

# Families and social security

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December 2012

## Abstract

The present paper quantifies the importance of family structures for the analysis of social security. For this reason we introduce home production as well as stable and unstable families into the standard stochastic overlapping generation model and simulate with each model version a move from a unfunded towards a funded pension system in Germany.

The simulation exercise computes intergenerational welfare changes and isolates aggregate efficiency effects by means of compensating transfers. Comparing the macroeconomic and welfare consequences resulting from the elimination of social security in the standard and in two-earner family models indicates two major conclusions. First, the consideration of home production has significant effects on labor supply and economic efficiency. Second, the impact of family insurance is fairly weak and can hardly substitute for social security.

JEL Classifications: J12, J22

Keywords: stochastic general equilibrium, home production, family insurance, marital risk

Financial support from Netspar and the Fritz Thyssen Stiftung is gratefully acknowledged.

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# 1 Introduction

This study aims to highlight the interaction of family structure and social security. In the past, numerous papers have analyzed the redistributive and efficiency consequences of existing social security systems. Typically, they quantify the insurance benefits of social security and the cost arising from labor supply distortions and rising liquidity constraints applying the standard overlapping generations model in which cohorts are represented by unmarried individuals. This household status is predefined and remains fixed during the life course. In reality, however, the household structure changes when people get married, move together in a cohabitation arrangement or get divorced again. Therefore, many decisions about labor supply, consumption and savings are made within a family context where husband and wife decide jointly.

There are reasons to believe that the explicit modeling of such family structures could be important especially in the context of social security analysis. First, couples realize economies of scale in consumption and benefit from efficient home production arrangements while at the same time the threat of divorce may constitute a specific new risk. This suggests that life-cycle labor supply and savings as well as liquidity constraints will be quite different for a couple than for a single agent. Second, as already discussed by Attanasio et al. (2005), Kotlikoff and Spivak (1981) and Brown and Poterba (2000), marriages may also provide insurance against labor market risk and longevity and therefore substitute (at least partly) for social security. Finally, specific features of the social security system such as survivors benefits or supplementary benefits to one-earner couples may especially redistribute resources from singles towards couples.

This all indicates that the single agent household might constitute a too drastic simplification. Explicit modeling of the changing household structure throughout the life cycle may be important for the quantitative consequences of social security reforms. Not surprisingly, recent studies have introduced family structures in order to analyze social security reform issues. Kaygusuz (2011) presents a life cycle model which explicitly distinguishes between single individuals and married partners. At the beginning of life, households are assigned a marital status which stays constant throughout the working phase. After retirement, each agent faces a gender and age dependent mortality risk. The study features a detailed modeling of the female spouses labor supply decisions. A married woman can thereby not only choose how much to work, but also whether to at all participate in the labor market or not. Households are disaggregated by educational and income levels and the resulting combinations for married couples. The calibration target is to match participation rates for these different combinations. In this framework, Kaygusuz (2011) examines the consequences of an abolition of progression as well as marital subsidies in the US pension system. He finds that such a reform would strengthen labor market participation rates especially of less skilled married women. In addition, married couples with high educational background will gain the most from this reform.

Sanchez-Martin and Sanchez-Marcos (2010) quantify the consequences of recent pension

reforms in Spain for single-earner and double-earner households of different educational backgrounds. In their set-up, male labor supply is fixed at the intensive margin while women face a participation decision on the labor market. Compared to the previous study, both genders select their retirement age endogenously. In addition, the study does not focus on steady states before and after the reform but computes the entire transition path including the population aging in Spain. Their results indicate that neglecting survival pensions may significantly underestimate future financial burdens of the Spanish pension system.

Kaygusuz (2011) as well as Sanchez-Martin and Sanchez-Marcos (2010) assume a deterministic income process and therefore abstract from insurance aspects against labor income shocks during the employment phase. Various recent approaches that analyze social security systems in a family context include this specific feature. Nishiyama (2010) quantifies the consequences of an elimination of spousal and survivor benefits in the US system using a model with stochastic wages. The study includes the transition path but only considers married households who decide jointly on their intensive labor supply. The removal of spousal and survivor benefits induces a strong increase in market work hours for women in the long run which could be transformed into a welfare gain for all current and future cohorts.

