Substitution Elasticities for CGE Models

An Empirical Analysis on the Basis of Non-Linear Least Squares Estimations*

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Abstract

Effectiveness, cost-efficiency and distribution issues are crucial for any form of future regulation. This results in the need for reliable instruments to assess regulations ex ante. Elasticities are key parameters for such instruments. We consistently estimate substitution elasticities for a three level nested CES KLEM production structure on the basis of non-linear least squares estimation procedures. Thereby we take advantage of the new World-Input-Output Database. This allows us for the first time to use one consistent dataset for the estimation process and gives us the opportunity to derive elasticities from the same data which researchers can use to calibrate their simulations. On the basis of our estimations, we demonstrate that the common practice of using Cobb-Douglas or Leontief production functions in economic models must be rejected for the majority of sectors. In response to this result, we provide a comprehensive set of consistently estimated substitution elasticities covering a wide range of different sectors. Our results suggest additionally, that no significant change in input substitutability takes place over during the time period we consider. Moreover, that there is no significant variation in substitution elasticities across regions.

Index Terms

Keywords:	Substitution elasticity
	CES production function
	Policy evaluation
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I. Introduction

Many of today's challenges require regulative interventions by policymakers. As a consequence, researchers as well as policymakers are discussing worldwide how polices should be designed to deal with a designated problem. From an economic perspective, effectiveness, distribution issues and cost-efficiency are crucial for any form of future regulation. This hold particularly true in times of turbulent economic outlook and scarce financial resources. Ultimately, this results in the need for capable and above all reliable instruments to asses environmental motivated regulation ex ante. In modern applied economics, Computable General Equilibrium (CGE) models have proven to be one of the leading instruments to evaluate alternative policy measures (Devarajan and Robinson , 2002; Böhringer et al. , 2003; Sue Wing , 2004). As is true also for other policy-oriented models, elasticities are key parameters for CGE models since they are crucial for determining the comparative static behaviour and thereby strongly influence the results of any counterfactual policy analysis undertaken with the help of these models (Dawkins et al. , 2001). A good illustration of this is provided by Jacoby et al. (2006), who perform a sensitivity analysis of structural parameters of their MIT-EPPA model. They conclude that assumptions with respect to technical progress and in particular elasticities of substitution between energy and value added are the main drivers of model results.

But despite the central role of elasticities within the framework of applied quantitative simulations, the current situation of elasticities is rather unsatisfying and although the lack of adequate elasticities has been acknowledged for a surprisingly long time (Mansur and Whalley , 1984; Dawkins et al. , 2001) the problem seems to persist. This holds particularly true for the constant elasticity of substitution (CES) framework commonly employed in CGE modelling and substitution elasticities (Okagawa and Ban , 2008). In this context, only few consistent estimates of the required elasticities exist. Those available are limited to a narrow set of sectors, rely on a combination of data from different sources, build on standard linear estimation procedures or focus on the substitutability between specific production inputs. Moreover their results are in parts contradictory.

Examples of studies having estimated substitution elasticities designated for the use in quantitative models building on a CES framework are Kemfert (1998), Balistreri et al. (2003), van der Werf (2008) and Okagawa and Ban (2008). Kemfert (1998) studies whether the CES framework is adequate to characterise the German industry and estimates the substitution elasticities between capital, energy and labour inputs for three CES production functions, each having a different nesting structure. Her findings suggest that CES production functions, ideally having a (KL)E nesting structure, can be used to describe Germany's industrial production behaviour. Balistreri et al. (2003) focus on the input substitutability between capital and labour and estimates the respective substitution elasticity for 28 US sectors. For the majority of sectors their results support the usage of Cobb-Douglas specification in the nest including capital and labour. van der Werf (2008) supplies estimated parameters for a set of two-level nested CES function with capital, labour and energy as inputs. Regarding substitution elasticities he also comes to the conclusion that the usage of a (KL)E nesting structure is justified and criticises the widespread use of Cobb-Douglas functions as his results imply that substitution elasticities are commonly smaller than one. Okagawa and Ban (2008) estimate CES production functions using panel data from the EU KLEMS dataset. They argue that higher values for substitution elasticities are closely related to energy inputs for energy-intensive industries. Moreover, according to them, substitution elasticities for other sectors are commonly overestimated in existing models evaluating climate policy.

Resulting from the lack of adequate estimates, modellers frequently feel impelled to use in their models elasticities from various originally unrelated sources, thereby exposing themselves to criticism with respect to the usage of potentially inconsistent parameters estimates. Another issue regarding the problematic usage of elasticity estimates in CGE models relates to the inappropriate usage of elasticities and the conceptual mismatch between the estimation results and the policy experiment explored in the CGE framework. McKitrick (1998) for example deplores the usage of elasticities estimated for commodity classifications which are in disaccord with those represented in the model or for countries the model does not cover. Browning et al. (1999) in turn highlight the difficulties possibly arising due to the mismatch of definitions, for instance the disregard of the differences between short-term and long-term substitution elasticities. In some extreme cases, when estimates are not available altogether, modellers even resort to the usage of rather arbitrary values. In this regard Dawkins et al. (2001) most fittingly term the frequent usage of elasticities of unity the "idiot's law of elasticities" or the usage of rather arbitrary values as "coffee table elasticities".

In this paper we seek to contribute to the solution of this problem and aim at overcoming the lack of adequate estimates. To this end, we consistently estimate substitution elasticities specifically for the usage in CGE models building on CES production functions. More specifically, we estimate elasticities of substitution for the well-established three level nested KLEM production structure on the basis of non-linear least squares estimation procedures. In the process we take advantage of the new World Input-Output Database (WIOD). The new WIOD database allows us for the first time to use one consistent dataset for the estimation process and gives us the opportunity to derive elasticities from the same data which researchers can use to calibrate their simulations.

The remainder of this paper is organised as follows. After presenting in Section II the production structures for which the elasticities of substitution are estimated, we describe the data and outline the estimation procedure in Section III. The estimation results are presented and discussed in Section IV. Finally, we summarise and conclude in Section V.

II. Specification of Production Structures

Not only in general equilibrium models but also in other economic applications with a micro-consistent basis, so called Constant Elasticity of Substitution (CES) functions have become very popular among programmers. The question to what extent factors of production are substitutable in a production process has become a main issue of economic research. It originates in the fundamental work of Solow (1956). Solow has considered three cases of production functions. He called the first "Harrod-Domar"

(Solow , 1956, p. 73) with an elasticity of substitution equal to zero, the "Cobb-Douglas" case (Solow , 1956, p. 76) with an elasticity of one and a third, not explicitly named possibility with a flexible elasticity (Solow , 1956, p. 77). Solow elaborated the idea of CES production functions concept for the first time, and, five years later, together with his co-authors (Arrow et al. (1961)) he conceptualized the general form of the two-factor constant-elasticity-of-substitution (CES) production function (see e.g. Klump and de La Grandville (2000)). This new-developed CES production function can be seen as a generalization of the two older concepts of the Harrod-Domar-Leontief production function, which is based on the assumption that there is no substitutability between factors, and the Cobb-Douglas production function, which assumes unitary factor substitution elasticity. Since the introduction of the CES production function in 1956, a multitude of extensive studies on the elasticities of substitution between production inputs have been published. One of the latest analysis in this regard is the work of León-Ledesma et al. (2010), who investigate if a simultaneous identification of the capital-labour substitution elasticity and the direction of technical change is feasible. For the n-input case the basic CES function takes the form:

$$y = \gamma \left(\sum_{i=1}^{n} \alpha_i x_i^{-\rho} \right)^{\frac{1}{-\rho}}, \tag{1}$$

where y is the output, x_i is input i, $0 \leq \alpha_i \leq 1$ with $\sum_{i=1}^{n} \alpha_i$ is the distribution parameter related to input i, $\gamma \geq 0$ represents the efficiency parameter and $\rho = \frac{1-\sigma}{\sigma}$ the substitution parameter whereas $\sigma = \frac{1}{1+\rho} \geq 0$ gives the elasticity of substitution and $\rho \geq -1$ must hold.

