 1 is characterized by a low supply of crude oil (a shift parameter of 4.08), a low supply of coking coal (a shift parameter of 3.82), a low supply of steam coal (a shift parameter of 3.33) and a high level of aggregate GDP (a GPD index of 1.20). All these shift parameters, as well as the GDP index, suggest that the price of oil will be high in scenario 1.

In our model investors know in period 1 (2010) that there are 10 scenarios, and they also know the probability of each scenario. In addition, for each scenario they correctly predict the equilibrium prices if this scenario is materialized (rational expectations). Because the oil and coal inverse supply functions and also the GDP growth rates differ across scenarios, *all equilibrium prices* will in general differ between the scenarios. Hence, at the time of investment *all* future prices are uncertain.

In examining the importance of uncertainty we compare the equilibrium of the stochastic LIBEMOD model with the equilibrium of the deterministic LIBEMOD model. We also compare the equilibrium of the stochastic LIBEMOD model with the output from a *C*omplete Monte Carlo *S*imulation (*CMCS*): Let k_s be a vector that characterizes each scenario, that is, it contains the three shift parameters and the GDP index in Table 1. With *CMCS* we run the *deterministic* LIBEMOD model once for each of the *s* (10) scenarios. For run *s* we use the parameter values in k_s.

The LIBEMOD model is solved on a state of the art Intel-based application server with the GAMS modeling language and the PATH solver. For efficiency, the variables are initialized with their 2000 calibration values when solving the 2030 equilibrium under no uncertainty – the deterministic case. This solution is then used as the starting point for each of the independent Monte Carlo simulations, which further provides initialization for the stochastic equilibrium.

5 Simulations

Using the stochastic LIBEMOD version we now discuss the impact of uncertainty on investment decisions and how that translates into equilibrium prices. To this end we compare the stochastic equilibrium with the deterministic outcome. The latter follows from running the deterministic LIBEMOD model using the expected values of the stochastic parameters; these are the slopes in the inverse supply functions of oil, coking coal and steam coal of the non-model countries, and country-specific growth rates (the latters are summarizes by the GDP index), see Section 4. We also compare these two outcomes with Monte Carlo simulations. Here, we run the deterministic LIBEMOD model for each scenario, that is, each time we use one set of values for the slopes and the GDP index – these correspond to one specific scenario, see Table 1.

5.1 The importance of uncertainty on investment decisions

In the deterministic case 5 GW is invested in international electricity transmission capacity between the model countries, whereas under uncertainty the level of investment is 16 GW, that is, three times higher, see Table 2. This is in line with the result from the theory model in Section 2.

[Table 2]

To understand why equilibrium investment differs between the stochastic and the deterministic case, remember that under uncertainty some countries will experience high growth rates whereas others will experience low growth rates. In countries with high growth rates, demand for energy will increase significantly, which, *cet. par.*, tends to increase domestic energy prices, thereby providing an incentive for other countries to export energy to these countries. This suggests that international transmission capacity is of great importance.

Also the stochastic slope in the inverse supply functions of coal and oil for the non-model countries generates demand for transmission services of electricity, but not to the same extent as the stochastic growth rates. In general, the impact of uncertain fossil fuel prices may not differ that much between countries because under perfect competition all countries will face the same producer prices of oil and coal net of transport costs and taxes.

However, the impact of uncertain producer prices does differ somewhat between countries. First, the *percentage change* in end-user prices of oil and coal will differ between countries because costs of domestic transport and distribution, and also taxes, differ across countries. Second, the *impact* of a percentage change in the end-user price of oil and coal will differ between countries because the market shares of oil and coal differ across countries, both in end-user demand and in electricity generation. If coal power has a large market share and the price of coal turns out to be high, production of coal power may decrease significantly. This tends to increase the domestic price of electricity, thereby providing an incentive for increased production from other domestic electricity technologies. This effect will, however, be dampened through international trade in electricity. The effect through international trade resembles the effect of differences in GDP growth rates, but it may be much lower in magnitude because it is a derived (indirect) effect.

