ECOMOD 2012, SEVILLE, JULY 4 – 7, 2012

FAIR QUALIFIED MAJORITIES IN WEIGHTED VOTING BODIES

František Turnovec[1]

Abstract. In parliaments elected by proportional systems the seats are allocated to the political parties roughly proportionally to the shares of votes for the party lists obtained in elections. Assuming that members of the parliament representing the same party are voting together, it has sense to require that distribution of the influence of the parties in parliamentary decision making is proportional to the distribution of seats. There exist measures (so called voting power indices) reflecting an ability of each party to influence outcome of voting. Power indices are functions of distribution of seats and voting quota (where voting quota means a minimal number of votes required to pass a proposal). By a fair voting rule we call such a quota that leads to proportionality of influence to relative representation. Usually simple majority is not a fair voting rule. That is the reason why so called qualified or constitutional majority is being used in voting about important issues requiring higher level of consensus. Qualified majority is usually fixed (60% or 66.67%) independently on the structure of political representation. In the paper we use game-theoretical model of voting to find a quota that defines the fair voting rule as a function of the structure of political representation. Such a quota we call a fair majority. Fair majorities can differ for different structures of the parliament. Concept of a fair majority is illustrated on the data for the Lower House of the Czech Parliament elected in 2010.

Keywords: fair majority, power indices, qualified majority, quota interval of stable power, simple weighted committee, voting power.

JEL Classification: C71, D72, H77
AMS Classification: 91A12, 91A40, 05C65

1 Fairness in voting

A qualified majority in committee voting is a requirement for a proposal to gain a specified level or type of support which exceeds a simple majority (over 50%). In some jurisdictions, for example, parliamentary procedure requires that any action that may alter the rights of the minority has a qualified majority support. Particular designs of qualified majority (such as 60% or two-thirds majority) are selected "ad hoc", without quantitative justification. In this paper we try to provide such a justification, defining qualified majority by a "fair quota", providing each legislator with (approximately) the same influence, measured as an a priori voting power.

Let us consider a committee with n members. Each member has some voting weight (number of votes, shares etc.) and a voting rule is defined by a minimal number of weights required for passing a proposal (model of such a committee is called a weighted voting committee). Given a voting rule, voting weights provide committee members with voting power. Voting power means an ability to influence the outcome of voting. Voting power indices are used to quantify the voting power.

The concept of fairness is being discussed related to the distribution of voting power among different actors of voting. This problem was clearly formulated by Nurmi (1982), p.

[1] Charles University in Prague, Faculty of Social Sciences, Institute of Economic Studies, Opletalova 26, 110 00 Prague 1, Czech Republic, e-mail: turnovec@fsv.cuni.cz
.

204: "*If one aims at designing collective decision-making bodies which are democratic in the sense of reflecting the popular support in terms of the voting power, we need indices of the latter which enable us to calculate for any given distribution of support and for any decision rule the distribution of seats that is 'just'. Alternatively, we may want to design decision rules that – given the distribution of seats and support – lead to a distribution of voting power which is identical with the distribution of support.*"

Voting power is not directly observable: as a proxy for it voting weights are used. Therefore, fairness is usually defined in terms of voting weights (e.g. voting weights are proportional to the results of an election or to the shares in a share-holding company). Assuming that a principle of fair distribution of voting weights is selected, we are addressing the question of how to achieve equality of voting power (at least approximately) to relative voting weights. For evaluation of voting power we are using concepts of a priori power indices (a comprehensive survey of power indices theory see in Felsenthal and Machover (1998)). The concepts of optimal quota, introduced by Słomczyński and Życzkowski (2006), (2007) for the EU Council of Ministers distribution of national voting weights (weights equal to square roots of population and quota that provides each citizen of the EU with the same indirect voting power measured by Penrose-Banzhaf index independently on her national affiliation), and of intervals of stable power (Turnovec (2011) are used to find, given voting weights, a fair voting rule minimizing the distance between actors' voting weights and their voting power.

