

# A Growth Accounting-based Productivity Analysis for the Flemish Economy: A First Exercise

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## Abstract

Productivity is a key indicator in the assessment of the economic performance of countries, regions and industries. Different productivity measures exist. A more comprehensive productivity measure is total factor productivity, using an aggregate input measure as denominator. A comparison of different productivity measures is made for the construction sector in the three Belgian regions.

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## 1. Introduction

Productivity is a key indicator in the assessment of the economic performance of countries, regions and industries and as such it has been and is an important research theme. There are different productivity measures for different purposes and/or because of different empirical reasons. An often used measure is labour productivity, being an output measure – such as gross domestic product or value added – per employee or, if available, per hour worked. The latter is to be preferred since the number of hours worked per employee can differ significantly over regions and countries.

A more comprehensive productivity measure is total factor productivity. Total factor productivity is defined as an output measure divided by an aggregate input measure, thus not only considering labour but also other production factors. Although not strictly necessary, total factor productivity is most of the times defined in a production function framework thereby imposing some neo-classical assumptions, more in particular it is assumed that the production process is subject to constant returns to scale and that there is perfect competition.

Over the years, both growth accounting and productivity analysis by industry branch have attracted a lot of attention on a global scale. A first application of the decomposition into the different inputs capital, labour, energy and materials (KLEM) in order to carry out a detailed productivity analysis by sector for the US economy can be found in Jorgenson et al. (1987). More recently and on the European level, Timmer et al. (2010) did a rather comprehensive analysis on the base of the EU KLEMS growth accounting database.

The ultimate aim of this analysis is to carry out a full growth accounting exercise for the three Belgian regions (the Brussels Capital Region, the Flemish Region and the Walloon Region) on a sectoral level. In this paper, however, the analysis is limited to only one sector, namely the construction sector. For this sector the productivity measures discussed above will be explored. Moreover, an output growth decomposition will guide us through the main drivers of growth in the construction sector during 1980-2006.

This paper continues by explaining in full detail the technique of growth accounting in both a general setting and the more specific textbook case of a Cobb-Douglas production function (Section 2). Section 3 then applies the growth accounting analysis on the construction sector in the three Belgian regions. Section 4 concludes.

## **2. Growth Accounting**

### **2.1. Definition and remarks**

Total factor productivity growth is the difference between the growth of output in volume and the growth of the combined inputs in volume. So, total factor productivity growth measures the increase in the output that can be produced with a given quantity of the different inputs. Total factor productivity growth measures disembodied technical change under neo-classical assumptions.

Aside from the assumption that there exists a production function linking inputs and outputs, the main assumption is that the production factors are rewarded by their marginal product. This is the neo-classical assumption of perfect

competition. It permits a representation of output growth as a weighted sum of the growth rates of the inputs. The weights for the input growth rates are the respective shares in total input payments.

Among the outputs of the growth accounting calculation the one to receive most attention is usually the difference between output growth and input growth. Although this measure has an interesting interpretation, it is also fraught with some difficulties, as underscored by the multitude of phrases used to refer to this difference: besides total factor productivity growth, multi-factor productivity growth, Solow residual, measure of ignorance, rate of technical change, ... Basically total factor productivity growth is a residual measure, and as such it may also include a variety of other effects, as is explained in Timmer (2010)<sup>2</sup>.

Firstly, since total factor productivity growth is calculated under the assumptions of neo-classical theory, it takes along any deviations from these assumptions. Secondly, total factor productivity growth does not only include technological change, but also e.g. organisational innovation. In the long run, the latter will lead in general to higher total factor productivity growth. In the short run, however, it might lead to lower measured total factor productivity growth rates since some of the inputs are used for the reorganisation process itself. More generally, total factor productivity comprises the effects from changes in unmeasured inputs, such as R&D. Thirdly, total factor productivity is calculated at the sectoral level, not at the firm level. Since productivity levels can differ greatly between firms, changes in total factor productivity on the industry level can also be caused by reallocation of market shares across firms. Lastly, total factor productivity growth is also influenced by measurement errors in inputs and outputs.

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<sup>2</sup> Or, see also e.g. Hulten (2010).

## 2.2. Technique

To explain the technique of growth accounting in some more detail, consider the following production function defined in terms of:

- capital ( $K$ ),
- labour ( $L$ ),
- energy inputs ( $E$ ),
- other intermediate inputs ( $M$ ), and
- an index of the level of technological progress ( $A$ ),

in function of time and hereby using a general form:

$$(1) \quad Y_{it} = f(A_{it}, K_{it}, L_{it}, E_{it}, M_{it}),$$

where  $Y_{it}$  is the output from branch  $i$  on time  $t$ . In the following the subscripts  $i$  and  $t$  will be dropped as much as possible for reasons of simplification.