But in such a basic CES framework the production structure is limited to feature equal substitution elasticities between all inputs. To overcome this Sato (1967) extended the CES functional form and suggests the usage of nested CES functions. The general idea behind Sato's approach is to construct a separate CES function for each group of inputs that share the same substitution elasticity and to combine the different CES functions in different levels or nests of the overall CES function. This allows to easily implement even complicated production structures and is one of the main advantages of the CES functional form. Following Sato a four-input three-level nested CES function can be specified as:

$$y = \gamma \left[\alpha_1 x_1^{-\rho_1} + (1-\alpha_1) \left(\left(\alpha_2 x_2^{-\rho_2} + (1-\alpha_2) \left(\left(\alpha_3 x_3^{-\rho_3} + (1-\alpha_3)\, x_4^{-\rho_3} \right)^{\frac{1}{-\rho_3}} \right)^{-\rho_2} \right)^{\frac{1}{-\rho_2}} \right)^{-\rho_1} \right]^{\frac{1}{-\rho_1}}, \tag{2}$$

where α_n and ρ_n are the distribution and substitution parameters on the n-th nest of the CES function.

Moreover, the basic CES functional form can easily be extended to be able to account for technological change in the CES framework. In this spirit, for example Henningsen and Henningsen (2011) suggests the CES function

$$y_t = \gamma e^{t\lambda} \left(\sum_i \alpha_i (x_{i,t})^{-\rho} \right)^{\frac{1}{-\rho}} \tag{3}$$

to account for Hicks-neutral technological change and the CES function

$$y_t = \gamma \left(\sum_i \alpha_i (e^{t\lambda_i} x_{i,t})^{-\rho} \right)^{\frac{1}{-\rho}} \tag{4}$$

to incorporate factor augmenting (non-neutral) technological change. In both equations t is a time variable and $\lambda \geq 0$ is the rate of technological change, although in the case of factor augmenting technological change λ_i is specific for input i.

In the estimation exercise in this paper, we focus on estimating elasticity of substitutions for a three-level CES approach including the inputs capital (K), labour (L), energy (E) and other intermediates (M). Besides, during our analysis we concentrate on a $((KL)\,E)\,M$ nesting structure. This structure is probably the most popular CES form employed in CGE models evaluating environmental and climate policy and has been confirmed to be a good approximation of the production behaviour in several studies (e.g. Kemfert , 1998; van der Werf , 2008). With regard to technological progress, we estimate two specifications, one including Hicks-neutral technological change and one on condition that $\lambda = 0$.[1]

A three-level CES nesting structure with capital and labour in the lowest nest, where energy joins the capital-labour composite in the middle nest and intermediates enter in the top nest has the functional form:

$$\begin{aligned} Y_t = \gamma e^{t\lambda} \Big(\alpha_{KLEM} (M_t)^{-\rho_{KLEM}} + (1-\alpha_{KLEM}) \Big(\big(\alpha_{KLE} (E_t)^{-\rho_{KLE}} + \\ (1-\alpha_{KLE}) \, (VA_t)^{-\rho_{KLE}} \big)^{\frac{1}{-\rho_{KLE}}} \Big)^{-\rho_{KLEM}} \Big)^{\frac{1}{-\rho_{KLEM}}} \end{aligned} \tag{5}$$

with

$$VA_t = \left(\alpha_{KL} (K_t)^{-\rho_{KL}} + (1-\alpha_{KL}) \, (L_t)^{-\rho_{KL}} \right)^{\frac{1}{-\rho_{KL}}} \tag{6}$$

III. Data and Estimation Procedure

A. Data

For our analysis we make use of the World Input-Output Database (WIOD).[2] The WIOD database has been constructed on the basis of national accounts data and harmonisation procedures were applied in order to ensure international comparability

[1] In practice, however, it is sometimes hard to distinguish between factor price induced innovation (technological change) and factor substitution. Suppose for example the "putty-clay" situation where a firm is unable to substitute factors for each other in the short run, for instance because of high costs of changing the production technology, and also research and development takes time so that factor input relations remains constant despite changing relative input prices. von Weizsäcker (1966, p. 245) argues that in such a case "[...] substitution takes time and it can therefore not strictly be distinguished from technical progress." Salter (1966) arrives at even a stronger conclusion, stating that "it is simply a matter of words whether one terms new techniques of this character inventions or a form of factor substitution" (Salter , 1966, p. 43).

[2] The WIOD database is available at http://www.wiod.org. We use data from March 2012 in this paper.

of the basic data. The dataset covers 40 regions (27 EU countries and 13 other major countries), which together account for approximately 85 % of world's GDP in 2006. The WIOD data is disaggregated in 35 industries and provides detailed information on primary (raw materials), secondary (manufacturing) as well as tertiary (services) sectors. In addition, it offers annual data which ranges from 1995 to 2009. Beside its broad country coverage, detailed sectoral disaggregation and time period character, the dataset has another important feature: it covers various aspects of economic activity and for example involves accounts for energy and environmental issues or socioeconomic and bilateral trade data. Employing the WIOD dataset in our estimation process involves three main benefits. We can estimate substitution elasticities using one consistent dataset and do not have to merge potentially incompatible data. The comprehensive sectoral coverage of WIOD allows us to estimate substitution elasticities for a broad set of sectors. Last but not least, for the first time we can derive elasticities from the same data which researchers can use also to calibrate their simulations.

In our analysis, we use in particular the WIOD Socio-Economic Accounts (SEA files) and the WIOD Energy Use tables (EU files). Taken together, they form a balanced panel covering 40 regions and 34 sectors plus an economy-wide sector aggregate over a period of 15 years (1995 to 2009) and include detailed information on production in- and outputs.[3] More specifically WIOD supplies us with data regarding the number of total hours worked by persons engaged for the independent variable labour L, capital stock for the independent variable K , gross value added at basic prices for the independent variable value added VA, intermediate inputs at purchasers' prices for the independent variable materials M, gross energy use for the independent variable energy E and finally gross output at basic prices for the dependent variable output Y. Even though WIOD includes data up to the year 2009, in order to avoid drawing conclusions from a period of economic turmoil we drop the years 2008 and 2009 for our analysis and focus on the period from 1995 to 2007. For the estimation, all monetary values have been transformed to U.S. Dollars using the Penn World Table (Heston et al. , 2011) and are reported at 1995 prices. Energy is in Terajouls. Labour is given in million hours worked. Table I gives an overview of the variables used in the estimation process. A complete list of the regions and sectors covered by this analysis is given in the Appendix.