Up to now, marital status was assumed to remain constant throughout the whole life cycle. However, given the existing divorce rates in Western societies, such an assumption does not seem very realistic. Thus, Domeij and Klein (2002) as well as Hong and Ríos-Rull (2007) try to overcome this problem by taking into account exogenous, stochastic changes in family status over the life cycle, i.e. marriage, divorce and remarriage are modeled as idiosyncratic shocks. Domeij and Klein (2002) study the impact of the Swedish pension system on the wealth distribution using a model with income and marital risk over the life cycle. The initial equilibrium of the model replicates the enormous difference between income and wealth inequality in Sweden. When the pension system is removed income and wealth inequality converge significantly. Hong and Ríos-Rull (2007) quantify the welfare effects of the US pension system in a model with stochastic families. As already mentioned above, the family can substitute at least partly for both income and longevity insurance aspects of a public pension system. Hence, it is not surprising that Hong and Ríos-Rull (2007) report tremendous welfare gains from its privatization. Nevertheless, they only compare long-run equilibria and therefore are not able to disentangle insurance and distortive effects from intergenerational redistribution.

All studies mentioned so far do not take into account the distinction between home work and market work. But the latter seems to be important in order to capture gender differences in labor markets. Among others, Olivetti (2006) and Greenwood et al. (2005) present calibrated models with two-earner households where husband and wife decide on home work and market work jointly. They show that the rising labor market participation of women in the past could be explained by changes in the returns to experience and the introduction of labor-saving consumer durables. But home production not only changes labor supply behavior, it also introduces additional insurance possibilities. For this reason Dotsey, Li and Yang (2012) have simulated social security reforms in a standard stochastic overlapping generation model (i.e. without families) that incorporates home production. Their results

indicate that due to self insurance effects the long-run welfare gains from pension funding rise when home production is taken into account.

The present study builds on all these previous approaches. We start with a base model featuring the traditional single agent approach and simulate a pension funding experiment as in Fehr, Habermann and Kindermann (2008). Then we extend the model structure step by step by incorporating home production, singles and couples with constant marital status and finally marital risk. Compared to previous studies we do not only compute the transition path towards the long-run equilibrium and the resulting short and long-run welfare consequences, but also isolate the aggregate efficiency consequences of the considered reforms by means of a separate compensation mechanism.

Our numerical exercises indicate three major results. First, home production as well as family transitions dampen the savings increase and the associated capital accumulation induced by pension funding. Consequently they reduce the intergenerational redistribution towards future cohorts. Second, due to the stronger labor supply reaction, home production significantly increases the efficiency losses from pension funding. Third, the consideration of families reduces the insurance benefits of the pension system due to (implicit) annuitization. However, in contrast to previous studies our results indicate that insurance provision by families is surprisingly small.

## 2 The model economy

This section provides a description of the base model version we use to quantify our results. In later sections we will extend this base model step by step.

### 2.1 Demographics and intracohort heterogeneity

Our model economy is populated by  $J$  overlapping generations. At any discrete point  $t$  in time, a new generation – populated in equal size by men  $M$  and women  $F$  – is born. Individuals face gender-specific lifespan uncertainty, where  $\psi_j^g \leq 1$  denotes the conditional survival probability of gender  $g \in \mathcal{G} = \{M, F\}$  from age  $j - 1$  to age  $j$  with  $\psi_{j+1}^g = 0$ .

Our model is solved recursively. The individual state vector of an age- $j$  agent is

$$z_j = (g, s, \eta_j, a_j, p_j). \quad (1)$$

Entries two and three of this vector describe the labor market status of the individual.  $s \in \mathcal{S} = \{1, \dots, S\}$  thereby is agent's skill level and  $\eta_j$  an idiosyncratic shock to labor income. Finally,  $a_j \in \mathcal{A} = [0, \infty]$  and  $p_j \in [0, \infty]$  are agents' beginning of period asset holdings and pension claims. While assets and pension claims are influenced by individual decisions, the other state variables are determined exogenously. Gender and skill level can be interpreted as one-time persistent shocks, the realization of which is revealed at the beginning of

the life-cycle. While the two realizations of gender occur with equal probability, there is a probability distribution  $\pi_g^s$  which assigns the skill level  $s$  conditional on gender  $g$ . Labor productivity is transitory and by assumption follows a first-order Markov process. Therefore the probability distribution of future labor productivity  $\eta_{j+1}$  only depends on the current productivity  $\eta_j$ , i.e. there exists a probability distribution  $\pi_{g,s}^\eta(\eta_{j+1}|\eta_j)$  which by assumption depends on gender and skill level.

