

# CONNECTEDNESS CYCLES IN EQUITY MARKETS: A WAVELET APPROACH

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## Abstract

The connectedness of international equity markets can be measured building on the well-established forecast error variance decomposition framework. This approach permits the assessment of the propagation of shocks (spillovers) across equity markets on a day-to-day basis. The focus of our contribution is on detecting cycles in the intensity of spillovers. As it is vital for enterprise managers to track business cycles, it is vital for investors seeking to diversify their portfolios to track the cyclical nature of spillovers. Our approach provides cycle information almost in real time, while business cycles are identified not before a cycle has been completed. We apply forecast error variance decomposition in a vector autoregression (VAR) model to rolling time windows to derive indices of spillover of shocks between markets. The time series of spillover indices is then analyzed by means of continuous Morlet wavelet transforms in order to obtain dynamic insight into the spillover's composition of cycles. Our empirical basis consists of daily equity market data from 1988 through 2012 from the USA, Germany, France, and Japan. International connectedness has increased significantly during the last two decades, while its variability has diminished. We find patterns consisting of superpositions of cycles in the spillover series, where the prevailing cycles have recently become longer. The composition of different cycles in the spillover series has changed as well, and is now confined to a narrower band of frequencies than in the late 1980s.

**Key words:** Connectedness of equity markets; spillovers; wavelets; cycles

## 1 Introduction

The beginning of the modern study of business cycles could be attributed to the formalization of the notion by Burns and Mitchell [5]. Their definition of the business cycle as "... a type of fluctuation found in aggregate economic activity..." is the starting point for any business cycle analysis. Much of the early work on business cycles was implemented for the US economy. However, European or euro-area business cycles have also been the topic of much recent study. The National Bureau of Economic Research (NBER) Business Cycle Dating Committee maintains a chronology of the U.S. business cycle. The Committee examines and compares the behavior of various measures of broad activity such as real GDP measured on the product and income sides, economy-wide employment, and real income as well as indicators that do not cover the entire economy, such as real sales and the Federal Reserve's index of industrial production (IP).<sup>1</sup>

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<sup>1</sup>[www.nber.org/cycles/recessions.html](http://www.nber.org/cycles/recessions.html). Accessed on 2012-04-09.

Altug [2] gives an overview of the modern theory and empirics of business cycles. Apart from identifying the business cycles for individual countries, the research on business cycle fluctuations has shifted its focus on the cross-country differences and similarities of macroeconomic fluctuations in developed countries. Kose et al. [12] investigate the common dynamic properties of business-cycle fluctuations across countries and find that a common world factor is an important source of volatility for aggregates in most countries, providing evidence for a world business cycle. Besides, Kose et al. [13] analyze the evolution of the degree of global cyclical interdependence using a 106 country sample divided into three groups — industrial countries, emerging markets, and other developing economies. They find some convergence of business cycle fluctuations among the groups of industrial and emerging market economies but divergence (or decoupling) between them.

Altug and Bildirici [3] study business cycle phenomena in a sample of 27 developed and developing economies and document the importance of heterogeneity of individual countries' experiences. From the point of view of an enterprise with operations spread over many countries, the importance of synchronization of business cycles lies in the fact that if countries have asymmetric business cycles, then it may not be optimal to have the same policies applied to every country. The key decisions should be based on how synchronized as well as how connected the countries are. Even though there exists significant heterogeneity among the countries' business cycle characteristics, as Imbs [11] finds, regions with strong financial links are significantly more synchronized. The recent global financial crisis which was initiated by spreading of risks due to "toxic-assets" mainly embedded in the multinational financial institutions' balance sheets also provided evidence that not only business cycles but also the ever increasing interconnectedness of markets should be a key ingredient in the decision making processes of enterprises as well as investors for their day-to-day as well as long-term decisions. The interconnection of financial markets has been associated with the justification for intervening and bailing out institutions that are "too big to fail" by the policy makers and central banks around the world.

In this project, we aim to identify the type of relationships between the economies' overall performances summarized by their GDPs coupled with their synchronization in their equity markets connectedness measures. To analyze connectedness of markets we follow the approach defined by Diebold and Yilmaz [9], which quantifies the connectedness based on forecast error variance decompositions and reverts to the network literature. Using the Diebold and Yilmaz [9] methodology, we compute daily connectedness measures for the US, German, French and Japanese stock markets with daily market return data from 1988 to 2012, and study these measures by analyzing their cyclicalities using wavelet transformations.