In the second section we provide simple illustrative examples, in the third section basic definitions are introduced and the power indices methodology is shortly resumed. The fourth section introduces concepts of quota intervals of stable power and fair quota. The fifth section applies the concept of fair quota for the Lower House of the Czech Parliament elected in 2010.

2 Simple illustrative examples

Let us consider a committee with 5 members: one strong member 1 with relative weight 0.6 and four weak members 2, 3, 4, 5 with relative weights 0.1.

There are 5 groups of majority voting coalitions:

One strong member for majority quota 0.6
One strong and one of the weak members for majority quota 0.7
One strong and two of the weak members for majority quota 0.8
One strong and three weak members for majority quota 0.9
One strong and four weak members for unanimity quota 1

Shapley-Shubik power indices, measuring probability that the member will be in a pivotal situation (if she votes YES, the outcome is YES, if she votes NO the outcome is NO):

(1, 0, 0, 0, 0) for quota 0.6
(0.8, 0.05, 0.05, 0.05, 0.05) for quota 0.7
(0.6, 0.1, 0.1, 0.1, 0.1) for quota 0.8
(0.4, 0.15, 0.15, 0.15, 0.15) for quota 0.9
(0.2, 0.2, 0.2, 0.2, 0.2) for quota 1

The fair majority quota is 0.8, because the relative power of each member equals exactly to relative representation.

Consider now the same committee with different voting weights: weight of the strong member 1 is 0.62, and weights of the weak members 2, 3, 4, and 5 are 0.095. There are 5 groups of majority voting coalitions:

One strong member for majority quota 0.62

One strong and one of the weak members for majority quota 0.715
One strong and two weak members for majority quota 0.81
One strong and three weak members for majority quota 0.905
One strong and four weak members for unanimity quota

Shapley-Shubik power indices corresponding to different majority quotas are (as before):

(1, 0, 0, 0, 0)	for quota 0.62
(0.8, 0.05, 0.05, 0.05, 0.05)	for quota 0.715
(0.6, 0.1, 0.1, 0.1, 0.1)	for quota 0.81
(0.4, 0.15, 0.15, 0.15, 0.15)	for quota 0.905
(0.2, 0.2, 0.2, 0.2, 0.2)	for unanimity quota 1

Considering distances between vector of relative weights and corresponding vectors of relative power for different majority quotas we obtain:

Quota	relative weights	relative power	distance
0.62	(0.62, 0.095, 0.095, 0.0.95, 0.095)	(1, 0, 0, 0, 0)	0.403051
0.715	(0.62, 0.095, 0.095, 0.095, 0.095)	(0.8, 0.05, 0.05, 0.05, 0.05)	0.190919
0.81	**(0.62, 0.095, 0.095, 0.095, 0.095)**	**(0.6, 0.1, 0.1, 0.1, 0.1)**	**0.021213**
0.905	(0.62, 0.095, 0.095, 0.095, 0.095)	(0.4, 0.15, 0.15, 0.15, 0.15)	0.233345
1	(0.62, 0.095, 0.095, 0.095, 0.095)	(0.2, 0.2, 0.2, 0.2, 0.2)	0.445477

In this case the exact fair majority does not exist, but the best approximation of fair quota, minimizing Euclidean distance between relative weights and relative power is 0.81.

3 Committees and voting power

A simple weighted committee is a pair [N, w], where N be a finite set of n committee members $i = 1,2,...,n$, and w = (w_1, w_2, …, w_n) be a nonnegative vector of committee members' voting weights (e.g. votes or shares). By 2^N we denote the power set of N (set of all subsets of N). By voting coalition we mean an element S ∈ 2^N, the subset of committee members voting uniformly (YES or NO), and $w(S) = \sum_{i \in S} w_i$ denotes the voting weight of coalition S. The voting rule is defined by quota q satisfying $0 < q \le w(N)$, where q represents the minimal total weight necessary to approve the proposal. Triple [N, q, w] we call a simple quota weighted committee. The voting coalition S in committee [N, q, w] is called a winning one if $w(S) \ge q$ and a losing one in the opposite case. The winning voting coalition S is called critical if there exists at least one member k ∈ S such that w(S\k) < q (we say that k is critical in S). The winning voting coalition S is called minimal if any of its members is critical in S.