Differentiating (1) with respect to time results in

$$(2) \quad \dot{Y} = \frac{\partial f}{\partial A} \dot{A} + \frac{\partial f}{\partial K} \dot{K} + \frac{\partial f}{\partial L} \dot{L} + \frac{\partial f}{\partial E} \dot{E} + \frac{\partial f}{\partial M} \dot{M},$$

where

$$(3) \quad \dot{X} = \frac{dX}{dt},$$

or,  $\dot{X}$  is the continuous time representation of  $\Delta X$ , the first difference of  $X$ :

$$(4) \quad \Delta X_t = X_t - X_{t-1}.$$

Take  $\lambda_x$  to be

$$(5) \quad \lambda_x = \frac{dX/dt}{X} = \frac{d \ln X}{dt},$$

the (continuous) growth rate of  $X$ , with its discrete counterpart:

$$(6) \quad \frac{X_t - X_{t-1}}{X_{t-1}} = \frac{\Delta X}{X},$$

Now, dividing the equation for the rate of change of  $Y$ , i.e. equation (2), by  $Y$  and hereby using (5), this becomes

$$(7) \quad \lambda_Y = \frac{\partial f}{\partial A} \frac{A}{Y} \lambda_A + \frac{\partial f}{\partial K} \frac{K}{Y} \lambda_K + \frac{\partial f}{\partial L} \frac{L}{Y} \lambda_L + \frac{\partial f}{\partial E} \frac{E}{Y} \lambda_E + \frac{\partial f}{\partial M} \frac{M}{Y} \lambda_M.$$

So, output growth is equal to a function of the growth rates of capital, labour, energy, other intermediate inputs and the remaining term, giving the effect of proportional changes in  $A$ , or the effect of productivity improvements on output, is defined as the Solow residual or total factor productivity growth. In the following this term is denoted by  $\Omega$  (where the subscripts  $i$  and  $t$  are dropped).

Equation (7) can be simplified and made more useful for calculation in practice by adopting the neo-classical assumption of perfect competition. By assuming that the production factors are traded in competitive markets, it follows that the production factors are paid their respective marginal products. For example, the

marginal product of labour equals the real wage, here denoted by  $\sigma_L$ . So, for the four production factors it holds then that

$$(8) \quad \frac{\partial F}{\partial X} = \sigma_X,$$

with  $X = K, L, E, M$ .

Substituting the marginal products in (7) by equation (8), it follows that

$$(9) \quad \lambda_Y = \Omega + \frac{\sigma_K K}{Y} \lambda_K + \frac{\sigma_L L}{Y} \lambda_L + \frac{\sigma_E E}{Y} \lambda_E + \frac{\sigma_M M}{Y} \lambda_M.$$

Again, this equation can be simplified by noting that

$$(10) \quad \Sigma_X = \frac{\sigma_X X}{Y},$$

with  $X = K, L, E, M$ , corresponds to the share of total income spent on payments to the production factor  $X$ . For example,  $\Sigma_L = (\sigma_L L)/Y$  is the share of total income spent by the economy on payments to the production factor labour and hence is called the labour share of income. Equation (9) is modified into

$$(11) \quad \lambda_Y = \Omega + \Sigma_K \lambda_K + \Sigma_L \lambda_L + \Sigma_E \lambda_E + \Sigma_M \lambda_M,$$

which leads to a simplified expression for the total factor productivity growth or Solow residual  $\Omega$ :

$$(12) \quad \Omega = \lambda_Y - (\Sigma_K \lambda_K + \Sigma_L \lambda_L + \Sigma_E \lambda_E + \Sigma_M \lambda_M).$$

The Solow residual equals thus the difference between the output growth rate and the weighted sum of factor growth rates, with the weights given by the factor income shares. Or, the Solow residual equals the growth of output that can not be attributed to the growth of the input of capital, labour, energy or other intermediate inputs.

A further simplification arises by assuming that the production function is constant returns to scale, in which case the sum of the factor income shares equals one, i.e.

$$(13) \quad \Sigma_K + \Sigma_L + \Sigma_E + \Sigma_M = 1.$$

Total factor productivity growth can be written then as follows

$$(14) \quad \Omega = \lambda_Y - (\Sigma_K \lambda_K + \Sigma_L \lambda_L + \Sigma_E \lambda_E + (1 - \Sigma_K - \Sigma_L - \Sigma_E) \lambda_M).$$

This is the equation which will be used in the calculations further on in the paper, thereby using both standard neo-classical assumption of perfect competition and constant returns to scale.