Wavelets have been used for the analysis of cyclical phenomena since the early 1980s. Applications in signal processing, medicine, physics, meteorology and astronomy abound, but uses of wavelets in economic applications are more recent, and the use of wavelets for business cycle analysis is still even scarcer.

Gencay et al. [10] provide a unified view of filtering techniques with a special focus on wavelet analysis in finance and economics. Yogo [19] finds that the business cycle component of the wavelet-filtered real US GDP series closely resembles the series filtered by the approximate band-pass filter proposed by Baxter and King [4]. Raihan et al. [17], using wavelets, try to characterize the timing of shocks that trigger the business cycle, as well as situations where the frequency of the business cycle shifts in time.

Crowley et al. [7] analyze growth cycles of the core of the euro area in terms of frequency content and phasing of cycles and find that coherence and phasing between the three core members of the euro area (France, Germany and Italy) have increased since the launch of the euro. They conclude that "... ECB might be acting through monetary policy to 'couple' the synchronicity of cycles within the euro area". Aguiar-Conraria and Soares [1] use wavelet analysis

to study business cycle synchronization across the EU-15 and the Euro-12 countries. Among their conclusions is that France and Germany form the core of the Euro land, being the most synchronized countries with the rest of Europe. They also find that the French business cycle has been leading the German business cycle as well as the rest of Europe.

In applying the techniques of wavelet analysis to spillover measures between stock markets, our hypotheses are:

- Higher-frequency phenomena, invisible to classical business cycle methods, can be discovered in market connectedness.
- Lower-frequency results from GDP-based business cycle analysis are in line with market connectedness characteristics.

The first hypothesis is suggested by the data structure underlying classical business cycle research, which is mostly given as quarterly time series, and hence provides little possibility of insight into high-frequency (with periodicity less than three months) phenomena. Insight into this kind of periodicity is of potential use for policy makers as well as for investors.

The second hypothesis states that the insight we gain with respect to lower-frequency, and hence higher period length, does not contradict the findings of classical business cycle theory.

This paper is organized as follows. Section 2 describes some properties of the data on which the study is based, and reviews the methodology developed by Diebold and Yilmaz [9] as far as relevant for our purposes. Section 3 reviews some aspects of wavelet analysis of a time series. Empirical results concerning the overall connectedness of the four stock markets considered are presented in Section 4, followed by Section 5, which presents results concerning pairwise spillovers. Section 6 concludes and discusses suggestions for further research.

## 2 Data and fevds

The empirical starting point of the present study consists of four time series of daily closing quotations of the stock indices Dow Jones Industrial Average (New York Stock Exchange, in the following called *dji*), the CAC 40 (*fchi*, Euronext Paris), DAX (*gdaxi*, Frankfurt Stock Exchange), and Nikkei 225 (*n225*, Tokyo Stock Exchange) in the time period beginning 1988-01-05 and ending 2012-01-24 (6255 observations). The level series, normed such that 1988-05-23  $\equiv 1$ , are shown in Figure 1. (The reason for selecting 1988-05-23 as base day is that a sequence of windows, each 100 days long, is used for fitting VAR models to the data, so that this base day is the first day for which results are available.) Directly used are daily simple returns in percent, which are plotted in Figure 2.

A visual inspection of the return series in Figure 2 suggests a simultaneous occurrence of periods of high volatility in the four markets considered, and the impression that returns are somehow “connected” is reinforced in Figure 3. However, it appears that *n225* is detached from the other three stock indices in the sense of lower correlation of daily returns. The main tool to measure the amount of connectedness is based on a decomposition of the forecast error variance (*fevd* stands for *forecast error variance decomposition*), which will be briefly described next.

Given a multivariate (here: 4-dimensional) empirical time series, the *fevd* results from the following steps:

1. Fit a standard VAR (vector autoregressive) model to the series.
2. Using series data up to, and including, time  $t$ , establish an  $n$  period ahead forecast (up to time  $t + n$ ).