Variable	**Definition**	**Source**	**Unit**
Output	Gross output at basic prices	WIOD SEA Files	million 1995 USD
Capital	Fixed capital stock	WIOD SEA Files	million 1995 USD
Labour	Total hours worked by persons engaged	WIOD SEA Files	million hours
Value Added	Gross value added at basic prices	WIOD SEA Files	million 1995 USD
Energy	Gross energy use	WIOD EU Files	Terajouls
Materials	Gross output at basic prices	WIOD SEA Files	million 1995 USD

Table I: List of Variables used in the Estimation

[3] While originally the WIOD dataset features information for 35 sectors, entries for the sector private households with employed persons remain empty in the SEA and EU files. Consequently we undertake the analysis only for the 34 remaining sectors plus the economy-wide sector total.

B. Estimation Procedure

CES functions are non-linear in parameters and hence parameters can initially not be estimated using standard non-linear estimation techniques. For this reason and due to the so far rather tricky implementation of non-linear estimation procedures, most researchers estimating elasticities of substitution within a CES framework work with CES functions that have been linearised in some form or the other. Thereby, the so-called Kmenta approximation (Kmenta , 1967) has been very popular. However, the original CES function cannot be linarised analytically and using approximation methods to linearise the CES function can have drawbacks. Kmenta (1967) himself notes that if in the production function under investigation the input ratio as well as the elasticity of substitution are either very high or very low, his approximation method may not perform well. Maddala and Kadane (1967) and Thursby and Lovell (1978) confirm this problem and shows that the standard Kmenta procedure may not lead to reliable estimates of parameters in a CES framework.

To avoid issues related to Kmenta approximations without having to use cumbersome non-linear estimation procedures, researchers also make use of the cost function approach (e.g. van der Werf , 2008; Okagawa and Ban , 2008). Thereby one can take advantage of the cost function associated with a specific production function and derive a linear system of equations from the corresponding optimal input demand. This can subsequently be used to estimate the function coefficients in question. But this approach requires comprehensive price data, which in most cases is rather difficult to come by, especially when undertaking sector specific analysis.

In contrast to the majority of other studies investigating the substitutability of inputs within a CES production structure, we estimate substitution elasticities directly from the CES production function and primarily building non-linear least-squares estimation procedures. Thereby we employ a set of different optimisation algorithms, namely the Levenberg-Marquardt algorithm (LM) (Marquardt , 1963), PORT routines (Gay , 1990), the Differential Evolution algorithm (DE) (Storn and Price , 1997; Price et al. , 2005), Nelder-Mead routines (NM) (Nelder and Mead , 1965), the Simulated Annealing algorithm (SANN) (Kirkpatrick et al. , 1987; Cerny , 1985) and the so called BFGS algorithm (Broyden , 1970; Fletcher , 1970; Goldfarb , 1970; Shanno , 1970). In some estimation runs we make use of starting values compiled by means of a preceding grid search for the substitution parameter ρ involving LM.[4] A detailed overview of all the estimations we run is given in Table X in the Appendix. However, after having shown that except for SANN and DE our results are robust with regard to the choice of the employed optimisation algorithm, we continue our analysis on the basis of the estimation process producing the best fit to the our data. Id est estimations relying on LM and PORT with starting values.

For the actual estimation process we use the programming environment R with the package micEconCES developed by Henningsen and Henningsen (2011). But the micEconCES package in its current version only allows to estimate parameters for a two-level nested CES production function. Yet we would like to derive the substitution elasticities for a three-level nested CES function. To overcome this lim-

4 For more information on how adequate starting values are derived applying a preceding grid search, the interested reader is kindly referred to Henningsen and Henningsen (2011).

itation, we benefit from the separability implied by the CES framework and split the originally three-level nested KLEM CES function we would like to investigate given by Equation (5) into two individual CES functions. Accordingly we estimate the substitution elasticities first for the non-nested CES function

$$VA_t = \gamma_{KL} e^{t\lambda} \left(\alpha_{KL} (K_t)^{-\rho_{KL}} + (1 - \alpha_{KL}) (L_t)^{-\rho_{KL}} \right)^{\frac{1}{-\rho_{KL}}}, \tag{7}$$

with the substitution elasticity $\sigma_{KL} = \frac{1}{1+\rho_{KL}}$. Subsequently we do the same for the two-level CES function

$$Y_t = \gamma_{KLEM} e^{t\lambda} \left(\alpha_{KLEM} (M_t)^{-\rho_{KLEM}} + (1 - \alpha_{KLEM}) \left(\left(\alpha_{KLE} (E_t)^{-\rho_{KLE}} + (1 - \alpha_{KLE}) (VA_t)^{-\rho_{KLE}} \right)^{\frac{1}{-\rho_{KLE}}} \right)^{-\rho_{KLEM}} \right)^{\frac{1}{-\rho_{KLEM}}}, \tag{8}$$

with the substitution elasticities $\sigma_{KLE} = \frac{1}{1+\rho_{KLE}}$ and $\sigma_{KLEM} = \frac{1}{1+\rho_{KLEM}}$. Taken together, Equation (7) and Equation (8) represent the overall CES function in question, whereas, as already indicated by Equation (5), Equation (7) is the bottom nest and Equation (8) corresponds to the middle and upper nests of the production function under investigation.

The substitution elasticities are estimated specifically for each of the 34 sectors and one sector aggregate representing the total of all industries available in the WIOD dataset. Thereby, we first pool all sectoral data across all regions. At a later stage we then evaluate if input substitutability varies across regions. As indicated by Equation (7) and (8), initially we assume that elasticities are constant over time. Hence, in our setting technological progress can only take place through changes in overall productivity. Though, this assumption is relaxed at a later stage.

IV. Estimation Results

Unsurprisingly, the estimates for the substitution parameters ρ_{KL}, ρ_{KLE} and ρ_{KLEM} and hence for σ_{KL}, σ_{KLE} and σ_{KLEM} differ across different estimation processes. But all in all and in view of the respective standard errors, deviations are rather minor. Nevertheless we observe a small divergence between gradient-based local optimisation algorithms (BFGS, LM and PORT) and algorithms targeting global minima (e.g. NM). Robustness across estimation techniques decreases for smaller estimated values of ρ_{KL}, ρ_{KLE} and ρ_{KLEM} and increases when adequate starting values from a prior grid search are used in the estimation process. The SANN, DE and in parts CG technique however are exceptions and lead to notable different results in several cases, mainly suggesting smaller values for ρ_{KL}, ρ_{KLE} and ρ_{KLEM} than other methods. Convergence is tends also to be an issue when applying these solvers. Given the overall robustness of the different estimations, we choose to continue our analysis on the basis of only one estimation process. Evaluated on the basis of R-squared, the estimations relying on PORT routines or LM and BFGS methodologies perform best. Without starting values from a preceding grid search, CG and DE generates the poorest fit. When using starting values, CG appears to be the least powerful method. Furthermore, by the same measure, estimations using starting values from a preceding grid search generally have a better fit than estimations without. This holds true for the investigation of Equation (7) as well as for Equation (8). Given the benefit of the usage of starting values and on the whole very similar estimation results, in the following we focus on the estimations with the best fit to the data, id est estimations relying on PORT routines and which use starting values. The corresponding estimation results for the substitution parameter ρ for the time period 1995 to 2007 with pooled data including all regions are summarised in Table II. Note that for the bottom nest of sector 8 (coke, refined petroleum and nuclear fuel) we do not achieve convergence for any acceptable convergence criteria and do not report a value for ρ_{KL}. Moreover, some of the estimates for ρ feature high standard deviations, these estimates are reported in brackets.