A priori voting power analysis seeks an answer to the following question: Given a simple quota weighted committee $[N, q, \mathbf{w}]$, what is an influence of its members over the outcome of voting? The absolute voting power of a member i is defined as a probability $\Pi_i[N, q, \mathbf{w}]$ that i will be decisive in the sense that such a situation appears in which she would be able to decide the outcome of voting by her vote (Nurmi (1997) and Turnovec (1997)), and a relative voting power as $\pi_i[N, q, \mathbf{w}] = \dfrac{\Pi_i[N, q, \mathbf{w}]}{\sum_{k \in N} \Pi_k[N, q, \mathbf{w}]}$.

Two basic concepts of decisiveness are used: swing position and pivotal position. The swing position is an ability of an individual voter to change the outcome of voting by a unilateral switch from YES to NO (if member j is critical with respect to a coalition S, we say that he has a swing in S). The pivotal position is such a position of an individual voter in a permutation of voters expressing a ranking of attitudes of members to the voted issue (from the most preferable to the least preferable) and the corresponding order of forming of the winning coalition, in which her vote YES means a YES outcome of voting and her vote NO means a NO outcome of voting (we say that j is pivotal in the permutation considered).

Let us denote by i the member of the simple quota weighted committee [N, q, w], W(N, q, w) the set of all winning coalitions and by W_i(N, q, w) the set of all winning coalitions with i, C(N, q, w) the set of all critical winning coalitions, and by C_i(N, q, w) the set of all critical winning coalitions i has the swing in, by P(N) the set of all permutations of N and P_i(N, q, w) the set of all permutations i is pivotal in. By card(S) we denote the cardinality of S, card(∅) = 0.

Assuming many voting acts and all coalitions equally likely, it makes sense to evaluate the a priori voting power of each member of the committee by the probability to have a swing, measured by the absolute Penrose-Banzhaf (PB) power index (Penrose (1946), Banzhaf (1965)) $\Pi_i^{PB}(N,q,\mathbf{w}) = \dfrac{card(C_i)}{2^{n-1}}$, where (card($C_i$) is the number of all winning coalitions the member i has the swing in and 2^{n-1} is the number of all possible coalitions with i). To compare the relative power of different committee members, the relative form of the PB power index $\pi_i^{PB}(N,q,\mathbf{w}) = \dfrac{card(C_i)}{\sum_{k\in N} card(C_k)}$ is used.

While the absolute PB is based on a well-established probability model (see e.g. Owen (1972)), its normalization (relative PB index) destroys this probabilistic interpretation, the relative PB index simply answers the question of what is the voter i's share in all possible swings.

Assuming many voting acts and all possible preference orderings equally likely, it makes sense to evaluate an a priori voting power of each committee member by the probability of being in pivotal situation, measured by the Shaply-Shubik power index (Shapley and Shubik (1954)): $\Pi_i^{SS}(N,q,\mathbf{w}) = \dfrac{card(P_i)}{n!}$, where card($P_i$) is the number of all permutations in which the committee member i is pivotal, and n! is the number of all possible permutations of committee members. Since $\sum_{i\in N} card(P_i) = n!$ it holds that $\pi_i^{SS}(N,q,\mathbf{w}) = \dfrac{card(P_i)}{\sum_{k\in N} card(P_k)} = \dfrac{card(P_i)}{n!}$, i.e. the absolute and relative form of the SS-power index is the same.

It can be easily seen that for any α > 0 and any power index based on swings or pivots or MWC positions it holds that $\Pi_i[N,\alpha q,\alpha\mathbf{w}] = \Pi_i[N,q,\mathbf{w}]$. Therefore, without the loss of generality, we shall assume throughout the text that $\sum_{i\in N} w_i = 1$ and 0 < q ≤ 1, using only relative weights and relative quotas in the analysis.

