Frequency aspects of information transmission in networks of equity markets

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Abstract

The dating of cyclical phenomena in economies, such as business cycles, is at the core of economic policy research. Moreover, policy decisions which are due to affect interacting economies should take into account the economies’ connectedness and synchronicity. The cross-country analysis of business cycles is conceptually close to the study of “contagion”, focusing on determinants of an economy’s susceptibility to, respectively responsibility for, shocks or more general spillover effects in both times of crisis and non-crisis.

Our analysis is based on VAR models in stock index return series and forecast error variance decomposition, resulting in return-to-volatility spillovers. This methodology allows to identify a stock market’s potential to act as a news disseminator, and we investigate frequency aspects of information transmission in a network of three Western equity markets: Dow Jones Industrial Average (New York), FTSE 100 (London) and Euro Stoxx 50 (euro area).

We find evidence that the range of relevant frequencies has become narrower, which may have an explanation in terms of the increasing intensity of information exchange and shrinking holding times of stocks. Furthermore, we find that the U.S. market is in anti-phase with the European markets, while the European markets are in phase.

Key words: Equity market connectedness; propagation values; cycles; synchronization; wavelets; phase-difference

1 Introduction

The study of frequency aspects in economic time series dates back to efforts to forecast the future of economies in the 19th century. Among the first to identify economic cycles and their synchronicity was Juglar [14] in 1862. He proposed 7–11 year cycles of fixed capital investments which were roughly synchronous for France, the UK and the US. The early 20th century witnessed a series of further proposals: the Kitchin [15] cycles with 3–5 years of periodicity associated with fluctuations of inventories; the Kuznets [19] swings of 15–25 years associated with infrastructure investments; the Kondratieff [16] “Long Waves” of 40–60 years as well as smaller cycles of 3–4 and 7–10 years. The beginning of the modern analysis could be attributed to the formalization of the notion by Burns and Mitchell [4], who define a business cycle to be “...a type of fluctuation found in aggregate economic activity”.

With its focus on GDP, the prime target of classical business cycle analysis is to investigate domestic aspects of an economy, see Altug [3], but business cycle research has recently also been

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undertaken to study cross-country differences and similarities of macroeconomic fluctuations. For example, Imbs [13] finds that regions with strong financial links are significantly more synchronized. Kose et al. [17] investigate the common dynamic properties of business-cycle fluctuations across countries and find that a common world factor is an important source of volatility for aggregates in most countries, providing evidence for a world business cycle. Besides, Kose et al. [18] analyze the evolution of the degree of global cyclical interdependence using a 106 country sample divided into three groups — industrial countries, emerging markets, and other developing economies. They find some convergence of business cycle fluctuations within the groups of industrial and emerging market economies, but divergence (or decoupling) between them. Based on wavelets applied to quarterly GDP growth data, Crowley and Mayes [7] analyze growth cycles of the core of the euro area in terms of frequency content and phasing of cycles and find that coherence and phasing between the three core members of the euro area (France, Germany and Italy) have increased since the launch of the euro. They conclude that “... ECB might be acting through monetary policy to ‘couple’ the synchronicity of cycles within the euro area”. Among the conclusions of Aguiar-Conraria and Soares [1]’s analysis (also wavelet-based, with monthly industrial production index data) is that France and Germany, being the most synchronized economies, form the core of the euro area. They also find that the French business cycle has been leading the German business cycle as well as other economies within the euro area.

The cross-country analysis of business cycles is conceptually close to the notion of contagion, which is “the cross-country transmission of shocks or the general cross-country spillover effects,” according to the World Bank.¹ In a similar vein, the present paper is an effort to investigate frequency aspects of information transmission in a network of equity markets.

One approach to analyze cross-country return-to-volatility spillovers between financial markets (or assets) has been proposed by Diebold and Yilmaz [8, 9]. It builds on the idea that the risk associated with investing in a market (an asset) can be estimated by the variance of the error when forecasting a future return on the market index (asset) price. A vector autoregressive (VAR) model can be used to derive the forecast error variance for each market (asset) in a set, and the decomposition with respect to its origin. This approach provides a framework to discuss pairwise spillovers, arranged in so-called spillover tables, and Diebold and Yilmaz [8] suggest a summary measure for the degree of market connectedness, the so-called spillover index. In a recent article, Diebold and Yilmaz [9] revert to the network literature, drawing a network perspective of this approach, with markets (assets) as nodes and pairwise spillovers as edge weights. Schmidbauer et al. [24] suggest two perspectives broadening the scope of this methodology: a focus on the current state of a network of markets, permitting an assessment of the vulnerability of the network to unforeseen shocks and the identification of a market’s potential to act as news disseminator which they call “propagation value”, and secondly, a focus on the network’s interday dynamics and the process of news creation.