Sector	N	ρ_{KL}-Est.	Std. Dev.	ρ_{KLE}-Est.	Std. Dev.	ρ_{KLEM}-Est.	Std. Dev.
1	520	-0.0652	0.0691	1.7926	0.8193	-0.3981	0.0747
2	520	0.2696	0.1283	(0.6173)	>10	5.8104	1.8030
3	520	3.5627	1.1909	(3.6182)	>10	-0.8772	0.2589
4	520	8.6122	4.2814	2.3552	1.1856	0.7798	0.1087
5	507	-1.6696	0.2706	3.4489	1.1924	0.9046	0.1042
6	520	7.3715	4.8299	(5.3491)	>10	-1.1113	0.1147
7	520	10.1202	4.5846	2.2040	2.2719	0.8621	0.1988
8	502	NA		-0.8024	4.4814	1.4299	0.3098
9	520	3.0891	0.4629	0.2873	0.2080	0.1431	0.2675
10	520	7.6535	2.1274	3.2905	0.4920	0.5610	0.1459
11	520	4.1269	0.9708	2.7814	4.4653	-0.0404	0.1057
12	520	4.5200	1.2438	-0.0880	0.1482	6.6645	0.8791
13	520	1.0962	0.1476	(2.6770)	>10	-1.0707	0.1039
14	520	6.8617	5.5543	5.1445	2.4021	-1.6300	0.1753
15	520	4.4527	0.7621	(3.2122)	>10	1.0076	0.2713
16	520	3.2720	0.5253	(9.1267)	>10	-0.6243	0.1162
17	520	-1.2951	0.4679	1.3441	0.2532	0.4590	0.1221
18	520	4.7623	1.1559	(3.3779)	>10	0.6699	0.2263
19	482	-1.1464	0.4433	(3.3588)	>10	0.6339	0.1413
20	520	11.0342	4.6822	1.9928	2.9571	1.3476	0.0906
21	520	(18.7635)	>10	(6.2982)	>10	-0.1299	0.0755
22	520	13.9728	4.4939	0.6246	1.5113	0.6322	0.1520
23	520	6.2963	0.9537	2.7383	1.1057	0.1031	0.1840
24	506	-0.7027	0.1219	-1.6242	0.1339	0.6856	0.1858
25	519	3.3042	1.0377	2.1739	0.6831	-0.1913	0.0778
26	520	2.8367	0.4624	1.2960	0.2650	0.6695	0.0348
27	520	-0.5790	0.2753	(19.4998)	>10	-0.6906	0.3398
28	520	4.5632	1.0965	(14.3869)	>10	-0.9412	0.1727
29	516	2.2850	0.3494	-0.0524	0.1539	0.1093	0.1089
30	520	3.0716	0.3976	3.4990	1.1617	0.4070	0.2264
31	507	4.4847	1.2736	2.2639	1.6622	2.0595	0.2582
32	520	-1.7683	3.6457	1.4266	0.8185	-0.6047	0.0512
33	520	0.7311	0.1222	(7.6346)	>10	-0.3502	0.0536
34	520	5.0364	2.7887	(3.1208)	>10	0.4730	0.2408
36	520	6.4792	1.1071	1.9972	0.5056	-0.3195	0.1747

Table II: Estimation Results for ρ (Unrestricted PORT Routine with Starting Values, 1995-2007, all Regions)

For several sectors $\rho_{KL} < 1$, $\rho_{KLE} < 1$ or $\rho_{KLEM} < 1$ and thus violate the basic assumptions of the standard CES framework which requires $\sigma \geq 0$ respectively $\rho \geq -1$. While so far we have applied an unrestricted estimation approach, this indicates the need to incorporate the three parameter constraints implied by the CES framework $\gamma > 0$, $0 \geq \alpha \geq 1$ and $\sigma \geq 0$ into our estimations. Table III summarises the results for ρ when applying the restricted estimation. The corresponding results for σ are given in Table XI in the Appendix. For obvious reasons the fit for the restricted model is not as good as before and again we do not achieve convergence for the bottom nest of sector 8. For six out of the 105 estimated elasticities, the condition that $\rho \geq -1$ is binding. This could be an indication that for a small set of sectors, the assumption of CES production structures provides only a poor fit to the actual prevailing production structure of these sectors. However, as for the big majority of sectors and nests our estimation results seem to be reliable with regard to fit to the data and standard deviations, and as the usage of of CES functions has proven to be very popular in particular in CGE models, we proceed with our estimation process and continue including the constraints on γ, α and σ given by the CES framework.

Sector	N	ρ_{KL}-Est.	Std. Dev.	ρ_{KLE}-Est.	Std. Dev.	ρ_{KLEM}-Est.	Std. Dev.
1	520	-0.0652	0.0691	1.5202	0.6513	0.0201	0.0875
2	520	0.2696	0.1283	1.3936	>10	3.5275	0.7814
3	520	3.5627	1.1909	4.2954	5.9377	0.5956	0.2764
4	520	8.6122	4.2814	2.6266	0.8898	0.7009	0.1066
5	507	-1.0000	0.1456	4.2824	1.1147	0.7887	0.0989
6	520	7.3715	4.8299	3.8764	1.5318	0.4029	0.1649
7	520	7.6868	2.6765	2.9515	1.5165	0.5129	0.1871
8	502	NA		-1.0000	5.8118	1.4086	0.3112
9	520	3.2330	0.4957	0.3942	0.2231	0.0600	0.2625
10	520	7.6535	2.1274	4.4137	0.6568	0.4711	0.1364
11	520	4.1269	0.9708	3.0187	2.5446	0.2388	0.1134
12	520	4.5200	1.2438	-0.0055	0.1417	8.0932	1.0911
13	520	1.0962	0.1476	4.0343	1.2505	0.8336	0.2055
14	520	9.0265	9.6388	-0.0528	0.0900	0.5583	>10
15	520	4.4527	0.7621	5.2611	3.8258	1.6428	0.3077
16	520	3.2720	0.5253	4.7058	2.0801	0.8868	0.1513
17	520	-1.0000	0.3691	1.1978	0.2380	0.4700	0.1242
18	520	4.7623	1.1559	5.7383	>10	0.6273	0.2233
19	482	-1.0000	0.3675	5.5439	9.5810	0.6571	0.1437
20	520	9.1375	3.1941	3.0928	2.5246	1.2898	0.0890
21	520	16.9923	9.8950	3.1064	1.5105	0.2890	0.0866
22	520	11.3644	3.0463	3.6164	2.3832	0.5994	0.1416
23	520	6.2963	0.9537	1.3576	3.3938	3.5116	0.3801
24	506	-0.7027	0.1219	-0.2131	0.0837	1.2093	0.2884
25	519	3.2201	0.9924	1.6778	0.5826	-0.0299	0.0852
26	520	2.8593	0.4695	1.5706	0.2701	0.7223	0.0382
27	520	-0.5816	0.2768	5.5188	>10	0.4779	0.4978
28	520	4.5632	1.0965	10.6398	>10	0.9370	0.3613
29	516	2.2436	0.3415	0.4655	0.1014	0.1930	>10
30	520	3.0716	0.3976	4.5351	1.3563	0.5918	0.2358
31	507	4.4847	1.2736	1.0137	0.2113	-1.0000	>10
32	520	-1.0000	1.2390	6.7918	4.5896	-0.0132	0.0756
33	520	0.7311	0.1222	5.4437	2.7086	0.2432	0.0719
34	520	5.0364	2.7887	3.6985	>10	0.4987	0.2461
36	520	6.7382	1.1876	1.6383	0.4412	-0.1277	0.1934

