

# 1 Introduction

The aim of this paper is to study how government behaviour - regarding government size and budget composition as well as debt accumulation - impinges on several inequality indicators.

Among the extensive empirical and theoretical literature on fiscal policy, very few papers focus on the impact of alternative fiscal instruments on inequality. Even studies focusing on debt consolidation efforts hardly assess redistributive costs (Giavazzi and Pagano [1990, 1995], Alesina and Perotti [1995], Alesina and Ardagna [1998], Ardagna [2004]).

Apparently, government size and expenditure composition is expected to impinge, both directly and indirectly, on inequality. While transfers, subsidies and a progressive tax system correct for inequality, directly, expenditure on the provision of public services such as education, health, or even R&D may, indirectly affect inequality via the effects on earning abilities and growth. In fact, efficient spending on education and health, among others, may bring a more homogenous distribution of income as these human-capital-augmenting services are delivered below market prices (see, for instance Ayala et al. [1999], Chu et al. [2000], Smeeding [2000], Afonso et al. [2008]). Several authors have also studied the relationship between the quality of public finances (size, expenditure composition, governance and legal framework) and growth (see, for a comprehensive review, Barrios and Schaechter [2008]) which, together with the vast amount of literature relating growth with inequality (Tanzi and Chalk [2002], Scully [2008], Kuznets [1955], Barro [2000], Benabou [1996, 2002], among others) provide intuition on the mechanisms through which fiscal policy may impinge on inequality.

Moreover, deficit financing, through taxes or debt issuing is not irrelevant, especially if Ricardian equivalence does not hold (arguments for the failure of the Ricardian equivalence proposition are studied, among others, in Barro [1974], Barsky et al. [1986], Bernheim [1987], Hubbard and Judd

[1986]). In this case, government indebtedness puts upward pressure on real interest rate, improving the relative wealth of the richer ones. On the other hand, debt lessens private sector credit constraints that are especially binding for the poorer, potentially improving welfare inequality.

Finally, regardless of the fiscal instrument, the way governments perform their stabilization role may also have redistributive effects. Given that the poorer depend strongly on labour income and that they are less protected against economic shocks (especially in countries with no or reduced public safety net systems), economic slowdowns affect more the poorer and, thus, increase inequality. Empirical evidence also points that the lowest income classes are increasingly vulnerable relative to the richer (Forster and Pearson [2002]) and to the positive relationship between how business cycle influences growth and wage/income dispersion (Afonso and Furceri [2008]).

Therefore, in this paper we built a general equilibrium model with heterogeneous agents capable of exploring, theoretically, the relationship between fiscal policy variables and inequality. Given the now widely established view that the robustness of any macroeconomic model relies on its microeconomic roots, heterogeneous-agents models improve on those using the representative-agent when the existence of significant cross-sectional dispersion across individuals (firms and workers) is considered relevant. In this kind of models, the determination of the stationary equilibrium implies solving iteratively dynamic optimization problems with large dimensional state vectors, through another large dimensional vector of prices. This became possible with the outcome of fast computers and new numerical methods. Seminal works using heterogeneous-agent models date from late 80s-early 90s and were developed by Bewley [1983], Imrohoroglu [1989], Huggett [1993], Aiyagari [1995] and Krusell and Smith [1998]. For a survey on heterogeneous-agents models, see Storesletten et al. [2009], Rios-Rull [1995], Ljungqvist and Sargent [1994] and Cagetti and Nardi [2008]. For technical and mathematical aspects see Huggett [1993], Krusell and Smith [2006], Rios-Rull [1995] and Rios-Rull [1999].

In particular, we build a micro-founded model based on Aiyagari and McGrattan [1998], and also used in Floden [2001]. This is a dynastic model that includes a continuum of infinitely-lived rational agents who are hit by idiosyncratic wage shocks in an incomplete capital market. The model includes government, with policy captured by a dynamic budget constraint, and optimizing firms endowed with a neoclassical Cobb-Douglas productive function. In order to smooth consumption/leisure, private agents optimally accumulate savings in "good times" spending them during "bad times". Besides allowing for taxes levied on labour and capital, we decompose government expenditure into transfers to private sector, and productive and unproductive spending. While productive expenditure is included in the production function and, through this channel, augments the global productivity of the economy, unproductive spending is solely utility-augmenting. With our model we aim to a better understand of the channels through which government influences household's behavior and to assess the aggregate and the distributive consequences of its policies. Based on the welfare criterion presented in Floden [2001], and Aiyagari and McGrattan [1998], we conduct a welfare analysis comparing different combinations of debt, expenditures and transfer, and considering different public spending compositions (static analysis across steady-states). We also explore the decomposition of the welfare effects according to the methodology proposed by Floden [2001]: the consumption/leisure level effect, the uncertainty effect and the inequality effects.

Measurement of inequality is not standard and usually involves the use of complement indicators (e.g., income or wealth Gini coefficients, income share-poorest 2 quintiles, poverty rate,), given the difficulty in finding a variable compact enough to capture individual welfare<sup>1</sup>. Another major problem in inequality assessment is the choice of the time period over which a chosen variable is measured. Enlarging the reference period will *ceteris paribus* reduce inequality (Cowell [2009]). Normally the fiscal year is used as the

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<sup>1</sup>For a broad survey about inequality measure see Cowell [2009], Jenkins and Kerm [2008]

reference period because it constitutes the natural accounting period for several income sources (Jenkins and Kerm [2008]). The third and last problem concerns the sample unit on which the chosen variable is measured. Normally, income or consumption can be measured on households, families or at an individual level: while labour earnings pose no problem since wages are paid on a individual basis, social transfer, tax payments or even capital earnings, should be included on a household basis . The standard practice is to adopt the household unit assuming that welfare, income or consumption are equally spread among household's members (Jenkins and Kerm [2008]).

In our essay, which is based on a theoretical model, many of these practical problems do not apply. We treat households on a yearly basis. As for inequality measures, we detail the dynamics of the distribution by calculating additional inequality measures based not only on wealth (asset holdings), disposable income (labour and capital net of tax income plus social transfers) and consumption, but also on leisure to complement the results obtained for the overall welfare inequality effect. For each of these variables we calculate the corresponding Gini coefficient.

On the one hand, the model predicts that a rise in unproductive expenditure and in transfers improve utility up until a certain point as they impinge positively on inequality and uncertainty, but negatively on the welfare level. On the other hand, debt brings a positive insurance effect but impinges negatively on welfare level and inequality. We also find that, given government consumption, optimal combination of debt and social transfer levels are smaller than the values observed in the EU countries during the last decades; moreover, the optimal debt-to-output level rises with the size of government (as measured by the expenditures on output ratio) and, thus, implies larger inequality. Finally, for a given level of public debt, (i) substituting unproductive spending by transfers is welfare enhancing and improves inequality but only up to a lower bound of unproductive spending; (ii) substituting unproductive by productive spending is always welfare enhancing and has no impact on any inequality measure; and (iii) shifting transfers

for productive expenditure is always welfare enhancing for a sufficiently high output elasticity of public investment and impacts negatively on inequality welfare.

The rest of this paper is organized as follows. In section 2 we describe the model, the welfare decomposition and define the social (aggregate) welfare criterion. The model is solved for different parameterization regarding fiscal variables and the main results are discussed thoroughly section 3. In section 4 we present the final remarks.

## 2 Model Description

The model is built from a standard growth model modified to include a role for government together with an uninsured idiosyncratic risk and liquidity/borrowing constraints.

We modify the original Aiyagari and McGrattan [1998], Floden [2001] model, breaking up the government expenditure into productive and unproductive. The former is introduced in the utility function and the latter in the production function. We also use a different approach for the calibration of the idiosyncratic shock. The model is composed by three sectors: households, firms and the government.

### 2.1 Households

There is a continuum of infinitely-lived agents of unit mass who receive after tax wage payments,  $\bar{w}$ , after tax interest from savings,  $\bar{r}a$ , and transfers,  $TR$ , from the government. Following Barro [1973], Floden [2001] and Floden [2003], we consider that beside private consumption  $c$ , and leisure,  $l$ , unproductive government spending  $G_u$  also increases households' utility at decreasing returns and according to a parameter  $\vartheta$ . In each period, agents are hit by idiosyncratic shocks  $e_t$  which determines the productivity level. Borrowing is allowed only up to a certain limit  $b$  and complete capital market is ruled out. This implies that agents have to ensure themselves by saving during "good times" ( $a_{t+1} - a_t > 0$ ) while, during "bad times", savings are negative ( $a_{t+1} - a_t < 0$ ). Each agent is endowed with one unit of time and solves the double problem of choosing between labor and leisure, and between consumption and saving.

Each household solves the following optimization problem:

$$\max_{c_t, l_t, a_{t+1}} E \left[ \sum_{t=0}^{\infty} \beta^t (u_1(c_t, l_t) + \vartheta u_2(G_{ut})) | a_0, e_0 \right] \quad (2.1)$$

Subject to:

$$c_t + a_{t+1} = \bar{w}_t(1 - l_t)e_t + (1 + \bar{r}_t)a_t + TR_t, \quad c_t \geq 0, \quad a_t \geq -b_t \quad (2.2)$$

where the household's instant utility functions are specified as:

$$u_1(c_t, l_t) = \frac{c_t^{1-\mu} \exp(-(1-\mu)\zeta(1-l_t)^{1+\gamma})}{1-\mu} \quad (2.3)$$

where  $\mu$  represents the risk aversion,  $\zeta$  is constant calibrated in order to match an average labor supply of 0.3, and  $\frac{1}{\gamma}$  represents the labor supply elasticity. The unproductive expenditure  $G_u$  utility is given by the function:

$$u_2(G_u) = \frac{G_u^{1-\mu}}{1-\mu} \quad (2.4)$$

The productivity shock  $e_t$  is an idiosyncratic shock that evolves stochastically over time according to the following process: the natural logarithm of  $e_t$  is represented by an AR(1) process with a serial correlation coefficient  $\rho$  and a standard deviation  $\sigma$ :

$$\log(e_t) = \rho \log(e_{t-1}) + \epsilon_t \quad (2.5)$$

The procedure of Tauchen [1986] is used to approximate the auto regression of  $\log(e_t)$  with a first-order Markov chain with seven states. The two main components of the Markov process are the productivity level vector  $edu$  and the  $7 \times 7$  probability transition matrix  $prob$ .

## 2.2 Firms

The firms are characterized by the following neoclassic production function:

$$Y_t = F(K_t, N_t, G_{pt}) = (K_t)^\alpha (N_t)^{1-\alpha} (G_{pt})^\eta \quad (2.6)$$

where:

$Y$ : *per capita* output

$K$ : *per capita* capital stock

$N$ : *per capita* labour supply

$G_p$ : productive government spending.

Productive government spending is identified with the share of public gross investment on output, in line with Barro [1990], Auschauer [1989]<sup>2</sup>, and enters as an input to private function.

The parameters  $\alpha$  and  $\eta$  represent, respectively, the output elasticities of private capital and productive government expenditure. The production function exhibits constant returns to scale over private inputs but increasing returns over all inputs. Assuming competitive markets of goods and inputs, private factors are paid according to their marginal productivity and output is exhaustively distributed.

$$\bar{w}_t = (1 - \tau_t)w_t = (1 - \tau_t)F_N(K_t, N_t, G_{pt}) \quad (2.7)$$

$$\bar{r}_t = (1 - \tau_t)r_t = (1 - \tau_t)(F_K(K_t, N_t, G_{pt}) - \delta) \quad (2.8)$$

where  $\tau$  is a proportional income tax rate levied on labour and capital,  $\delta$  is the depreciation rate of capital and  $w$  and  $r$  stand respectively, for gross real wage and gross real interest rate.

### 2.3 Government

The government promotes both productive and unproductive expenditures, collects taxes and pays lump-sum transfers to households, facing the following budget constraint in real terms:

$$G_{ut} + G_{pt} + TR_t + r_t D_t = D_{t+1} - D_t + \tau_t(w_t N_t + r_t A_t) \quad (2.9)$$

where,  $G_{ut}$ , represents government final consumption (unproductive expenditure),  $G_{pt}$ , public gross investment (productive expenditure),  $TR_t$ , government transfers to households, and  $D_t$ , government debt

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<sup>2</sup>R. Barro, in a seminal paper (Barro [1990]) incorporates a public sector into a simple, constant return model of economic growth. The ratio of real public gross investment to real GDP, which is assumed to correspond to a flow of services identified as the measure of infrastructure services enters directly to the production function



$$A_t = D_t + K_t \quad (2.10)$$

represents the asset market clearing condition: asset holdings equalize private plus public debt

## 2.4 Solving the model

First we transform the model in order to work with variables defined relative to output. Afterwards, we define the steady state equilibrium and describe the algorithm used for its computation following McGrattan [1996, 2003].

### 2.4.1 The model with variable defined relative to output

In order to transform the model we define:  $k_t = \frac{K_t}{Y_t}$ ,  $\tilde{w}_t = \frac{\bar{w}_t}{Y_t}$ ,  $\tilde{c}_t = \frac{c_t}{Y_t}$ ,  $\tilde{a}_t = \frac{a_t}{Y_t}$ ,  $d_t = \frac{D_t}{Y_t}$ ,  $\bar{a}_t = \frac{A_t}{Y_t}$ ,  $tr_t = \frac{TR_t}{Y_t}$ ,  $g_{ut} = \frac{G_{ut}}{Y_t}$ ,  $g_{pt} = \frac{G_{pt}}{Y_t}$  and  $\tilde{b}_t = \frac{b_t}{Y_t}$ .

In steady-state, the model reaches a balanced growth equilibrium in which all variables, namely output ( $Y$ ) private capital ( $K$ ), public debt ( $D$ ), and all policy variables ( $TR, G_u, G_p$ , and the tax burden), evolve at the same growth rate.