4 Quota intervals of stable power and the fair quota

Let us formally define a few concepts we shall use later in this paper:

Definition 1. Let $\mathbf{w} = (w_1, w_2, ..., w_n)$ be a fair distribution of voting weights (with whatever principle is used to justify it) in a simple weighted committee [N, **w**], **π** is a relative power index, (**π**[N, q, **w**] is a vector valued function of q), and d is a distance function, then the voting rule q_1 is said to be *at least as fair* as voting rule q_2 with respect to the selected **π** and distance d if d(**w**, π(N, q_1, **w**)) ≤ d(**w**, π(N, q_2, **w**)).

Intuitively, given **w**, the voting rule q_1 is preferred to the voting rule q_2 if q_1 generates a distribution of power closer to the distribution of weights than q_2.

Definition 2. The voting rule q* that minimizes a distance d between **π**[N, q, **w**] and **w** is called a fair voting rule (*fair quota*) for the power index π with respect to the distance d.

Proposition 1. Let [N, q, **w**] be a simple quota weighted committee and C_{is} be the set of critical winning coalitions of the size s in which i has a swing, then $card(P_i) = \sum_{s\in N} card(C_{is})(s-1)!(n-s)!$ is the number of permutations with the pivotal position of i in [N, q, **w**].

Proof follows directly from Shapley and Shubik (1954). □

From Proposition 1 it follows that the number of pivotal positions corresponds to the number and structure of swings. If in two different committees sets of swing coalitions are identical, then the sets of pivotal positions are also the same.

Proposition 2. *Let [N, q_1, **w**] and [N, q_2, **w**], $q_1 \neq q_2$, be two simple quota weighted committees such that W(N, q_1, **w**) = W(N, q_2, **w**), then* $C_i(N,q_1,\mathbf{w}) = C_i(N,q_2,\mathbf{w})$ *and* $P_i(N,q_1,\mathbf{w}) = P_i(N,q_2,\mathbf{w})$ *for all i ∈ N.*

Proof: Without a loss of generality suppose $q_1 > q_2$. Assuming that the statement of the proposition is not true, i.e. a member *k* from *S* has a swing in *S* for quota q_2 and does not have a swing for quota q_1, we obtain $w(S\backslash k) - q_2 < 0$ and $w(S\backslash k) - q_1 \geq 0$, hence $S\backslash k \notin W(N, q_2, \mathbf{w})$ and $S\backslash k \in W(N, q_1, \mathbf{w})$, which contradicts the assumption of Proposition 2 that the sets of winning coalitions are equal. From Proposition 1 it follows that $C_i(N, q_1, \mathbf{w}) = C_i(N, q_2, \mathbf{w})$ implies $P_i(N, q_1, \mathbf{w}) = P_i\{N, q_2, \mathbf{w})$. □

From Proposition 2 it follows that in two different committees with the same set of members, the same weights and the same sets of winning coalitions, the PB-power indices and SS-power indices are the same in both committees, independently of quotas.

Proposition 3. *Let [N, q, **w**] be a simple quota weighted committee with a quota q,* $\mu^+(q) = \min_{S\in W[N,q,w]} (w(S)-q)$ *and* $\mu^-(q) = \min_{S\in 2^N \backslash W(N,q,w)} (q-w(S))$. *Then for any particular quota q we have W(N, q, **w**) =W(N, γ, **w**) for all* $\gamma \in (q-\mu^-(q), q+\mu^+(q)]$.