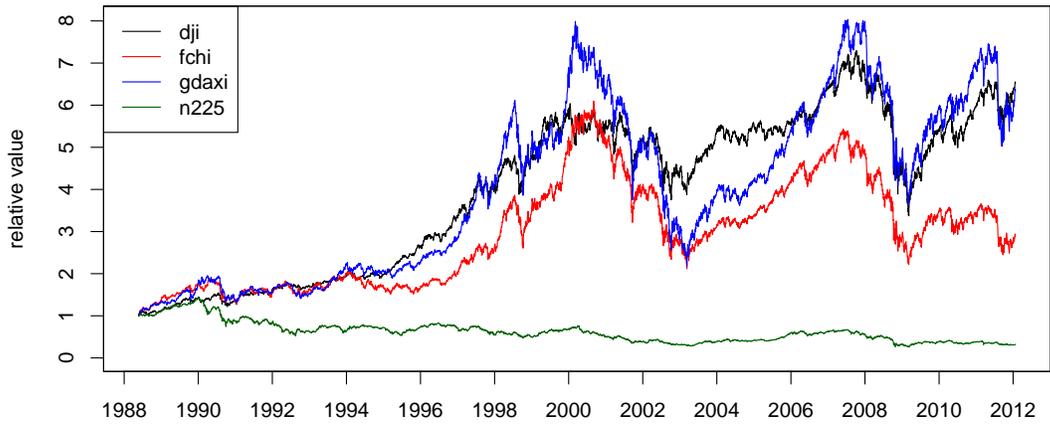


Figure 1: The level series (with 1988-05-23  $\equiv$  1)