Since our model abstracts from annuity markets, individuals that die before the maximum age of  $J$  may leave accidental bequests that will be distributed in a lump-sum fashion across all working individuals. Agents retire at age  $J_R$  and start to receive pension benefits which are financed by proportional payroll taxes paid up to to the double of average labor income. In the following, we will, for the sake of simplicity, omit the indices  $t, g$  and  $s$  wherever possible.

## 2.2 The household decision problem

All agents value streams of consumption  $x_j$  and leisure  $\ell_j$  according to the standard expected utility function

$$E \left[ \sum_{j=1}^J \beta^{j-1} u(x_j, \ell_j) \right], \quad (2)$$

where  $\beta$  is a time discount factor. Due to additive separability over time, we can formulate the decision problem recursively so that

$$V(z_j) = \max_{x_j, \ell_j} u(x_j, \ell_j) + \beta \psi_{j+1}^g E[V(z_{j+1})]. \quad (3)$$

Since lifespan is uncertain, future utility is weighted with the gender-specific survival probability  $\psi_{j+1}^g$ . Future utility is computed over the distribution of future states of productivity  $\eta_{j+1}$ . Agents maximize (3) subject to the budget constraint

$$a_{j+1} = [1 + (1 - \tau_r)r] a_j + (1 - \tau_w)(1 - \tau_p)y_j + (1 - \tau_w)\tilde{p}_j + b_j - (1 + \tau_x)x_j. \quad (4)$$

At the beginning of life households are endowed with zero assets  $a_1 = 0$ . Throughout the whole life cycle assets are restricted to be greater or equal to zero, i.e. agents might be liquidity constrained and do not value bequests, i.e.  $a_{J+1} = 0$ . In addition to net interest income from savings  $(1 - \tau_r)ra_j$ , they receive gross income from supplying labor to the market  $y_j = we_j\eta_j l_j$  during their working period as well as public pensions  $\tilde{p}_j$  during retirement.<sup>1</sup>  $w$  defines the wage rate for effective labor while  $e_j$  denotes gender- and skill-specific productivity at age  $j$ . Market labor is given by  $l_j = 1 - \ell_j$ . Households may inherit accidental bequests  $b_j$  from their parents' generation. They contribute at a rate  $\tau_p$  to the public pension system. Taxes on labor and pensions income are paid at constant rate  $\tau_w$ , whereby pension contributions are exempt from taxation. Finally, the price of market goods  $x_j$  includes consumption taxes  $\tau_x$ .

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<sup>1</sup> Note that  $\tilde{p}_j = p_j$ , if  $j \geq J_R$  and  $\tilde{p}_j = 0$  otherwise.

Pension claims are fully earnings related. Specifically, they evolve according to

$$p_{j+1} = p_j + \kappa y_j, \quad (5)$$

where  $\kappa$  denotes the accrual rate and  $p_1 = 0$ .<sup>2</sup>

Finally, the period utility function is defined as

$$u(x_j, \ell_j) = \frac{1}{1 - \frac{1}{\gamma}} \left( x_j^{1 - \frac{1}{\rho}} + \alpha \ell_j^{1 - \frac{1}{\rho}} \right)^{\frac{1 - \frac{1}{\gamma}}{1 - \frac{1}{\rho}}}, \quad (6)$$

where  $\gamma$  denotes the intertemporal elasticity of substitution between consumption at different ages,  $\rho$  defines the intratemporal elasticity of substitution between consumption and leisure at each age  $j$  and  $\alpha$  is an age-independent leisure preference parameter.

## 2.3 Technology

Firms in this economy use capital and labor to produce a single good according to a Cobb-Douglas production technology. Capital depreciates at a rate  $\delta$ . Firms maximize profits renting capital and hiring labor from households such that net marginal product of capital equals the interest rate for capital  $r$  and the marginal product of labor equals the wage rate for effective labor  $w$ , i.e.

$$\max_{K_t, L_t} \{ \theta K_t^\varepsilon L_t^{1-\varepsilon} - wL_t - (r + \delta)K_t \} \quad (7)$$

where  $K_t$  and  $L_t$  are aggregate capital and labor, respectively,  $\varepsilon$  is capital's share in production and  $\theta$  defines a technology parameter.