Proof: a) Let $S \in W(N, q, \mathbf{w})$, then from the definition of $\mu^+(q)$

$$w(S) - q \geq \mu^+(q) \geq 0 \Rightarrow w(S) - q - \mu^+(q) \geq 0 \Rightarrow S \in W(N, q+\mu^+(q), \mathbf{w}),$$

hence *S* is winning for quota $q+\mu^+(q)$. If *S* is winning for $q+\mu^+(q)$, then it is winning for any quota $\gamma \leq q+\mu^+(q)$.

b) Let $S \in 2^N \backslash W(N, q, \mathbf{w})$, then from the definition of $\mu^-(q)$

$$q - w(S) \geq \mu^-(q) \geq 0 \Rightarrow q - \mu^-(q) - w(s) \geq 0 \Rightarrow S \in 2^N \backslash W(N, q-\mu^-(q), \mathbf{w}),$$

hence *S* is losing for quota $q-\mu^-(q)$. If *S* is losing for $q-\mu^-(q)$, then it is losing for any quota $\gamma \geq q-\mu^-(q)$.

From (a) and (b) it follows that for any $\gamma \in (q-\mu^-(q), (q-\mu^+(q)]$

$$S \in W(N,q,\mathbf{w}) \Rightarrow S \in W(N,\gamma,\mathbf{w})$$

$$S \in \{2^N \backslash W(N,\gamma,\mathbf{w})\} \Rightarrow S \in \{2^N \backslash W(N,q,\mathbf{w})\}$$

which implies that $W(N,q,\mathbf{w}) = W(N,\gamma,\mathbf{w})$. □

From Propositions 2 and 3 it follows that swing and pivot based power indices are the same for all quotas $\gamma \in (q-\mu^-(q), q+\mu^+(q)]$. Therefore the interval of quotas $(q-\mu^-(q), q+\mu^+(q)]$ we call an *interval of stable power* for quota q. Quota $\gamma^* \in (q-\mu^-(q), q+\mu^+(q)]$ is called the marginal quota for q if $\mu^+(\gamma^*) = 0$.

Now let us define a partition of the power set 2^N into equal weight classes $\Omega^{(0)}, \Omega^{(1)}, \ldots, \Omega^{(r)}$ (such that the weight of different coalitions from the same class is the same and the weights of different coalitions from different classes are different). For the completeness set $w(\emptyset) = 0$. Consider the weight-increasing ordering of equal weight classes $\Omega^{(0)}, \Omega^{(1)}, \ldots, \Omega^{(r)}$ such that for any $t < k$ and $S \in \Omega^{(t)}$, $R \in \Omega^{(k)}$ it holds that $w(S) < w(R)$. Denote $q_t = w(S)$ for any $S \in \Omega^{(t)}$, t = 1, 2, …, r.

Proposition 4. *Let $\Omega^{(0)}, \Omega^{(1)}, \ldots, \Omega^{(r)}$ be the weight-increasing ordering of the equal weight partition of 2^N. Set $q_t = w(S)$ for any $S \in \Omega^{(t)}$, t = 0, 1, 2, …, r. Then there is a finite number $r \leq 2^{n-1}$ of marginal quotas qt and corresponding intervals of stable power $(q_{t-1}, q_t]$ such that $W(N, q_t, \mathbf{w}) \subset W(N, q_{t-1}, \mathbf{w})$.*

Proof follows from the fact that card $(2^N) = 2^n$ and an increasing series of k real numbers $a_1, \ldots, a_k$ subdivides interval $(a_1, a_k]$ into *k-1* segments. An analysis of voting power as a function of the quota (given voting weights) can be substituted by an analysis of voting power in a finite number of marginal quotas. □

From Proposition 4 it follows that there exist at most r distinct voting situations generating r vectors of power indices.

Proposition 5. *Let [N, q, w] be a simple quota weighted committee and $(q_{t-1}, q_t]$ is the interval of stable power for quota q. Then* $card(C_i(N,q,\mathbf{w})) = card(C_i(N,\gamma,\mathbf{w}))$ *and* $card(P_i(N,q,\mathbf{w})) = card(P_i(N,\gamma,\mathbf{w}))$ *for any $\gamma = 1 - q_t + \varepsilon$, where $\varepsilon \in (0, q_t - q_{t-1}]$ and for all $i \in N$.*