Table III: Estimation Results for ρ (Restricted PORT Routine with Starting Values, 1995-2007, all Regions)

Given our estimates, we continue and investigate whether the common simplification of using Cobb-Douglas or Leontief functions in CGE models can be rejected by our estimation results. Table IV presents our findings to this regard, sectors with no convergence or binding restrictions for ρ are marked with NA.

Sector	H0: $\sigma_{KL} = 0$	H0: $\sigma_{KL} = 1$	H0: $\sigma_{KLE} = 0$	H0: $\sigma_{KLE} = 1$	H0: $\sigma_{KLEM} = 0$	H0: $\sigma_{KLEM} = 1$
1	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
2	<0.01	<0.01			<0.01	<0.01
3	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
4	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
5	NA	NA	<0.01	<0.01	<0.01	<0.01
6	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
7	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
8	NA	NA	NA	NA	<0.01	<0.01
9	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
10	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
11	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
12	<0.01	<0.01	<0.01		<0.01	<0.01
13	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
14	<0.01	<0.01	<0.01	<0.01		
15	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
16	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
17	NA	NA	<0.01	<0.01	<0.01	<0.01
18	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
19	NA	NA	<0.01	<0.01	<0.01	<0.01
20	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
21	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
22	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
23	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
24	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
25	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
26	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
27	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
28	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
29	<0.01	<0.01	<0.01	<0.01		
30	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
31	<0.01	<0.01	<0.01	<0.01	NA	NA
32	NA	NA	<0.01	<0.01	<0.01	<0.01
33	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
34	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
36	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01

Table IV: Evaluation of Cobb-Douglas and Leontief Specification for CGE models (two-sided p-values for H0)

For all three nests the assumption of a Cobb-Douglas function ($\sigma_{KL} = 1$, $\sigma_{KLE} = 1$ or $\sigma_{KLEM} = 1$) can be dismissed for almost all sectors. A similar picture emerges for the assumption of a Leontief functional form ($\sigma_{KL} = 0$, $\sigma_{KLE} = 0$ or $\sigma_{KLEM} = 0$). To be exact, in the bottom nest the Leontief and the Cobb-Douglas framework must be rejected for the all sectors. While in the middle nest the assumption of a Leontief-like production structure can not be discarded for sector 2 which represents the mining and quarrying industry, a Cobb-Douglas production function can not be excluded in sectors 2 and 12 (basic metals and fabricated metal). A Leontief framework in the top nest can be rejected for all sectors but sectors 14 (electrical and optical equipment) and 29 (real estate activities). The same hold true for a Cobb-Douglas production structure. Overall, this strongly suggests that a simplified approach to the choice of substitution elasticities including only Cobb-Douglas or Leontief production functions is not appropriate and will eventually lead to misguiding results of any counterfactual analysis.

Table V compares the result of our estimations to the findings of Okagawa and Ban (2008), van der Werf (2008) and Kemfert (1998). It must be noted however that for several reasons a direct comparison of the results is difficult. First, non of the studies uses the same data. Second, all of researchers undertake estimations for a different set of sectors. Hence their findings can only be compared on the basis of a specific (possibly arbitrary) sectoral mapping. Third, Okagawa and Ban (2008) as well as Kemfert (1998) do not supply information on the standard error of their results. Fourth, the studies employ different estimation techniques. As a consequence we can not truly test whether our results differ from their findings. Nevertheless, keeping this in mind, we can observe that at large our estimates for σ_{KL}, σ_{KLE} and σ_{KLEM} are neither consistently higher nor smaller than the substitution elasticities supplied by Okagawa and Ban . With few exceptions, compared to the elasticities derived by van der Werf (2008) or Kemfert (1998), our estimates seem to be systematically smaller. Besides the more fundamental issues mentioned above, there are potentially several reasons for the differences between our estimates and those from other studies. Our data may not be up to the task or our choice of variables as illustrated in Table I is inappropriate. However, as Okagawa and Ban (2008), van der Werf (2008) as well as Kemfert (1998) use similar data and variables, these issues do not immediately suggest themselves as the main reasons for the deviations. Alternatively, the deviations may arise due to the usage of different estimation approaches, in particular with regard to linear or and non-linear estimation techniques. In effect, while Okagawa and Ban (2008) and van der Werf (2008) estimate substitution elasticities using linear estimation processes and applying a cost function approach, the elasticities derived in this paper stem from a non-linear estimation process using the original functional form of a CES production function. Only Kemfert (1998) applies also a non-linear estimation to the problem.