**First**, we rewrite the household's budget constraint in the per product units form:

$$\frac{c_t}{Y_t} + \frac{a_{t+1}}{Y_t} = \frac{\bar{w}_t}{Y_t}(1 - l_t)e_t + (1 + \bar{r}_t)\frac{a_t}{Y_t} + \frac{TR_t}{Y_t}$$

$\Leftrightarrow$

$$\frac{c_t}{Y_t} + \frac{a_{t+1}}{Y_{t+1}}(1 + g) = \frac{\bar{w}_t}{Y_t}(1 - l_t)e_t + (1 + \bar{r}_t)\frac{a_t}{Y_t} + \frac{TR_t}{Y_t}$$

where  $g$  is the steady state growth rate of output.

Using the definitions  $c_t = \tilde{c}_t \times Y_t$  and  $G_{ut} = g_{ut} \times Y_t$ , the consumer's problem becomes:

$$\max_{\tilde{c}_t, l_t, \tilde{a}_{t+1}} E \left[ \sum_{t=0}^{\infty} \beta^t Y_t^{1-\mu} (u_1(\tilde{c}_t, l_t) + \vartheta u_2(g_{ut})) | \tilde{a}_0, e_0 \right] \quad (2.11)$$

Subject to:

$$\tilde{c}_t + (1+g)\tilde{a}_{t+1} = \tilde{w}_t(1-l_t)e_t + (1+\bar{r}_t)\tilde{a}_t + tr_t, \quad \tilde{c}_t \geq 0, \quad \tilde{a}_t \geq -\tilde{b} \quad (2.12)$$

$$\tilde{w}_t = (1-\tau_t) \frac{F_N(K_t, N_t, G_{pt})}{Y_t} \quad (2.13)$$

$$\bar{r}_t = (1-\tau_t)(F_K(K_t, N_t, G_{pt}) - \delta) \quad (2.14)$$

**Second** by introducing the asset market clearing condition into the government budget constraint, and by using the definition of  $w_t$  and  $r_t$  together with the property of first degree homogeneity of the production function, we can rewrite the government budget constraint as:

$$\begin{aligned} & G_{ut} + G_{pt} + TR_t + (F_K(K_t, N_t, G_{pt}) - \delta)D_t = \\ & D_{t+1} - D_t + \tau_t \left( F_N(K_t, N_t, G_{pt})N_t + (F_K(K_t, N_t, G_{pt}) - \delta)(K_t + D_t) \right) \\ & \iff \\ & G_{ut} + G_{pt} + TR_t + (1-\tau_t)(F_K(K_t, N_t, G_{pt}) - \delta)D_t = \\ & D_{t+1} - D_t + \tau_t \left( F_N(K_t, N_t, G_{pt})N_t + F_K(K_t, N_t, G_{pt})K_t - \delta K_t \right) \\ & \iff \\ & G_{ut} + G_{pt} + TR_t + (1+\bar{r}_t)D_t = D_{t+1} + \tau_t(Y_t - \delta K_t) \end{aligned}$$

Now dividing by  $Y_t$ , we get the stationary government budget constraint:

$$g_{ut} + g_{pt} + tr_t + (\bar{r}_t + 1)d_t - (1+g)d_{t+1} = \tau_t(1 - \delta k) \quad (2.15)$$

**Third** We divide the asset market clearing condition by the output to obtain its per product version:

$$\bar{a}_t = k_t + d_t \quad (2.16)$$

### 2.4.2 Solving the steady state equilibrium

During the stationary equilibrium, by definition, the economy moves at a constant rate. For a given level of  $d, tr, g_u$  and  $g_p$ , the steady state of this economy is characterized by:

- A tax rate:  $\tau$
- A government debt:  $d = \frac{D}{Y}$
- An interest rate:  $r$
- An after tax wage:  $w$
- Two time invariant decision rules: for asset holdings by households (asset demand) and for labour supply, respectively:

$$\tilde{a}_{t+1} = \alpha(\tilde{a}_t, e_t) \quad (2.17)$$

$$(1 - l_t) = h(\tilde{a}_t, e_t) \quad (2.18)$$

- A stationary distribution of households across asset holdings and productivity shocks:

$$\lambda(\tilde{a}, e) \quad (2.19)$$

- An aggregate level of effective hours worked and asset holdings, respectively:

$$N = \int e_t h(\tilde{a}_t, e_t) d\lambda \quad (2.20)$$

$$\bar{a} = \int \alpha(\tilde{a}_t, e_t) d\lambda \quad (2.21)$$

...such that

1. Decision rules are the solution maximization problems for the household;
2. Government budget constraint is fulfilled;
3. Input markets clear;
4. Aggregate saving (asset demand) equals demand for capital from firms plus government debt (asset supply).

$$\int \alpha(\tilde{a}_t, e_t) d\lambda = k(r) + d \quad (2.22)$$

The steady state of this economy is characterized by a vector of prices  $\{r, w\}$  which solve (2.22) and (2.20)

The expression  $\int \alpha(\tilde{a}_t, e_t) d\lambda$  represents the *per capita* assets wanted by consumers (relative to *per capita* output) ; The expression  $k(r^*) + d$  is the *per capita* supply of assets (capital plus government debt) relative to *per capita* output, expressed as a function of the interest rate; finally  $\int e_t h(\tilde{a}_t, e_t) d\lambda$  is the *per capita* effective labor supplied by households.

### Algorithm for solving the steady state equilibrium

1. Inputs:  $\beta, \mu, \zeta, \vartheta, \gamma, \rho, \sigma_e, edu, prob, \alpha, \eta, g_u, g_p, tr$  and  $d$
2. Start with an initial guess for  $N$  (aggregate labour supply).
3. With  $N$  fixed, apply the bisection method Aiyagari [1994] to calculate the interest rate that clears the asset market as follows: we try a first guess for interest rate within some interval  $[r_l, r_u]$ .
4. Given the guess for interest rate, back out the private capital output ratio, the after tax wage and the tax rate, using the government budget constraint and the fact that input markets are perfectly competitive.

⇒ *Level of private capital per unit of output:*

$$\begin{aligned}
Y &= F(K, N, G_p) = K^\alpha N^{1-\alpha} G_p^\eta \\
r + \delta &= \alpha K^{\alpha-1} N^{1-\alpha} G_p^\eta \iff \frac{r + \delta}{Y} = \frac{\alpha K^{\alpha-1} N^{1-\alpha} G_p^\eta}{K^\alpha N^{1-\alpha} G_p^\eta} \\
\frac{r + \delta}{Y} &= \frac{\alpha}{K} \iff k = \frac{\alpha}{r + \delta}
\end{aligned}$$

⇒ *Wage per unit of output:*

$$\begin{aligned}
Y &= F(K, N, G_p) = K^\alpha N^{1-\alpha} G_p^\eta \\
w &= (1 - \alpha) K^\alpha N^{-\alpha} G_p^\eta \iff \frac{w}{Y} = \frac{(1 - \alpha) K^\alpha N^{-\alpha} G_p^\eta}{K^\alpha N^{1-\alpha} G_p^\eta} \\
\tilde{w} &= (1 - \tau) \frac{w}{Y} = (1 - \tau) \frac{1 - \alpha}{N}
\end{aligned}$$

⇒ *Tax rate:* tax rate is determined by combining the former results with the government budget constraint, under the assumption of  $g = 0$ :

$$\tau = \frac{g_u + g_p + tr + rd}{1 - \delta \frac{\alpha}{r + \delta} + rd}$$

5. We compute time invariant decision rules for the asset holdings by households (asset demand), for the labour supply and the stationary distribution of household: (2.17), (2.18) and (2.19).

In order to obtain an numerical approximation for the decision rules that solve the first order conditions of the households maximisation problem and respective restrictions, we use the finite elements technique (McGrattan [1996])

6. We update interest rate using the bisection method (Aiyagari [1994]) until (2.22) is verified
7. We update aggregate labour supply,  $N$  (step 2) until (2.20) is verified.

We do it using a Newton-Raphson iterative scheme (McGrattan [2003]):

$$N^{m+1} = N^m - J(N^m)^{-1}f(N^m)$$

where

$f(N) = \int e_t h(\tilde{a}_t, e_t) d\lambda - N$  and  $J(N)$  is the corresponding jacobian matrix.

## 2.5 Social welfare computation

The stationary form also allows us to define household's problem in a recursive form. Define  $V(e_0, \tilde{a}_0)$  as the optimal value for the expected life time utility maximization problem starting from an initial state  $(e_0, \tilde{a}_0)$ . The value function  $V$  satisfies the following functional Bellman equation<sup>3</sup>:

$$V(e, \tilde{a}) = \max_{\tilde{c}, l, \tilde{a}'} \left[ Y^{1-\mu}(u_1(\tilde{c}, l) + \vartheta u_2(g_u)) + \beta Y^{1-\mu} \sum_{e'} V(e', \tilde{a}') \right] \quad (2.23)$$

Subject to: (2.6), (2.12), (2.13), (2.14), (2.15) and (2.16).

The utilitarian social welfare is defined as the solution of (2.23) across all households (i.e, conforming the stationary distribution):

$$U = \int E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t, G_{ut}) d\lambda(a, e) \quad (2.24)$$

The utilitarian social welfare increases with consumption, leisure or government unproductive expenditure. Since the utility function is concave, the social welfare is influenced by the distribution, and then, more inequality or uncertainty will reduce welfare.

In order to measure the impact of policy in welfare across steady states, we decompose, following Floden [2001], the global welfare into three particular

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<sup>3</sup> $x'$  means next period value of  $x$

effects: the consumption level effect, the insurance effect and the inequality effect.

### 2.5.1 Decomposition of the steady state global welfare gain

Consider a policy change that moves an economy from steady state A to steady state B. The global welfare gain is measured as a percentage of lifetime consumption that households gain (or lose) from moving from economy instantly from A to B, and is defined by  $w_u$  in:

$$\int E_0 \sum_{t=0}^{\infty} \beta^t u((1+w_u)c_t^A, l_t^A, G_{ut}^A) d\lambda^A(a, e) = \int E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^B, l_t^B, G_{ut}^B) d\lambda^B(a, e) \quad (2.25)$$

### 2.5.2 Certainty-equivalence

The certainty-equivalent levels of consumption  $\bar{c}$  and leisure  $\bar{l}$  represent the constant levels of consumption and leisure that would ensure an utility level equivalent to that expected under the uncertain utility flows in the future. The certainty-equivalent consumption and leisure bundle must solve, for each household:

$$\sum_{t=0}^{\infty} \beta^t u(\bar{c}_t, \bar{l}_t, G_{ut}) = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t, G_{ut}) \quad (2.26)$$

As pointed in Floden [2001] the last equation does not define a unique combination of  $\bar{c}$  and  $\bar{l}$ . To get an unique combination, we opt to set leisure at the level chosen by each household at  $t = 0$ . A second hypothesis was to use the average leisure level of the whole economy. The results are very similar and don't affect the conclusions.

### 2.5.3 Insurance effect

The insurance effect explores the time dimension of the utility function concavity. In order to remove the inequality effect, we compare the average

level of consumption,  $C$ , and leisure,  $L$ , with the average of the certainty-equivalent corresponding levels,  $\bar{C}$  and  $\bar{L}$ .

$$C = \int c d\lambda(a, e) \quad \bar{C} = \int \bar{c} d\lambda(a, e) \quad L = \int l d\lambda(a, e) \quad \bar{L} = \int \bar{l} d\lambda(a, e)$$

We define,  $p_{unc}$ , the cost of uncertainty and calculate it from the difference (in percent of life time consumption) between the utility evaluated at the average consumption and leisure, and that evaluated at the corresponding certainty-equivalence levels:

$$\sum_{t=0}^{\infty} \beta^t u((1 - p_{unc})C, L, G_{ut}) = \sum_{t=0}^{\infty} \beta^t u(\bar{C}, \bar{L}, G_{ut}) \quad (2.27)$$

We define,  $w_{unc}$ , as the insurance effect, associated with moving from A to B:

$$w_{unc} = \frac{1 - p_{unc}^B}{1 - p_{unc}^A} - 1 \quad (2.28)$$

If uncertainty increases,  $\bar{C}$  decreases and moves away from  $C$  and therefore  $p_{unc}$  increases. The insurance welfare effect will be negative ( $w_{unc} < 0$ ), and a rise in uncertainty impacts negatively on global welfare.

#### 2.5.4 Level effect

Defining the leisure-compensated consumption (Floden [2001]) in economy B,  $\hat{C}^B$ , as the consumption increment (or decrease) necessary to reach the level of utility in B, *ceteris paribus*, namely, leaving  $L^A$  and  $G_u^A$  unchanged.

$$\sum_{t=0}^{\infty} \beta^t u(\hat{C}^B, L^A, G_u^A) = \int \sum_{t=0}^{\infty} \beta^t u(C^B, L^B, G_u^B)$$

Let,  $w_{lev}$ , the welfare level effect associated with moving from A to B:

$$w_{lev} = \frac{\hat{C}^B}{C^A} - 1 \quad (2.29)$$



### 2.5.5 Inequality effect

The inequality effect explores the space dimension of the utility function concavity. We now use the certainty-equivalence variables to remove uncertainty from welfare. We define  $p_{ine}$  as the difference (in percent of life time consumption) between the utility of average certainty-equivalence of consumption and leisure, and the utility evaluated at the corresponding certainty-equivalence levels. It is equivalent to the level of consumption that people are willing to give up in order to promote an equal distribution of consumption and leisure, maintaining the same level of social welfare.

$$\sum_{t=0}^{\infty} \beta^t u((1 - p_{ine})\bar{C}, \bar{L}, G_{ut}) = \int \sum_{t=0}^{\infty} \beta^t u(\bar{c}, \bar{l}, G_{ut}) \quad (2.30)$$

Finally define  $w_{ine}$  as the inequality welfare effect associated with moving from A to B:

$$w_{ine} = \frac{1 - p_{ine}^B}{1 - p_{ine}^A} - 1 \quad (2.31)$$

A more unequal utility distribution will decrease the right side of (2.30) and raises  $p_{ine}$ . Thus a rise in inequality impacts negatively on global welfare ( $w_{ine} < 0$ ). Floden [2001] notes that, an increase of uncertainty leaves the inequality unchanged because  $\bar{C}$  and  $\bar{L}$  change together with  $\bar{c}$  and  $\bar{l}$ . Likewise inequality does not affect  $\bar{C}$  and  $\bar{L}$ , and so, uncertainty is not affected by inequality.