Our discussion suggests that stochastic growth rates tend to increase the level of investment in international electricity transmission capacity: In the future (2030), a country *A* may experience high demand for electricity because of high growth rates. Rather than investing a lot in power plants so that future domestic demand for sure can be met by domestic production, electricity can partly be imported from the neighboring country *B* through existing and new transmission capacities if it turns out that future demand (in 2030) is low in country *B*. These capacities can alternatively be used to transport electricity from *A* to *B* if it turns out that demand (in 2030) is high in country *B*, but low in country *A*.

Alternatively, future production of electricity may be high in country *A* but low in country *B* because of structural differences in electricity production capacities. If, for example, country *A* invests in coal power whereas country *B* invests in oil power, and the price of coal (in 2030) turns out to be low whereas the price of oil turns out to be high, then country *A* may use its entire new coal power capacity whereas oil power production in country *B* may not be profitable because oil is too expensive. In such a case it would have been profitable (in 2010) to build a new transmission line between country *A* and *B,* which can now (in 2030) be used to export electricity from country *A* to country *B*. Likewise, if it turns out that the price of coal (in 2030) is high whereas the price of oil is low, a new transmission line can be used to transport electricity from *B* to *A*.

The discussion above may be considered as a general rationalization of higher investment in transmission capacity under uncertainty than in the deterministic case. However, as seen from Table 2 the difference in investment in international gas transmission capacity between the case of uncertainty and the deterministic outcome is marginal (2 percent). The difference in transmission investment between the stochastic and the deterministic equilibrium – 200 percent for electricity versus 2 percent for natural gas - reflects that whereas electricity transmission provides flexibility because electricity can be transported one way or the other, natural gas is mainly transported one way, that is, exported from the big extractors. Which scenario that materializes may have significant impact on the export magnitudes, but not much impact on which countries that are exporters of natural gas. Thus, for natural gas two-way flexibility is not a big issue. This tends to lower investment in transmission capacity.

In the deterministic case, total investment in electricity production capacity is 365 GW, whereas under uncertainty investment in electricity production capacity is slightly lower; 358 GW. The tiny difference reflects that demand for electricity in the deterministic case is - by construction - equal to expected demand under uncertainty and also that in the stochastic model agents are risk neutral. The distribution of investment by technology differs, however, between the two cases: In the deterministic equilibrium, investment in (steam) coal power is 304 GW, which is 54 GW higher than under uncertainty. Under certainty investors know for sure the profitability of new coal power plants and undertake all projects with non-negative profitability. Under uncertainty, investors know the cost of investment, but the cost of operating a new coal power plant is uncertain because the future price of coal is uncertain. In addition, the price of electricity is uncertain. Thus, (part of the) new coal power capacity will not be used if the input price turns out to be too high (or the output price turns out to be too low).

While this gives intuition to why investment in coal power in a non-linear model like LIBEMOD is lower under uncertainty than in the deterministic case, it does not tell the whole story because all prices are uncertain in the stochastic equilibrium. To explain the low level of investment in coal power capacity, note that the price of (steam) coal is "more uncertain" than most other prices: Table 3 provides information on how energy prices differ between scenarios. In the table price variation is measured by the coefficient of variation (CV), that is, the expected price relative to the standard deviation of the price. The CV of steam coal is 0.56, whereas the CV of natural gas is only 0.17. This difference reflects that supply of steam coal (from the non-model countries) is one of the sources of uncertainty, whereas uncertain natural gas prices, that is, the fact that the equilibrium price of natural gas differs between the scenarios, is a derived effect in the model. As seen from Table 3, the CV of all prices that are directly linked to the sources of uncertainty (coal and oil) are higher than the CVs of the remaining prices (natural gas, biomass and electricity).

[Table 3]

The discussion about the degree of price uncertainty, measured by the coefficient of variation, suggests that there may also be a significant reduction in oil power investment in the stochastic equilibrium relative to the deterministic case. However, as seen from Table 2 both in the deterministic case and under uncertainty there is no investment in oil power. In the deterministic case, investment in oil power capacity is simply not profitable. Under uncertainty, there is a chance that the oil price will turn out to be so low that investment in oil power will be profitable. However, at the point in time of investment (2010) this probability is too low to ensure a positive expected profit. Hence, also under uncertainty there will be no investment in oil power.

