Rules of thumb for estimating changes in industry level employment

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Abstract

We develop four rules of thumb for estimating how industry level employment responds to changes in production. These rules can be used to estimate employment effects when labor and capital are not explicitly modeled. Many employment estimates from these types of analyses assert that labor increases in proportion to output without specifying the underlying assumptions about labor and capital. Other estimates assume that either labor or capital is fixed. We develop a simple set of calculations that embody four alternative assumptions for factor demand and supply. For the proportional returns rule, we find that employment grows at the same rate as output if the technology is Leontief or both the wage rate and rental rate of capital are fixed and technology is homothetic and exhibits constant returns to scale. If instead the capital stock is fixed, the diminishing returns rule indicates that employment increases at a greater rate than output due to the diminishing returns to capital. The value of marginal product rule shows that wages increase in proportion to labor productivity and the change in price if capital and labor are fixed. When technology is homothetic and exhibits returns to scale, the returns to scale rule shows that changes in employment are inversely proportional to the degree of the returns to scale.

Introduction

The impact on employment is important to the analysis of trade and other policies. Partial equilibrium models typically estimate changes in output and prices but do not directly model the impact on factors of production such as labor and capital. In many instances, employment estimates have simply assumed that employment changes in proportion to the volume of output without specifying what underlying assumptions have been made about the factors of production.

We describe several rules of thumb for estimating the impact on employment from a change in the volume of output. The rules specify the underlying assumptions about the supply and demand of labor and capital, focusing on how price elastic supply and the type of technology. The estimated changes in employment and wages from these four rules forms a range of values to provide a sense of the impact on employment and wages for the more general case where both wages and labor change.

First, we review other work that has estimated employment without incorporating labor and capital into the overall modeling framework. We then describe the rules and the intuition behind them (we also include the theoretical details in an appendix). Finally we use the rules to estimate the employment impact of the 35 percent safeguard duty placed on passenger car and light truck tires imported from China to the United States in 2009.
**Other work on estimating employment**

Many analyses of trade policy and employment do not explicitly model labor and capital markets and effectively assume that a one percentage change in output translates into a one percentage change in employment. For example, Hufbauer and Elliot (1994) make this assumption when estimating jobs lost in United States from the removal of tariffs and quotas on specific products. The petitioning labor union in the tires safeguard investigation (Button (2009)) assumed that employment would increase in the same proportion as output (keeping labor productivity constant) when estimating the effect of petitioners recommended quota in the Chinese tire safeguard investigation in 2009. This approach assumes a fixed proportions (Leontief) technology or both fixed input prices and a constant returns to scale homothetic technology.

However, it is not always realistic to expect that supply for labor and capital supply are perfectly elastic. In the short run, at least one factor of production (typically capital) is fixed. Francois (2009) assumes that the capital is fixed and therefore employment needs to increase at a greater rate than production because of diminishing returns to labor. Assuming that value added is a Cobb-Douglas function of capital and labor, he finds that the percentage change in employment is equal to the percentage change in output divided by the share of labor in value added.

The Cobb-Douglas assumption is restrictive because it assumes that the substitution elasticity between capital and labor is equal to one. This means that a one percentage increase in capital productivity relative to labor productivity will increase employment relative to capital by one percent. Gibson (2011) suggests estimating a range of employment effects assuming first Cobb-Douglas technology and then Leontief technology (which assumes that there is no substitution between capital and labor) when uncertain about the degree of substitutability between labor and capital. Although this does not account for the possibility that the substitution elasticity is greater than one, it does account for a wide range of substitutability.

If input prices are fixed and a homothetic technology exhibits returns to scales, the response of labor will be inversely proportional to the degree of the returns to scale. Returns to
scale occur when production consistently increases by either a larger or smaller proportion of labor and capital when labor and capital are increased by the same proportion. This is different than economies of scale which indicates that the minimum costs from any combination of capital and labor decreases as output increases.

We must also keep in mind that the gross change in jobs may be much larger than the net change in a particular industry’s employment. Jansen and Lee (2007) indicate that empirical work has shown that there is a large amount of employment reshuffling within sectors in response to trade rather than across sectors as one might expect. However, displaced workers who can find similar type of employment may be able to preserve their income. Ebenstein, et al. (2011) find that manufacturing workers who reshuffled into another manufacturing job experience almost no decline in wages, while manufacturing workers who relocate to the service sector experience a significant wage loss.