Building on the methodology outlined above, the present study investigates the relative importance of three Western equity markets with respect to information transmission: Dow Jones Industrial Average (U.S.), FTSE 100 (U.K.) and Euro Stoxx 50 (proxy for euro area).

The average holding period of stocks has dropped secularly in all markets studied over our analysis period. Haldane [12] reports that the mean duration for the US equity holdings has dropped from around 7 years (in 1940) to around 7 months (in 2007), for the UK market, the similar trend is observed with average holding period of stocks around 5 years (in mid-1960s) to 7.5 months (in 2007). Furthermore, at the international level this trend is also confirmed for the major equity markets, for the Shanghai stock index, the mean duration is closer to 6 months. Decreasing transaction costs as well as advances in High-Frequency-Trading (HFT)

technology, which allows transactions in milli- or micro-seconds, are believed to have impact on the decreasing average holding periods (see Haldane [12]).

However, not all perceive the fact positively. Regulators have reported increasing concerns on the issue. And it was reported that some global investors have plans “...to discuss whether companies should offer special shares to reward long-term holders in a short-term world.”, (see The WSJ, 2013-03-22).

For the three equity markets in our study, we obtain propagation values, updated on a daily basis, to serve as proxies for their relative importance in the network with respect to information transmission. We undertake an analysis in the frequency domain, with a focus on detecting which frequencies have occurred jointly in pairs of propagation value series. To identify the most influential player at a given time in a network of financial markets and to detect cyclical phenomena is vital for investors seeking to diversify their portfolios, as it is vital for policy makers and enterprise managers in order to evaluate the future state of the economy by anticipating business cycles.

Generally, we hypothesize that the frequency structure of stock-price related observed series was richer in the past. More specifically, this means that frequency aspects have partially disappeared, leaving less predictable structure. In more detail:

- The range of relevant frequencies in propagation values has become narrower.
- Especially the importance of high frequencies is diminished. (Short-term trading can balance out market inequilibria so fast that short periods — length typically several days — disappear.)
- The difference between markets with respect to cycle phase is diminished, even for those frequencies that are still highly significant; reflecting the idea that all markets in the network approach a similar level of importance.

To study the frequency structure, we use wavelet methodology, based on continuous Morlet wavelets and cross-wavelets.

This paper is organized as follows. Section 2 describes some properties of the data on which the study is based. The methodology, as far as relevant to obtain a market’s series of propagation values, is reviewed in Section 3. Section 4 introduces some concepts of wavelet and cross-wavelet analysis. Empirical results of our study are presented in Section 5, followed by a discussion in Section 6. All computations and plots are carried out with R [22].

2 Data

The empirical starting point of the present study consists of daily closing quotations of three Western equity market indices: Dow Jones Industrial Average (New York Stock Exchange, in the following called dji), FTSE 100 (London Stock Exchange, ftse) and Euro Stoxx 50 (proxy for Euro Area equity markets, sx5e) in the time period from January 1987 through September 2013 (6932 observations).

The time series of daily simple returns in percent are plotted in Figure 1.

A visual inspection of the return series in Figure 1 suggests a simultaneous occurrence of periods of high volatility in the three markets considered, and the impression that returns are somehow “connected”. Our approach of how to assess, in this network of markets, proportions of importance with respect to information transmission, will be briefly described next.

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2 2012-02-22, The Washington Post: “The chairman of the Securities and Exchange Commission is worried about the rise of high-frequency trading, but two years after the agency flagged the phenomenon as a potential problem, Chairman Mary L. Schapiro says regulators still don't know enough to do much more about it.”
3 A market’s propagation value

The goal of the present study is to analyze, in the frequency domain, the time pattern of the relative importance of an equity market as a news disseminator in a network of equity markets. The relative importance of a market in this respect is quantified in a series of propagation values.\footnote{Similar concepts were developed in population science under the name \textit{reproductive value} in a Leslie-model framework, and under the name \textit{eigenvector centrality} in social network analysis.} This procedure will be outlined briefly in the following (for a more comprehensive discussion, see Schmidbauer et al. \cite{24}).