Whereas investment in coal power is lower under uncertainty than in the deterministic case, investment in gas power is higher under uncertainty than in the deterministic case, see Table 2. The difference reflects partly that the price of coal is "more uncertain" than the price of natural gas, see discussion above. Because of non-linearities in the LIBEMOD model, this difference in degree of uncertainty has powerful implications even though agents are risk neutral. Another difference between coal power and gas power is cost of investment, which is much higher for coal power than for gas power. At the point in time of investment (period 1), that is, prior to the uncertainty is revealed (period 2), a higher expenditure is required for a new coal power station (with a specific capacity) than for a new gas power station (with identical capacity). Hence, also with respect to the cost structure of electricity technologies, coal power is “more uncertain” than gas power.

In the simulations, expected demand under uncertainty is by construction equal to demand for electricity in the deterministic case. Because investment is lower under uncertainty than in the deterministic case only for coal power, that is, the electricity technology with the highest degree of uncertainty, there is room for additional investment in electricity production capacity under uncertainty relative to the deterministic case. Table 2 shows that under uncertainty, investment in hydro power is only marginally higher than in the deterministic case, investment in bio power is around 20 percent higher than in the deterministic case, investment in gas power is around 60 percent higher than in the deterministic case, and investment in wind power is roughly 200 percent higher than in the deterministic case. In general, investment in different electricity technologies depends on their long-run marginal cost of production. In LIBEMOD these are increasing in the equilibrium quantities, either because it is assumed that the best locations are taken first (reservoir hydro and wind), or because there are increasing costs of providing more of the input (biomass, coal and natural gas). Our results reflect that reservoir hydro has a much steeper long-run marginal cost curve than wind power.

Monte Carlo simulations

We now turn to the Monte Carlo simulations. As explained above, in the Monte Carlo simulations there is no uncertainty: agents know for sure in 2010 (period 1) which scenario that will materialize in 2030 (period 2). Hence, if agents know that there will be high growth rates, they will tend to investment more; if they know the price of coal will be low, they will tend to invest in coal power; and if they know the price of oil will be low, they will tend to invest in oil power – all solutions are tailor-made.

Under Complete Monte Carlo Simulations (*CMCS*) investment in coal power varies between 54 GW and 367 GW, see Table 2. The weighted average of the *CMCS* is 250 GW,[6] which is almost identical to investment in coal power under uncertainty (249 GW). For some of the other electricity technologies the (weighted) average of the *CMCS* does not, however, provide a good estimate of the level of investment under uncertainty: for gas power, the difference is around 30 percent, whereas for wind power the difference is roughly 10 percent.

In the *CMCS* there is investment in oil power in scenario 5 only; this scenario is characterized by a very low oil price. Here the level of investment is as high as 211 GW. The average *CMCS* of oil power investment (13 GW) therefore exceeds the optimal level under uncertainty (zero). However, the average *CMCS* overshoots oil power investment by almost the same magnitude as it undershoots gas power and wind power investments. Therefore, the average *CMCS* of total investment in electricity production capacity differs by only one percent from the optimal solution under uncertainty. Finally, investment in oil power in scenario 5 is localized in a few countries only. In these countries it is profitable to invest in international electricity transmission capacity in order to export part of the domestic oil power production. This is the main reason why average *CMCS* investment in electricity transmission lines is about 20 percent higher than the solution under uncertainty, see Table 2.

5.2 Equilibrium prices and supply of electricity

As explained above, the equilibrium prices in each scenario reflect the realization of the stochastic parameters in each scenario. For example, from Table 1 we see that the slope in the inverse supply function of steam coal (from the non-model countries) is high in scenario 10 (parameter value of 27.37), scenario 4 (parameter value of 9.55) and 1 scenario 1 (parameter value of 3.33), which, *cet. par.*, suggests a high price of steam coal in these scenarios. As seen from Table 4, in the stochastic equilibrium the price of steam coal is in fact highest in these three scenarios; the price of steam coal is 252, 247 and 193 USD per toe in scenario 10, 4 and 1,

[6] We use the scenario probabilities as weights.

respectively.