At least part of the increase in employment may come in the form of existing employees working more hours than new employees being hired. Bivens (2011) found that during the Great Recession, the aggregate hours worked fell by 30 percent more than employment, suggesting that 30 percent of the change in GDP would be absorbed by changes in working hours by existing workers and the other 70 percent would change the number of jobs.

When labor productivity increases, employment will increase at a slower rate than production. For example, Bivens (2011) notes that during the early part of an economic recovery, output can be increased through productivity gains and more work by current employees to some extent rather than new employment. Labor hoarding may also dampen changes in employment.

**The rules of thumbs**

We develop rules of thumb for estimating employment for different assumption about factor supply and demand. Factor demand is the amount of labor and capital that firms wish to use at a particular input price (wage or rental rate of capital). This demand is determined by the production function or technology used to produce the product. Factor supply is the amount of
labor and capital that workers and owners of capital wish to supply at a given input price. Here we focus on the extreme cases where either factor prices are fixed or the factor is fixed.

For the **proportional returns** rule, we assume that either the technology is Leontief or both the wage rate and rental rate of capital are fixed and technology is homothetic and exhibits constant returns to scale. If technology is Leontief, production is based on fixed proportions of value added (capital and labor) and intermediate goods (such raw materials), employment increases at the same rate as output. Employment will also change in proportion output if factor supply is perfectly elastic and technology is homothetic and exhibits constant returns to scale. Homothetic technology implies that the capital labor ratio depends solely on relative factor prices, and not their levels. Perfectly elastic factor supply implies that product supply is also perfectly elastic. This scenario would be appropriate for instances where there is unemployed capital (such as unused capacity) that can be used and unemployed labor that is mobile enough to use the unemployed capital.

For the **diminishing returns** rule, we follow Francois (2009) and assume that capital and the wage rate are fixed and labor supply is perfectly elastic. Employment increases at a higher rate than production because of the diminishing returns to labor when capital is fixed. The increase in employment depends on the increase in production and the share of labor in value added if we assume Cobb-Douglas production for value added. The smaller labor’s share of value added, the more employment increases. For example, this scenario would apply when there is unemployed labor skilled in producing the product, but little or no unused capacity.

For the **value of marginal product** rule, assume that the supply of both capital and labor are fixed, but the wage rate can change. Any increase in demand for U.S. production will increase the wage rate. Profit maximizing firms will pay higher wages in proportion to the change in the price as a multiple of marginal labor productivity. Product supply with also be perfectly inelastic because the factors of production will be fixed. This assumption is appropriate when there is no
unemployed labor skilled in producing the product or any available skilled labor is not able move to where it can be employed.

For the **returns to scale** rule, we assume that wages and the rental rate of capital are fixed so the capital labor ratio is unchanged as output changes and there are either increasing or decreasing returns to scale (and technology is homothetic). If there are increasing returns to scale, employment increases by a smaller proportion than output, depending on the degree of the returns to scale. With decreasing returns to scale, employment increases in a greater proportion than output. This would be appropriate when there is unemployed labor and capital and changing the scale of industry will either increase or decrease the degree to which production responds to changes in labor and capital.

It is also possible that labor supply increases at higher wages. As the wage rate increases, additional workers may decide to work in the industry and existing employees may decide to work additional hours. The estimated impact on employment and the wage rate from all relevant scenarios provide an estimate of the range of the impact on employment and wages when both employment and wages can vary.

The estimated changes in employment and wages are for a specific industry and do not include the impact of the import duty on any other industries or any economy-wide feedback effects from those industries. Therefore, these rules would be less helpful in estimating the change employment or wages for the entire economy.¹ Similarly, these rules estimate net changes in employment for the industry being examined and account for job turnover that most likely will occur within the industry.

The estimated changes in employment are best translated into an increase in hours worked rather than an increase in the number of workers because of the fixed costs associated

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¹ Using panel data of manufacturing industries from 1977 to 1987, Revenga (1992) finds that most of the adjustment of an adverse trade shock occurs through wages rather than employment, suggesting that labor is mobile across industries. However, the estimates represent averages for a panel of industries and may hide substantial variation across sectors.
with hiring additional workers (such as medical insurance). An estimate of the number of jobs can be estimated from historical changes in the hours worked and jobs and applying it to the estimates. Also, these rules may overestimate changes in employment to the extent that labor hoarding occurs or technical change is contributing to increases in output.