Proof. Let *S* be a winning coalition, member *k* has the swing in *S* and $(q_{t-1}, q_t]$ is an interval of stable power for *q*. Then it is easy to show that $N\backslash S \cup k$ is a winning coalition, *k* has a swing in $N\backslash S \cup k$ and $(1-q_t, 1-q_{t-1}]$ is an interval of stable power for any quota $\gamma = 1 - q_t + \varepsilon$ $(0 < \varepsilon \leq q_t - q_{t-1})$. Let *R* be a winning coalition, *j* has a swing in *R*, and $(1-q_t, 1-q_{t-1}]$ is an interval of stable power for quota $\gamma = 1 - q_t + \varepsilon$ $(0 < \varepsilon \leq q_t - q_{t-1})$. Then $N\backslash R \cup j$ is a winning coalition, *j* has a swing in $N\backslash R \cup j$ and $(q_{t-1}, q_t]$ is an interval of stable power for any quota $q = q_{t-1} + \tau$ where $0 < \tau \leq q_t - q_{t-1}$. □

While in [N, q, **w**] the quota q means the total weight necessary to pass a proposal (and therefore we can call it a *winning quota*), the *blocking quota* means the total weight necessary to block a proposal. If q is a winning quota and $(q_{t-1}, q_t]$ is a quota interval of stable power for q, then any voting quota $1 - q_{t-1} + \varepsilon$ (where $0 < \varepsilon \leq q_t - q_{t-1}$), is a blocking quota. From Proposition 5 it follows that the blocking power of the committee members, measured by swing and pivot-based power indices, is equal to their voting power. It is easy to show that voting power and blocking power might not be the same for power indices based on membership in minimal winning coalitions (HP and DP power indices). Let r be the number of marginal quotas, then from Proposition 4 it follows that for power indices based on swings and pivots the number of majority power indices does not exceed *int (r/2)+1*.

Proposition 6. *Let [N, q, w] be a simple quota weighted committee, d be a distance function and $\pi i(N, q_t, w)$ be relative power indices for marginal quotas q_t, and q_{t*} be the majority marginal quota minimizing the distance* $d\left[\boldsymbol{\pi}(N,q_j,\mathbf{w}),w_i\right]$ *where j = 1, 2, …, r, r is the number of intervals of stable power such that q_j are marginal majority quotas, then the fair quota for a particular power index used with respect to distance d is any $\gamma \in (q_{t*-1}, q_{t*}]$ from the quota interval of stable power for q_{t*}.*

Proof follows from the finite number of quota intervals of stable power (Proposition 5).□

From Proposition 6 it follows that the voting rule based on quota q_{t*} minimizes selected distance between the vector of relative voting weights and the corresponding vector of relative voting power. The problem of fair quota has an exact solution via the finite number of majority marginal quotas

5 Fair quota in the Lower House of the Czech Parliament

The Lower House of the parliament has 200 seats. Members of the Lower House are elected in 14 electoral districts from party lists by proportional system with 5% threshold. Seats are allocated to the political parties that obtained not less than 5% of total valid votes roughly proportionally to fractions of obtained votes (votes for parties not achieving the required threshold are redistributed among the successful parties roughly proportionally to the shares of obtained votes). Five political parties qualified to the Lower House: left centre Czech Social Democratic Party (Česká strana sociálně demokratická, ČSSD), right centre Civic Democratic Party (Občanská demokratická strana, ODS), right TOP09 (Tradice, Odpovědnost, Prosperita – Traditions, Responsibility, Prosperity 2009), left Communist Party of Bohemia and Moravia (Komunistická strana Čech a Moravy, KSČM) and supposedly centre (but not very clearly located on left-right political dimension) Public Issues (Věci veřejné, VV).

In Table 1 we provide results of the 2010 Czech parliamentary election (by relative voting weights we mean fractions of seats of each political party, by relative electoral support fractions of votes for political parties that qualified to the Lower House, counted from votes that were considered in allocation of seats). Three parties, ODS, TOP09 and VV, formed right-centre government coalition with 118 seats in the Lower House.

