# Storage and Investments in a Combined Energy Network Model

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### DRAFT VERSION

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#### Abstract:

Natural gas plays an important role in the future development of electricity markets as it is the least emission intensive fossil generation option while additionally providing the needed flexibility in plant operation to deal with intermitted renewable generation. As both the electricity and the natural gas market rely on networks, congestion on one market may lead to changes on another. This influence has been analyzed by Abrell and Weigt (2010) for a static partial equilibrium market model setting showing upstream and downstream feedback effects in a stylized European framework. The objective of this paper is to extend the static model by incorporating dynamic restrictions – particularly seasonal and daily demand variations, natural gas storage, and pumped hydro storage – and an investment representation to evaluate the interaction between both network markets under realistic market conditions.

Key words: Electricity network, Natural gas network, Europe, MCP

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# **1** Introduction

A transition of existing energy systems is supposed to take place in the coming decades. Furthermore, in developing regions of the world a significant increase of energy demand will occur. Both developments will require a large amount of investments in energy production and transport infrastructure. Energy markets are interlinked with each other as different energy products can be seen as substitutes (eg. heating oil/gas vs. heating with biomass vs. electricity) and in the electricity sector primary and secondary energy inputs are transformed. In addition energy markets often rely on network structures. The relations of different energy markets and network congestion effects must be considered when analyzing the development of the energy system as a whole.

Abrell and Weigt (2010) show this combination of energy network models for a static market setting using the MCP format. They show that changes in both the supply in the natural gas market and the generation dispatch in the electricity market impact the respective downstream and upstream markets beyond the pure price connection. In this paper we extend the static model of Abrell and Weigt (2010) by firstly incorporating the time dimension: The natural gas market is largely characterized by seasonal patterns whereas the electricity market is defined by daily load levels which requires a matching of the two time frames. Furthermore, the storage options for the two markets are included as storage operators: seasonal storage for natural gas and pumped hydro for electricity. Given this basic dynamic setting in a second step investment options are included. The investment options include the extension of natural gas and electricity network capacities as well as investment in new natural gas generation capacities. The dynamic model will be applied to a stylized model representation.

The remainder of this paper is structured as follows. Section 2 describes the modeling framework for the inclusion of time periods and storage. In Section 3, the investment formulation is presented. Following, in Section 4, the basic numerical example is initiated and the first results are obtained. Section 5 summarizes the current state of the model and highlights future steps.

## **2** Dynamic Market Formulation

Following the basic model setting including time periods and storage representations for the natural gas market, the electricity market and the combined model are presented. The models are formulated as Mixed Complementarity Problems (MCP) and are based on Abrell and Weigt (2010). We provide the optimization setting for each market participants as well as the market clearing conditions equalizing demand and supply. The full MCP formulation is provided in the Annex; Table 1 provides the underlying notation for all models. We assume perfect competition, i.e. all market participants take prices as given. However, the equilibrium concept allows an easy adjustment of the underlying competition assumptions. The MCP model is formulated in the General Algebraic Modeling System (GAMS; Brook et al., 2008) and solved using the PATH (Ferris and Munson, 2000) solver.

#### **Table 1: Notation**

Indexes and Set		Variables
<i>e C E</i> Node in the electricity network	CAP	Capacity
g, $h \in G$ Nodes in natural gas network	DEM	Demand
$\tilde{g} \in G$ Origin node of natural gas	F	Flow
<i>i C I</i> Plant types	P	Price
k C K Load segmets	PC	Scarcity prices of capacity
$l \in L$ Lines in the electricity network	PD	Demand price
$t \in T$ Time periods	PF	Fuel price (endogenous)
GE Mapping between G and E	PHUB	Marginal system price in the electricity market
	PN	Nodal price
	PS	Supply price
	PT	Transport service price
	<u>S</u>	Storage Level
	T	Transporter and traded volume
	W	Storage withdrawal
	V	Storage injection
	X	Natural gas extraction
	Y	Power injected/withdrawn into electricity grid
Parameters		Superscripts
α LNG liquefaction loss	+/-	Positive or negative direction
$\beta$ LNG regasification loss	buy	Bought quantity
$\eta$ Plant efficiency	el	Electricity
$\sigma$ Storage related efficiencies	gas	Natural gas
a Demand function intercept	inv	Investment
<i>b</i> Demand function slope	line	Lines in electricity network
c Marginal cost	liq	Liquefaction
<i>cinv</i> Investment costs (annualized)	LNG	Liquefied natural gas
<i>cap</i> Capacity	new	New investments
<i>pf</i> Fuel price (exogenous)	pipe	Pipeline
<i>PTDF</i> Power Transmission Distribution Factor	reg	Regasification

#### 2.1 Natural Gas Market

In the natural gas market we explicitly model five market participants and final consumers. Natural gas producers extract the gas and sell it either to a LNG operator or to trader transporting via pipelines. LNG operators transport the gas from liquefaction to regasification plants and sell it then to the trader using pipelines. Only the trader serves final demand by buying natural gas and the pipeline transport services necessary to transport to the final consumers. The pipeline operator provides associated transport services. The storage operator buys the gas from the pipeline trader and sells it to again to this trader in a later period. Three markets in the natural gas market are explicitly modeled: the supply market, the pipeline transport service market, and the final demand market.