As stated in the introduction we use several inequality measures. We specifically calculate the Gini coefficients for wealth, disposable income, consumption and leisure. Naturally, the specific inequality indexes about each of those variables do not always reveal the same tendency of the more complete welfare inequality in which composition reflects all source of utility (consumption and leisure).

Instead of the simple formula presented in Floden [2001] to decompose  $w_u$  into  $w_{level}$ ,  $w_{unc}$  and  $w_{ine}$ , due to the inclusion of unproductive expenditures

in the utility function, we get a much more complex formula<sup>4</sup>.

## 2.6 Calibration

The model presented above follows closely Aiyagari and McGrattan [1998] and Floden [2001]. The model period is one year. Parameters calibration uses relevant literature while policy variables are calibrated using average EU countries values (EU15<sup>5</sup> in AMECO data source for the period between 1990 and 2008).

**Preferences:** for the risk-aversion parameter  $\mu$ , we use a value of 1.5 which is commonly used in literature. For the inverse of labour supply elasticity,  $\gamma$  we follow Floden [2001] and set it to 2 which is equivalent to a wage elasticity of labour supply equal 0.5. The parameter  $\zeta$  determines the fraction of time devoted to labour and is set in order to match an average labour supply of around 0.3 ( $\zeta = 9.145$ ). Finally  $\vartheta$  represents preferences toward public goods and services relative to private goods; for the baseline calibration we set  $\vartheta = 0.1$ . The use of larger value of  $\vartheta = 0.1$  is not compatible with the level of policy variables observed in EU and developed countries.

**Technology:** the production function is inspired in Barro [1990] to incorporate productive government spending. For our baseline model we follow Auschauer [1989] and use a value of 0.3 for the output elasticity of productive government spending,  $\eta$ . For the capital share,  $\alpha$  we use 0.3 (Aiyagari and McGrattan [1998], Floden [2001] )

**Discount factor and interest rate :** according to our model,  $r = \frac{\alpha}{k} - \delta$ . We set  $\delta = 7.5\%$  as Aiyagari and McGrattan [1998]. The variable  $k$  represents the capital output ratio and the steady state value is calibrated as to match

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<sup>4</sup>The formula is available upon request.

<sup>5</sup>Belgium, Denmark, Germany, Ireland, Greece, Spain, France, Italy, Luxembourg, Netherlands, Austria, Portugal, Finland, Sweden and United Kingdom

the average value of the capital to output ratio of the EU15 countries (1990-2008)<sup>6</sup>. Thus, the steady state value for the real interest rate yields 2.8%. We calibrate the discount factor in order to reach an equilibrium with this real interest rate level, which implies  $\beta = 0.981$ .

**Government:** the government is characterized by a set of fiscal indicators  $\{D, TR, G_u, G_p\}$ . These are differently calibrated to capture different aspects of the public sector. Nevertheless, in our theoretical exercises, we use values close to the empirical reality of the UE15. Specific values will be released during simulations below in section 3.

**The idiosyncratic shock:** the idiosyncratic shock is described as Markov chain specification with seven states to match a first order autoregressive representation (Aiyagari [1994]). The relevant parameters for the process are the coefficient of variation,  $\sigma$ , and the serial correlation coefficient,  $\rho$ . Aiyagari [1994], Aiyagari and McGrattan [1998], Floden [2001] base their values of  $\rho$  and  $\sigma$  on empirical data for earnings and annual hours worked. Due to data unavailable to the EU15 average, we follow a different path. As Rios-Rull et al. [2003] we fix both parameters as to match the existent inequality in the EU15. Specifically, we use the income Gini index as a reference. Knowing that, in EU15 the income Gini index varies between 0.26 and 0.34, we will fix both parameters so that simulation outcome equals disposable income distribution (or at least the same Gini index). For the present paper we set  $\rho = 0.8$  and  $\sigma = 0.27$  which leads to an income Gini index of around 0.28.

### 3 Optimal Government Expenditure and Financing - Steady State Analysis

This section gives insight on optimal government size and financing. In particular we want to assess how fiscal policy variables  $\{g_u, g_p, tr, d\}$  impinge on

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<sup>6</sup>Source: AMECO database,  $k = 2.9$

welfare as well as how they affect inequality. In order to make such assessment, we produce a continuum of steady state equilibria that are characterized by different government behaviours. First, we consider different endowment on each of the fiscal policy instruments, allowing for a corresponding tax adjustment. Second we evaluate the impacts of a changing composition on policy instruments regarding fiscal policy inter-temporal structure: how do welfare and inequality indicators behave in face of more expenditure, financed through current taxes, relative to higher public debt? Third, for a given debt-to-output ratio, we study how a changing composition of government expenditure (intra-temporal substitution, for a fixed tax burden) impinges on welfare and inequality. Thus we find the optimal government composition of fiscal instruments (inter-temporal and intra-temporal) while assessing on how government expenditure and financing impacts on welfare inequality, as well as on several complementary inequality measures.

To solve for the steady state equilibrium, one must find the equilibrium interest rate that clears the asset market, i.e. when aggregate asset demand from households (in order to save and insure themselves against the idiosyncratic earning shocks) equals the asset supply by government and firms (in order to search for funds to finance deficits and investment).

The equilibrium is represented by the intersection between asset supply/capital demand curve (government and firms) and asset demand/capital supply curve (household) shown in figure 1.

The supply curve is composed by a private share plus public debt. Increases (decreases) in public debt shift the asset supply curve to the right (left). The asset demand curve is positively sloped relative to interest rate. In order to understand what shifts in the asset demand curve we must examine each of its determinants individually, namely idiosyncratic shocks, debt, transfer and public expenditures (see Ljungqvist and Sargent [1994], for the details).

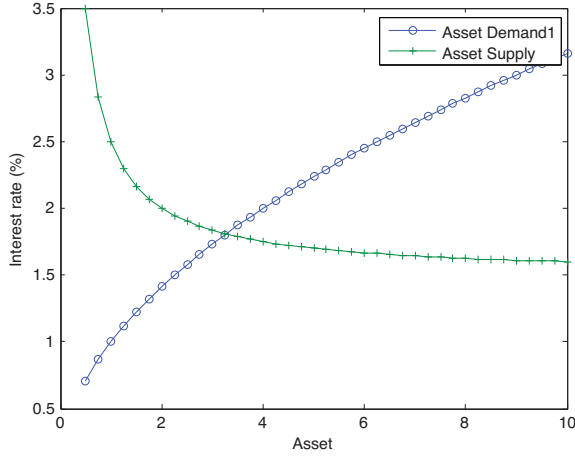


Figure 1: Asset market equilibrium

The idiosyncratic shock is characterized by the variance of the shock and by its persistence . When the variance of the shock or its persistence increases, assets demand increases induced by an increased need for insurance and shifts the asset demand curve to the right. Higher public debt or public expenditure requires more taxes to satisfy debt constraint; as such, this implies less disposable income and shifts the asset demand curve to the left. Finally, the transfer effect is ambiguous. Transfers increase directly disposable income and pushes the demand curve to the right. But at the same time, the fiscal cost of transfers reduce disposable income, shifting the asset demand leftwards. If we derive disposable income in respect to transfer we find that the net effect depends on the sign of the expression (in steady-state):

$$\frac{\partial income}{\partial tr} = 1 - \frac{r\tilde{a}_t - we_t}{1 + rd - \frac{\delta\alpha}{r+\delta}} \quad (3.1)$$

Also, as real interest rate rises the household asset demand (capital supply) increases, which means that the asset demand curve has an upward slope. On the other side, asset supply from firms and government (capital demand) decreases with the interest rate, which means that the asset supply has a downward slope.

Combining all these effects on asset supply and asset demand curves makes it impossible to predict a final equilibrium. The solution is to solve the model numerically for all possible combination of idiosyncratic shock, debt, transfer and government expenditures using the algorithm presented in section (2).

### 3.1 Individual impact of policy variables

In order to better understand the channels through which fiscal variables  $\{d, tr, g_u\}$  affect welfare and inequality, we calculate a sequence of steady state equilibria considering alternative values for a single instrument, leaving the others unchanged. It is worthwhile to note that, throughout this exercises, taxes adjust to fulfill the government budget constraint and thus, also impact on welfare. However tax impacts on inequality are only indirect, namely through their effect on labour supply, as they are not progressive in the model.

#### 3.1.1 Public debt: $d$

In the following exercise, we set  $tr = 7.5\%$  and  $g_u + g_p = 20\%$ . The choice is somehow arbitrary, but nevertheless, these values are very close to the optimal levels obtained in the next exercises below, and they are compatible with average values for the EU countries. We calculate the steady state equilibria for a continuum of debt to output ratios between 0 and 100%. We decompose the welfare measure into the three effects (level, insurance and inequality) and follow the impact on main macro-variables across equilibria

In a standard deterministic representative agent growth model, the impact of government debt on welfare depends on the tax regime. With lump-sum tax, debt is neutral. While, with distorting tax, debt helps to smooth tax burden over time (Barro [1974, 1979], Aiyagari and McGrattan [1998]). In a heterogeneous-agent framework, government debt has an additional impact on welfare by providing further means to smooth consumption. By issuing

debt, the government lessens agent's borrowing constraint (Aiyagari and McGrattan [1998]). Larger debt puts upward pressure on interest rates, making assets more profitable to hold and, thus, households become better insured against earning fluctuations. Naturally, higher debt (and interest rates) have also negative impacts on welfare: they imply higher taxes, crowding out private investment.

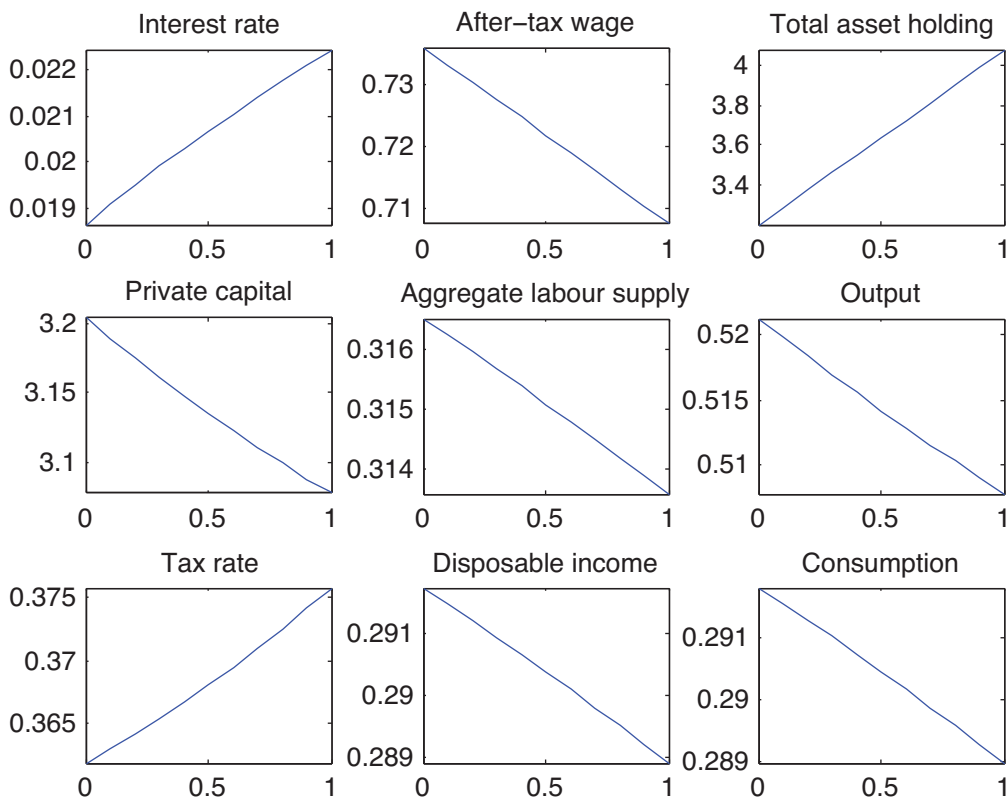


Figure 2: Macroeconomic variables across debt-to-output ratio

Figure 2 shows how the model adjusts several macroeconomic variables (vertical axis) for different values of the debt-to-output ratio (horizontal axis). Larger debt-to-output ratios (i.e. larger capital demand) imply larger interest and tax rates and lower after-tax wages. Total asset holdings increase but private capital is crowded out by public debt. Consumption, disposable income and output follow a similar decreasing path as debt-to-output ratio

increase. In the labour market, the substitution effect dominates as aggregate labour supply decreases when net wage falls.

Figure 3 depicts the welfare assessment for different debt-to-output ratios, as well as its decomposition into level ( $w_{level}$ ), insurance ( $w_{unc}$ ) and inequality ( $w_{ine}$ ) effects. As debt-to-output ratio increases, insurance increases (line with circles). As government issues debt, the consumer's borrowing constraint loosens (Aiyagari and McGrattan [1998] show analytically why) and the interest rate increases. Saving becomes more profitable and the insurance capacity improves.

The level effect takes into consideration the consumption made, independently of how it is distributed. When debt output ratio increases, the welfare level effect (line with asterisk) is negative. This rules out the over-accumulation of private capital beyond the golden rule<sup>7</sup>.

Finally the inequality effects (lines with crosses) are negative due to the interest rate increase which benefits more the assets owners in relation to the lower wealth classes. Welfare becomes more unequally distributed across households. Combining the three effects, the global welfare is hump shaped and peaks around  $d = 50\%$ .