	Seats	Votes in % of valid votes	Relative voting Weight	Relative electoral support
ČSSD	56	22,08	0,28	0,273098
ODS	53	20,22	0,265	0,250093
TOP09	41	16,7	0,205	0,206555
KSČM	26	11,27	0,13	0,139394
VV	24	10,58	0,12	0,13086
∑	200	80,85	1	1

Table 1 Results of 2010 election to the Lower House of the Czech Parliament
Source: http://www.volby.cz/pls/ps2010/ps?xjazyk=CZ

We assume that all Lower House members of the same party are voting together and all of them are participating in each voting act. Two voting rules are used: simple majority (more than 100 votes) and qualified majority (at least 120 votes). There exist 16 possible winning coalitions for simple majority voting (12 of them are winning coalitions for qualified majority), 16 marginal majority quotas and 16 majority quota intervals of stable power (see Table 2).

Parties of possible winning coalitions	Absolute marginal majority quota	Relative marginal majority quota	Intervals of stable power
ODS+KSČM+VV	103	0.515	(0.485, 0.515]
CSSD+KSČM+VV	106	0.53	(0.515, 0.53]
ČSSD+ODS	109	0.545	(0.53, 0.545]
ODS+TOP09+VV	118	0.59	(0.545, 0.59]
ODS+TOP09+KSČM	120	0.6	(0.59, 0.6]
ČSSD+TOP09+VV	121	0.605	(0.6, 0.605]
ČSSD+TOP09+KSČM	123	0.615	(0.605, 0.615]
ČSSD+ODS+VV	133	0.665	(0.615, 0.665]
ČSSD+ODS+KSCM	135	0.675	(0.665, 0.675]
ODS+TOP09+KSČM+VV	144	0.72	(0.675, 0.72]
ČSSD+TOP09+KSČM+VV	147	0.735	(0.72, 0.735]
ČSSD+ODS+TOP09	150	0.75	(0.735, 0.75]
ČSSD+ODS+KSČM+VV	159	0.795	(0.75, 0.795]
CSSD+ODS+TOP09+VV	174	0.87	(0.795, 0.87]
ČSSD+ODS+TOP09+KSČM	176	0.88	(0.87, 0.88]
ČSSD+ODS+TOP09+KSČM+VV	200	1	(0.88, 1]

Table 2 Possible winning coalitions in the Lower House of the Czech Parliament (own calculations)

For analysis of fair voting rule we selected Shapley-Shubik power index and Euclidean distance function. In Table 3 we provide Shapley-Shubik power indices (distribution of relative voting power) for all of marginal majority quotas.

Party	Seats	Relative voting weight	SS power for q=0.515	SS power for q=0.53	SS power for q=0.545	SS power for q=0.59	SS power for q=0.6	SS power for q=0.605	SS power for q=0.615	SS power for q=0.665
ČSSD	56	0,28	0,3	0,35	0,3167	0,2667	0,3167	0,3667	0,3333	0,3
ODS	53	0,265	0,3	0,2667	0,3167	0,2667	0,2333	0,2	0,25	0,3
TOP09	41	0,205	0,1333	0,1833	0,2333	0,2667	0,2333	0,2	0,1667	0,1333
KSČM	26	0,13	0,1333	0,1	0,0667	0,1	0,15	0,1167	0,1667	0,1333
VV	24	0,12	0,1333	0,1	0,0667	0,1	0,0667	0,1167	0,0833	0,1333
∑	200	1	0,9999	1	1,0001	1,0001	1	1,0001	1	0,9999
Distance			0,08339	0,08169	0,10802	0,07271	0,07996	0,01195	0,08501	0,08339
Party	Seats	Relative voting weight	SS power for q=0.675	SS power for q=0.72	SS power for q=0.735	SS power for q=0.75	SS power for q=0.795	SS power for q=0.87	SS power for q=0.88	SS power for q=1
ČSSD	56	0,28	0,2667	0,2333	0,4333	0,3833	0,35	0,3	0,25	0,2
ODS	53	0,265	0,2667	0,2333	0,1833	0,3833	0,35	0,3	0,25	0,2
TOP09	41	0,205	0,1833	0,2333	0,1833	0,1333	0,1	0,3	0,25	0,2
KSČM	26	0,13	0,1833	0,15	0,1	0,05	0,1	0,05	0,25	0,2
VV	24	0,12	0,1	0,15	0,1	0,05	0,1	0,05	0	0,2
∑	200	1	1	0,9999	0,9999	0,9999	1	1	1	1
Distance			0,06238	0,07271	0,17874	0,20275	0,15637	0,14816	0,17875	0,14816