The gas network is defined by nodes g and  $h \in G$  and pipelines denoted as directed and ordered pairs  $(g,h) \in GxG$  with capacity  $cap_{gh}^{pipe}$ . Natural gas transport via LNG is not restricted by arc capacities, but by the technical characteristics of the connected nodes. Time periods are given by  $t \in T^G$ , where  $T^G$  is the set of time periods in the natural gas market.

Final demand is represented by a linear demand function which varies over time:

$$DEM_{gt}^{gas} = a_{gt}^{gas} + b_{gt}^{gas} PD_{gt}^{gas} \quad \forall \ g \in G, t \in T$$

$$\tag{1}$$

The natural gas producer is assumed to maximize profits by selling production ( $X^{gas}$ ) at its production site *g* for the supply price ( $PS^{gas}$ ) given his extraction capacity restriction ( $cap^{gas}$ ):

$$\max_{X_{gt}^{gas}} \pi = \sum_{gt} PS_{gt}^{gas} X_{gt}^{gas} - c_{gt}^{gas} X_{gt}^{gas}$$
(2)

$$X_{gt}^{gas} \le cap_{gt}^{gas} \ \forall \ g \in G, t \in T^G$$
(3)

The LNG trader maximizes its profit by buying and selling gas  $(T^{LNG})$  on the on the supply market accounting for liquefaction  $(cap^{liq})$  and regasification  $(cap^{reg})$  constraints as well as the corresponding efficiencies ( $\alpha$  and  $\beta$ ) and transport costs on LNG routes  $(c^{LNG})$ :

$$\max_{T_{ght}^{LNG}} \pi = \sum_{ght} PS_{ht}^{gas} \beta T_{hgt}^{LNG} - PS_{gt}^{gas} \frac{T_{ght}^{LNG}}{\alpha} - c_{ght}^{LNG} T_{ght}^{LNG}$$
(4)

$$\sum_{h} T_{hgt}^{LNG} \le cap_{gt}^{reg} \quad \forall \ g \in G, t \in T^G$$
(5)

$$\sum_{h} T_{ght}^{LNG} \le cap_{gt}^{liq} \quad \forall \ g \in G, t \in T^G$$
(6)

The pipeline trader maximizes its profits by buying gas  $(T^{gas})$  on the supply market at a supply node  $\tilde{g}$ , transporting it through the network accounting for transport fees  $(PT^{pipe})$ , and selling on the demand market while accounting for a nodal mass balance (flow conservation constraint):

$$\max_{T_{\tilde{g}t}^{buy}T_{\tilde{g}ht}^{gas}F_{\tilde{g}ght}^{gas}}\pi = \sum_{\tilde{g}ht} \left( PD_{ht}^{gas}T_{\tilde{g}ht}^{gas} - PS_{\tilde{g}t}^{gas}T_{\tilde{g}t}^{buy} \right) - \sum_{\tilde{g}ght} PT_{ght}^{pipe}F_{\tilde{g}ght}^{gas}$$
(7)

$$\sum_{g} F_{\tilde{g}ght}^{gas} + \left(T_{\tilde{g}t}^{buy}\right)_{if \; \tilde{g}=h} = \sum_{g} F_{\tilde{g}hgt}^{gas} + T_{\tilde{g}ht}^{gas} \; \forall \; \tilde{g}, h \in G, t \in T^{G}$$

$$\tag{8}$$

The storage operator buys and sells gas inter-temporal accounting for capacity restrictions of its storage facilities (the overall capacity ( $cap^{Sgas}$ ), and the injection ( $cap^{Vgas}$ ) and withdrawal ( $cap^{Wgas}$ ) capacities) and the inter-temporal storage balance linking past periods storage level ( $s_{g,t-1}$ ) with the current level ( $s_{g,t}$ ):

$$\max_{W_{gt}^{gas}V_{gt}^{gas}} \pi = \sum_{gt} PS_{gt}^{gas} \sigma_{gt}^{Wgas} W_{gt}^{gas} - PD_{gt}^{gas} V_{gt}^{gas}$$
(9)

$$S_{gt}^{gas} = S_{gt-1}^{gas} + \sigma_{gt}^{Vgas} V_{gt}^{gas} - W_{gt}^{gas} \quad \forall \ g \in G, t \in T^G$$

$$\tag{10}$$

$$S_{gt}^{gas} \le cap_{gt}^{Sgas} \quad \forall \ g \in G, t \in T^G$$

$$\tag{11}$$

$$V_{gt}^{gas} \le cap_{gt}^{Vgas} \quad \forall \ g \in G, t \in T^G$$

$$\tag{12}$$

$$W_{gt}^{gas} \le cap_{gt}^{Wgas} \quad \forall \ g \in G, t \in T^G$$
(13)