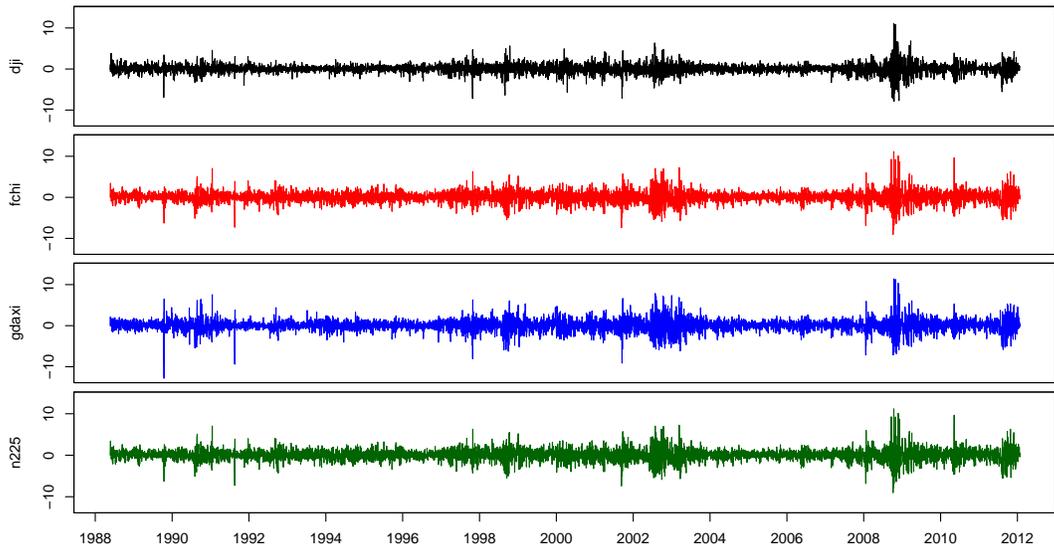


Figure 2: The series of daily returns

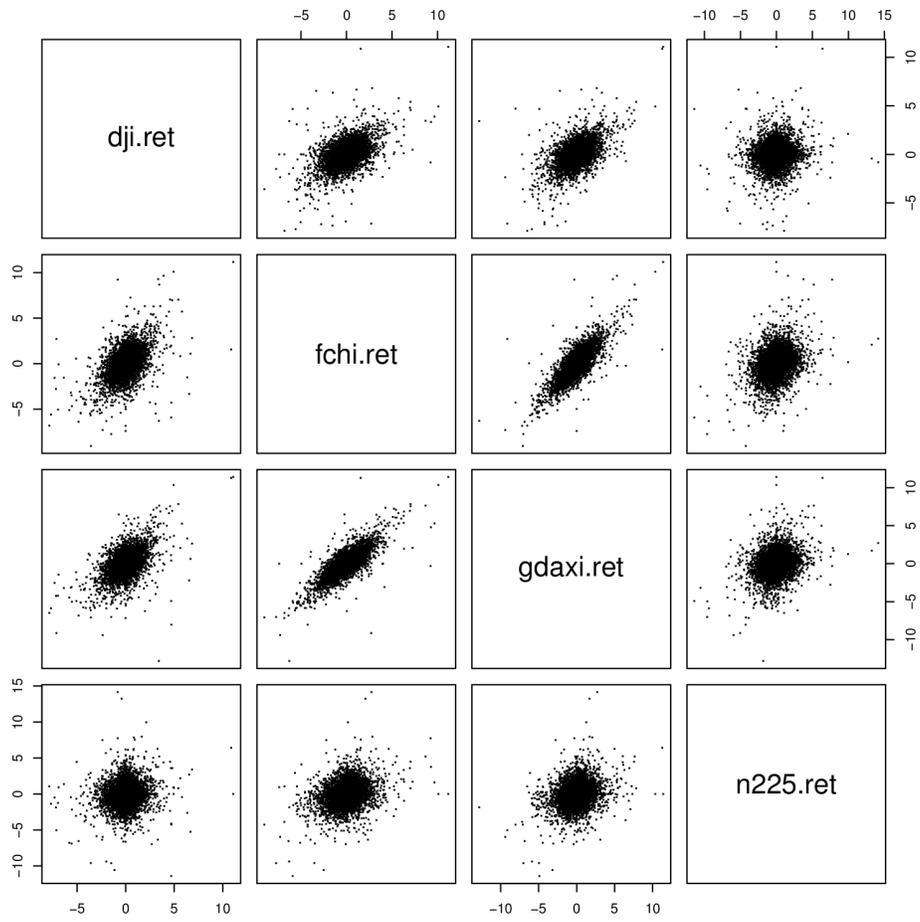


Figure 3: Scatterplots of daily returns

- Decompose the error variance of the forecast for each component with respect to shocks from the same or other components at time  $t$ .

Diebold and Yilmaz [8] propose several connectedness measures derived from variance decompositions, and they argue that these measures are intimately related to key measures of connectedness as used in the network literature. The decomposition of forecast error variance is given in terms of the structural VAR. This needs to be identified on the basis of the standard VAR by imposing suitable restrictions in the form of an ordering of the variables. To circumvent the undesirable dependence on the ordering, we revert to the so-called generalized fevd, proposed by Pesaran and Shin [16], which remedies the dependence on the ordering by giving each component of the series in question first priority. For example, the fevd for returns on dji is given as

$$\text{var} \left\{ \sum_{i=0}^{n-1} (\Phi_{\text{dji}}^{\text{dji}}(i), \Phi_{\text{dji}}^{\text{fchi}}(i), \Phi_{\text{dji}}^{\text{gdaxi}}(i), \Phi_{\text{dji}}^{\text{n225}}(i)) \times \begin{pmatrix} \epsilon_{\text{dji}, t+n-i} \\ \epsilon_{\text{fchi}, t+n-i} \\ \epsilon_{\text{gdaxi}, t+n-i} \\ \epsilon_{\text{n225}, t+n-i} \end{pmatrix} \right\}, \quad (1)$$

which equals

$$\begin{aligned} & \sigma_{\text{dji}}^2 \cdot \sum_{i=0}^{n-1} (\Phi_{\text{dji}}^{\text{dji}})^2(i) + \sigma_{\text{fchi}}^2 \cdot \sum_{i=0}^{n-1} (\Phi_{\text{dji}}^{\text{fchi}})^2(i) + \\ & \sigma_{\text{gdaxi}}^2 \cdot \sum_{i=0}^{n-1} (\Phi_{\text{dji}}^{\text{gdaxi}})^2(i) + \sigma_{\text{n225}}^2 \cdot \sum_{i=0}^{n-1} (\Phi_{\text{dji}}^{\text{n225}})^2(i) \end{aligned}, \quad (2)$$

where each  $\Phi_l^k$  designates an impulse response function from series  $k$  to series  $l$ . Schematically, the spillover index is then obtained as

$$\frac{\sum \blacksquare}{\sum \blacksquare + \sum \square}, \quad (3)$$

where the symbols are defined according to the following so-called spillover table of component-wise fevds (each square in a row equals one of the terms in the sum in Equation 2, for each row replacing dji with the index in the corresponding row):

		from (time $t$ )			
		dji	fchi	gdaxi	n225
to (time $t+n$ )	dji	□	■	■	■
	fchi	■	□	■	■
	gdaxi	■	■	□	■
	n225	■	■	■	□

(4)