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<sup>7</sup>Except for the nonexistence of over-accumulation of private capital all the findings are in line with Floden [2001]



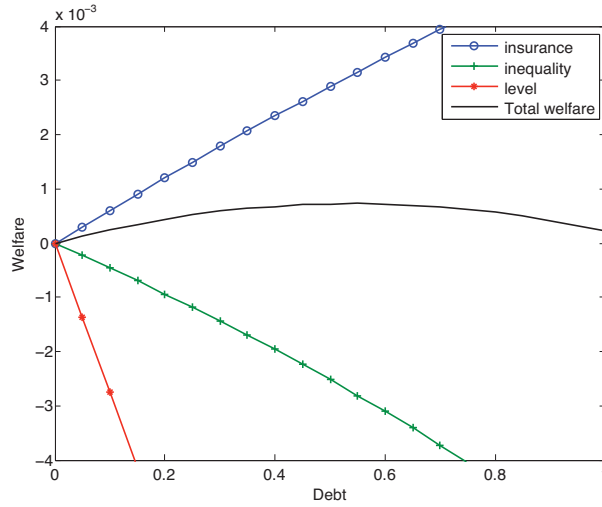


Figure 3: Welfare decomposition across debt-to-output ratio.

Figure 4 provides additional measures to assess impacts on inequality. The inequality measures reflect mostly the debt-to-output ratio effect on interest rate and also the labour supply response. Debt lessens credit constraint and, thus, reduces the layer of households with zero or negative asset position, improving the wealth distribution (the wealth Gini index decreases). According to figure 4, the income Gini index goes down as debt climbs. The improvement on income distribution is due to the capital earnings and also to the labour market response. As debt increases, tax rate goes up affecting the after tax wage. The substitute effect dominates on labour market, but the elasticity of labour supply is higher among the wealthier who tend to switch more labour for leisure. Therefore, the income Gini index decreases.

As interest rate rises with debt-to-output ratio, capital earnings become more unequally distributed since wealthier households possess more assets (despite the improved asset distribution). However, the corresponding tax rise reduces net wages and, prevailing substitute effects, labour supply falls more among the wealthier households to whom labour supply elasticity relative to wage is higher. In balance, income dispersion improves with debt. In spite of the improvement of income and asset holding distribution, we have just seen that the overall inequality effect contributes negatively to welfare

(figure 3 above)

This example shows how difficult it may be to find adequate definitions and measures of inequality. A theoretical model, where a welfare function is defined, enables a comprehensive identification of the three relevant sources of welfare inequality: consumption, leisure and unproductive expenditures (collective consumption):  $\{c, l, g_u\}$ . Collective consumption is not relevant because it is defined as a fraction of output distributed equally across households. As debt increases, consumption becomes more equally distributed in line with the disposable income and the asset holding. On the other hand, leisure distribution becomes more unequal, as a counterpart of the stronger labour supply response to the fall in the after tax wage by the wealthier. Imbalance of the two effects determines the sign of the overall inequality effect; in this case increase in leisure inequality dominates.

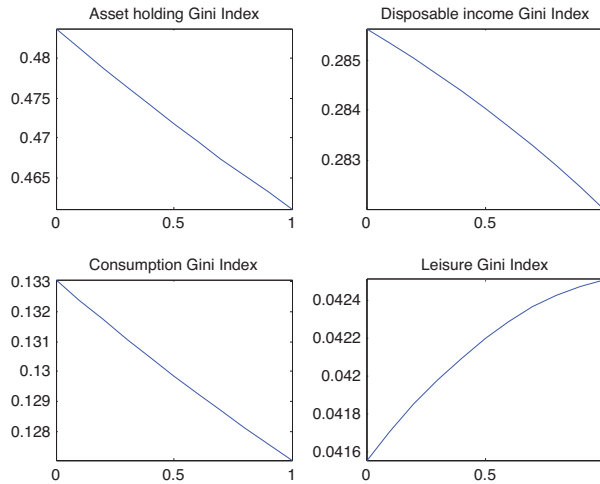


Figure 4: Gini Indexes across debt-to-output ratio.

### 3.1.2 Social Transfers: $tr$

Consider now  $d = 50\%$ ,  $g_u + g_p = 20\%$  and the steady state equilibria for a continuum transfer to-output-ratios between 0 and 15%. Figure 5 depicts how macroeconomic variables (vertical axis) behaves for different transfers-to-output ratios (horizontal axis). Larger transfers-to-output ratios imply higher interest and tax rates. While larger debt-to-output ratios put pressure

on the demand for capital (larger asset supply), larger transfers-to-output ratios reduce asset demand (reduces capital supply): total asset holdings fall with social transfers, not only because income is lower, but also because the need to hold assets for insurance motive is reduced as transfers become larger. Higher tax effort and interest rate depress private capital and net wages. Thus, consumption, disposable income and output follow a similar decreasing path with transfer-to-output rise. As with debt increase, the substitution effect dominates in the labour supply adjustment to a fall in net wages. Note that social transfers seem to cause more tax distortions in the economy when compared to debt. While transfers enter directly and fully in the government budget constraint, only the debt service enters the government budget constraint. Naturally, a one percent increase in transfer will be more tax demanding when compared to an identical increase in debt.

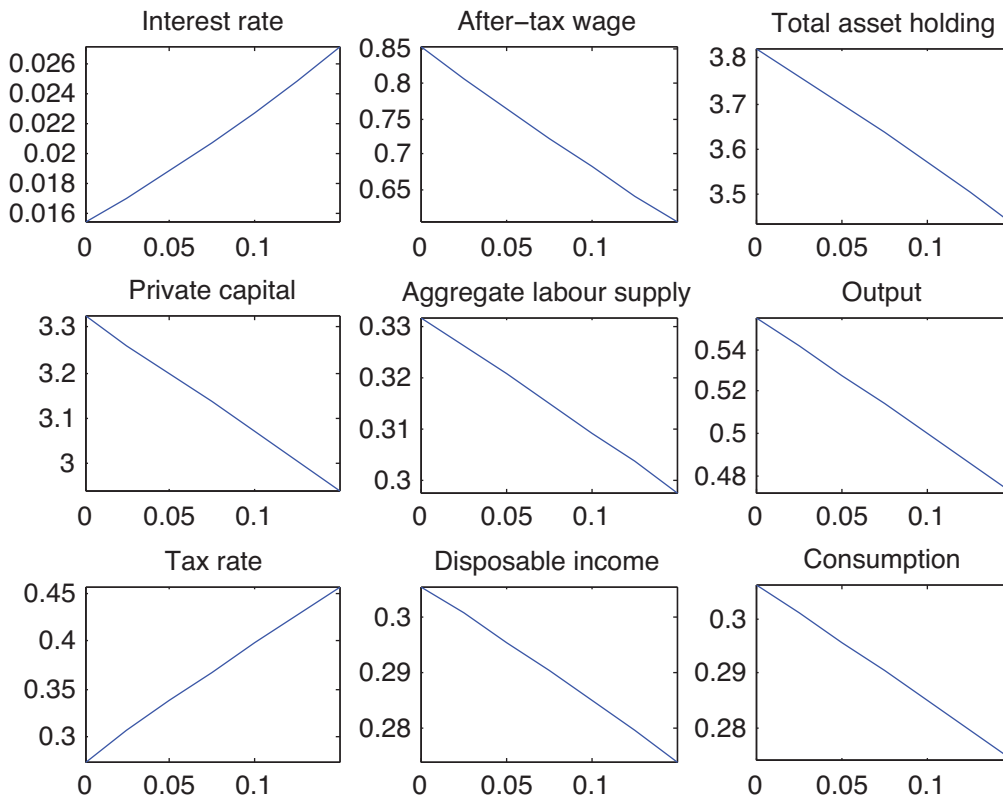


Figure 5: Macroeconomic variables across transfers-to-output ratio.

Figure 6 shows how insurance (line with circles), consumption level (line with asterisk) and inequality (line with crosses) affect welfare across different transfer-to-output ratios. It is clear that larger transfer-to-output ratios imply a negative welfare level effect reflecting a lower labor supply and smaller savings. Insurance and inequality effects are positive. Larger social transfer means that a larger portion of income is granted (independent of the idiosyncratic shock) and thus uncertainty is lower. In this case the positive insurance effect comes directly through the income channel. Lump-sum social transfers benefit all population, but the poorest benefit proportionally more because they hold a lower amount of assets. Therefore the inequality effect on welfare is now positive. Combining the three effects, the total welfare measure maximizes with  $tr = 8\%$ .

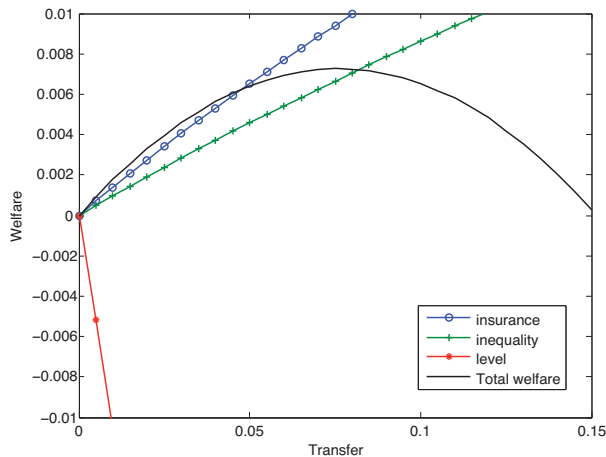


Figure 6: Welfare decomposition across transfer-to-output ratio.

For a closer inspection on the impacts on inequality, figure 7 show how the Gini coefficient on asset holdings, disposable income, consumption and leisure, change across transfer-to-output ratios. Concerning the wealth (asset holdings) distribution, the Gini index is larger (higher inequality) for larger transfer-to-output ratios. Transfers, by reducing the need for insurance, affect especially the poorer: the fraction of households with negative or no wealth increases and the asset distribution becomes flatter. The effects on

disposable income inequality are direct: transfers represent a lump sum element that make disposable income more homogeneous across households. As accorded before, overall inequality effect is positive while wealth and income inequality move in opposite ways with transfers. Apparently, disposable income effect on inequality dominates to reduce consumption inequality (figure 7). As for leisure, and in contrast with debt, inequality is smaller, the larger the transfer-to-output ratio is - as transfers rise, the poorer face stronger disincentive to work, increasing leisure relatively to the wealthier. Both consumption and leisure inequality is lower with larger transfers, making overall inequality (figure 6) welfare enhancing.

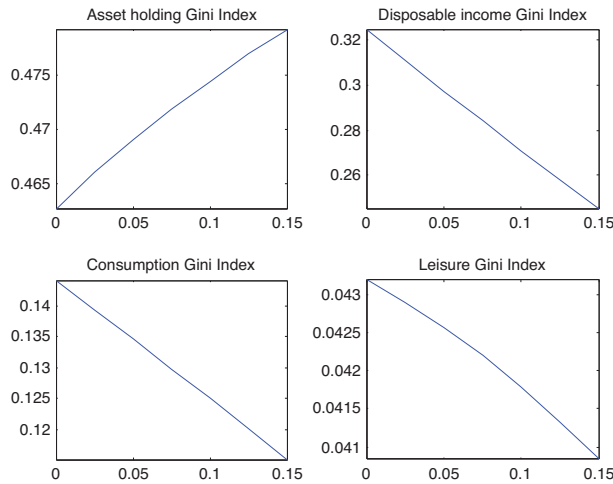


Figure 7: Gini Indexes across transfer-to-output ratio.

### 3.1.3 Unproductive expenditure: $g_u$

Unproductive spending,  $g_u$  affects welfare directly as it delivers utility for private agents (e.g. public health education or law and order). In the following exercise, we set  $d = 50\%$ ,  $tr = 7.5\%$  and  $g_p = 1.5\%$ . As before, we calculate the steady state equilibria for a continuum of unproductive government expenditure in percentage of output (between 8.5% and 18.5%).

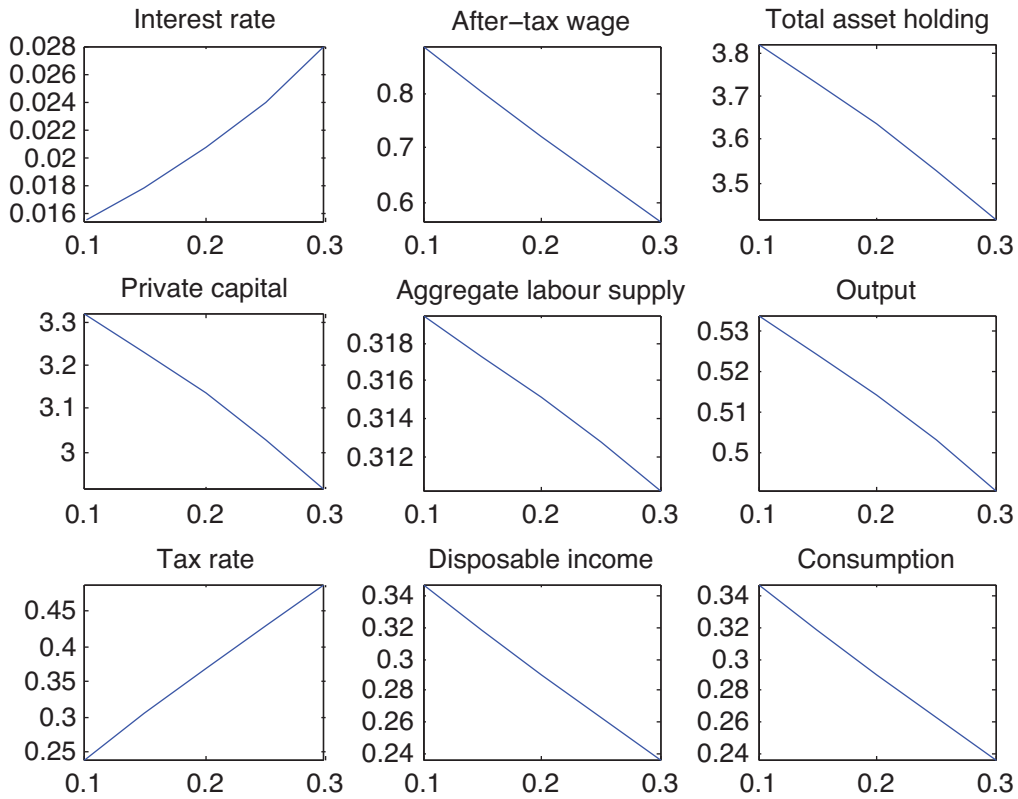


Figure 8: Macroeconomic variables across government expenditure:  $g_u + g_p$ .