**Applying the rules to the tire safeguard duties**

To see how these rules may be applied, we estimate the employment effects of the imposition of a safeguard duty. For example, consider the 35 percent safeguard duty placed on tires imported from China in September 2009. We first use the COMPAS model and elasticity assumptions about the U.S. tire market used in the tires safeguard investigation from Fetzer (2009) to estimate changes in output. The estimates from the COMPAS model indicate the 35 percent safeguard duty would increase shipments by U.S. producers by 2 to 5 percent, which translates into a 2 to 4 increase in production assuming that exports by U.S. producers remain unchanged.2

Given the availability of some unused capacity and the decline in employment in the U.S. tire producing industry, the proportional returns rule where both labor and capital can change seems be the most appropriate of the four rules to use here.3 Using data available in September 2009, the *proportional returns rule* suggests that the duty is estimated to increase tire production and employment by 3 to 6 percent in the U.S. tire producing industry if both labor and capital can change. This is greater than the 2 to 4 percent increase in production estimated using the elasticities in the remedy memo because the supply elasticity for U.S. has been increased from 2 to 4 to infinity (actually 10,000) to account for labor and capital having perfectly elastic supply.

We also estimate the impact on employment using the other rules to illustrate their use and to provide a sense of the sensitivity of estimates to various assumptions. Many tire producers are multinational firms, suggesting that tire production may exhibit increasing returns to scale.

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2 The COMPAS model examines one market at a time, so exports to other markets are effectively assumed to be fixed. Exports made up about 14 percent of U.S. production of tires in 2009. USITC (2009), Table C-1.

3 USITC (2009), pp. III-9 and V-5.
For example, if tire production increases by twice the proportion as proportional changes in capital and labor, the **returns to scale rule** indicates that employment will increase by half as much as output increases (2 to 3 percent) if input prices are fixed as we assumed above. If the degree of returns to scale is less, employment would increase by a greater amount. If tire production technology exhibits decreasing returns to scale, then employment would increase by a greater proportion than output.

Production capacity may not be available at production facilities that would benefit from increased demand for U.S. production. If capital and the wage rate are fixed, employment is estimated to increase by 5 to 10 percent by the **diminishing returns rule**. This is greater than the 2 to 4 percent estimated increase in production due to the diminishing returns to fixed level capital.⁴

It is also possible that unemployed workers may not be located near the production facilities that would benefit from increased demand for U.S. production from an import duty. If employment and capital are both fixed, the wage rate for workers in the U.S. tire producing industry is estimated to increase by 9 to 12 percent or $2.11 to $3.11 per hour by the **value of marginal product rule**. The price of U.S. production is estimated to increase by 3 to 4 percent or $1.84 to $2.71 per tire, but output does not change since U.S. supply is assumed to not respond to price changes.⁵ This rule is measured in levels of prices and wages, rather than percentage changes.

Manufacturing workers in other industries may be enticed to work in the tire industry in exchange for a higher wage. Also, unionized tire workers may decide to use their bargaining power to negotiate some combination of higher wages and higher employment.⁶ If employment and wages are both flexible, employment would be expected to increase up to 10 percent and

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⁴ Labor is estimated to have a 45.3 percent share of value added (capital and labor).
⁵ Labor productivity is estimated to be 2.54 tires per hour worked. USITC (2009), table III-4.
⁶ However, we do not consider the possibility that unionized workers could negotiate lower wages for a higher level of employment or lower employment levels for higher wages as suggested in Revenga (1992) and Gaston and Trefler (1995).
wages would be expected to increase by up to 12 percent. Since additional workers insist on higher wages, employment and wages will both increase less than the highest estimated impact of the three rules. If capital is also flexible, we can limit the expected change in employment to be what is estimated by the proportion returns rule, or up to 6 percent.

The estimated change in new jobs in the tire industry created by the safeguard measures should be very close to the change in employment. Since the safeguard was implemented, total work hours in the rubber manufacturing sector typically increased more than new jobs, but the difference was at no point more than 10 percent, and on average less than 4 percent.

**Comparison to data trends**

We compare the estimates from these rules with actual changes in employment and wages in tire related industries. This comparison is limited because of other changes in market conditions such as the increase in the rubber prices probably also affected the tire industry. Also, the lag of the impact of the safeguard duties is not known. The actual rate of change in employment and wages in related manufacturing sectors was generally more modest than the estimated changes in employment and wages from these rules.

Between 2004 and 2008, production of tires subject to the safeguards fell by 27 percent while hours worked fell by only 17 percent and the number of production and related workers fell by only 14 percent.\(^7\) This possibly could be due to labor hoarding (tire companies retaining skilled workers in anticipation of future increases in output) or labor contracts. It is difficult to see if this is a cyclical effect since output decreased in each year. Whatever the cause, the less than proportionate response of employment to output dampen any increase in production in response to the safeguard tariffs.