An interesting analysis instrument arises when equation (14) is derived in per capita terms. To that end, the following notations are used:

$$(15) \quad \lambda_x = \frac{d(X/L)/dt}{X/L} = \frac{d \ln(X/L)}{dt},$$

where  $X = Y, K, E, M$  and  $x = y, k, e, m$ . Or,  $k, e, m$  are the factor intensities with respect to the production factor labour  $L$ , e.g.  $e$  is energy intensity or the amount

of energy per unit of labour ( $E/L$ ) and  $k$  capital intensity or the amount of capital per unit of labour ( $K/L$ ).  $y$  of course represents labour productivity, i.e. output per unit of labour ( $Y/L$ ).

Rewriting equation (14) in per capita terms results in:

$$(16) \quad \Omega = \lambda_y - \Sigma_K \lambda_k - \Sigma_E \lambda_e - \Sigma_M \lambda_m .$$

This equation can be rearranged to decompose the labour productivity growth into the contributions from the growth of respectively capital intensity, energy intensity and intermediate inputs intensity on the one hand and the contribution of the growth of total factor productivity on the other hand:

$$(17) \quad \lambda_y = \Sigma_K \lambda_k + \Sigma_E \lambda_e + \Sigma_M \lambda_m + \Omega .$$

A production function which is often used in the context of growth accounting is the Cobb-Douglas function, which is written as follows:

$$(18) \quad Y_{it} = A_{it} K_{it}^{\alpha} L_{it}^{\beta} E_{it}^{\chi} M_{it}^{\delta} .$$

In the case of a Cobb-Douglas production function, equation (7) simplifies to

$$(7') \quad \lambda_Y = \lambda_A + \alpha \lambda_K + \beta \lambda_L + \chi \lambda_E + \delta \lambda_M ,$$

such that total factor productivity growth  $\Omega = \lambda_A$  is written as follows:

$$(12') \quad \Omega = \lambda_A = \lambda_Y - (\alpha \lambda_K + \beta \lambda_L + \chi \lambda_E + \delta \lambda_M) .$$

By assuming perfect competition and constant returns to scale, the parameters  $\alpha, \beta, \chi, \delta$  equal the respective factor income shares and they furthermore sum to one:

$$(10') \quad \alpha = \frac{\sigma_L L}{Y}, \beta = \frac{\sigma_K K}{Y}, \chi = \frac{\sigma_E E}{Y} \text{ and } \delta = \frac{\sigma_M M}{Y},$$

and

$$(13') \quad \alpha + \beta + \chi + \delta = 1.$$

### 3. A case study: the construction sector in the Belgian regions

HERMREG is a regional econometric model developed by a cooperation of the Federal Planning Bureau and the three regional statistical institutions, among which the Research Centre of the Flemish Government. On this moment, the HERMREG database is being extended to include also series with respect to production, capital stock, energy inputs and other intermediate inputs as well as labour volume (expressed in working hours). The goal of this extension is to implement a production function methodology in the HERMREG model.

The data used in this paper are sourced from the HERMREG database and run from 1980 to 2007 and include series with respect to production, labour volume and number of persons, capital stock, energy input, other intermediate inputs, wages, capital cost and prices of energy and intermediate inputs; and this of course for the three Belgian regions in the construction sector.

Table 1 contains the shares of the production factors used in the calculation of the weights for the growth decomposition for which it is supposed that both assumptions of perfect competition and constant returns to scale hold.

**Table 1: Production factor income shares (in %, averages 1980-2007)**

	Brussels	Flanders	Wallonia
capital stock [ $K$ ]	24.8%	17.6%	19.0%
labour [ $L$ ]	17.4%	19.0%	20.3%
energy input [ $E$ ]	0.1%	1.4%	1.5%
other intermediate inputs [ $M$ ]	57.7%	62.0%	59.2%

Source: Calculations based on HERMREG.

It is clear from Table 1 that in the construction sector, other intermediate inputs play an important role in the production process in the three regions reaching shares of around 60%, and even more in the Flemish Region (62.0%). The production factors labour and capital have more or less an equal important share in total production costs. The share of wages is the lowest in the Brussels Region and (17.4%), followed by the Flemish Region (19.0%) and the Walloon Region (20.3%). The share of the capital cost is somewhat lower than the wage share in the two major regions (Flanders, 17.6% and Wallonia, 19.0%), and somewhat higher in the Brussels Capital Region (24.8%).