Finally, the pipeline operator maximizes its profit by selling transmission capacity for a transport fee  $(PT^{pipe})$  accounting for the networks capacity restriction  $(cap^{pipe})$ :

$$\max_{F_{ght}^{pipe}} \pi = \sum_{ght} PT_{ght}^{pipe} F_{ght}^{pipe} - c_{ght}^{pipe} F_{ght}^{pipe} \quad \forall \ g \in G, t \in T^G$$
(14)

$$F_{ght}^{pipe} \le cap_{ght}^{pipe} \quad \forall \ g, h \in G, t \in T^G$$
(15)

Market prices are determined by market clearing conditions. On the supply market total supply from producers, LNG traders, and storage operators at that node has to be at least as high as total wholesale demand:

$$X_{gt}^{gas} + \sum_{h} \beta T_{hgt}^{LNG} + \sigma_{gt}^{Wgas} W_{gt}^{gas} \ge T_{\tilde{g}t}^{buy} + \sum_{h} \frac{T_{ght}^{LNG}}{\alpha} \quad \forall \ g \in G, t \in T^G$$
(16)

On the demand market total provision of traded gas has to cover residual demand and storage demand:

$$\sum_{h} T_{hgt}^{gas} \ge DEM_{gt}^{gas} + V_{gt}^{gas} \qquad \forall g \in G, t \in T^G$$
(17)

And for a pipeline the physical flow has to cover all trades on that pipeline:

$$F_{ght}^{pipe} \ge \sum_{\tilde{g}} F_{\tilde{g}ght}^{gas} \qquad \forall g \in G, t \in T^G$$
(18)

#### 2.2 Electricity market

In the electricity market we assume that all trading is managed by the system operator with generators selling their power plant output and storage operators buying and selling at their respective nodes. Similar to the gas market final demand is supplied by system operator only. The market formulation follows a hub price approach as presented in Hobbs (2001) thus the price at a specific node is given by the overall market clearing price of the whole system (*PHUB*) and the transmission fee of the node (*PT<sub>e</sub>*). Nodes in the electricity network are given by  $e \in E$ . A node is characterized by the generators at these node and electricity demand. Generators are furthermore differentiated by their respective technologies type  $i \in I$ . Nodes are connected via lines  $l \in L \subseteq ExE$  of a given capacity  $cap_l^{line}$ . Lines are not ordered pairs of nodes as electricity can flow in both directions. Each time period  $t \in T^E$ , with  $T^E$  being the set of time period in the electricity model, is characterized by different load segment  $k \in K$  to cover the off-peak and peak characteristic.

Demand is assumed to be time varying and linear:

$$DEM_{ekt}^{el} = a_{ekt}^{el} + b_{ekt}^{el} \left( PT_{ekt}^{el} + PHUB_{kt} \right) \quad \forall \ e \in E, k \in K, t \in T^E$$
<sup>(19)</sup>

The electricity producer is assumed to maximize profits given his plant capacity restriction:

$$\max_{X_{ekt}^{el}} \pi = \sum_{eikt} \left( PT_{ekt}^{el} + PHUB_{kt} \right) X_{eikt}^{el} - \frac{pf_{eit}}{\eta_{ei}} X_{eikt}^{el}$$
(20)

$$X_{eikt}^{el} \le cap_{eikt}^{el} \quad \forall \, e \in E, i \in I, k \in K, t \in T^E$$

$$\tag{21}$$

The storage operator buys and sells electricity inter-temporal during load segments but not during periods accounting for capacity restrictions of its storage facilities and the inter-temporal storage balance:

$$\max_{W_{ekt}^{el}V_{ekt}^{el}} \pi = \sum_{ekt} \left[ \left( PT_{ekt}^{el} + PHUB_{kt} \right) \left( \sigma_{et}^{Wel}W_{ekt}^{el} - V_{ekt}^{el} \right) \right]$$
(22)

$$S_{ekt}^{el} = S_{ek-1t}^{el} + \sigma_{et}^{Vel} V_{ekt}^{el} - W_{ekt}^{el} \quad \forall \ e \in E, k \in K, t \in T^E$$

$$\tag{23}$$

$$S_{ekt}^{el} \le cap_{ekt}^{Sel} \quad \forall \ e \in E, k \in K, t \in T^E$$

$$\tag{24}$$

$$V_{ekt}^{el} \le cap_{ekt}^{Vel} \quad \forall \ e \in E, k \in K, t \in T^E$$

$$\tag{25}$$

$$W_{ekt}^{el} \le cap_{ekt}^{Wel} \quad \forall \ e \in E, k \in K, t \in T^E$$

$$\tag{26}$$

The network operator accounts for all trades based on the net injections and secures line capacities limits. Power flows are obtained using a power transfer distribution matrix (*PTDF*):

$$\max_{Y_{ekt}^{el}} \pi = \sum_{ekt} PT_{ekt}^{el} Y_{ekt}^{el}$$
(27)

$$\left|\sum_{e} PTDF_{le} Y_{ekt}^{el}\right| \le cap_{l}^{line} \quad \forall \ l \in L, k \in K, t \in T^{E}$$

$$\tag{28}$$

Total supply at a node minus local demand equals the net injection at each node:

$$X_{ekt}^{el} - DEM_{ekt}^{el} = Y_{ekt}^{el} \quad \forall e \in E, k \in K, t \in T^E$$

$$\tag{29}$$

In the overall system supply has to equal demand:

 $\sum_{e} X_{ekt}^{el} \ge \sum_{e} DEM_{ekt}^{el} \quad \forall k \in K, t \in T^{E}$ (30)

#### 2.3 Combined Market Representation

For the combination of both markets the underlying network topologies need to be matched, identifying which gas and electricity nodes are identical. This mapping between the set of natural gas and electricity nodes is denoted as GE(g,e) and associate each electricity node to exactly one natural gas node.

In a similar manner, the set  $\Gamma \subseteq T^E \times T^G$  denotes whether a electricity period is associated with a period in the natural gas model. This set represents the mapping of electricity to natural gas periods.

The actual combination of both market environments is then implemented via the fuel price element in the profit function of the electricity generators (equation 20). Whereas in a single electricity market formulation *pf* is an externally defined parameter it needs to become an endogenous variable for the combined model. In order to obtain the price the market clearing for natural gas demand (equation 17) has be complemented by including the demand of natural gas fired power generation. Thus, the interaction of the two markets is depicted by an additional fuel market in which natural gas traders are suppliers and electricity producers represent the demand:

$$\sum_{h} T_{hgt}^{gas} \ge DEM_{gt}^{gas} + V_{gt}^{gas} + \sum_{\substack{k,\tau \in \Gamma \\ e \in GE \\ i = \prime gas \prime}} \frac{X_{eik\tau}^{ei}}{\eta_{ei}} \qquad \forall g \in G, t \in T$$
(31)

The profit function of the generator is then adjusted as follows:

$$\max_{X_{ekt}^{el}} \pi = \sum_{eikt} \left( PT_{ekt}^{el} + PHUB_{kt} \right) X_{eikt}^{el} - \sum_{\substack{i \neq gas, \\ i \neq gas, }} \frac{pf_{eit}}{\eta_{ei}} X_{eikt}^{el} - \sum_{\substack{eikt \\ i = gas, }} \frac{\sum_{\tau \in \Gamma} PD_{et}^{gas}}{\eta_{ei}} X_{eikt}^{el}$$
(32)

# **3** Investment representation

For the investment representation we rely on an annualized cost estimation to prevent price spikes in the year of investment and dependence of investment decisions on the modeled time frame. The investments are implemented in the respective profit optimizations of the market participants adding a second cost element in addition to the variable costs. Furthermore, the respective capacity value becomes a variable and an intertemporal balance equation is added. The latter is similar to the storage balance representation: the actual capacity in period t is given by the capacity in the former period t-1 and the added new capacity in the current period.

In a first approach we consider the possibility of extending generation capacity. Furthermore, two infrastructure investment opportunities exist: extensions in the natural gas and electricity network and investments into electricity generation. Natural gas production, LNG facilities and storage are excluded from investment. For electricity transmission extensions we neglect the feedback effect on

the underlying power distribution, consequently the PTDF matrix remains unchanged.<sup>2</sup> Following the adjusted profit objectives are presented, the First-Order-Conditions for the MCP formulation are provided in the Annex.

For the natural gas network operator the pipeline capacity is the respective investment choice and the profit optimization is given by:

$$\max_{F_{ght}^{pipe}} \pi = \sum_{ght} PT_{ght}^{pipe} F_{ght}^{pipe} - c_{ght}^{pipe} F_{ght}^{pipe} - cinv_{ght}^{pipe} CAP_{ght}^{pipe} \quad \forall \ g \in G, t \in T$$
(1)

$$F_{ght}^{pipe} \le CAP_{ght}^{pipe} \quad \forall \ g, h \in G, t \in T$$

$$\tag{2}$$

$$CAP_{ght}^{pipe} = CAP_{ght-1}^{pipe} + CAP_{ght}^{pipe\_new} \quad \forall \ g \in G, t \in T$$
(3)

For the electricity network operator the line capacity is the investment choice:  $\max_{Y_{ekt}^{el}} \pi = \sum_{ekt} PT_{ekt}^{el} Y_{ekt}^{el} - cinv_{lt}^{line} CAP_{lt}^{line}$ (4)

$$\left|\sum_{e} PTDF_{le} Y_{ekt}^{el}\right| \le CAP_{lt}^{line} \quad \forall l \in L, k \in K, t \in T$$
(5)

$$CAP_{lt}^{line} = CAP_{lt-1}^{line} + CAP_{lt}^{line\_new} \quad \forall \ l \in L, t \in T$$
(6)