The starting point is to fit a VAR (vector autoregressive) model to \( N \) series \((x_l)_t, l = 1, \ldots, N, \) of daily returns on stock indices (or any other assets), using the past 100 days. The method to assess return-to-volatility spillovers is then to decompose the forecast error variance with respect to its origin, and the resulting shares of forecast variability in \( x_l \) due to shocks in \( x_k \) can be arranged in a spillover table (or matrix) \( M, \)\footnote{The spillover matrix is summarized into the \textit{spillover index} by Diebold and Yilmaz \cite{8, 9}.} for example with \( N = 3 \) return series:

\[
\begin{array}{l}
\begin{array}{c}
\text{from return } (x_k) \\
\hline
\end{array} \\
\begin{array}{ccc}
x_1 & x_2 & x_3 \\
\hline
x_1 & \square & \square & \square \\
x_2 & \square & \square & \square \\
x_3 & \square & \square & \square \\
\end{array}
\end{array}
\tag{1}
\]

Each row of \( M \) thus sums up to 1 (or 100\%) and provides a breakdown of the forecast error variance of the corresponding stock index return with respect to its origin. The columns of \( M \) provide the key to analyzing the propagation of a shock: An initial shock to market \( k \) can be represented by a unit vector \( n_0 \) with 1 in its \( k \)-th component. The subsequent repercussions of this shock across markets can then be modeled by the transmission equation

\[
n_{s+1} = M \cdot n_s, \quad s = 0, 1, 2, \ldots
\tag{2}
\]

The ultimate (as \( s \to \infty \)) relative impact level of a shock to market \( k \) is given by the \( k \)-th component of the (normed) left eigenvector \( v \) of \( M, \) that is, the vector satisfying \( v' = v' \cdot M. \) We call this the \textit{propagation value} of market \( k \) on the day in question. For example, let \( v' = (1/6, 1/3, 1/2), \) or 1:2:3. Then, the impact of a shock to market 3 on this day will ultimately
have three times the size (in terms of seed for future variability in the network) of an impact to market 1. A high propagation value indicates high importance of a market as news disseminator in this sense.

These steps will then be repeated for each day, with a rolling window of 100 days, resulting in N daily series of propagation values reflecting the markets’ daily relative importance as news disseminators.

4 Wavelets, cross-wavelets and phase differences

With the time series of daily propagation values for the three markets at hand, cyclical phenomena and synchronization among markets with respect to their importance as news disseminators can be studied in the next step. Cycles of different frequencies and of limited duration may overlap across time which necessitates decomposition of the time series into the time and frequency domain simultaneously. The arising time and frequency resolution dilemma (resulting from the Heisenberg uncertainty principle) can appropriately be resolved by employment of wavelet methodology. We will briefly outline the concepts adopted in our study.

We use the Morlet wavelet, which is a continuous wavelet transform and complex-valued, therefore is information-preserving with any careful selection of time and frequency parameters and provides information on both amplitude and phase — a prerequisite for the study of synchronicity between equally periodic time series. The “mother” Morlet wavelet is defined by

\[ \psi(\eta) = \pi^{-1/4} e^{i\eta} e^{-\eta^2/2} \]  

(with a particular choice of 6 oscillations, which implies that for computational purposes the wavelet can be treated as analytic), and depicted in Figure 2.

![Figure 2: The Morlet mother wavelet — real part (black line) and imaginary part (green line)](image)

The Morlet wavelet transform of a daily (i.e. discrete) time series \( x_t \) is then defined as the convolution of the series with a scaled and time-translated and — to ensure direct comparability.
of wavelet transform results — suitably normalized version of \( \psi \), resp. “wavelet daughter”:

\[
W_x(\tau, s) = \sum_t x_t \frac{1}{\sqrt{s}} \psi^* \left( \frac{t - \tau}{s} \right)
\]

(4)

with \( \ast \) denoting the complex conjugate; \( \tau \) is the localized time parameter which determines the position of the wavelet in the time domain, while the scale parameter \( s \) gives its position in the frequency domain.\(^6\) The scale is set to a fractional power of 2 (a “voice” in an “octave”). The mother wavelet corresponds to \( s = 1 \), higher values translate into lower frequency content. In the given setting, frequencies are related to scales according to the formula \( f = 6/(2\pi s) \), and the Fourier factor \( 2\pi/6 \) can also be used to convert scales to periods \( 1/f \).

The local amplitude of any periodic component of \( (x_t) \) and how it evolves with time can then be retrieved from the modulus \( |W_x(\tau, s)| \) of the wavelet transform, while displacements relative to a localized origin in the time domain are given by the local wavelet phase, which is an angle in the interval \([-\pi, \pi]\):

\[
\phi_x(\tau, s) = \text{Arg}(W_x(\tau, s)) = \tan^{-1} \left( \frac{\text{Im}(W_x(\tau, s))}{\text{Re}(W_x(\tau, s))} \right).
\]

(5)

The square of the modulus, \( |W_x|^2(\tau, s) \), has an interpretation as time-frequency (resp. time-scale) wavelet energy density, which is called the wavelet power spectrum (c.f. Carmona et al. [5]). In case of a white noise process, its expectation value at each time and scale equals the process variance. Therefore, in applications of wavelet methodology, it is conventional to standardize the time series at hand, after detrending,\(^7\) to obtain a measure of the wavelet power which is relative to unit-variance white noise and directly comparable to results of other time series.