The cost shares given in Table 1 are used as weights of the volume growth rates of each individual production factor in the growth accounting. Results of the growth accounting analysis are given in Table 2 and Figures 1,2 and 3.

**Table 2: Productivity, average growth rates (in %, 1981-2007)**

	1981-2007	1981-1990	1991-2000	2001-2007
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TFP KLEM				
Brussels Capital Region	0.91	0.50	0.63	1.92
Flemish Region	-0.15	-0.25	-0.23	0.11
Walloon Region	-0.09	-0.30	-0.17	0.34
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labour productivity per hour				
Brussels Capital Region	3.44	2.53	2.50	6.09
Flemish Region	1.98	2.28	1.71	1.93
Walloon Region	1.80	2.38	0.91	2.25
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labour productivity per head				
Brussels Capital Region	2.71	2.47	1.55	4.72
Flemish Region	1.75	2.44	1.00	1.83
Walloon Region	1.38	2.48	0.70	0.77
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Source: Calculations based on HERMREG.

Table 2 contains the average productivity growth rates for the entire sample period and different subperiods, hereby comparing alternative productivity measures: labour productivity per head, labour productivity per hour and total factor productivity growth in a KLEM setting.

A first observation is that the average total factor productivity growth over the entire sample period is negative in both the Flemish Region and the Walloon Region (respectively -0.15% and -0.09%), whereas it is positive in the Brussels Capital Region (0.91%). Although not given in Table 2, the average production growth rate is positive in all three regions (0.31% in the Brussels Capital Region, 0.62% in the Flemish Region and 0.37% in the Walloon Region). This implies that the combined input of production factors grew faster than the production output during 1981-2007.

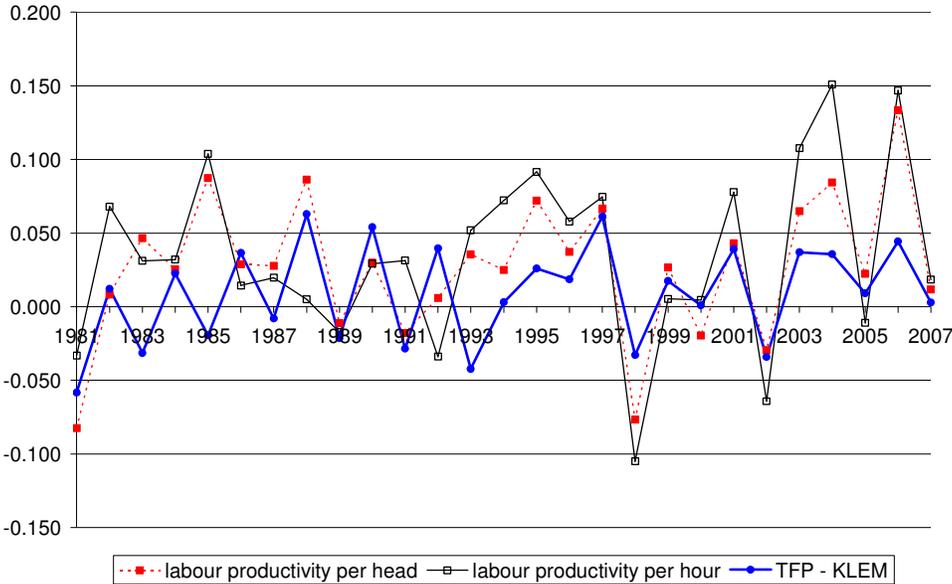
Total factor productivity growth is increasing over time in the three regions, reaching positive growth rates in the last subperiod 2001-2007, indicating that the production factors were used in a more efficient way in the production process.

The average growth rate over the entire sample period of labour productivity per hour and per worker are positive. Since in all three regions the number of working hours decreased during the sample period, labour productivity per hour is higher than the labour productivity per worker. Whereas in the first subperiod (1981-1990) labour productivity per hour is more or less equal in the three regions, this is less the case in the two subsequent periods (1991-2000 and 2001-2007). In the latter two periods it is clearly the Brussels Capital Region which has a higher labour productivity per hour.

Since the labour productivity measures do not take into account the input of the other production factors capital, energy and other intermediate input, their growth rates are significantly higher than the total factor productivity growth rate.