Employment for workers producing all tires (including ones not subject to the safeguard measures), increased by almost 3 percent in the first three months after the safeguard duty.

7 USITC (2009), table III-4.
safeguard was enacted, and changed by about 2 percent overall in October 2011. By comparison, overall manufacturing employment decreased by just fewer than two percent in the first three months after the safeguard and has increased by only 1 percent through November 2011.8

Wages have appeared to have decreased for most of the first year after the safeguard, and not increased or remained the same as suggested by the rules of thumb. Wages and hours worked are not available for the tire industry, but are available for the rubber manufacturing industry.9 Hourly earnings in the rubber producing industry fell by about 3 percent in first few months after the safeguards and did not return to their pre-safeguard level until July 2010.10

Some of the increased employment may be due to increased hours worked by existing employees. Average weekly hours worked by production and nonsupervisory workers in the rubber producing industry increased by 5 percent in the four months after the safeguards before decreasing and fluctuating, not returning to this level until October 2011.11

Also, the price of U.S. produced passenger car tires increased by about 4 percent in first month after the safeguard was in place and then fluctuated, increasing by 9 percent in the first year after the safeguard and by another 9 percent in the second year.12 Part of this increase was most likely due to higher prices for rubber which increased in both the month after the safeguard and the first year after the safeguard. According to annual data reported by the Rubber Manufacturers Association, total U.S. shipments (including imports) of passenger car and light truck tires increased by 9 percent in 2010 and declined by 1 percent in 2011.13

The price of synthetic rubber increased by 29 percent between 2009 and 2010, while the price of natural rubber increased by 88 percent during the same period.14 A weighted average of

8 BLS data for employment, downloaded March 13, 2012.
9 Employment in the tire industry is about 41 to 44 percent of the rubber manufacturing industry.
10 BLS data for wages and hours worked, downloaded March 13, 2012.
11 Ibid.
these prices taking into account the proportion of synthetic and natural rubber used in producing passenger and light truck tires suggests that the average price of rubber increased by 50 percent between 2009 and 2010 and doubled between 2009 and 2011.\textsuperscript{15} Since rubber makes up about one half of the material costs for tires and raw material costs as a whole accounted for approximately half of the total cost tires sold, the supply shock from higher rubber prices is about 13 percent.\textsuperscript{16}

The increase in rubber prices after the safeguard may have reduced U.S. tire production and increased tire prices after the safeguard and therefore disguised the impact of the safeguard duties in the data. If the tire industry exhibited proportional or increasing returns to scale, there would be no impact from increased rubber prices because supply would is perfectly elastic and therefore not responsive to changes in input price prices. Assuming that the increase in rubber prices caused a 13 percent supply shock on the U.S. tire industry and using the default elasticities in the COMPAS model would imply that output would decline by up to 2 percent or increase by up to 1 percent despite the safeguard duty (see table 3). The value of marginal product rule implies employment declining up to 5 percent or increasing up to 3 percent. If instead supply is perfectly inelastic, the COMPAS model estimated prices to increase by 10 to 16 percent due to the safeguard with output fall by 11 percent due to the increased price of rubber. The value of margin product rule suggests that wages increased by 31 to 49 percent. The limited observed changes in price and wages and signs of increased production and employment suggest that supply is fairly elastic in the U.S. tire industry with a limited impact from rubber prices.

\textsuperscript{15} It is assumed that a typical passenger car tire is made up of almost twice as much synthetic rubber (measured by weight) as natural rubber, while a typical truck tire is made up of almost twice as much natural rubber as synthetic rubber. It is also assumed that passenger car tires make up 87 percent of the share of passenger car and light truck tires.