### 3 Wavelets

Connectedness cycles of different frequencies and of limited duration may overlap in time. A time and frequency resolution dilemma (resulting from the Heisenberg uncertainty principle) arises which can be resolved to some extent by means of wavelet analysis. Wavelets were first introduced in signal analysis to retrieve information on the phase and amplitude of seismic signals, and how these characteristics may change versus propagation time; cf. Morlet et al. [14], [15]. Unlike Fourier transforms, they allow the decomposition of a time series into the time and frequency domain simultaneously. We apply a continuous complex wavelet transform, the so-called Morlet wavelet, which is defined by

$$\psi_0(\eta) = \pi^{-1/4} e^{i6\eta} e^{-\eta^2/2}$$

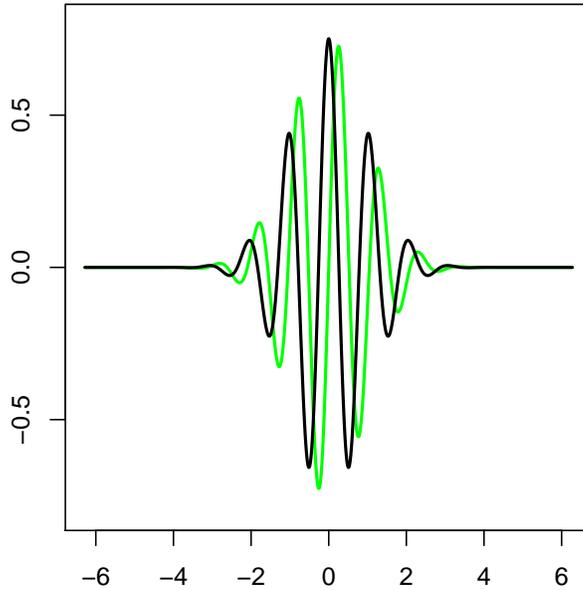


Figure 4: The Morlet mother wavelet (Morlet et al. [14])

(the mother wavelet; the number of oscillations is set to 6), and depicted in Figure 4.

We use a discretized version of the Morlet transform, matching the daily data structure on which the current study is based. The Morlet transform  $W_n(s)$  at scale  $s$  and translated by the localized time index  $n$  is then defined as the convolution of the data series with the accordingly scaled and translated version of  $\psi_0(\eta)$ . By choosing a set of different scales, usually fractional powers of 2, and translating the wavelet along time, the amplitude  $|W_n(s)|$  of any periodic signal versus the scale can be retrieved, as well as how this amplitude evolves with time. Torrence and Compo. [18] have provided the R package “dplR” performing this transformation; a typical contour plot of the local wavelet power spectrum  $|W_n(s)|^2$  as output is shown in Figure 6, with the period instead of scale on the vertical axis. The authors show that the power spectrum is essentially  $\chi^2$ -distributed; 95% confidence contour lines (the black lines) are added to the plot.

While the amplitudes  $|W_n(s)|$  show the oscillations of individual components of the decomposed time series, the phases  $\tan^{-1} \left( \frac{\text{Im}(W_n(s))}{\text{Re}(W_n(s))} \right)$  of the transform carry information about the location of local extrema across scales; cf. Carmona et al. [6]. In the time domain, they measure the displacements of the signal components relative to a fixed origin of time, allowing conclusions about their synchronicity.

## 4 Empirical results, part 1: overall spillover

A spillover index series was obtained by proceeding along the steps outlined in Section 3, where a moving window of 100 days was used for fitting a sequence of VARs, resulting in a spillover table for every day, which leads to the spillover index according to Equation (3). The result of this operation is shown in Figure 5, together with a smoothed version (obtained by local polynomial regression fitting) of the series. This series, and hence market connectedness, has been strongly increasing since 1994, with intermittent shorter periods of decline. The smoothed series was used to detrend the spillover index series, and the wavelet transform was then applied to the detrended series. Figures 6, 7 and 8 display the results of the wavelet transform. It turned out that these results are robust with respect to the degree of smoothing applied when detrending