Table 3 Shapley-Shubik power of political parties for majority marginal quotas (own calculations)

For any quota from each of intervals of stable power is Shapley-Shubik relative power identical with relative power in corresponding marginal majority quota. Entries in the row "distance" give Euclidean distance between vector of relative voting weights and relative power for each quota interval of stable power

The fair relative majority quota in our case is q = 0.675 (with respect to Euclidean distance between relative voting weights and relative voting power 0.06238), or any quota from interval of stable power (0.665, 0.675]. It means that minimal number of votes to approve a proposal is 135 (in contrast to 101 votes required by simple majority and 120 votes required by qualified majority). Voting rule defined by this quota minimizes Euclidean distance between relative voting weights and relative voting power (measured by Shapley-Shubik power index) and approximately equalizes the voting power (influence) of the members of the Lower House independently on their political affiliation.

The measure of fairness follows the same logic as measures of deviation from proportionality used in political science, evaluating the difference between results of an election and the composition of an elected body - e.g. Gallagher (1991) based on the Euclidean distance, or Loosemore-Hanby (1971) based on the absolute values distance. Using in our particular case the absolute values distance we shall get the same fair quota.

Acknowledgements

This research was supported by the Grant Agency of the Czech Republic, project No. 402/09/1066 "Political economy of voting behavior, rational voters' theory and models of strategic voting".

References

Banzhaf, J.F.(1965): Weighted Voting Doesn't Work: A Mathematical Analysis, *Rutgers Law Review*, **19**, 317-343.

Felsenthal, D. S., and M. Machover (1998): *The Measurement of Voting Power, Theory and Practice,* Edward Elgar, Cheltenham.

Gallagher, M.(1991): Proportionality, Disproportionality and Electoral Systems, *Electoral Studies*, **10** (1991), 33-51.

Loosemore, J., and V.J.Hanby (1971): The Theoretical Limits of Maximum Distortion: Some Analytical Expressions for Electoral Systems, *British Journal of Political Science*, **1**, 467-477.

Nurmi, H.(1982): The Problem of the Right Distribution of Voting Power. *I*n: *Power, Voting and Voting Power* (Holler M.J., ed.), Physica Verlag, Würzburg, Wien, 203-212.

Nurmi, H. (1997): On Power Indices and Minimal Winning Coalitions, *Control and Cybernetics*, **26**, 609-611.

Owen, G. (1972): Multilinear Extensions of Games. *Management Science*, **18**, 64-79.

Penrose, L.S.(1946): The Elementary Statistics of Majority Voting. *Journal of the Royal Statistical Society*, **109**, 53-57.

Shapley, L.S., and M. Shubik (1954): A Method for Evaluation the Distribution of Power in a Committee System. *American Political Science Review*, **48**, 787-792.

Słomczyński, W., and K. Życzkowski (2007): From a Toy Model to Double Square Root System. *Homo Oeconomicus*, **24**, 381-400.

Słomczyński, W., and K. Życzkowski (2006): Penrose Voting System and Optimal Quota. *Acta Physica Polonica* **B37**, 3133-3143.

Turnovec F.(2011): Fair Voting Rules in Committees, Strict Proportional Power and Optimal Quota. *Homo Oeconomicus*, **27/4**, 463-479.

Turnovec, F.(1997): Power, Power Indices and Intuition. *Control and Cybernetics*, **26**, 613-615.