For purpose of comparison of frequency content between the markets’ time series of daily propagation values in this study, and conclusions about their synchronicity, we adopt the concepts of cross-wavelet analysis. The cross-wavelet transform of two time series, \( (x_t) \) and \( (y_t) \), with wavelet transforms \( W_x \) and \( W_y \) respectively, decomposes the Fourier co- and quadrature-spectra in the time-frequency (resp. time-scale) domain:

\[
W_{xy}(\tau, s) = W_x(\tau, s) \cdot W^*_y(\tau, s).
\]

(6)

Its modulus \( |W_{xy}(\tau, s)| \) has the interpretation as cross-wavelet power and enables an assessment of the similarity of wavelet power between the two time series considered. Furthermore, the cross-wavelet transform carries information about the series’ synchronization in terms of the local phase advance of any periodic component of \( (x_t) \) with respect to the correspondent component of \( (y_t) \), viz. the so-called phase difference of \( x \) over \( y \) at each time and scale:

\[
\phi_{xy}(\tau, s) = \text{Arg}(W_{xy}(\tau, s)),
\]

(7)

which equals the difference of individual phases \( \phi_x - \phi_y \) when converted into an angle in the interval \([-\pi, \pi]\). An absolute value less (larger) than \( \pi/2 \) indicates that the two series move in phase (anti-phase) at the respective frequency, while the sign of the phase difference shows which is the leading series in this relationship. Figure 3 (in the style of a diagram by Aguiar-Conraria and Soares [2]) illustrates the range of possible phase differences and their interpretation.

\(^6\)Fast Fourier Transform algorithms can be used to evaluate formula (4) efficiently, see e.g. Carmona et al. [5], Torrence and Compo [25].

\(^7\)The time series in our study are detrended by means of local polynomial regression with a span of 75%.
Figure 3: Phase-differences and their interpretation

A different definition of phase difference uses a smoothed cross-wavelet transform, in which
smoothing in both time and scale directions is performed by convolution with appropriate win-
dows, see e.g. Cazelles et al. [6], Aguiar-Conraria and Soares [2]. Cross-wavelet results become
less noisy then. To this effect, we apply Bartlett windows of length 3 both in time and scale.

In order to assess the statistical significance of the patterns emerging from wavelet analysis
concerning the time series in our study, we employed bootstrapping methods following the
approach by Aguiar-Conraria and Soares [2]. To this end, the following technical null hypotheses
are installed and tested:

H1 There is no periodicity in any time series of propagation values.

H2 Neither market is prominent with respect to its propagation value at any time. All prop-
gagation values are equal.

As white noise series meet this hypothesis, a set of white noise series surrogates (2000 for
each series) is simulated and subject to wavelet analysis. P-values in the time-scale domain are
derived from proportions of exceedances of levels attained by the time series to be tested.

All computations and plots are carried out with R [22], combining, modifying and extending
functionality from two sources: the (currently archived) R package “WaveletCo” by Tian and
Cazelles, and R code from the wavelet toolbox by Aguiar-Conraria and Soares.8

5 Empirical results

A three-dimensional time series was obtained by proceeding along the steps outlined in Section 3,
where a moving window of 100 days was used for fitting a sequence of VARs, resulting in a
spillover table for every day (according to scheme (1)), the normed left eigenvector of which
provided the markets’ propagation values at that day. The result of this operation is shown in
Figure 4, visualizing a continuous process of relocation of the “news balance” among the three

8The ASToolbox is available at URL http://sites.google.com/site/aguiarconraria/joanasoares-wavelets; the
WaveletCo package was retrieved from the R archive http://cran.r-project.org/src/contrib/Archive/WaveletCo/.
markets considered in this study, which can be related to economic and geopolitical events: The two most distinct peaks of news propagation potential in the U.S. equity market coincide with the “Friday the 13th mini-crash” of October 1989, and with the crash of the “dot-com bubble” in March 2000.

Further peaks in the U.S. news propagation potential apparently bear traces from other events, however, there is evidence for an enduring structural change since the turn of the millennium: The U.S. market appears to have taken over a less prominent role as news disseminator, at least from a day-by-day perspective, with propagation values levelling off, and moving on a level with the European markets.