Conclusion

We develop rules of thumb describing how industry specific employment responds to an import duty that increases domestic production. These rules do not replace the need to model labor and capital in economywide models. However, they provide a guide to how much employment changes when employment is not explicitly modeled given the supply and demand for labor and capital. Future work may better identify where in practice the assumptions about labor supply and production technologies apply. It would also be useful to compare the calculations from these rules to employment estimates for economywide models.
<table>
<thead>
<tr>
<th>Name</th>
<th>Rule</th>
<th>Factor supply</th>
<th>Factor demand</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Proportional returns</strong></td>
<td>%Δ Labor = %Δ Output</td>
<td>Flexible capital and labor</td>
<td>Leontief/Fixed proportions</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Perfectly elastic labor and capital</td>
<td>Homothetic/CRS (ex. CES, Cobb-Douglas)</td>
</tr>
<tr>
<td><strong>Diminishing returns</strong></td>
<td>%Δ Labor = %Δ Output / Labor share</td>
<td>Fixed capital, flexible labor</td>
<td>Cobb-Douglas</td>
</tr>
<tr>
<td><strong>Value of marginal product</strong></td>
<td>ΔWage = ΔPrice * Labor share * Output / Labor</td>
<td>Fixed labor and capital</td>
<td>Cobb-Douglas</td>
</tr>
<tr>
<td><strong>Returns to scale</strong></td>
<td>%Δ Labor = %Δ Output / Returns to scale</td>
<td>Perfectly elastic labor and capital</td>
<td>Log-linear</td>
</tr>
</tbody>
</table>
Table 2-Percentage change in employment, wages, and use of capital from the 35 percent safeguard duty on tires under various assumptions of labor and capital supply.

<table>
<thead>
<tr>
<th>Rule</th>
<th>Price</th>
<th>Output</th>
<th>Employment</th>
<th>Wages</th>
<th>Change to product elasticity assumptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportional returns</td>
<td>Fixed</td>
<td>3.4 to 6.0</td>
<td>3.4 to 6.0</td>
<td>Fixed</td>
<td>Infinite U.S. supply</td>
</tr>
<tr>
<td>Diminishing returns</td>
<td>Fixed</td>
<td>0.8 to 1.4</td>
<td>4.6 to 9.7</td>
<td>Fixed</td>
<td>Standard</td>
</tr>
<tr>
<td>Value of marginal product</td>
<td>2.7 to 3.9</td>
<td>Fixed</td>
<td>Fixed</td>
<td>8.5 to 12.5</td>
<td>Fixed U.S. supply</td>
</tr>
<tr>
<td>Increasing returns to scale</td>
<td>Fixed</td>
<td>3.4 to 6.0</td>
<td>1.7 to 3.0</td>
<td>Fixed</td>
<td>Infinite U.S. supply</td>
</tr>
<tr>
<td>Range of estimates</td>
<td>up to 3.9</td>
<td>up to 6.0</td>
<td>up to 9.7</td>
<td>up to 12.5</td>
<td>---</td>
</tr>
<tr>
<td>Actual changes</td>
<td>4 to 18</td>
<td>8 to 9</td>
<td>2 to 5</td>
<td>Decreased/unchanged</td>
<td>---</td>
</tr>
</tbody>
</table>

1Returns to scale factor of 2; output changes twice as much as labor and capital

Source: Authors’ calculations.

Table 3-Percentage change in employment, wages, and use of capital from the 35 percent safeguard duty on tires under various assumptions of labor and capital supply and a 13 percent negative supply shock due to higher prices of rubber.

<table>
<thead>
<tr>
<th>Rule</th>
<th>Price</th>
<th>Output</th>
<th>Employment</th>
<th>Wages</th>
<th>Change to product elasticity assumptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportional returns</td>
<td>Fixed</td>
<td>4 to 6</td>
<td>3 to 6</td>
<td>Fixed</td>
<td>Infinite U.S. supply</td>
</tr>
<tr>
<td>Diminishing returns</td>
<td>Fixed</td>
<td>-2 to 1</td>
<td>-5 to 3</td>
<td>Fixed</td>
<td>Standard</td>
</tr>
<tr>
<td>Value of marginal product</td>
<td>31 to 49</td>
<td>Fixed</td>
<td>Fixed</td>
<td>51 to 87</td>
<td>Fixed U.S. supply</td>
</tr>
<tr>
<td>Increasing returns to scale</td>
<td>Fixed</td>
<td>4 to 6</td>
<td>2 to 3</td>
<td>Fixed</td>
<td>Infinite U.S. supply</td>
</tr>
<tr>
<td>Actual changes</td>
<td>4 to 18</td>
<td>8 to 9</td>
<td>2 to 5</td>
<td>Decreased/unchanged</td>
<td>---</td>
</tr>
</tbody>
</table>

1Returns to scale factor of 2; output changes twice as much as labor and capital

Source: Authors’ calculations.
Theoretical appendix

For estimating the effects on employment and capital, we assume that good $x_j$ is produced from a Leontief production function of value added $VA_j$ and intermediate inputs $I_j$ such that

\[(1) \quad x_i = \min[VA_i, I_i].\]