Sector				σ_{KL}-Est.				σ_{KLE}-Est.				σ_{KLEM}-Est.		
Own	OB	W	K	Own	OB	W	K	Own	OB	W	K	Own	OB	
1	AGR				1.0697	0.023			0.3968	0.516			0.9803	0.392
2	MIN		Stone and earth		0.7876	0.139		0.21	0.4178	0.553		0.56	0.2209	0.729
3	FOO	Food and tob.	Food		0.2192	0.382	0.4597	0.66	0.1888	0.395	0.399	0.78	0.6267	0.329
4	TEX	Textiles etc.			0.1040	0.161	0.2737		0.2757	0.637	0.2944		0.5879	0.722
5					(Inf)				0.1893				0.5591	
6	WOO				0.1195	0.087			0.2051	0.456			0.7128	0.695
7	PPP	Paper etc.	Paper		0.1151	0.381	0.4103	0.35	0.2531	0.211	0.4489	0.73	0.6610	0.187
8					NA				(Inf)				0.4152	
9	CHM		Chemical industry		0.2362	0.334		0.37	0.7172	-0.065		0.97	0.9434	0.848
10					0.1156				0.1847				0.6798	
11	NMM	Non-metallic minerals			0.1950	0.358	0.4541		0.2488	0.411	0.2546		0.8072	0.306
12	BME	Basis metals	Iron		0.1812	0.22	0.619	0.5	1.0055	0.644	0.6454	0.4	0.1100	1.173
13	MAC				0.4771	0.295			0.1986	0.292			0.5454	0.13
14	EEQ				0.0997	0.163			1.0557	0.524			0.6417	0.876
15	TEQ	Transport eq.	Vehicle		0.1834	0.144	0.4638	0.1	0.1597	0.519	0.1705	0.35	0.3784	0.548
16	MAN				0.2341	0.046			0.1753	0.529			0.5300	0.406
17	EGW				(Inf)	0.46			0.4550	0.256			0.6803	-0.04
18	CON	Construction			0.1735	0.065	0.2242		0.1484	0.529	0.2892		0.6145	1.264
19					(Inf)				0.1528				0.6034	
20					0.0986				0.2443				0.4367	
21					0.0556				0.2435				0.7758	
22					0.0809				0.2166				0.6252	
23	TRN				0.1371	0.31			0.4242	0.281			0.2217	0.352
24	TRN				3.3637	0.31			1.2709	0.281			0.4526	0.352
25	TRN				0.2370	0.31			0.3734	0.281			1.0308	0.352
26	TRN				0.2591	0.31			0.3890	0.281			0.5806	0.352
27	TEL				2.3901	0.37			0.1534	0.518			0.6766	0.654
28	FBS				0.1798	0.264			0.0859	0.32			0.5163	0.492
29					0.3083				0.6823				0.8382	
30					0.2456				0.1807				0.6282	
31					0.1823				0.4966				(Inf)	
32					(Inf)				0.1283				1.0133	
33					0.5777				0.1552				0.8044	
34	PSE				0.1657	0.316			0.2128	0.784			0.6673	0.902
36					0.1292				0.3790				1.1464	
			Non-ferrous					-0.22				0.83		

Table V: Comparison of Results (Restricted PORT Routine with Starting Values, 1995-2007, all Regions), OB: Okagawa and Ban (2008), W: van der Werf (2008), K: Kemfert (1998)

The time series character of our data allows us to engage in an additional analysis and makes it possible to investigate whether substitution elasticities change over time. In the economic literature, technological progress within the CES framework is mainly understood as a change in input productivity and researchers focus primarily on determining λ in Equation 4. But in principle the CES framework for production functions leaves room for technological change affecting not only productivity but also the substitutability between different production inputs. In this case a modified CES function which takes into account changes of the substitution parameter over time and incorporates Hicks-neutral technological change would take the form:

$$y = \gamma e^{\lambda t} \left(\sum_i \alpha_i (x_i)^{-\rho_t} \right)^{\frac{1}{-\rho_t}} . \tag{9}$$

The textile industry at the end of the 18th century provides an excellent example of this form of technological change. As looms became more and more advanced, human labour could be replaced more easily in the production process. Eventually this had a huge effect on business and society in that period.
Embarking on a simple approach, we test whether we can observe a change in input substitutability over time by reestimating Equations (7) and (8) and comparing σ for two different time periods (1995 to 1997 and 2005 to 2007). Table VI summarises the results to this regard. Note that for some sectors convergence or CES constraint issues arise when using a restraint time period, the respective sectors are marked with an NA value. In the bottom, middle nest and top nest, the hypothesis that the substitution elasticities do not change over time can be rejected for about two thirds of the sectors under investigation. When evaluating a less stringent comparison between the two periods, the picture becomes even clearer and we can reject a significant change in input substitutability for all but a handful of sectors.[5] To allow for a more detailed analysis investigating if there have been any structural changes of the substitution elasticities over time, we group the results in five groups depending on the evaluated sector, namely Basic Materials, Energy, Manufacturing, Services and Transport, the underlying mapping is outlined in Table **??** in the Appendix. But even when investigating only specific sectors groups, we do not observe any significant changes over time. Moreover, for all sector groups, the hypothesis that there has been an increase in elasticities has to be rejected equally often as the hypothesis that elasticities have decreased. Hence our results suggest, that there has been no structural change in elasticities over time. This implies also that changing substitution elasticities appear not to be a problem for our estimations, which originally consider the complete time period between 1995 to 2007. But nevertheless, the issue is potentially important. As a consequence, in future research this particular dimension of technological progress needs to be taken into account and should be investigated with more rigour. Ultimately this will require studying longer time periods as those under investigation so far in studies on the substitutability of inputs and also a formalisation of the issue within the CES framework.

5 We assume that a significant change would imply a production structure changing from Leontief to Cobb Douglas, i.e. a change resulting in $|\sigma_{95-97} - \sigma_{05-07}| > 1$.

Sector	H0 for σ_{KL}-Est.: $\sigma_{95-97} \leq \sigma_{05-07}$	$\sigma_{95-97} \geq \sigma_{05-07}$	$\lvert\sigma_{95-97} - \sigma_{05-07}\rvert > 1$	H0 for σ_{KLE}-Est.: $\sigma_{95-97} \leq \sigma_{05-07}$	$\sigma_{95-97} \geq \sigma_{05-07}$	$\lvert\sigma_{95-97} - \sigma_{05-07}\rvert > 1$	H0 for σ_{KLEM}-Est.: $\sigma_{95-97} \leq \sigma_{05-07}$	$\sigma_{95-97} \geq \sigma_{05-07}$	$\lvert\sigma_{95-97} - \sigma_{05-07}\rvert > 1$
1	<0.01		<0.01		<0.01	<0.01		<0.1	<0.01
2		<0.01	<0.01				<0.01		<0.01
3			<0.01			<0.01			<0.01
4			<0.01	<0.01		<0.01		<0.01	<0.01
5	NA	NA	NA	NA	NA	NA	NA	NA	NA
6			<0.01			<0.01		<0.01	<0.01
7	<0.05		<0.01			<0.01	<0.01		<0.01
8	NA	NA	NA	NA	NA	NA	<0.01		<0.01
9	<0.1		<0.01		<0.01	<0.01		<0.01	<0.01
10			<0.01			<0.01		<0.01	<0.01
11		<0.01	<0.01	<0.01		<0.01			<0.01
12		<0.05	<0.01		<0.01	<0.01	<0.01		<0.01
13		<0.01	<0.01		<0.01	<0.01	<0.01		<0.01
14			<0.01	<0.05		<0.05			
15			<0.01			<0.01			<0.01
16	<0.01		<0.01	<0.01		<0.01		<0.01	<0.01
17	NA	NA	NA	<0.01		<0.01			<0.01
18		<0.01	<0.01			<0.01	<0.05		<0.01
19	NA	NA	NA			<0.01	<0.05		<0.01
20			<0.01			<0.01	<0.01		<0.01
21		<0.01	<0.01	<0.01		<0.01	<0.05		<0.01
22	<0.01		<0.01	<0.01		<0.01		<0.01	<0.01
23		<0.01	<0.01			<0.01	<0.01		<0.01
24				<0.01		<0.01		<0.01	<0.01
25	<0.01		<0.01	<0.01		<0.01		<0.01	<0.01
26	<0.01		<0.01		<0.01	<0.01	<0.01		<0.01
27									<0.01
28	<0.01		<0.01			<0.01		<0.01	<0.01
29		<0.01	<0.01	<0.01		<0.01			
30	<0.05		<0.01	<0.01		<0.01	<0.01		<0.01
31	<0.01		<0.01		<0.05	<0.01	<0.01		<0.01
32	NA	NA	NA		<0.01	<0.01	<0.01		<0.01
33	<0.01		<0.01			<0.01		<0.05	<0.01
34			<0.01			<0.1		<0.05	<0.01
36	<0.01		<0.01		<0.01	<0.01		<0.01	<0.01