[Table 4]

A high price of steam coal should, *cet. par.*, imply low supply of steam coal power. Table 4 confirms that this is the case: both supply of steam coal power from plants that existed in the data year 2000 (“old” plants) and supply from steam coal plants that came online in 2030 because of investment in 2010 (“new” plants) are particularly low in scenarios 10 and 4 in the stochastic equilibrium. In scenario 1 supply of steam coal power is in the same range as the remaining scenarios, which reflects a particularly high demand for energy in this scenario; the GDP index in scenario 1 is 1.20, see Table 1, which is the highest value of the index across the scenarios. This explains why the price of electricity is highest in scenario 1 (74 USD per MWh), see Table 4.

By construction, the slope in the inverse supply function of steam coal (from the non-model countries) is also with Monte Carlo simulations highest in scenarios 10, 4 and 1. Again this suggests that the price of steam coal is high in these three scenarios, and this tends to lower supply of coal power. These predictions are roughly confirmed in Table 4: supply of steam coal power from new plants is particularly low in scenario 10 (428 TWh) and also low in scenario 4 (1088 TWh). The relatively high supply of steam coal power in scenario 1 reflects high demand for energy in this scenario due to a particularly high GDP index (1.20).

Note that in scenario 10, that is, the scenario with the highest shift parameter of steam coal, supply of new coal power is 428 TWh with Monte Carlo simulations compared to 1177 TWh in the stochastic equilibrium. This difference reflects that with Monte Carlo simulations investment is scenario-specific and there is no uncertainty. Therefore, when the price of steam coal is high in a Monte Carlo simulation there will not be much investment in new steam coal power because of low profitability, but the new production capacity will be maximally utilized.

6 Conclusions

Our framework for stochastic equilibrium modeling can be used to solve numerical equilibrium models with several agents simultaneously maximizing their payoff under uncertainty. The method has been implemented in a stochastic version of LIBEMOD, a numerical multi-market model of the Western European energy market. We find that replacing the uncertainty with the expected value, which here is referred to as the deterministic outcome, leads to large deviations from the optimal solution under uncertainty. The weighted average of the Monte Carlo simulations often approximates the optimal solution better, at least for aggregate numbers like total production of electricity.

The current version of LIBEMOD is not dynamic. However, by requiring that investments are identical across scenarios while energy consumption is scenario specific, we impose a structure where investments are decided *before* the uncertainty is revealed while use of energy is decided *after* the uncertainty is revealed.

Our contribution builds on stochastic programming. This literature mostly examines a single optimizing agent facing uncertainty, see, for example, Kall and Wallace (1994). Choosing a planner as the optimizing agent, one can find the efficient outcomes of an economy, see Kolstad (1996). The assumption of a single agent also characterizes most of the real option theory; in that literature a key topic is when to invest and how much to invest assuming that the firm takes as exogenous a stochastic process of the price, see, for example, Dixit and Pindyck (1994). However, under simplifying assumptions the stochastic process that clears the market at each point in time can be determined as part of the competitive equilibrium. In our approach only how much to invest is examined, but we need no simplifying assumptions to identify the competitive equilibrium, and we allow for many agents making decisions.

Moreover, our approach can be extended to dynamic multi-period models with learning, see section 2.4. The information available in different periods would then be represented by partitions of the set of scenarios; the decision makers in a period do not yet know the exact scenario that will materialize in the future, only which subset the true scenario will belong to. Typically, decisions made in the first period will have to be the same in all scenarios, while decision in a later period will be the same within a subset of scenarios, but different across subsets. In the last period the exact scenario will be known. Learning is represented by the gradually finer partition of the set of scenarios.

We can also account for risk aversion, either by assuming that investment is decided by the firms' owners who are diversified in the financial market, or that investment is decided by risk-averse managers. In the first case, probabilities are replaced by weights derived from the prices of Arrow securities, that is, contingent claims that pay 1 $ if a particular scenario materializes, see Arrow (1964). After replacing probabilities with normalized prices of contingent claims, all agents will behave as if they were risk neutral. Risk aversion will be reflected in the prices for contingent claims. This approach is similar to the use of equivalent martingale measures in finance (Harrison and Kreps, 1979; Duffie, 1996). With risk averse managers, a similar approach can be used, but in this case the scenario-weights will be firm specific, and thus some changes in the first-order conditions would be required.[7]

[7] More details on how to account for risk aversion and to extend the method to cover dynamic models can be obtained from the authors upon request.