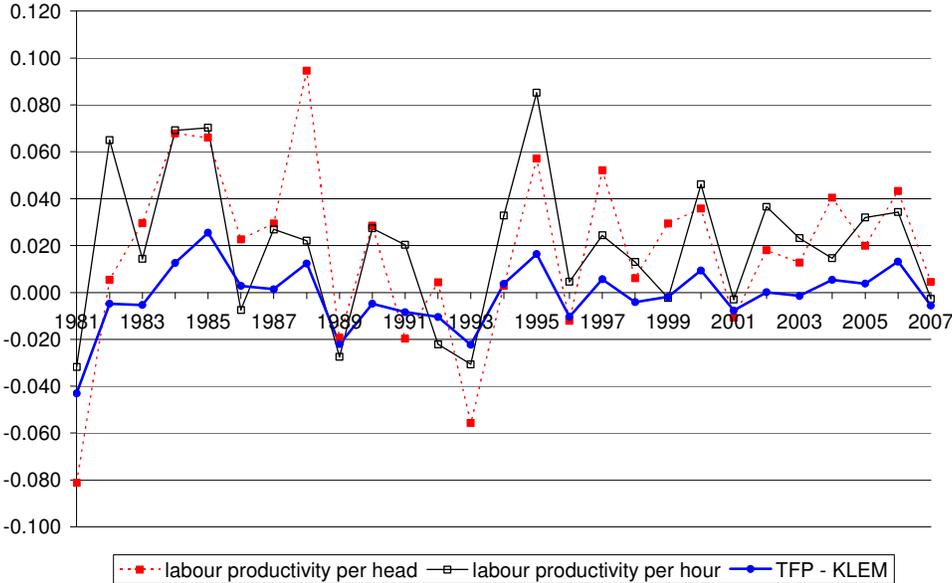
In Figures 1, 2 and 3 the different productivity measures are plotted over time for the three regions. It can e.g. be observed that productivity decreased in the first year of the sample period which was a recession year, certainly in the construction sector. It can moreover be observed that, although not perfect, the three measures tend to move in the same direction.

**Figure 1: Productivity growth rates for the Brussels Capital Region, 1981-2007**



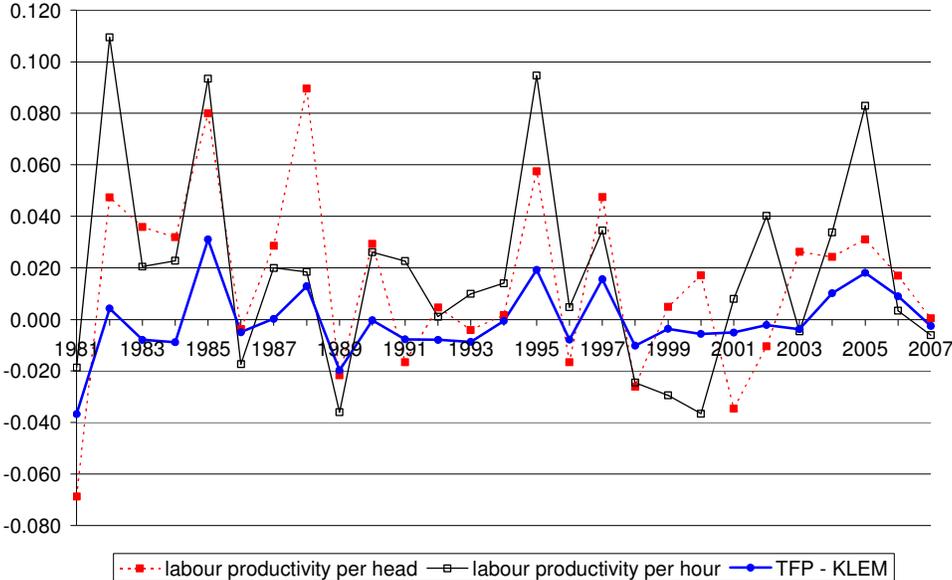
Source: Calculations based on HERMREG.

**Figure 2: Productivity growth rates for the Flemish Region, 1981-2007**



Source: Calculations based on HERMREG.

**Figure 3: Productivity growth rates for the Walloon Region, 1981-2007**



Source: Calculations based on HERMREG.

**4. Conclusion**

In the framework of the HERMREG project, the regional database is being extended to include series with respect to production, capital stock, energy input and other intermediate inputs, as well as their respective prices. The final goal is to implement a regional production function setting in the HERMREG model. An interesting by-product is a regional growth accounting analysis. In this paper, a first exercise is done for the construction sector in the three Belgian regions.

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