Finally, electricity generator can decide about new plant capacities:  

$$\max_{X_{ekt}^{el}} \pi = \sum_{ekt} (PT_{ekt}^{el} + PHUB_{kt}) X_{ekt}^{el} - \frac{pf_{fet}}{\eta_{fe}} X_{ekt}^{el} - cinv_{et}^{el} CAP_{et}^{el}$$
(7)

$$X_{ekt}^{el} \le CAP_{et}^{el} \quad \forall e \in E, k \in K, t \in T$$
(8)

$$CAP_{et}^{el} = CAP_{et-1}^{el} + CAP_{et}^{el\_new} \quad \forall g \in G, t \in T$$
(9)

### 4 Test Case

The developed dynamic and investment setting will be tested using a simple example system. The objective is to verify the model formulation. An applied analysis in the European market context will be included in future paper version.

#### 4.1 Data

The test example consists of a simple four node natural gas network with an additional LNG supply node; the electricity system is a three node setting. In both networks residual demand is only located at one node and the network capacities are limited. The topology is provided in Figure 1 including the mapping of gas and electricity nodes.

The production, generation, storage and LNG dataset is provided in Table 2 and Table 3. The demand function is assumed to be linear with a slope of -1 and varying intercepts depending on the time period (winter and summer in natural gas) or the load segment (off-peak, mid, peak in electricity). The only varying parameter over the periods is the domestic natural gas demand at node g4 growing by one unit for each consecutive season (i.e. starting with an intercept of 10 in the first summer period t1 and ending with an intercept of 15 in the last summer period t11; the same holds for the winter periods).

<sup>&</sup>lt;sup>2</sup> Implementing the feedback would require that the PTDF becomes a function of the underlying network topology and chosen line capacities which complicates the model formulation and is omitted at this stage of model design.

This will induce the potential for investments in the natural gas sector. In the electricity sector the initial parameters are kept fix for the considered periods. Thus any investments occurring in electricity infrastructure in the combined setting after the first two periods (covering the first summer and winter seasons) will be caused by the changed natural gas conditions.

#### **Figure 1: Network representations**



#### Table 2: Natural gas market characteristics

Gas node	Production capacity	Production costs	Lique. capacity	Regas. capacity	Demand intercept	Pipeline	Pipe capacity
g1	15	1				p1	2
g2					4	p2	2
g3				10		p3	4
g4					10-15 (summer) 20-25 (winter)	p4	10
g5	10	1	10			p5	10

Electricty node	Plant type	Plant capacity	Demand intercept	Line	Line capacity
e1	gas mid	2		11	10
	gas peak	2		12	10
e2	base	4		13	2
	mid	4			
e3			5 (off-pak) 10 (mid) 15 (peak)		

#### 4.2 Single Market Results

In a first step the single markets are simulated neglecting any endogenous price setting for the gas fuel in the electricity model. Implementing the dynamic setting with storage possibilities provides in both markets the option to increase production in the low demand times (summer season in natural gas, and off-peak segment in electricity) to store energy for following high demand times. In the natural gas setting we observe an increasing usage of the existing storage capacities due to the increasing demand level. In the electricity sector the storage utilization remains constant over the time periods with a high level of injection in the off-peak and a lower injection during the mid-load segment. The storage is completely depleted in the peak period.

Adding the investment option we observe a gradual investment in the natural gas setting extending the pipeline between producer and demand. The investment lead to a shift of the price development compared to a no-investment setting (Figure 2). In the latter case the demand price increases steadily over time due to the increasing demand level with storage as only option to influence prices over the periods. In the investment case the increased demand is compensated by increased investments leading to a greatly reduced price increase only covering the incremental investment cost increases. Figure 2 also highlights the price differences between the low demand summer seasons (odd time periods) and the high demand winter seasons (even periods). In the electricity market investments only occur in the first period as all remaining periods have constant demand parameters. Consequently the price level is constantly lower in the investment setting. Storage utilization is lower in both markets when investments are included as part of the necessary storage price differential is reduced by increased transport and/or generation capacities.





#### 4.3 Combined Market Results

Combining both markets in a dynamic setting without investments will lead to changes at the electricity market as the former exogenously gas price will now be determined endogenously. Furthermore, the natural gas demand at node g2 is now replaced by the endogenous gas demand by power producers at this node. Only if the exogenously defined and endogenously set prices and demand levels would be equivalent the combining of both models would have no impact. As single

market models often rely on stylized assumption for the input parameters it is unlikely that this will be achieved for most market representations.

Consequently, for the simple test example combining both markets leads to shifts in the obtained results. For the natural gas market the price level increases as the endogenously electricity demand is larger than parameterized one leaving less natural gas for the domestic demand at node g4. However, in the electricity market the actual gas based generation is lower as in the single market setting; the assumed parameterized demand interaction therefore was flawed in the first place. The lower gas based generation leads to network problems in the electricity sector as it reduces the potential for economic counter flow utilization leading to higher prices and nodal different prices in the electricity market. The results highlight the problem when setting up single market models that only account in a stylized way for input characteristics of other markets. A combination of models therefore provides a tradeoff between a more complicated model setting and less requirements for well-defined relevant interaction parameters.