A smoothed version of the series was used to provide detrended series of propagation values, which were then processed by wavelet transformation — both individually and pairwise, according to the methodology outlined in Section 4. Figure 5 displays the results of individual wavelet transformation in terms of their power spectra; cross-wavelet transforms of pairs of propagation values are depicted in Figure 6.

The power spectrum gives information on the relative power of a wavelet component at a certain period length (as denoted on the vertical axis) and at a certain location in time (on the horizontal axis). The white contour lines define the time/period domain of significance at the 5% level with respect to deviance from the null hypothesis of white noise; the palette of rainbow colors serves to indicate different levels of wavelet power, from lowest (violet) to highest (red). In Figure 5, ridges of wavelet power are marked by means of black lines. Figure 6 provides additional information about phase differences between the time/period components of the series involved: arrows have been inserted into the plot according to the scheme in Figure 3, into areas where both series show significant at the 5% level.

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Footnotes:

9 Major U.S. indices had seen all-time highs on Monday when they plunged on Friday 13th 1989, just minutes after the announcement of a deal’s failure for UAL, the parent company of United Airlines, resulting in a 6.91 percent drop for the Dow Jones Industrial Average. According to Shiller [23] the news event was “a ‘story’ that enhanced the feedback from stock price drops to further stock price drops, thereby preserving the feedback effect for a longer period than would otherwise have been the case. Yet it was unlikely to have been the cause of the crash.”


11 The smoothed version was obtained by local polynomial regression fitting with a span of 75%; it turned out that the results of wavelet transformation are robust with respect to the degree of smoothing applied.
Figure 5: Wavelet power spectra of propagation values
Figure 6: Cross-wavelet power spectra of propagation values
In our plots of wavelet spectra, see Figure 5, the area of high significance always corresponds to lower frequency components of the time series (i.e. to higher period lengths), the bandwidth of which clearly diminished between 1987 and 2013. While an overlap of a broad spectrum of wavelets seems to have constituted the ridge of oscillations in the propagation potential of all three markets during the late 1980s, the oscillatory characteristics lost power towards the early 2010s, confining to higher period lengths (below 1024 days, i.e. 3 to 4 years). Hypothesis H1 of lacking periodicity in any time series of propagation values has to be rejected though.

The impression of a narrowing range of significant frequencies as time proceeded is confirmed by inspection of cross-wavelet powers in Figure 6. A persistent pattern can be identified when looking at the arrows and their interpretation (according to the scheme in Figure 3): While in the upper two plots of Figure 6, the arrows, reflecting the local phase advances of dji over ftse (sx5e), consistently point to the left and indicate negative phase differences, in the bottom plot, the phase differences of ftse over sx5e are positive with arrows pointing to the right, at least for the last two decades. This means, that with respect to information transmission, dji is in anti-phase to both ftse and sx5e, while ftse and sx5e are in phase. Therefore, hypothesis H2 stating that neither market is prominent with respect to its propagation value at any time has to be rejected.

The particular synchronicity of transmission processes within the network of markets can be concluded from the arrows’ angle. A comparison of angles in the late 1980s and early 2010s suggests two distinct network structures which are contrasted schematically in Figures 7 and 8: In the late 1980s, there was a kind of carousel of information transmission between dji, ftse and sx5e, while today, ftse and sx5e appear highly synchronous as compared to dji.

6 Discussion

Our findings suggest, that the power of frequency information which could contribute to understand propagation potentials of the markets in our study (U.S., U.K., euro area) has diminished in the course of 25 years. The shrinking range of frequencies may have an explanation in terms of the intenseness of information exchange, which has become much higher than could be measured by the concept of directional spillovers. Information flows easily today. The observation
of shrinking holding times of stocks is an argument in the same vein. While three decades ago, the frequency structure of information transmission has been richer, and more effective so as to investors could have waited for similar patterns to occur again, they rather tend to act immediately today, and won’t bet on frequency. White noise became more important, and the frequency structure of information transmission less predictable to build one’s portfolio on it.

Though overall frequency appears to loose its importance, with respect to synchronicity of information transmission a pattern can be detected which is consistent insofar as it persists since decades: the U.S. market is in anti-phase to the European markets, while the European markets are in phase. Moreover, European markets are highly synchronous news propagators today as compared to 25 years ago, which, on the other hand, emphasizes the consistently particular role of the U.S. market for the network. There is no more evidence of an “informational divide” between the U.S. and European markets with respect to their potential of information propagation, the phasing of propagation though has evolved a distinguishing pattern.

References