Table VI: Comparison of the Substitution Elasticities for the Periods 1995-1997 and 2005-2007 (Restricted PORT Routine with Starting Values, all Regions, p-values for H0)

Having investigated the variability of input substitutability over time, we next evaluate whether elasticities vary across regions. We apply a similar approach as before and compare estimates for different regions with each other to test for regional variation. Although here we only present the results of a comparison between the estimates for the EU27 and the BRIC countries (Brazil, Russia, India and China), a similar picture emerges when contrasting the results for countries with a high productivity to those featuring a relatively low productivity.[6] Table **??** illustrates the results when comparing estimation results for the EU27 with those for BRIC. Again, we do not achieve convergence for the bottom nest of sector 8 and in particular in the bottom nest several estimates are driven by the constraints of the CES framework. These estimates are marked with NAs. For all three nests, there is no significant change in input substitutability across regions for the large majority of sectors. Only for the groups associated with service and manufacturing activities in the BRIC countries, we can not reject a higher input substitutability between the capital-labour-energy composite and materials compared to estimates for the EU for a fair number of sectors. For the other two nests and sector groups, one can not conclude that elasticities are higher or lower in the EU compared to the BRIC countries and vice versa. Hence overall, there appears to be no regional variation in elasticities of substitution.

[6] For this analysis, we rank the countries under investigation according to their score in the index GDP per person employed (constant 1990 PPP USD) from the World Bank

Sector	H0 for σ_{KL}-Est.: $\sigma_{EU} \leq \sigma_{BRIC}$	$\sigma_{EU} \geq \sigma_{BRIC}$	$\lvert\sigma_{EU} - \sigma_{BRIC}\rvert > 1$	H0 for σ_{KLE}-Est.: $\sigma_{EU} \leq \sigma_{BRIC}$	$\sigma_{EU} \geq \sigma_{BRIC}$	$\lvert\sigma_{EU} - \sigma_{BRIC}\rvert > 1$	H0 for σ_{KLEM}-Est.: $\sigma_{EU} \leq \sigma_{BRIC}$	$\sigma_{EU} \geq \sigma_{BRIC}$	$\lvert\sigma_{EU} - \sigma_{BRIC}\rvert > 1$
1	<0.01		<0.01		<0.01	<0.01		<0.01	<0.01
2	<0.01		<0.01			<0.01		<0.01	<0.01
3	<0.01		<0.01			<0.05			<0.01
4			<0.01		<0.05	<0.01		<0.01	<0.01
5	<0.01		<0.01			<0.01	<0.01		<0.01
6		<0.01	<0.01		<0.05	<0.01	<0.01		<0.01
7	NA	NA	NA		<0.01	<0.01		<0.01	<0.01
8	NA	NA	NA					<0.01	<0.01
9	NA	NA	NA		<0.01	<0.01		<0.01	<0.01
10	NA	NA	NA			<0.01	<0.01		<0.01
11						<0.01		<0.05	<0.01
12		<0.01	<0.01					<0.01	<0.01
13		<0.01	<0.01						<0.01
14	NA	NA	NA					<0.01	<0.01
15	NA	NA	NA			<0.01		<0.01	<0.01
16		<0.01	<0.01			<0.01		<0.01	<0.01
17	NA	NA	NA			<0.05			<0.01
18		<0.01	<0.01		<0.05	<0.01			<0.01
19		<0.01	<0.01		<0.01	<0.01	<0.01		<0.01
20	NA	NA	NA			<0.01		<0.01	<0.01
21	<0.01		<0.01	<0.01		<0.01		<0.01	<0.01
22	NA	NA	NA		<0.1	<0.01		<0.01	<0.01
23	<0.01		<0.01						<0.01
24	NA	NA	NA	NA	NA	NA			<0.05
25	<0.01		<0.01	NA	NA	NA			
26	<0.01		<0.01		<0.01	<0.01		<0.01	<0.01
27	NA	NA	NA			<0.01		<0.01	<0.01
28	<0.01		<0.01	<0.1		<0.01		<0.01	<0.01
29		<0.01	<0.01		<0.01	<0.01			
30		<0.01	<0.01			<0.01	<0.05		<0.01
31	NA	NA	NA		<0.01	<0.01		<0.01	<0.01
32	<0.01		<0.01		<0.01	<0.01			<0.01
33		<0.01	<0.01	<0.05		<0.01	<0.01		<0.01
34	NA	NA	NA		<0.01	<0.01	<0.01		<0.01
36	<0.01		<0.01			<0.05			<0.01

Table VII: Comparison of the Substitution Elasticities for the Regions EU and BRIC (Restricted PORT Routine with Starting Values, 1997-2009, p-values for H0)

V. Summary and Conclusion

Elasticities, in particular substitution elasticities, are vital parameters for any micro-consistent economic model and crucially influence the results of counterfactual policy analysis. But so far only few consistent estimates of elasticities exist. With this paper we aim at overcoming this problem. Building on a rich dataset based on the WIOD data, we systematically investigate input substitutability in a CES production framework for a KLEM production structure using non-linear estimation procedures. On the basis of our estimations, we demonstrate that the common practice of using Cobb-Douglas or Leontief production functions in economic models must be rejected for the majority of sectors. This calls for a more elaborate approach with regard to substitution elasticities. In particular in response to this result, we provide a comprehensive set of consistently estimated substitution elasticities covering a wide range of different sectors. Our results suggest additionally that no significant change in input substitutability takes place over during the time period we consider. Hence for most sectors we do not observe technological change through this channel. Although technological progress in the form of changing substitution elasticities may potentially be an issue when studying longer time periods. Moreover, there is no significant regional variation in substitution elasticities across regions. By providing an exhaustive set of substitution elasticities and with our analysis of input substitutability over time and across regions, we hope to make a valuable contribution to making instruments designed to evaluate policy measures ex-ante more reliable and support researchers as well as policy makers in their efforts to find solutions for today's challenges requiring regulative interventions.