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TABLES

Table 1 Definition of scenarios and sources of uncertainty.*

Scenario	Mimicked historic period	Probability	Shift parameter (β_{js}) in the slope of the inverse supply function for			GDP index for model countries ($\overline{\gamma}_s$)
			crude oil	coking coal	steam coal	
1	1970-79, 1970-79	0.0625	4.08	3.82	3.33	1.20
2	1970-79, 1980-89	0.125	1.13	1.56	1.33	1.11
3	1970-79, 1990-99	0.125	1.93	1.02	0.97	1.09
4	1970-79, 2000-2009	0.125	3.00	6.24	9.55	0.98
5	1980-89, 1980-89	0.0625	0.31	0.63	0.53	1.02
6	1980-89, 1990-99	0.125	0.54	0.41	0.39	1.01
7	1980-89, 2000-2009	0.125	0.83	2.54	3.80	0.91
8	1990-99, 1990- 1999	0.0625	0.92	0.27	0.28	1.00
9	1990-99, 2000-2009	0.125	1.43	1.66	2.78	0.89
10	2000-2009, 2000-2009	0.0625	2.21	10.19	27.37	0.81

* Shift parameters and the GDP index are measured relative to corresponding values in the deterministic case.

Table 2. Investments in transmission capacity and in electricity production capacity.

	New transmission capacity		New power capacity (GW)						
	Electricity (GW)	Gas (mtoe)	Total power	Hydro power	Gas power	Steam coal power	Oil power	Bio power	Wind power
Deterministic	5	157	365	9	30	304	-	13	9
Stochastic	16	154	358	11	49	250	-	17	31
Monte Carlo§	19	157	354	10	37	250	13	15	28
MC 1	*6*	*164*	*432*	*13*	*45*	*262*	-	*23*	*89*
MC 2	*3*	*173*	*405*	*10*	*27*	*330*	-	*16*	*22*
MC 3	*4*	*173*	*417*	*10*	*15*	*367*	-	*14*	*12*
MC 4	*8*	*156*	*323*	*13*	*71*	*138*	-	*22*	*79*
MC 5	*146*	*173*	*370*	*8*	*5*	*137*	*211*	*9*	*1*
MC 6	*25*	*165*	*377*	*8*	*10*	*348*	-	*10*	*1*
MC 7	*3*	*136*	*291*	*10*	*55*	*189*	-	*16*	*22*
MC 8	*32*	*162*	*398*	*7*	*3*	*379*	-	*9*	*0*
MC 9	*3*	*130*	*304*	*10*	*51*	*216*	-	*14*	*13*
MC 10	*26*	*150*	*238*	*12*	*90*	*54*	-	*21*	*61*

§ Weighted average of ten Monte Carlo scenario values.

Table 3 Coefficient of variation of energy prices. The stochastic equilibrium.

	Oil	Steam coal	Coking coal	Natural gas	Biomass	Electricity
CV	0.32	0.56	0.47	0.17	0.03	0.18

Table 4 Price of electricity (US dollar per MWh), price of steam coal (US dollar per toe), supply of old and new coal power (TWh) by scenario. Stochastic equilibrium and Complete Monte Carlo Simulations.

	Stochastic equilibrium				Complete Monte Carlo Simulation			
Scenario	Price of electricity	Price of steam coal	Supply of old steam coal power	Supply of new steam coal power	Price of electricity	Price of steam coal	Supply of old steam coal power	Supply of new steam coal power
1	74	193	322	1968	66	199	315	2066
2	60	106	322	1968	51	118	322	2598
3	61	89	322	1968	48	101	322	2891
4	59	247	27	1902	65	197	315	1088
5	46	73	321	1968	40	65	322	1080
6	48	62	322	1968	41	68	322	2745
7	43	136	244	1963	51	120	322	1487
8	50	55	322	1968	39	59	322	2991
9	42	108	306	1968	48	104	322	1702
10	58	252	19	1177	62	184	315	428