Allowing for investments the results change significantly. On both markets the price level decreases. Furthermore, due to investments in natural gas plants at node e2 now also a natural gas demand price at the corresponding node g3 occurs. Table 4 summarizes the differences between the single market model investments and the combined results. It is evident that the initial investment setting, especially in the natural gas market, varies considerably leading to a different market dynamic. Of course, the obtained results are based on the underlying test example assumptions and cannot be generalized but they show that it indeed is complicated to provide reasonable estimated on combined energy markets with only single market representations.

				-	-	-					-	
	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12
n_g1>n_g2		1.56		0.37		1		1		1		0.17
l_e3	0.22											
n_e1: mid	0.02											
n_e3: ccgt	1.38											
Combined mark	et mode	l										
	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12
n_g1>n_g2	3.63	4.39		•	•	0.36		0.93		0.94	•	
n_g1>n_g3		0.41		0.27		0.07						
l_e3	0.36	0.01										
n_e1: mid	0.54											
n e2: ccgt	0.43											

## Table 4: Investment values

Single market models

## **5** Conclusion

In the paper we set up a dynamic model of interacting natural gas and electricity network models. The formulation is based on Abrell and Weigt (2010) which is extended by introducing time scales for the

individual models as well as a mapping between the scales in order to combine the models. Furthermore, storage facilities are introduced in both sub-models. Finally, we allow for investment into electricity generation, electricity transmission, and natural gas pipeline capacities.

Setting up the small test case, shows that introducing storage facilities has a smoothing price effect as the storage operators use the possibility of intertemporal arbitrage leading to convergence of prices. Allowing for investments into transmission capacities reduces prices as increased transmission capacities reduce congestion cost. In combining the models, we show that there are interaction effects in the investment patterns which are induced by the endogenous demand and price of natural gas. However, the test case presented here is too simplistic in order to allow for a meaningful conclusions. Currently, we work on extending underlying data and the model parameterization procedure in order to include meaningful scenarios and conclusions in a future version of this work.

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# Annex

### Natural Gas Model

Zero-profit restrictions:

$DEM_{gt}^{gas} = a_{gt}^{gas} + b_{gt}^{gas} PD_{gt}^{gas}$	$\perp DEM_{gt}^{gas}$	$\forall g \in G, t \in T$	(1)
$c_{gt}^{gas} + PC_{gt}^{gas} \ge PS_{gt}^{gas}$	$\perp X_{gt}^{gas}$	$\forall g \in G, t \in T$	(2)
$c_{ght}^{LNG} + \frac{PS_{gt}^{gas}}{\alpha} + PC_{gt}^{LNG} + PC_{gt}^{reg} \ge \beta PS_{ht}^{gas}$	$\perp T_{ght}^{LNG}$	$\forall g, h \in G, t \in T$	(3)
$PN_{\tilde{g}ht}^{gas} \ge PD_{ht}^{gas}$	$\perp T^{gas}_{\tilde{g}ht}$	$\forall \; \tilde{g}, h \in G, t \in T$	(4)
$T_{\tilde{g}t}^{buy} \ge PS_{\tilde{g}gt}^{gas}$	$\perp T^{buy}_{\tilde{g}t}$	$\forall \; \tilde{g}, h \in G, t \in T$	(5)
$PT_{ght}^{pipe} \ge PN_{\tilde{g}ht}^{gas} - PN_{\tilde{g}gt}^{gas}$	$\perp F^{gas}_{\tilde{g}ght}$	$\forall \; \tilde{g}, g, h \in G, t \in T$	(6)
$\lambda_{gt}^{gas} + PC_{gt}^{STgas} \ge \lambda_{gt+1}^{gas}$	$\perp S_{gt}^{gas}$	$\forall g \in G, t \in T$	(7)
$\lambda_{gt}^{gas} + PC_{gt}^{Wgas} \ge \sigma_{gt}^{Wgas} PS_{gt}^{gas}$	$\perp W_{gt}^{gas}$	$\forall g \in G, t \in T$	(8)
$PD_{gt}^{gas} + PC_{gt}^{Vgas} \ge \sigma_{gt}^{Vgas} \lambda_{gt}^{gas}$	$\perp V_{gt}^{gas}$	$\forall g \in G, t \in T$	(9)
$c_{ght}^{pipe} + PC_{ght}^{pipe} \ge PT_{ght}^{pipe}$	$\perp F_{ght}^{pipe}$	$\forall g, h \in G, t \in T$	(10)