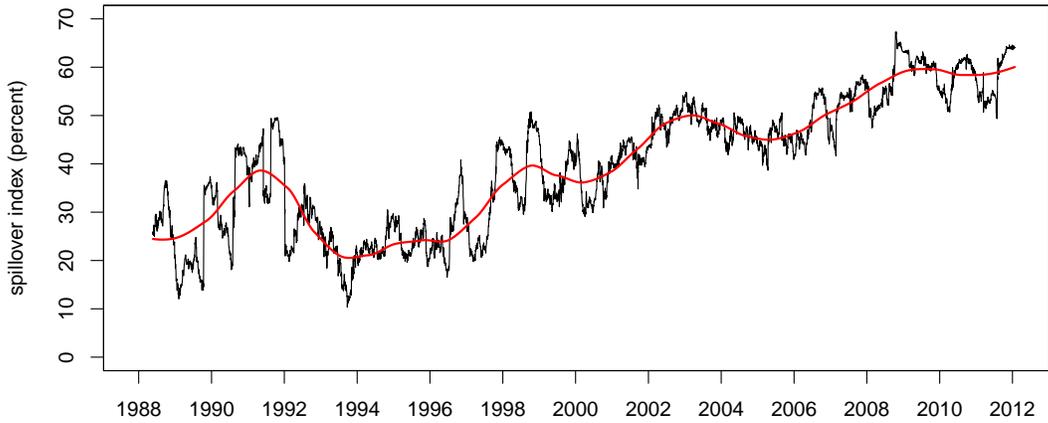


Figure 5: The spillover index series

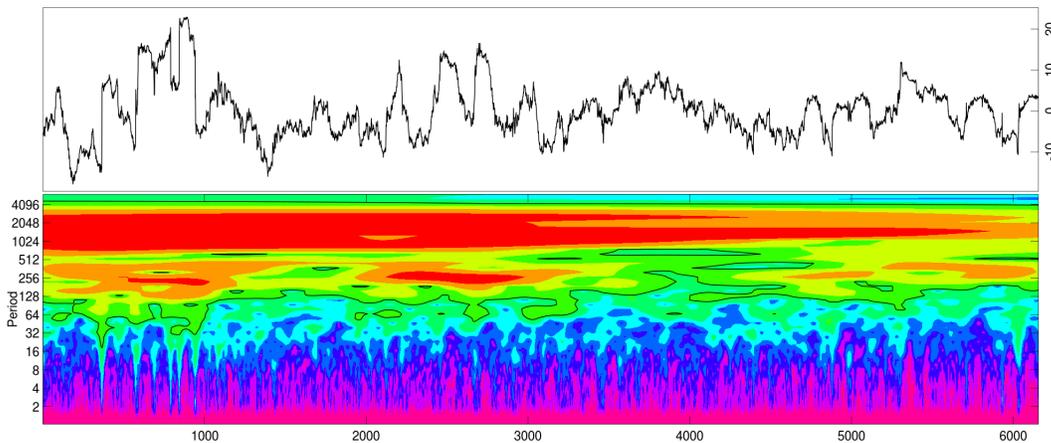


Figure 6: Power spectrum of the spillover index series

the spillover index series.

The detrended spillover series together with a contour plot of its wavelet power spectrum is depicted in Figure 6. The power spectrum gives information on the relative power of a wavelet component at a certain scale (or period length, the vertical axis) and at a certain location in time (the horizontal axis). The red area indicates highest power. In our plot, it corresponds to lower frequency components (i.e. with higher period lengths), the bandwidth of which clearly diminished between 1988 and 2012. While an overlap of a broad spectrum of wavelets seems to constitute the ridge of oscillations in the spillover series during the first half of the time interval considered, the oscillatory characteristics sharpen with a tendency to increasing period lengths (above 1024 days, i.e. 4 years) in the second half.

The displays of phases and amplitudes in Figures 7 and 8 confirm this impression. The amplitudes tend to decline, and so do the phases (and hence wavelet translations) as time proceeds.

## 5 Empirical results, part 2: pairwise spillovers

Pairwise spillovers (from one stock index to another) are given by the terms of the sum in Equation (2), with any index name (dji, fchi, gdaxi, or n225 substituted for dji in  $\Phi_{dji}^\bullet$ ); each of