Figure 8 plots several macroeconomic variables (vertical axis) against government expenditure ( $g_u + g_p$ ) as percentage of output (horizontal axis). Common to previous exercises, larger government expenditures imply higher tax and interest rates. Asset demand (supply of capital) lowers with government expenditures (the demand curve in figure 1 moves to the left, raising interest rate) mostly because  $g_u$  enables households with a constant (certain) flow of utility and the need for risk insurance, as with  $tr$ , is reduced<sup>8</sup>. As in previous cases, and for the same reasons, disposable income, output and labour supply is lower, the higher unproductive spending is.

<sup>8</sup>This income effect due to the income reduction also concurs to reduce the asset demand.

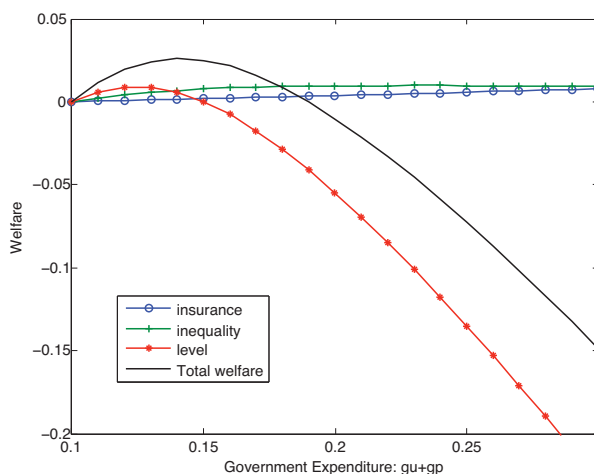


Figure 9: Welfare decomposition across government expenditure:  $g_u + g_p$ .

Figure 9 plots total welfare as well as its components against government unproductive spending as percentage of output. The welfare level effect is positive for small increments of government expenditures. This means that, as explained in Aiyagari and McGrattan [1998] and Floden [2001], there is an over-accumulation of capital beyond the gold rule level which maximizes consumption. From about 12% of government spending, the distortion effect on labour and saving choices<sup>9</sup> dominates and the level effect turns negative. As for inequality and uncertainty, they both have a slight positive effect on welfare as government spending increases. Government delivers a constant (certain) flow of utility to households, reducing uncertainty. As for inequality, the mechanism is also direct - a larger endowment of public services is distributed evenly across households, reducing disparity in welfare. Combining the three effects, the global welfare reaches a maximum for a government expenditure of 14%. The importance of both these effects depends crucially on the value for the utility parameter concerning the unproductive government expenditure  $\vartheta = 0.1$ . Notice that the model is not capturing positive indirect effects in the welfare and inequality from public services, namely those on growth resulting for instance from human capital accumulation, nor those affecting the idiosyncratic shock.

<sup>9</sup>due to the increased tax needs to finance the government expenditure increment

Figure 10 illustrates several inequality measures for different values of unproductive spending (as % of output). As with debt, disposable income and wealth (and thus, consumption) become more evenly distributed. The decrease of the wealth Gini coefficient reveals that asset selling affects more the wealthier, who are always in better condition to smooth consumption and leisure. The income Gini index decreases because of the dominance of the labour supply elasticity effect. Figure 11 plots labour supply across asset holdings for the two extreme values of government expenditures ( $g_u = 8.5\%$  and  $g_u = 18.5\%$ ). For larger values of  $g_u$ , after tax wage becomes smaller and the wealthier will work less relative to the poorer. thus, labour income becomes more equally distributed. Naturally, the opposite occurs concerning leisure in which distribution becomes less compressed (i.e. more unequally distributed) -see figure 10 . Unlike debt, consumption inequality effects slightly dominate and overall inequality has a modest positive impact on welfare as  $g_u$  rises.

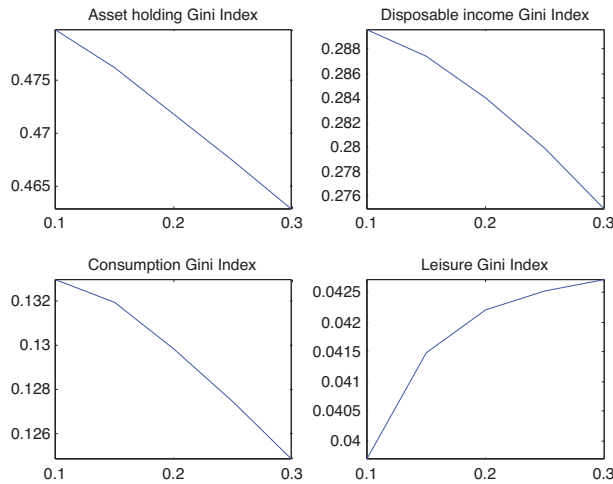


Figure 10: Gini Indexes across government expenditure:  $g_u + g_p$ .



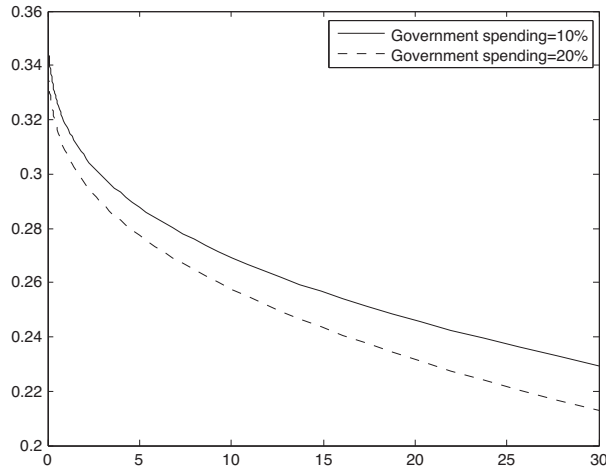


Figure 11: Labour supply against wealth

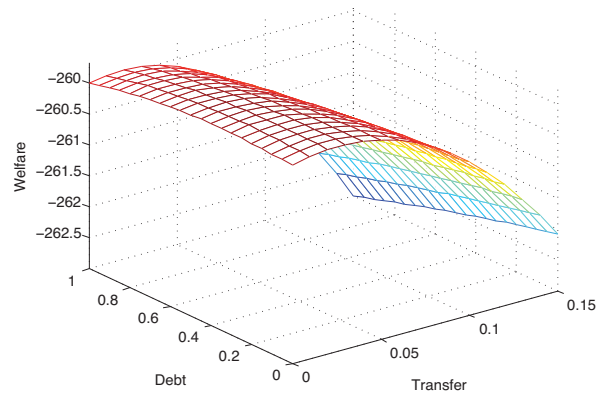
### 3.2 Optimal combination of social transfer government spending and debt

In the last section we assessed the long run effects of changing debt, social transfers and unproductive expenditures on welfare and inequality. We've concluded that the three instruments discussed are welfare enhancing up to a certain amount (i.e. up to a certain interest and tax rates). We have also concluded that all instruments have a positive welfare insurance effect but only transfers and unproductive expenditures improve welfare distribution; debt inequality effects contribute negatively to welfare. Moreover, the contributions of unproductive expenditure to welfare insurance and inequality effect are almost negligible. Table 1 resumes the individual effects of the policy variables on each inequality measures.

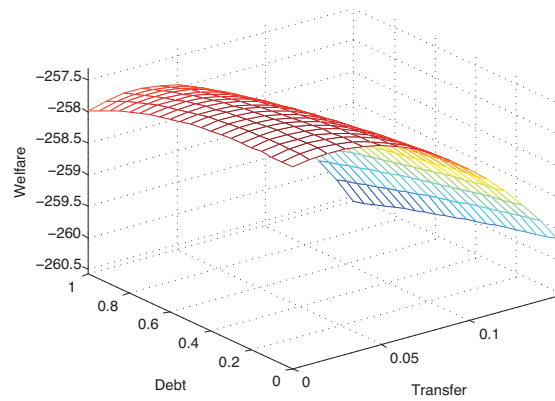
	$d$	$tr$	$g_u$
Wealth Gini	↘	↗	↘
Income Gini	↘	↘	↘
Consumption Gini	↘	↘	↘
Leisure Gini	↗	↘	↗
Welf. inequal.	⊖	⊕	⊕

Table 1: Inequality effect of our fiscal instruments

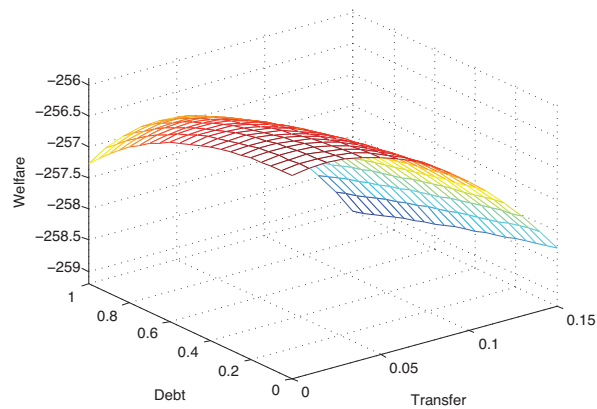
In this context, a crucial question arises: is the (inter-temporal) composition of government instruments meaningful to welfare and inequality? Does improving welfare precludes more tax-financed government expenditure or is public debt accumulation better? In the present section our exercises consists of assessing the optimal combination of debt and transfer for different level of government spending. We opt to study the inequality trade-off effects between transfers and debt because calibration of utility-enhancing in government spending is less robust - in most of heterogeneous-agent models, government expenditures are excluded from the utility function. In this case, unproductive expenditure has negative effects on both welfare and inequality. In this exercise we thus consider three government sizes: large,  $\{g_u + g_p = 0.20\}$ , medium,  $\{g_u + g_p = 0.175\}$  and small,  $\{g_u + g_p = 0.15\}$ , values taken among those observed for the EU countries. For each government size we calculate all steady states for a continuum of debt levels between  $[0, 1.5]$  and transfer levels between  $[0, 0.15]$  (all values are expressed in percent of output). Figure 12 plots combinations of  $d$  and  $tr$  for different government sizes in baseline scenario ( $\vartheta = 0.1$ ). Figure 13 exhibits similar results but under the extreme scenario ( $\vartheta = 0$ ), as in Aiyagari and McGrattan [1998], Floden [2001]. Table 2 sums up, for each scenarios, the optimal debt and transfer combination, the corresponding welfare value and the inequality indicators.



(a) Large-size government:  $g_u + g_p = 20.0\%$

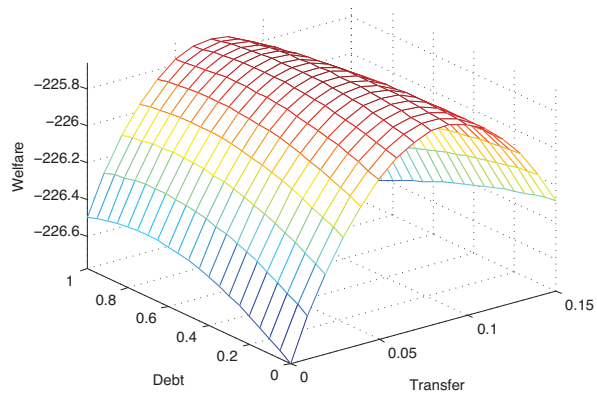


(b) Medium-size government:  $g_u + g_p = 17.5\%$

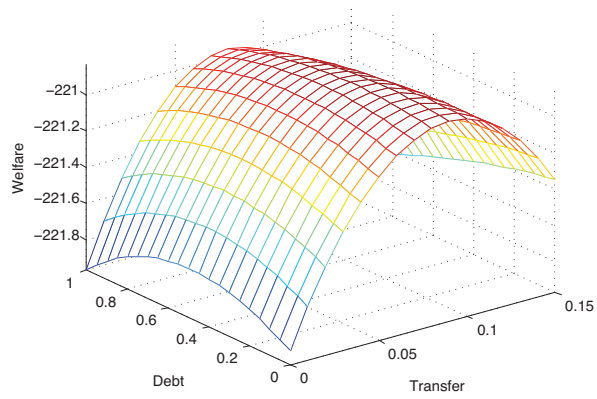


(c) Small-size government:  $g_u + g_p = 15.0\%$

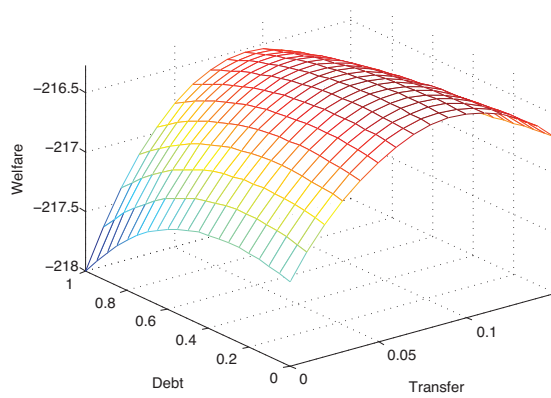
Figure 12: Unproductive expenditure in the utility function ( $\vartheta = 0.1$ )



(a) Large-size government:  $g_u + g_p = 20.0\%$



(b) Medium-size government:  $g_u + g_p = 17.5\%$



(c) Small-size government:  $g_u + g_p = 15.0\%$

Figure 13: Unproductive expenditure out of the utility function ( $\vartheta = 0$ )

In both scenarios, as government size decreases, the optimal level of public debt decreases and social transfer remains rather stable. The optimal level of debt is for the smaller, medium and large government sizes, of 0%, 10% and 30% respectively for the baseline scenario (otherwise, 20%, 40% and 50%, respectively). Public debt becomes more relevant with a large size government to compensate for the utility loss associated with a large public spending. Social transfer optimal level maintains a constant level of 2% for the baseline scenario for all government sizes. For the alternative scenario ( $\vartheta = 0$ ) the optimal level of social transfers is 9% for the small size government and 8% for the medium and large size government. Transfers end up being less elastic with government size than debt because the latter implies a smaller tax distortion (tax effort). As such, (as table 2 shows), government-size impinges negatively on welfare inequality. The insurance effect is welfare enhancing (due to the increased level of interest rate) and the level effect depresses welfare. In both scenarios tax and interest rates raise as government expenditure and debt augment. Both welfare and inequality measures reflect mostly the debt effect. According to the results condensed in table 2, wealth, income and consumption Gini coefficients decrease and the leisure Gini index increases<sup>10</sup>. Unproductive government expenditure distorts incentives significantly, especially from a determined value<sup>11</sup>. Debt, in which distortion effect is much smaller, can accommodate part of government spending increase. The tax increase necessary to finance the growing expenditure affects labour and saving decisions. Household will supply less labour, specially the upper wealth class, compressing the income distribution and pushing up the leisure Gini coefficient.