Capacity restrictions:

$$\begin{aligned} cap_{gt}^{gas} \geq X_{gt}^{gas} & \perp PC_{gt}^{gas} & \forall g \in G, t \in T \end{aligned} \tag{11} \\ cap_{gt}^{reg} \geq \sum_{h} T_{hgt}^{LNG} & \perp PC_{gt}^{reg} & \forall g \in G, t \in T \end{aligned} \tag{12} \\ cap_{gt}^{liq} \geq \sum_{h} T_{ght}^{LNG} & \perp PC_{gt}^{liq} & \forall g \in G, t \in T \end{aligned} \tag{13} \\ cap_{ght}^{pipe} \geq F_{ght}^{pipe} & \perp PC_{ght}^{pipe} & \forall g, h \in G, t \in T \end{aligned} \tag{14} \\ cap_{gt}^{STgas} \geq S_{gt}^{gas} & \perp PC_{gt}^{STgas} & \forall g \in G, t \in T \end{aligned} \tag{15} \\ cap_{gt}^{Wgas} \geq W_{gt}^{gas} & \perp PC_{gt}^{Wgas} & \forall g \in G, t \in T \end{aligned} \tag{16} \\ cap_{gt}^{Vgas} \geq V_{gt}^{gas} & \perp PC_{gt}^{Vgas} & \forall g \in G, t \in T \end{aligned}$$

#### Balances:

$$\begin{split} & \sum_{g} F_{\tilde{g}ght}^{gas} + \left(T_{\tilde{g}t}^{buy}\right)_{if\;\tilde{g}=h} = \sum_{g} F_{\tilde{g}hgt}^{gas} + T_{\tilde{g}ht}^{gas} \perp PN_{\tilde{g}ht}^{gas} \qquad \forall \;\tilde{g}, h \in G, t \in T \quad (18) \\ & S_{gt}^{gas} = S_{gt-1}^{gas} + \sigma_{gt}^{Vgas} V_{gt}^{gas} - W_{gt}^{gas} \perp \lambda_{gt}^{gas} \qquad \forall \; g \in G, t \in T \quad (19) \end{split}$$

### Market clearing:

$$X_{gt}^{gas} + \sum_{h} \beta T_{hgt}^{LNG} + \sigma_{gt}^{Wgas} W_{gt}^{gas} \ge T_{\tilde{g}t}^{buy} + \sum_{h} \frac{T_{ght}^{LNG}}{\alpha} \perp PS_{gt}^{gas} \quad \forall g \in G, t \in T$$
(20)  
$$\sum_{h} T_{hgt}^{gas} \ge DEM_{gt}^{gas} + V_{gt}^{gas} \qquad \perp PD_{gt}^{gas} \quad \forall g \in G, t \in T$$
(21)  
$$F_{ght}^{pipe} \ge \sum_{\tilde{g}} F_{\tilde{g}ght}^{gas} \qquad \perp PT_{gt}^{pipe} \quad \forall g \in G, t \in T$$
(22)

### **Electricity Model**

Zero-profit restrictions:

$$\begin{split} DEM_{ekt}^{el} &= a_{ekt}^{el} + b_{ekt}^{el} \left( PT_{ekt}^{el} + PHUB_{kt} \right) &\perp DEM_{ekt}^{el} &\forall e \in E, k \in K, t \in T \quad (23) \\ \frac{pf_{fet}}{\eta_{fe}} + PC_{ekt}^{el} &\geq \left( PT_{ekt}^{el} + PHUB_{kt} \right) &\perp X_{ekt}^{el} &\forall e \in E, k \in K, t \in T \quad (24) \\ \lambda_{ekt}^{el} + PC_{ekt}^{STel} &\geq \lambda_{ek+1t}^{el} &\perp S_{ekt}^{el} &\forall e \in E, k \in K, t \in T \quad (25) \\ \lambda_{ekt}^{el} + PC_{ekt}^{Wel} &\geq \sigma_{ekt}^{Wel} \left( PT_{ekt}^{el} + PHUB_{kt} \right) &\perp W_{ekt}^{el} &\forall e \in E, k \in K, t \in T \quad (26) \\ \left( PT_{ekt}^{el} + PHUB_{kt} \right) + PC_{ekt}^{vel} &\geq \sigma_{ekt}^{Vel} \lambda_{ekt}^{el} &\perp V_{ekt}^{el} &\forall e \in E, k \in K, t \in T \quad (27) \\ PT_{ekt}^{el} &= \sum_{e} PTDF_{le} \left( PC_{lkt}^{-} - PC_{lkt}^{+} \right) &\perp Y_{ekt}^{el} &\forall l \in L, k \in K, t \in T \quad (28) \\ \end{split}$$

Capacity restrictions:

$cap_{ekt}^{el} \ge X_{ekt}^{el}$	$\perp PC_{ekt}^{el}$	$\forall \ e \in E, k \in K, t \in T$	(29)
$cap_{ekt}^{STel} \ge S_{ekt}^{el}$	$\perp PC_{ekt}^{STel}$	$\forall \ e \in E, k \in K, t \in T$	(30)
$cap_{ekt}^{Wel} \ge W_{ekt}^{el}$	$\perp PC_{ekt}^{Wel}$	$\forall \ e \in E, k \in K, t \in T$	(31)
$cap_{ekt}^{Vel} \ge V_{ekt}^{el}$	$\perp PC_{ekt}^{Vel}$	$\forall \ e \in E, k \in K, t \in T$	(32)
$cap_l^{line} \geq \sum_e PTDF_{le} Y_{ekt}^{el}$	$\perp PC_{lkt}^+$	$\forall \ l \in L, k \in K, t \in T$	(33)
$cap_l^{line} \ge -\sum_e PTDF_{le} Y_{ekt}^{el}$	$\perp PC_{lkt}^{-}$	$\forall \ l \in L, k \in K, t \in T$	(34)

Balances:

$S_{ekt}^{el} = S_{ek-1t}^{el} + \sigma_{et}^{Vel} V_{ekt}^{el} - W_{ekt}^{el}$	$\perp \lambda_{ekt}^{el}$	$\forall \ e \in E, k \in K, t \in T  (35)$
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Market clearing:

$X_{ekt}^{el} - DEM_{ekt}^{el} = Y_{ekt}^{el}$	$\perp PT_{ekt}^{el}$	$\forall \ e \in E, k \in K, t \in T$	(36)
$\sum_{e} X_{ekt}^{el} \ge \sum_{e} DEM_{ekt}^{el}$	$\perp PHUB_{kt}$	$\forall \ k \in K, t \in T$	(37)

### **Investment formulation**

Natural Gas, network operator:			
$c_{ght}^{pipe} + PC_{ght}^{pipe} \ge PT_{ght}^{pipe}$	$\perp F_{ght}^{pipe}$	$\forall g, h \in G, t \in T$	(38)
$CAP_{ght}^{pipe} \ge F_{ght}^{pipe}$	$\perp PC_{ght}^{pipe}$	$\forall g, h \in G, t \in T$	(39)
$cinv_{ght}^{pipe} + PC_{ght}^{pipe\_inv} \geq PC_{ght}^{pipe} + PC_{ght+1}^{pipe\_inv}$	$\perp CAP_{ght}^{pipe}$	$\forall g, h \in G, t \in T$	(40)

$$0 \ge PC_{ght}^{pipe\_inv} \qquad \qquad \perp CAP_{ght}^{pipe\_new} \forall g, h \in G, t \in T$$

$$CAP_{ght}^{pipe} = CAP_{ght-1}^{pipe} + CAP_{ght}^{pipe\_new} \qquad \qquad \perp PC_{ght}^{pipe\_inv} \quad \forall g \in G, t \in T$$

$$(41)$$

$$\begin{aligned} Electricity, network operator: \\ PT_{ekt}^{el} &= \sum_{e} PTDF_{le} \left( PC_{lkt}^{-} - PC_{lkt}^{+} \right) & \perp Y_{ekt}^{el} & \forall l \in L, k \in K, t \in T \quad (43) \\ CAP_{l}^{line} &\geq \sum_{e} PTDF_{le} Y_{ekt}^{el} & \perp PC_{lkt}^{+} & \forall l \in L, k \in K, t \in T \quad (44) \\ CAP_{l}^{line} &\geq -\sum_{e} PTDF_{le} Y_{ekt}^{el} & \perp PC_{lkt}^{-} & \forall l \in L, k \in K, t \in T \quad (45) \\ cinv_{lt}^{line} + PC_{lt}^{line\_inv} &\geq PC_{lkt}^{+} + PC_{lkt}^{-} + PC_{lt+1}^{line\_inv} & \perp CAP_{l}^{line} & \forall l \in L, t \in T \quad (46) \\ 0 &\geq PC_{lt}^{line\_inv} & \perp CAP_{lt}^{line\_inv} & \forall l \in L, t \in T \quad (47) \\ CAP_{lt}^{line} &= CAP_{lt-1}^{line\_new} & \perp PC_{lt}^{line\_inv} & \forall l \in L, t \in T \quad (48) \end{aligned}$$

Electricity, generator:

$\frac{pf_{fet}}{\eta_{fe}} + PC_{ekt}^{el} \ge \left(PT_{ekt}^{el} + PHUB_{kt}\right)$	$\perp X_{ekt}^{el}$	$\forall \ e \in E, k \in K, t \in T$	(49)
$CAP_{et}^{el} \ge X_{ekt}^{el}$	$\perp PC_{ekt}^{el}$	$\forall \ e \in E, k \in K, t \in T$	(50)
$cinv_{et}^{el} + PC_{et}^{el\_inv} \ge PC_{ekt}^{el} + PC_{et+1}^{el\_inv}$	$\perp CAP_{et}^{el}$	$\forall \ e \in E, t \in T$	(51)
$0 \geq PC_{et}^{el\_inv}$	$\perp CAP_{et}^{el\_new}$	$\forall \ e \in E, t \in T$	(52)
$CAP_{et}^{el} = CAP_{et-1}^{el} + CAP_{et}^{el\_new}$	$\perp PC_{et}^{el\_inv}$	$\forall g \in G, t \in T$	(53)