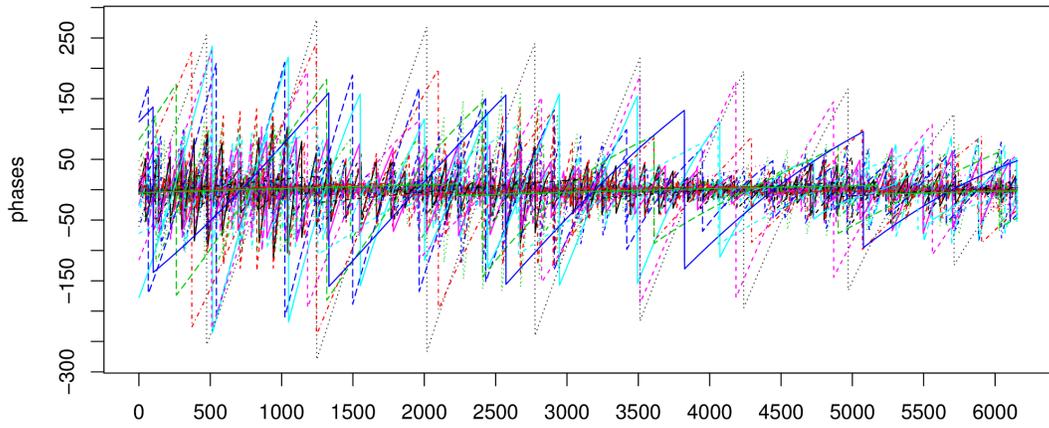


Figure 7: Phases of the spillover index series

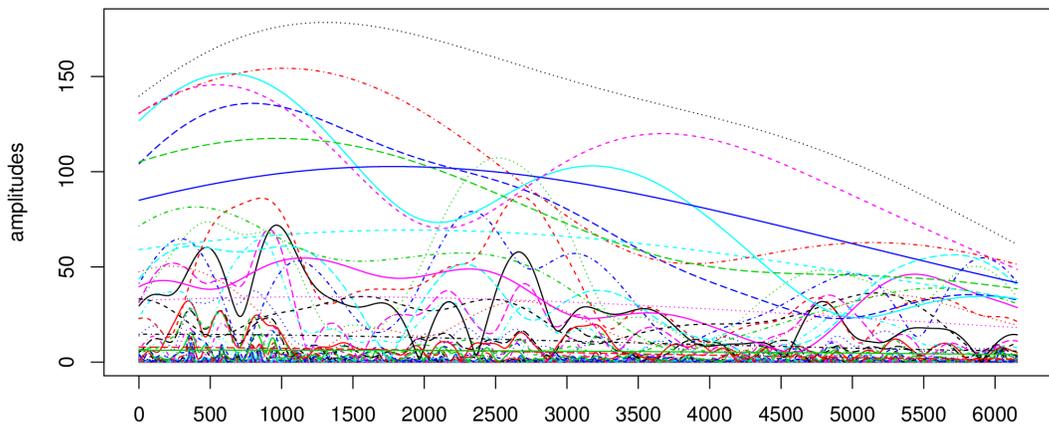


Figure 8: Amplitudes of the spillover index series

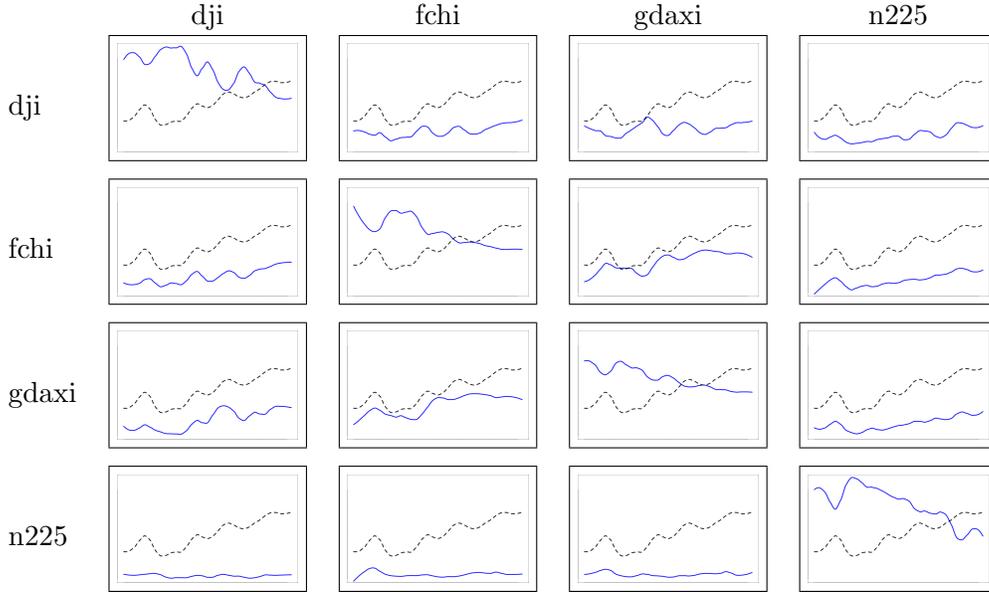


Figure 9: Pairwise spillover series, smoothed

the 16 squares in the spillover table (see (4)) stands for a pairwise spillover.