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<sup>10</sup>The exception concerns the income Gini index with  $\vartheta = 0$  and  $g_u + g_p = 175.0\%$ : it rises lightly because transfer decrease from 9 to 8%

<sup>11</sup>about 12% with a debt output ratio of 50% and a transfer output ratio of 7.5% as we saw above in figure 9

<b>Unprod. expend. in the utility function: <math>\vartheta = 0.1</math></b>			
	$g_u + g_p = 0.15$	$g_u + g_p = 0.175$	$g_u + g_p = 0.20$
Debt	0.00	0.10	0.30
Transfer	0.02	0.02	0.02
Tax rate	0.2289	0.2624	0.2968
Interest rate	0.0124	0.0139	0.0159
Wealth Gini	0.4814	0.4766	0.4697
Income Gini	0.3178	0.3165	0.3147
Consumption Gini	0.1471	0.1455	0.1431
Leisure Gini	0.0417	0.0422	0.0428
Welfare	0.0000	-0.0129	-0.0340
Wlevel	0.0000	-0.0225	-0.0560
Winsurance	0.0000	+0.0017	+0.0041
Winequality	0.0000	-0.0005	-0.0017
<b>Unprod. expend. off the utility function: <math>\vartheta = 0.0</math></b>			
	$g_u + g_p = 0.15$	$g_u + g_p = 0.175$	$g_u + g_p = 0.20$
Debt	0.20	0.40	0.50
Transfer	0.09	0.08	0.08
Tax rate	0.3200	0.3417	0.3742
Interest rate	0.0178	0.0192	0.0211
Wealth Gini	0.4847	0.4768	0.4724
Income Gini	0.2811	0.2837	0.2814
Consumption Gini	0.1327	0.1321	0.1304
Leisure Gini	0.0412	0.0418	0.0421
Welfare	0.0000	-0.0409	-0.0814
Wlevel	0.0000	-0.0449	-0.0981
Winsurance	0.0000	+0.0010	+0.0025
Winequality	0.0000	-0.0016	-0.0018

Table 2: The optimal combination of debt, transfers and government expenditure

### 3.3 Optimal composition of global government spending

We have concluded that the larger the size of government, the higher the optimal debt-to-output ratio is. However, this is achieved at higher inequality costs ( $w_{ine} < 0$  when debt increases). At this stage, a final exercise is in order: given debt level, optimal or not, one should expect expenditure composition to affect total welfare and, in particular, welfare inequality. Thus, this section closes the steady state analysis by assessing optimal composition of government expenditure, i.e., setting aside the impacts on taxes and the corresponding second-order effects on leisure and income inequality operating through the labour supply channel.

We calculate a new series of steady states equilibria considering unproductive expenditure versus transfer, unproductive expenditure versus productive expenditure and finally, transfer versus productive expenditure. We repeat the exercises for different government sizes: a large-size government where total spending ( $G$ )<sup>12</sup> represents 30% of output; a medium-size government with  $G = 25\%$  and a small-size government with  $G = 20\%$ . For all exercises we set a public debt output ratio, not necessary optimal, of 50%.

#### 3.3.1 Unproductive expenditure ( $g_u$ ) versus social transfers ( $tr$ )

Given a ceiling for total government expenditure, we substitute unproductive expenditure by social transfer. Productive expenditure remains constant ( $g_p = 1.5\%$ ).

**The large-size government case ( $G = 30\%$ ):** Table 3 shows the sequence of different  $(g_u, tr)$  combinations and the corresponding values for the welfare and for several inequality measures. Unproductive expenditures ( $g_u$ ) varies from 28.5% to 8.5% of output while social transfers evolve progressively from zero to 20% of output.

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<sup>12</sup> $G = g_u + g_p + tr$

$g_u$	$tr$	wu	WG	IG	CG	LG	wine
0.2850	0.0000	0.0000	0.4518	0.3225	0.1425	0.0446	0.0000
0.2600	0.0250	0.0433	0.4583	0.3076	0.1381	0.0438	0.0026
0.2350	0.0500	0.0815	0.4643	0.2942	0.1340	0.0430	0.0048
0.2100	0.0750	0.1138	0.4696	0.2820	0.1302	0.0424	0.0069
0.1850	0.1000	0.1392	0.4745	0.2709	0.1265	0.0418	0.0087
0.1600	0.1250	0.1564	0.4789	0.2607	0.1231	0.0412	0.0103
0.1350	0.1500	0.1629	0.4831	0.2513	0.1198	0.0407	0.0118
0.1100	0.1750	0.1550	0.4869	0.2426	0.1167	0.0402	0.0131
0.0850	0.2000	0.1259	0.4904	0.2345	0.1138	0.0397	0.0143

Table 3: Unproductive expenditure versus social transfers:  $G = 30\%$ .

Notes: wu = global welfare - WG = Wealth Gini index - IG = Income Gini index - CG = consumption Gini index - LG = Leisure Gini index - wine = inequality welfare effect

The welfare analysis is represented in figure 14. The global welfare, which reflects the combination of the three effects described above, reaches a maximum with a combination of 15% for social transfers and 13.5% for unproductive expenditure. As we substitute unproductive expenditure by transfers, the welfare decomposition shows slight positive effects on equality and insurance, in line with the results obtained in (3.1). The hump shape of the welfare curve is determined by the level component of the welfare decomposition. The level effect, described above, corresponds to the private consumption level necessary, *ceteribus paribus* (namely with the initial values of leisure, social transfer and non productive expenditure), to equalize the global welfare level of utility of the subsequent combinations of transfer and unproductive expenditures. As we reduce the collective consumption (by reducing  $g_u$ ), the household augments its private consumption in order to keep the same level of utility. The level effect is positive. However, for a sufficiently high transfer level, above 10%, the disincentive to work induced by higher transfer dominates and the level effect decreases and becomes negative. Endowed with more social transfers, households tend to save less and



supply less labour. Figure 15 shows the asset demand moving leftwards with interest rate rising and private capital falling<sup>13</sup>. The decreasing level of labour and private capital depresses output, affecting downwards the level component of welfare.

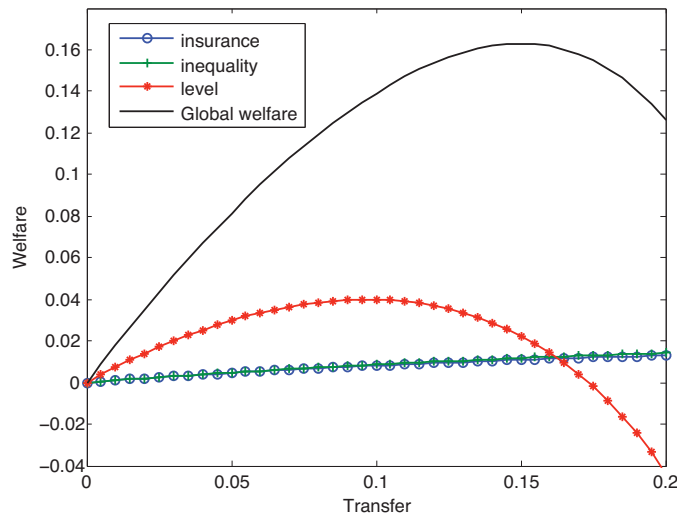


Figure 14: Welfare effects of alternative combinations of transfers and unproductive expenditures

The inequality impact reflects the dominant effect of social transfers (see table 1). Higher social transfers with less unproductive expenditure leads to higher wealth inequality increases because the need for insurance is smaller (asset holding is smaller). However, this affects strongly the lower wealth classes (wealth Gini index increases). The disposable income and consumption distributions become more equitable due to the direct effect of social transfers. The fall in the leisure Gini index indicates that the lower labour supply induced by more generous transfer payments, affects especially the poorest. By supplying less labour, they enjoy more leisure and the leisure distribution becomes more compressed. Naturally, the inequality component

<sup>13</sup>Asset demand and supply: the translation of the asset demand curve to the left results from the substitution of unproductive expenditure for social transfers. Therefore interest rate raises

of welfare is positive and it increases steadily for larger transfer-to-output ratios, representing a benefit of 1.18% of lifetime private consumption for the optimal combination ( $g_u = 0.1350, tr = 0.15$ ) in comparison to the initial case ( $g_u = 0.285, tr = 0.00$ ).

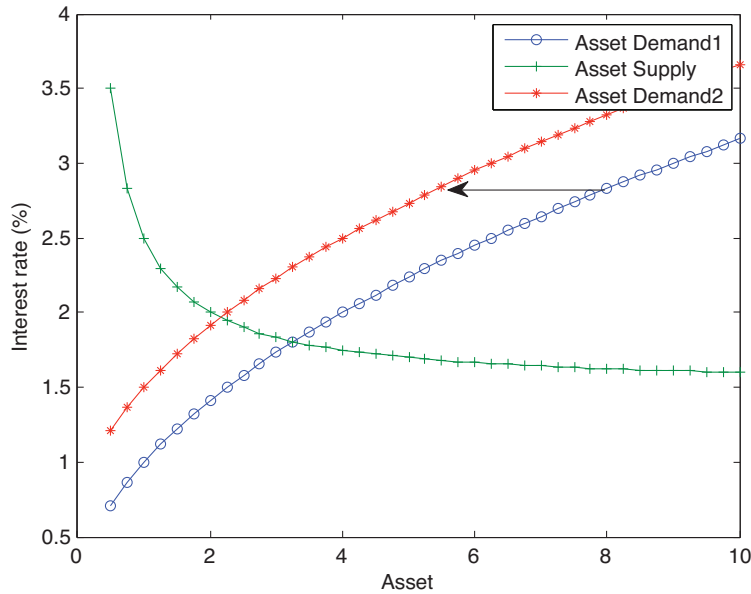


Figure 15: Impact on interest rate from substituting  $g_u$  for  $tr$ .

**The medium-size government ( $G = 25\%$ ):** Tables 4 show different combinations of  $g_u$  and  $tr$  for  $G = 25\%$  and the corresponding welfare and inequality indicators. Figure 16 also depict the welfare decomposition.

The analysis is very similar to that provided for  $G = 30\%$ . As we substitute unproductive expenditures by transfers, the welfare decomposition shows slight positive effects on insurance and inequality. The level effect shows a slight hump shaped curve indicating a smaller but effective substitute effect between private and public consumption when compared with the previous case. The global welfare reaches a maximum with a combination of 10.5% for social transfer and 13% for unproductive expenditure<sup>14</sup>. As before,

<sup>14</sup>We use a spline interpolation for the intermediate values of the table

Gini indicators improve as transfers substitute government unproductive expenditures, except for the wealth Gini index.

$g_u$	$tr$	wu	WG	IG	CG	LG	wine
0.2350	0.0000	0.0000	0.4574	0.3238	0.1440	0.0440	0.0000
0.2100	0.0250	0.0342	0.4635	0.3100	0.1399	0.0433	0.0021
0.1850	0.0500	0.0617	0.4690	0.2974	0.1361	0.0426	0.0038
0.1600	0.0750	0.0812	0.4740	0.2859	0.1324	0.0419	0.0053
0.1350	0.1000	0.0904	0.4786	0.2753	0.1289	0.0413	0.0066
0.1100	0.1250	0.0857	0.4829	0.2655	0.1256	0.0407	0.0077
0.0850	0.1500	0.0608	0.4868	0.2564	0.1224	0.0401	0.0086
0.0600	0.1750	0.0028	0.4904	0.2479	0.1193	0.0396	0.0092
0.0350	0.2000	-0.1212	0.4938	0.2400	0.1163	0.0390	0.0097

Table 4: Unproductive expenditure versus social transfers:  $G = 25\%$ .  
Note: see table 3.

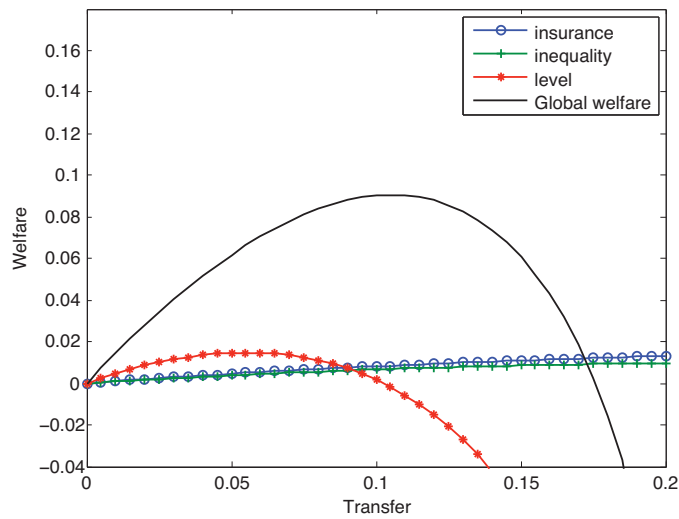


Figure 16: Unproductive expenditure versus social transfers:  $G = 25\%$

**The small-size government case ( $G = 20\%$ ):** Table 5) shows the sequence of the different combinations and the respective values for the welfare and the inequality measures.

The welfare analysis is represented in figure 17. The global welfare reaches a maximum with a combination of 6% for social transfer and 12.5% for unproductive expenditure. The welfare decomposition shows some qualitative differences. Concerning the level effect, the distortion effect dominates against the substitution effect (in which households substitute collective consumption for private consumption). The inequality effect, contrary to the precedent cases, achieves a maximum with a combination of  $g_u = 8.5\%$  and  $tr = 10\%$ . Both  $g_u$  and  $tr$  improve the inequality effect of welfare. By substituting unproductive expenditure for transfer, the effect of transfer dominates for large and medium size governments. For small size governments, the transfer effect dominates and the inequality welfare effect improves up to the combination of ( $g_u = 11\%$ ,  $tr = 7.5\%$ ). Thus, there is also a lower bound for  $g_u$  ( $g_u = 11\%$ ) below which unproductive expenditure effect dominates and the inequality welfare decreases for small-size government size.