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Appendix

Country	Countrycode	Country	Countrycode
Australia	AUS	Italy	ITA
Austria	AUT	Japan	JPN
Belgium	BEL	Latvia	LVA
Brazil	BRA	Lithuania	LTU
Bulgaria	BGR	Luxembourg	LUX
Canada	CAN	Malta	MLT
China	CHN	Mexico	MEX
Cypres	CYP	Netherlands	NLD
Czech Republic	CZE	Poland	POL
Denmark	DNK	Portugal	PRT
Estonia	EST	Republic of Korea	KOR
Finland	FIN	Romania	ROU
France	FRA	Russia	RUS
Germany	DEU	Slovakia	SVK
Great Britain	GBR	Slovenia	SVN
Greece	GRC	Spain	ESP
Hungary	HUN	Sweden	SWE
India	IND	Taiwan	TWN
Indonesia	IDN	Turkey	TUR
Ireland	IRL	United States of America	USA

Table VIII: List of Regions Included in the Analysis

Sector Description	NACE	Code	Group
Agriculture, hunting, forestry and fishing	AtB	1	Basic Materials
Mining and quarrying	C	2	Basic Materials
Food, beverages and tobacco	15t16	3	Manufacturing
Textiles and textile	17t18	4	Manufacturing
Leather, leather and footwear	19	5	Manufacturing
Wood and products of wood and cork	20	6	Basic Materials
Pulp, paper, paper, printing and publishing	21t22	7	Manufacturing
Coke, refined petroleum and nuclear fuel	23	8	Energy
Chemicals and chemical	24	9	Manufacturing
Rubber and plastics	25	10	Manufacturing
Other non-metallic mineral	26	11	Manufacturing
Basic metals and fabricated metal	27t28	12	Manufacturing
Machinery, nec	29	13	Manufacturing
Electrical and optical equipment	30t33	14	Manufacturing
Transport equipment	34t35	15	Manufacturing
Manufacturing nec; recycling	36t37	16	Manufacturing
Electricity, gas and water supply	E	17	Energy
Construction	F	18	Manufacturing
Sale, maintenance and repair of motor vehicles and motorcycles; retail sale of fuel	50	19	Services
Wholesale trade and commission trade, except of motor vehicles and motorcycles	51	20	Services
Retail trade, except of motor vehicles and motorcycles; repair of household goods	52	21	Services
Hotels and restaurants	H	22	Services
Inland transport	60	23	Transport
Water transport	61	24	Transport
Air transport	62	25	Transport
Supporting and auxiliary transport activities; activities of travel agencies	63	26	Transport
Post and telecommunications	64	27	Services
Financial intermediation	J	28	Services
Real estate activities	70	29	Services
Renting of m&eq and other business activities	71t74	30	Services
Public admin and defence; compulsory social security	L	31	Services
Education	M	32	Services
Health and social work	N	33	Services
Other community, social and personal services	O	34	Services
Total industries	TOT	36	Total

Table IX: List of Sectors Included in the Analysis

Region	Time Period	Solver	Starting Values from Grid Search	Restricted Coefficients ($\gamma \geq 0, 0 \leq \alpha_i \leq 1, \sum_{i=1}^{n} \alpha_i, \rho \geq -1$)	Technological Progress (Hicks-neutral)
All Countries	1995 to 2007	BFGS	no	no	yes
			yes	no	yes
		DE	no	no	yes
			no	yes	yes
		KM	no	no	no
		LM	no	no	yes
			yes	no	yes
			no	no	no
			yes	no	no
		NM	no	no	yes
			yes	no	yes
		PORT	no	no	yes
			no	yes	yes
			yes	no	yes
			yes	yes	yes
			no	no	no
			no	yes	no
			yes	no	no
			yes	yes	no
		SANN	no	no	yes
			yes	no	yes
All Countries	1995 to 1997	PORT	yes	yes	yes
All Countries	2005 to 2007	PORT	yes	yes	yes
All Countries	2008 to 2009	PORT	yes	yes	yes
High Productivity	1995 to 2007	PORT	yes	yes	yes
Low Productivity	1995 to 2007	PORT	yes	yes	yes
EU	1995 to 2007	PORT	yes	yes	yes
Western	1995 to 2007	PORT	yes	yes	yes
ROW	1995 to 2007	PORT	yes	yes	yes
BRIC	1995 to 2007	PORT	yes	yes	yes

Table X: List of Estimations Procedures Included in the Analysis

Sector	N	σ_{KL}-Est.	Std. Dev.	σ_{KLE}-Est.	Std. Dev.	σ_{KLEM}-Est.	Std. Dev.
1	520	1.0697	0.0791	0.3968	0.1026	0.9803	0.0841
2	521	0.7876	0.0796	0.4178	>10	0.2209	0.0381
3	522	0.2192	0.0572	0.1888	0.2117	0.6267	0.1086
4	523	0.1040	0.0463	0.2757	0.0677	0.5879	0.0369
5	524	(Inf)	NA	0.1893	0.0399	0.5591	0.0309
6	525	0.1195	0.0689	0.2051	0.0644	0.7128	0.0838
7	526	0.1151	0.0355	0.2531	0.0971	0.6610	0.0817
8	527	NA		(Inf)	NA	0.4152	0.0537
9	528	0.2362	0.0277	0.7172	0.1148	0.9434	0.2337
10	529	0.1156	0.0284	0.1847	0.0224	0.6798	0.0630
11	530	0.1950	0.0369	0.2488	0.1576	0.8072	0.0739
12	531	0.1812	0.0408	1.0055	0.1433	0.1100	0.0132
13	532	0.4771	0.0336	0.1986	0.0493	0.5454	0.0611
14	533	0.0997	0.0959	1.0557	0.1003	0.6417	>10
15	534	0.1834	0.0256	0.1597	0.0976	0.3784	0.0441
16	535	0.2341	0.0288	0.1753	0.0639	0.5300	0.0425
17	536	(Inf)	NA	0.4550	0.0493	0.6803	0.0575
18	537	0.1735	0.0348	0.1484	0.2952	0.6145	0.0843
19	538	(Inf)	NA	0.1528	0.2237	0.6034	0.0523
20	539	0.0986	0.0311	0.2443	0.1507	0.4367	0.0170
21	540	0.0556	0.0306	0.2435	0.0896	0.7758	0.0521
22	541	0.0809	0.0199	0.2166	0.1118	0.6252	0.0554
23	542	0.1371	0.0179	0.4242	0.6106	0.2217	0.0187
24	543	3.3637	1.3787	1.2709	0.1352	0.4526	0.0591
25	544	0.2370	0.0557	0.3734	0.0813	1.0308	0.0905
26	545	0.2591	0.0315	0.3890	0.0409	0.5806	0.0129
27	546	2.3901	1.5812	0.1534	0.3779	0.6766	0.2279
28	547	0.1798	0.0354	0.0859	0.1241	0.5163	0.0963
29	548	0.3083	0.0325	0.6823	0.0472	0.8382	>10
30	549	0.2456	0.0240	0.1807	0.0443	0.6282	0.0931
31	550	0.1823	0.0423	0.4966	0.0521	(Inf)	NA
32	551	(Inf)	NA	0.1283	0.0756	1.0133	0.0777
33	552	0.5777	0.0408	0.1552	0.0652	0.8044	0.0465
34	553	0.1657	0.0765	0.2128	1.0670	0.6673	0.1096
36	554	0.1292	0.0198	0.3790	0.0634	1.1464	0.2541

Table XI: Estimation Results for σ (Restricted PORT Routine with Starting Values, 1995-2007, all Regions)