Figure 9 shows the 16 smoothed spillover series, together with the spillover index series (the dashed line), for purposes of comparison. The spillover series located off the diagonal in Figure 9 are mostly increasing, while those on the diagonal are mostly decreasing, which indicates increasing importance of impacts across borders and therefore increasing connectedness among markets.

Periodic characteristics of the (unsmoothed, but detrended) spillover series in the form of power spectrum are displayed in Figure 10, where only areas indicating significant periods were retained. The lower bound of the red area, showing the lowest frequency still significant for a given time  $t$  ( $t = 1, \dots, 6255$ ), has a tendency to increase in most cases (with the notable exception of Japan): Significant periods in these spillover series have been increasing since the late 1980. At the beginning of the period considered, some of the significant periods were as low as 64 (for combinations dji-gdaxi, fchi-fchi, gdaxi-fchi, gdaxi-gdaxi); these periods have disappeared. No big trends could be observed for pairs with n225 as a component.

## 6 Summary and conclusions

The present study focuses on revealing the frequency structure of stock market connectedness on a daily basis, covering four major stock indices: DJIA, CAC 40, DAX, and Nikkei 225.

An analysis of the daily spillover index series based on Morlet wavelet transforms reveals that the shorter periods (and hence higher frequencies) have gradually lost their importance in the time period reaching from the late 1980s through 2012, and that the periods prevailing in this series have thus gradually become longer. This phenomenon could also be observed for pairwise spillovers, with the exception of the Japanese stock market, which seems not to have altered its frequency character during the time period considered. To be specific, significant periods of length less than 128 days can be found in pairwise spillovers before the year 2000, which remain important in the case of the Japanese stock market to date.

Our findings further suggest that cyclicity in the spillover series does not reduce to simple peaks and troughs, but is rather composed of an intricate superposition of frequencies. We could

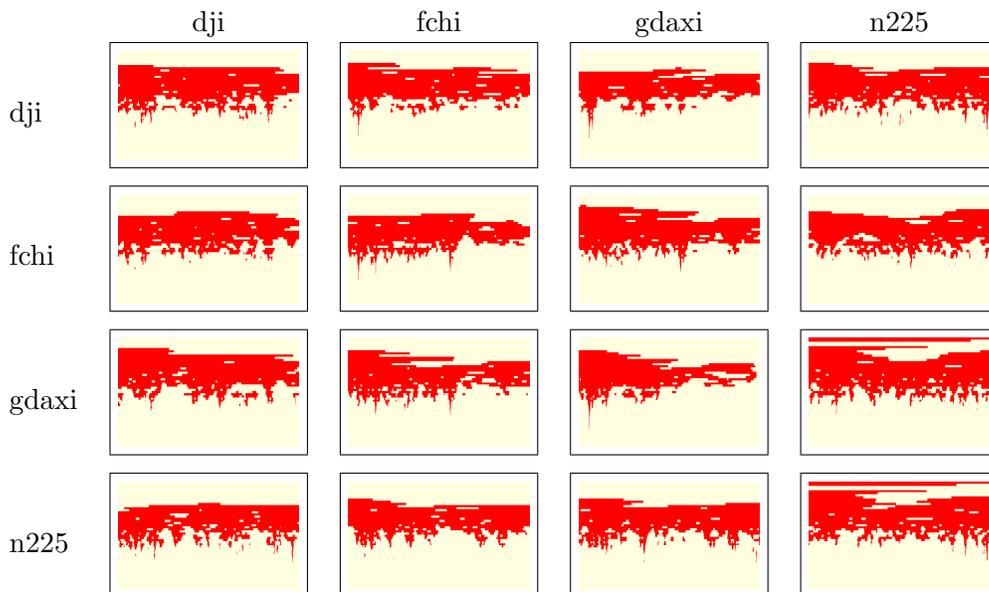


Figure 10: Pairwise power spectrums

identify certain secular trends in the spillover index series, namely:

- increasing connectedness among stock markets,
- amplitudes which are decreasing in magnitude,
- diminishing importance of high-frequency phenomena, especially between closely integrated markets.

These phenomena could be made visible owing to the timing of the data (namely daily returns on stock indices) on which this study is based; they would have to remain hidden if the empirical starting point had a longer spacing, as is the case for quarterly data.

An investigation of the question in how far our results are in line with classical business cycle theory remains a task to be accomplished.

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