$g_u$	$tr$	wu	WG	IG	CG	LG	wine
0.1850	0.0000	0.0000	0.4627	0.3249	0.1454	0.0432	0.0000
0.1600	0.0250	0.0220	0.4683	0.3118	0.1414	0.0424	0.0011
0.1350	0.0500	0.0339	0.4734	0.2998	0.1376	0.0416	0.0018
0.1100	0.0750	0.0322	0.4781	0.2887	0.1340	0.0408	0.0022
0.0850	0.1000	0.0108	0.4824	0.2785	0.1305	0.0400	0.0022
0.0600	0.1250	-0.0427	0.4863	0.2689	0.1271	0.0392	0.0017
0.0350	0.1500	-0.1599	0.4899	0.2599	0.1237	0.0384	0.0006

Table 5: Unproductive expenditure versus social transfers:  $G = 20\%$ .  
Note: see table 3.

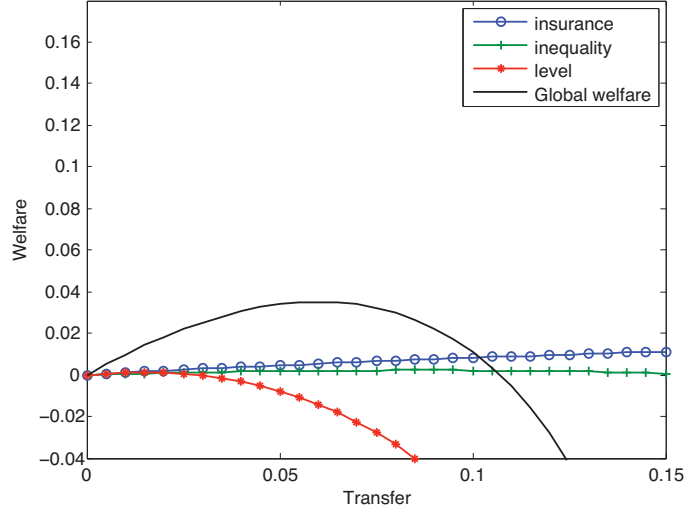


Figure 17: Unproductive expenditure versus social transfers:  $G = 20\%$

In the three examples used above we conclude that a more transfer intensive composition of government spending improves welfare until a certain level (see the optimal composition for each government size in table 6). Substituting unproductive expenditures for transfer induces a positive level effect, more intense in the large governments and almost null in the small ones. This re-enforces the result obtained above, according to which the unproductive expenditures are welfare enhancing specially for moderate levels around 10-15%. For higher levels, the unproductive expenditure penalizes strongly the global welfare due to its severe disincentive effect on labour market and saving. Decreasing the unproductive expenditure from high level, like the large and medium size government, implies significant efficient gains, and consequently positive effects of the level welfare component. In the small size government, the efficient gains of the expenditure cut are canceled by the distortion effect of the social transfer increase. More transfers also induce a positive inequality effect and improve the distribution of income, consumption and leisure. A more transfer intensive government spending with its negative impact on saving, pushes up the wealth Gini.

Comparing the three optimal combinations (table 6), the global spending decreases from 30% to 20% of GDP inducing a welfare improvement. The welfare decomposition shows that the cited improvement is due mostly to a level effect (the other two components, inequality and insurance, are negative). This is not surprising as we already know the distortion effect of government spending. The optimal spending composition varies significantly. The unproductive expenditures remain stable but the social transfers fall down. In a larger government the optimal level of transfer is proportionally higher. When  $G = 30\%$  (the larger government), the optimal level of social transfer represents 50% of global spending against 30% when  $G = 20\%$  (the smaller government). This means that the marginal rate of substitution of government expenditure for social transfer is rising. The model points to a minimum level of unproductive expenditure that households are not willing to easily give up.

Concerning the inequality measures, we have already seen that the welfare distribution gets worse with increased government sizes. The Gini index variation reflect mostly the reduction of transfers ( $g_u$  remains stable and debt is fixed). As pointed out in table 1, when social transfer reduces, income, consumption and leisure Gini index increases, which means more unequal distributions, while the wealth Gini index decreases. The income distribution becomes more sparse mainly due to the social transfer. The consumption Gini index follows closely the income distribution. The leisure distribution is interesting because it allow us to also understand the change in labour distribution. We have seen that a pure tax effect induces a response in labour supply more visible among the upper wealth classes<sup>15</sup>. In the present case, both the leisure Gini index and the aggregate labour supply rise. The transfer cut affects more the poor who must work harder to keep their utility levels. By working more, they necessarily enjoy less leisure and the leisure distribution becomes more sparse (leisure Gini index rises).

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<sup>15</sup>Section 3.1.3 explains why the Income Gini coefficient decreases when unproductive expenditures augments

	$G = 0.30$	$G = 0.25$	$G = 0.20$
Debt	0.5000	0.5000	0.5000
Transfer	0.1500	0.1050	0.0600
$g_u$	0.1350	0.1300	0.1250
Tax rate	0.3981	0.3365	0.2732
Interest rate	0.0234	0.0197	0.0164
Wealth Gini	0.4831	0.4795	0.4754
Income Gini	0.2513	0.2732	0.2953
Consumption Gini	0.1198	0.1282	0.1362
Leisure Gini	0.0407	0.0412	0.0413
Welfare	0.0000	0.0606	0.1148
Wlevel	0.0000	0.1663	0.3438
Winsurance	0.0000	-0.0043	-0.0093
Winequality	0.0000	-0.0041	-0.0098

Table 6: Unproductive expenditure versus social transfers: resume

### 3.3.2 Unproductive ( $g_u$ ) versus productive expenditure ( $g_p$ )

In this second exercise we set  $tr = 7.5\%$  and  $d = 50\%$  and calculate the equilibria for a series of combinations  $(g_u, g_p)$ . As the results are similar across government sizes, we only report results for  $G = 20\%$  in table 18. Figure 18 plots the corresponding welfare decomposition across productive expenditures,  $g_p$ . As it can be seen in figure 18 exchanging unproductive for productive expenditure induces a productivity welfare effect through the production function. The other two components of the welfare decomposition are completely neutral. Note that since productive and unproductive expenditures are equally distributed among all population, the former through the production function and the latter through the utility function, changing the proportion of both policy instruments in the government budget has no consequence on distribution as it can be seen in the three mentioned tables. Globally, substituting  $g_u$  for  $g_p$  is always welfare enhancing and has no impact on the inequality measures.

$g_u$	$g_p$	wu	WG	IG	CG	LG	wine
0.1100	0.0150	0.0000	0.4781	0.2887	0.1340	0.0408	0.0000
0.1050	0.0200	0.2883	0.4781	0.2887	0.1340	0.0408	0.0000
0.1000	0.0250	0.5699	0.4781	0.2887	0.1340	0.0408	0.0000
0.0950	0.0300	0.8456	0.4781	0.2887	0.1340	0.0408	0.0000
0.0900	0.0350	1.1151	0.4781	0.2887	0.1340	0.0408	0.0000

Table 7: Unproductive versus productive expenditure:  $G = 20\%$   
Note: see table 3.

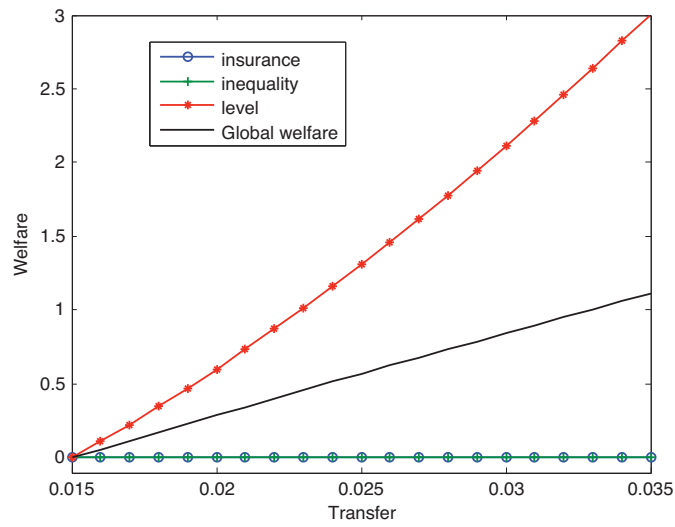


Figure 18: Welfare Decomposition across productive expenditures ( $g_p$ ):  $G = 20\%$



### 3.3.3 Social Transfer ( $tr$ ) versus productive expenditure ( $g_p$ )

We now test several combinations of social transfers and productive expenditures, maintaining constant the level of global government spending. We set  $G = 30\%$  and  $d = 50\%$ . Concerning the global spending, we fix,  $g_u = 18.5\%$  and we start with a combination of  $g_p = 1.5\%$  and  $tr = 10\%$ . From that starting point, we progressively switch transfers for productive expenditures. The results are similar for different levels of  $G$ , so we omit them. Table 8 and 9 exhibit the different  $(g_p, tr)$  combinations and the corresponding outcomes, for distinctive values of  $\eta$  ( $\eta = 0.3$ , (baseline case), and  $\eta = 0.03$ ).

As we referred to, the results depend strongly on output elasticity of government productive expenditures. Figures 19a and 19b show for  $\eta = 0.3$  and  $\eta = 0.03$  respectively the welfare decomposition effects through all combination of  $(g_p, tr)$ .

$g_p$	$tr$	wu	WG	IG	CG	LG	wine
0.0150	0.1000	0.0000	0.4745	0.2709	0.1265	0.0418	0.0000
0.0200	0.0950	0.2784	0.4735	0.2731	0.1272	0.0419	-0.0003
0.0250	0.0900	0.5493	0.4726	0.2753	0.1280	0.0420	-0.0007
0.0300	0.0850	0.8149	0.4716	0.2775	0.1287	0.0421	-0.0010
0.0350	0.0800	1.0760	0.4706	0.2797	0.1294	0.0423	-0.0014
0.0400	0.0750	1.3334	0.4696	0.2820	0.1302	0.0424	-0.0018
0.0450	0.0700	1.5874	0.4686	0.2844	0.1309	0.0425	-0.0021
0.0500	0.0650	1.8384	0.4675	0.2868	0.1317	0.0426	-0.0025
0.0550	0.0600	2.0865	0.4665	0.2892	0.1325	0.0428	-0.0029
0.0600	0.0550	2.3317	0.4654	0.2917	0.1332	0.0429	-0.0033
0.0650	0.0500	2.5742	0.4643	0.2942	0.1340	0.0430	-0.0038
0.0700	0.0450	2.8138	0.4631	0.2968	0.1348	0.0432	-0.0042

Table 8: Productive expenditure versus Transfers:  $G = 30\%$  and  $\eta = 0.3$ .  
Note: see table 3.

Under the baseline parameterization the welfare augments permanently until social transfers are exhausted (see figure 19a). The level effect dominates through the productivity effect.

$g_p$	$tr$	wu	WG	IG	CG	LG	wine
0.0150	0.1000	0.0000	0.4745	0.2709	0.1265	0.0418	0.0000
0.0200	0.0950	0.0069	0.4735	0.2731	0.1272	0.0419	-0.0003
0.0250	0.0900	0.0104	0.4726	0.2753	0.1280	0.0420	-0.0007
0.0300	0.0850	0.0117	0.4716	0.2775	0.1287	0.0421	-0.0010
0.0350	0.0800	0.0113	0.4706	0.2797	0.1294	0.0423	-0.0014
0.0400	0.0750	0.0098	0.4696	0.2820	0.1302	0.0424	-0.0018
0.0450	0.0700	0.0073	0.4686	0.2844	0.1309	0.0425	-0.0021
0.0500	0.0650	0.0040	0.4675	0.2868	0.1317	0.0426	-0.0025
0.0550	0.0600	0.0001	0.4665	0.2892	0.1325	0.0428	-0.0029
0.0600	0.0550	-0.0044	0.4654	0.2917	0.1332	0.0429	-0.0033
0.0650	0.0500	-0.0093	0.4643	0.2942	0.1340	0.0430	-0.0038
0.0700	0.0450	-0.0146	0.4631	0.2968	0.1348	0.0432	-0.0042

Table 9: Productive expenditure versus Transfers:  $G = 30\%$  and  $\eta = 0.03$   
Note: see table 3.

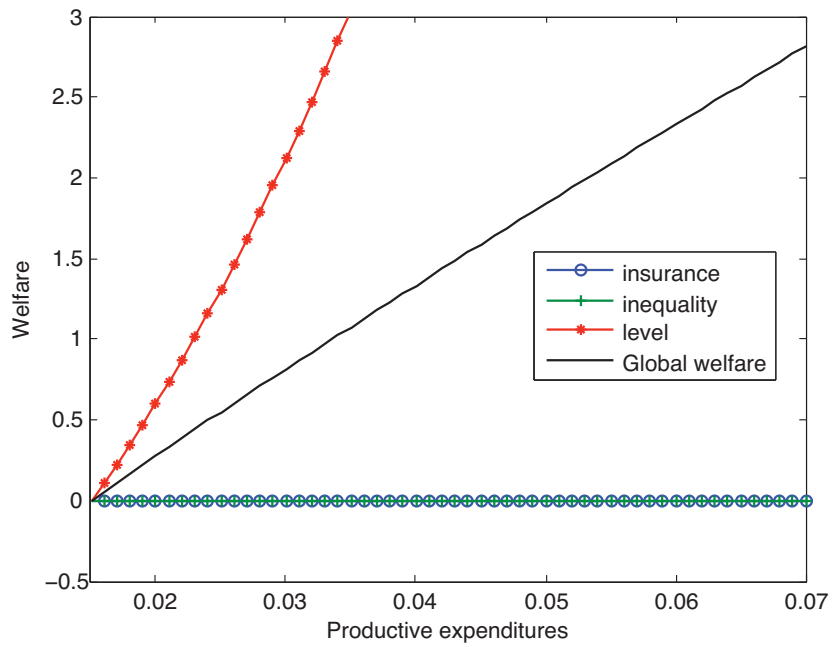
Although not visible in the graph, the simulation shows a negative effect on the insurance and inequality components of welfare, caused by the fall in the interest rate and transfers. A small interest rate decrease reflects a high asset demand, recalling figure 15 but considering now an opposite move of the asset demand curve to the left. As transfer is substituted by public investment, households will work harder, will save more and consume more since output improves. Given that the productive expenditure does not affect the distribution, the changes occurring in inequality measure must be justified only by the progressive reduction of the social transfer level. With lower transfers, households, specially the poorer, have more need for precautionary savings - wealth Gini index decreases. On the other hand, the transfers cut directly induces a more unequal distribution of income and con-

sumption. Finally, and as seen above, the transfer cut affects strongly the poorest<sup>16</sup> inducing the latter to increase the labour supply. That explains why the leisure Gini index rises, and also why the welfare distribution, including consumption and leisure, worsens.