It follows that for any change in output, if value added and intermediate inputs increase by the same proportion, the output will also increase by that proportion

\[(2) \quad \frac{\partial x_i}{x_i} = \frac{\partial VA_i}{VA_i} = \frac{\partial I_i}{I_i}.\]

If we assume further that value added from capital and labor come from a fixed proportions or Leontief technology, then labor must increase by at least the same proportion as output and by the same proportion if additional capital is also increased by the same proportion:

\[(3) \quad \frac{\partial x_i}{x_i} = \frac{\partial VA_i}{VA_i} = \frac{\partial L_i}{L_i} = \frac{\partial K_i}{K_i},\]

And therefore

\[(4) \quad \frac{\partial L_i}{L_i} = \frac{\partial K_i}{K_i}.\]

This would also be true if labor, capital, and other inputs are increased in the same proportions in any homothetic production function with constant returns to scale (such a Cobb-Douglas or CES production function). For example, if the value added function is assumed to take a Cobb-Douglas form as assumed in Francois (2008),

\[(4) \quad VA_i = A_i L_i^\alpha K_i^{1-\alpha}.\]

the percentage change in value added will be proportional to the percentage change in labor and labor’s share in value added plus the percentage change in capital and capital’s share in value,

\[(5) \quad \frac{\partial VA_i}{VA_i} = \alpha \frac{\partial L_i}{L_i} + (1 - \alpha) \frac{\partial K_i}{K_i}.\]
If wages and the rental rate of capital are fixed, labor and capital will grow at the same rate, and so will value added and output.

\[ \dot{L}_i = \dot{x}_i \]

(5)

If wages and the rental rate of capital change can change as can labor and capital, the growth rate in employment will be increasing in the growth rate of value added and therefore output, but decreasing the in growth rate of capital. Because of our Leontief production function, percentage changes in value added are proportional to percentage changes in production (see (2)). Therefore, percentage changes in employment will be proportional to percentage changes in production and inversely proportional to labor’s share in value added,

\[ \dot{L}_i = \frac{\dot{x}_i}{\alpha} - \left( \frac{1}{\alpha} - 1 \right) \dot{K}_i \]

(6)

The growth in output will be between the growth rates of labor and capital. Therefore, if labor grows at a higher rate than capital, it will also grow a higher rate than output due to diminishing returns to slower growing rate of capital. Labor growth will be smaller than output growth when capital grows at a greater rate. The smaller labor’s share of value added, the less an increase in labor will increase value added because of the diminishing returns to capital.

If capital is fixed in the short run, (6) reduces to

\[ \dot{L}_i = \frac{\dot{x}_i}{\alpha} , \]

(7)

as in Francois (2008).

If both capital and labor are fixed, but the wage rate is allowed to respond to changes in labor demand. For a given rental rate of capital, a profit maximizing firm will offer wages that are proportional to changes in labor productivity. Cobb-Douglas demand for labor for a profit maximizing firm takes the form of
(8) \[ L_i = \alpha \frac{P_i x_i}{w_i}. \]

Rearranging this to make the wage a function of the price:

(9) \[ w_i = \alpha \frac{x_i}{L_i} p_i. \]

Taking the derivative of wages with respect to the price level, we get:

(10) \[ \frac{\partial w_i}{\partial p_i} = \alpha \frac{x_i}{L_i}. \]

where the degree the wage rate increases depends on the change in value of the marginal product of labor. Even the prices of the good increase the price for intermediate goods, the average productivity of labor will be a lower bound on the change in wages from a change in price.

A value added function that is a log-linear form similar to Cobb-Douglas, but is not constant returns to

(11) \[ VA_i = A_i L_i^\alpha K_i^{1-\beta}. \]

Where \( \alpha + \beta \neq 1 \) the percentage change in value added will be proportional to the percentage change in labor and labor’s share in value added plus the percentage change in capital and capital’s share in value,

(12) \[ \frac{\partial VA_i}{VA_i} = \alpha \frac{\partial L_i}{L_i} + \beta \frac{\partial K_i}{K_i}. \]

If wages and the rental rate of capital are fixed, labor and capital will grow at the same rate and value added and output will grow in a \( \alpha + \beta \) proportion,

(12) \[ \hat{L}_i = \frac{\hat{x}_i}{\alpha + \beta}. \]

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If we have increasing returns to $\alpha + \beta > 1$ and labor will grow in smaller proportion than output. If there are decreasing returns to scale, $\alpha + \beta < 1$ and labor will grow in a larger proportion than output depending on the degree of the returns to scale.
Bibliography