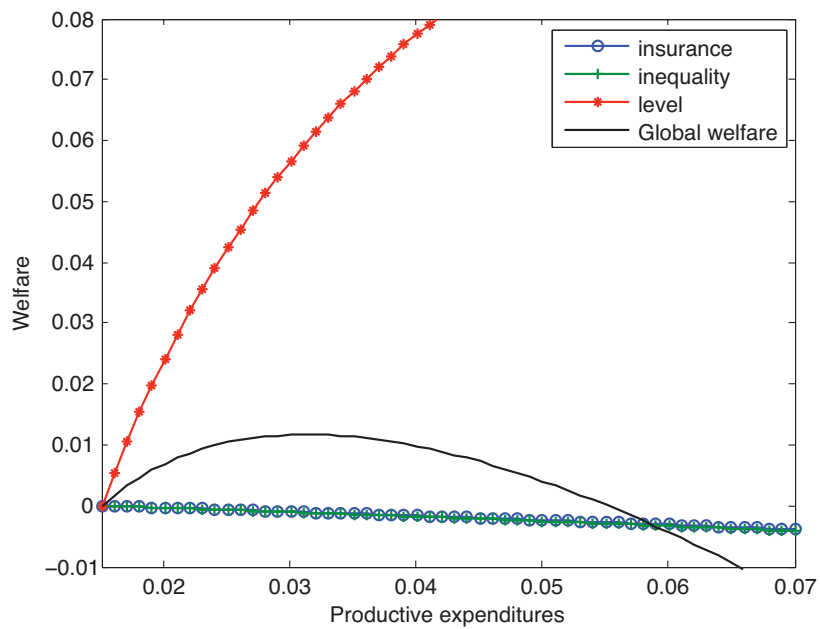
The results depend crucially on the productive expenditure elasticity in the production function. For  $\eta$  equal to 0.03, the productivity effect is much weaker, and the welfare level gain is not sufficiently high to compensate the negative effect on insurance and inequality. According to table 9 and as it can be observed in figure 19b the welfare is maximized with a combination of  $g_p = 3.1\%$  and  $tr = 8.4\%$ . The dynamics are similar except for the fact that the efficiency effect of the productive expenditures is not so high as in baseline. Above a certain level, it can no longer compensate the utility loss due to lower transfer. As predictable, different  $\eta$  leave the household distribution unchanged because the output elasticity affects all household equally. All the Gini coefficients as well as the inequality effect of welfare, present the same values.

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<sup>16</sup>The income reduction is proportionally stronger among the poor because by definition they own less assets



(a)  $\eta = 0.3$



(b)  $\eta = 0.03$

Figure 19: Welfare decomposition:  $tr$  versus  $g_p$

## 4 Conclusion

Using a general equilibrium model with heterogeneous agents, calibrated according to the EU empirical reality, we exemplify the channels by which debt, social transfer, collective consumption and public gross investment affect social welfare and its distribution. Moreover, we explore the optimal combination of these policy variables in terms of social welfare, decomposing it on a level effect, an insurance effect and an inequality effect. We complement welfare inequality measure by calculating the Gini index on several standard inequality variables such as wealth, disposable income and consumption, but also on leisure.

We find that a rise in unproductive expenditure and in transfers improve utility up until a certain point. Direct effects on utility and disposable income, respectively, impinge positively on inequality and uncertainty but (indirect) tax effects affect strongly labor supply, with negative impacts on the welfare level. On the other hand, debt brings a positive insurance effect through interest rate incentives on savings, but the dominance of this channel over alleviating credit constraint impinges negatively on welfare inequality. Welfare level effect is also negative due to side effects of taxes on labor supply. We also find that, for a calibration mimicking average unproductive and productive spending of the EU countries, optimal combination of debt and social transfer levels are smaller than the values observed in the EU countries during the last decades and, in the case of debt, optimal values are below the limits established by the Stability Growth Pact and they are larger, the larger the size of government (as measured by the expenditures on output ratio). Consequently, the larger government size, the worse is welfare inequality. Most of heterogeneous-agent models rule out the presence of collective consumption from the household utility function. We find that, by including the utility flow from in-kind government transfers in the household's utility function, the optimal level of debt and transfers decrease significantly. Finally, for a given level of public debt and government size, we assess how (intra-temporal) composition of government expenditures impinges on wel-

fare and related inequality measures: (i) substituting unproductive spending by transfers is welfare enhancing and improves inequality but only up to a lower bound of unproductive spending, rather inelastic; the smaller the government size is, unproductive expenditure cuts gradually dominate transfer welfare- and inequality-enhancing effects; (ii) substituting unproductive by productive spending is always welfare enhancing and has no impact on any inequality measure; and (iii), shifting transfers for productive expenditure is always welfare enhancing for a sufficiently high output elasticity of public investment; if not, there is an optimal maximum level of optimal productive expenditure. Since productive expenditure has no direct effects on inequality, transfer reduction impacts negatively on inequality welfare.

The present paper relies only on steady state analysis, i.e., we do not account for a transition period, with welfare and related consequences in inequality, in between steady-states (i.e., fiscal arrangements). An interesting and necessary extension of this paper should include the study, for instance, of an optimal fiscal consolidation strategy in terms of social welfare. Considering the actual stance of public finances in most of the EU countries, we believe that this can be an important issue in the near future.

Finally, the paper model considers a closed economy, isolated from any foreign influence and assuming no possibility of government default. We believe that the inclusion of more than one country, eventually with different sizes, will bring more robustness to our results and also new insights, especially if we can mimic actual EU environment. The absence of any sovereign risk is also a simplification hypothesis which can overestimate the optimal level of public debt. This can also represent another valuable extension of the model.

## References

- A. Afonso and D. Furceri. Government size, composition, volatility and economic growth. *European Central Bank Working Paper Series*, 849, 2008.
- A. Afonso, L. Schuknecht, and V. Tanzi. Income distribution: Determinants and public spending efficiency. *European Central Bank Working Paper Series*, 86, 2008.
- S. R. Aiyagari and E.R. McGrattan. The optimum quantity of debt. *Journal of Monetary Economics*, 42(3):447–69, 1998.
- S.R. Aiyagari. Uninsured idiosyncratic risk and aggregate saving. *Quarterly Journal of Economics*, 109(3):659–84, 1994.
- S.R. Aiyagari. Optimal capital taxation with incomplete markets, borrowing constraints, and constant discounting. *Journal of Political Economy*, 103(6):1158–1175, December 1995.
- A. Alesina and S. Ardagna. Tales of fiscal contractions. *Economic Policy*, 27, 1998.
- A. Alesina and R. Perotti. Fiscal adjustments in oecd countries: Compositions and macroeconomic effects. *IMF Staff Papers*, 44:210–48, 1995.
- S. Ardagna. Fiscal stabilizations: When do they work and why. *European Economic Review*, 5:1047–1074, October 2004.
- D. A. Auschauer. Is public expenditure productive? *Journal of Monetary Economics*, 23(2):177–200, 1989.
- L. Ayala, R. Martnez, and J. Ruiz-Huerta. Inequality, growth and welfare: An international comparison. *Luxembourg Income Study (LIS) Working Papers*, 215, 1999.
- S. Barrios and A. Schaechter. The quality of public finances and economic growth. *ECB Working Paper*, N 337, September 2008.

- R. Barro. The control of politicians: An economic model. *Public Choice*, 14(1):19–42, March 1973.
- R. Barro. Are government bonds net wealth? *Journal of Political Economy*, 82 (6):1095–1117, 1974.
- R. Barro. On the determination of public debt. *Journal of Political Economy*, 87, October 1979.
- R. Barro. Government spending in a simple model of endogenous growth. *Journal of Political Economy*, 98(5), 1990.
- R. Barro. Inequality and growth in a panel of countries. *Journal of Economic Growth*, 5:5–32, 2000.
- R.B. Barsky, N. G. Mankiw, and S.P. Zeldes. Ricardian consumers with keynesian propensities. *American Economic Review, American Economic Association*, 76(4):676–91, 1986.
- R. Benabou. Inequality and growth. *NBER Macroeconomics Annual 1996, Ben Bernanke and Julio Rotemberg eds. MIT Press*, 11(74), Spring 1996.
- R. Benabou. Tax and education policy in a heterogeneous agent economy: What levels of redistribution maximize growth and efficiency? *Econometrica*, 70:481–517, 2002.
- B. D. Bernheim. Does the estate tax raise revenue? *National Bureau of Economic Research*, 1:113–138, 1987.
- T. Bewley. A difficulty with the optimum quantity of money. *Econometrica*, 51(5):1485–504, 1983.
- M. Cagetti and M. De Nardi. Wealth inequality: Data and models. *Macroeconomic Dynamics*, 12(52):285–313, 2008.
- K. Chu, H. Davoodi, and S. Gupta. Income distribution and tax and government social spending policies in developing countries. *International Monetary Fund Working Papers*, WP/00/62, 2000.



- F. A. Cowell. *Measuring Inequality*. Oxford University Press, Oxford, December 2009.
- M. Floden. The effectiveness of government debt and transfers as insurance. *Journal of Monetary Economics*, 48(1):81–108, 2001.
- M. Floden. Public saving and policy coordination in aging economies. *Scandinavian Journal of Economics*, 105(3):379–400, 2003.
- M. Forster and M. Pearson. Income distribution and poverty in the ocde area:trends and driving forces. *OECD Economic Studies*, 34, 2002.
- F. Giavazzi and M. Pagano. Can severe fiscal contractions be expansionary? tales of two small european countries. *NBER Macroeconomics Annual*, 5: 75–111, Spring 1990.
- F. Giavazzi and M. Pagano. Non-keynesian effects of fiscal policy changes: International evidence and the swedish experience. *CEPR Discussion Paper*, 1284, 1995.
- R. G. Hubbard and K. L. Judd. Liquidity constraints, fiscal policy, and consumption. *Brookings Papers on Economic Activity, Economic Studies Program, The Brookings Institution*, 17(1896-1):1–60, 1986.
- M. Huggett. The risk-free rate in heterogeneous-agent incomplete-insurance economies. *Journal of Economic Dynamics and Control*, 17(5-6):953–69, 1993.
- A. Imrohoroglu. Cost of business cycles with indivisibilities and liquidity constraints. *Journal of Political Economy*, 97(6):1364–83, 1989.
- S. P. Jenkins and P. V. Kerm. The measurement of economic inequality. In *Oxford Handbook on Economic Inequality*. Brian Nolan, Wiermer Salverda and Tim Smeeding, 2008.
- P. Krusell and AA. Smith. Income and wealth heterogeneity in the macroeconomy. *Journal of Political Economy*, 106:5:867–896, 1998.

- P. Krusell and AA. Smith. Quantitative macroeconomic models with heterogeneous agents. In *Advances in Economics and Econometrics: Theory and Applications*. Blundell R, Newey W, T Persson. Cambridge: Cambridge University Press., 2006.
- S. Kuznets. Economic growth and income inequality. *American Economic Review*, 49(1):1–28, 1955.
- L. Ljungqvist and T.J. Sargent. *Recursive Macroeconomic Theory*. MIT Press, Cambridge, MA, 1994.
- Ellen R. McGrattan. Solving the stochastic growth model with a finite element method. *Journal of Economic Dynamics and Control*, 20, 1996.
- Ellen R. McGrattan. The optimum quantity of debt: Technical appendix. *Annals of Economics and Finance*, 4, 2003.
- J.V. Rios-Rull. Models with heterogeneous agents. In *Frontiers of Business Cycle Research*. edited by Cooley TF. Princeton, NJ: Princeton University Press, 1995.
- J.V. Rios-Rull. Computation of equilibria in heterogenous agent models. In *Computational Methods for the Study of Dynamic Economies: An Introduction*. Edited by Ramon Marimon and Andrew Scott, Oxford University Press., 1999.
- J.V. Rios-Rull, Ana Castaneda, and Javier Diaz-Gimenez. Accounting for earnings and wealth inequality. *Journal of Political Economy*, Vol. 111, no 4:818–857, 2003.
- G. W. Scully. Optimal taxation, economic growth and income inequality in the united states. *NCPA Policy Report*, 316, September 2008.
- T. Smeeding. Changing income inequality in oecd countries: Updated results from the luxembourg income study (lis). *Luxembourg Income Study Working Paper*, 252, 2000.

- Kjetil Storesletten, J. Heathcote, and G. L. Violante. Quantitative macroeconomics with heterogeneous households. *Annual Review of Economics*, 1: 319–354, 2009.
- V. Tanzi and N. Chalk. Impact of large public debt on growth in the eu: A discussion of potential channels. In *The behaviour of fiscal authorities. Stabilization, growth and institutions*, pages 186–211. Buti, M., J. von Hagen and C. Martinez-Mongay (eds.), 2002.
- G. Tauchen. Statistical properties of generalized method-of-moments estimators of structural parameters obtained from financial market data. *Journal of Business & Economic Statistics, American Statistical Association*, 4(4): 397–416, October 1986.