## 2.4 The government sector

Our model distinguishes between the tax and the pension system. In each period  $t$ , the government issues debt  $B_{G,t+1} - B_{G,t}$  and collects taxes from households in order to finance general government expenditure  $G$  which is fixed per capita as well as interest payments on existing debt,<sup>3</sup> i.e.

$$B_{G,t+1} - B_{G,t} + \tau_w [ wL_t - \tau_p PC_t + PB_t ] + \tau_r r A_t + \tau_x X_t = G + r B_{G,t}, \quad (8)$$

where  $PC_t$ ,  $PB_t$ ,  $A_t$  and  $X_t$  define the pension contribution base, aggregate pension benefits, assets and consumption, respectively. Since our focus is on pension reform, we model a

<sup>2</sup> Note that our model takes into account a contribution ceiling which fixes the maximum contribution and pension accrual base to the double of average income per year.

<sup>3</sup> Since we assume a population growth rate of zero, the government can't issue new debt in a long-run equilibrium.

simple proportional income tax system. Note, however, that we let pension contributions be exempt from tax and assume pension benefits to be fully taxed. The consumption tax rate balances the intertemporal budget so that debt becomes endogenous annually.

In each period  $t$ , the pension system pays old-age benefits and collects payroll contributions from labor income. In the initial long-run equilibrium there is no debt (i.e.  $B_P = 0$ ) and the contribution rate is computed endogenously. Starting in the reform period the pension system is phased-out by simply setting the accrual rate to  $\kappa = 0$  in equation (5). Consequently, additional pension claims could not be accumulated any more while at the same time existing claims of current and future pensioners are not affected. After the reform future benefits of the pension system are financed by a time-invariant payroll tax rate derived from the intertemporal budget constraint. Consequently, we allow for new pension debt  $B_{P,t+1} - B_{P,t}$  in the transitional periods to balance periodical budgets

$$B_{P,t+1} - B_{P,t} = r_t B_{P,t} + PB_t - \tau_p PC_t. \quad (9)$$

## 2.5 Equilibrium conditions

Given a specific fiscal policy, an equilibrium path of the economy has to solve the household decision problem, reflect competitive factor prices, and balance aggregate inheritances with unintended bequests. Furthermore aggregation must hold, and consumption tax and pension contribution rate have to balance the tax and pension system's budgets. Since we assume a closed economy setting, output has to be completely utilized for private consumption, public consumption  $G_t$  and investment purposes, i.e.

$$Y_t = X_t + G + K_{t+1} - (1 - \delta)K_t,$$

aggregate savings have to balance capital demands of firms and the government and aggregate labor supply has to be employed by firms.

# 3 Calibration of the initial equilibrium

## 3.1 Preference, technology and government parameters

Table 1 reports the central parameters of the model. In order to reduce computational time, each model period covers five years. Agents reach adulthood at age 20 ( $j = 1$ ), retire mandatorily at age 60 ( $J_R = 9$ ) and face a maximum possible life span of 100 years ( $J = 16$ ). Conditional survival probabilities  $\psi_j^S$  are computed from the year 2000 Life Tables for Germany reported in Bomsdorf (2002). However, in order to simplify the demographic transition, we restrict uncertain survival to retirement years, i.e.  $\psi_j^F = \psi_j^M = 1, j < J_R$ . We distinguish high-skilled and regular-skilled individuals (i.e.  $S = 2$ ). The initial distribution of men and women over these two groups is extracted from the German Socio-Economic Panel Study

(SOEP).<sup>4</sup> In this data base 23.5 percent of males and 15.1 percent of females are high-skilled.

Table 1: Parameter selection

<i>Demographic parameters</i>		<i>Preference parameters</i>	
(Adult) Life span ( $J$ )	16	Intertemporal elasticity of substitution ( $\gamma$ )	0.50
Retirement period ( $J_R$ )	9	Intratemoral elasticity of substitution between	
Skill levels ( $S$ )	2	... consumption and leisure ( $\rho$ )	0.60
		Coefficient of leisure preference ( $\alpha$ )	0.90
		Discount factor ( $\beta$ )	0.86
<i>Technology/Budget parameters</i>		<i>Government parameters</i>	
Factor productivity ( $\theta$ )	1.53	Debt-to-output ( $B_G/Y$ )	0.60
Capital share ( $\epsilon$ )	0.35	Consumption tax rate ( $\tau_x$ )	0.20
Depreciation rate ( $\delta$ )	0.26	Labor income tax rate ( $\tau_w$ )	0.10
		Capital income tax rate ( $\tau_r$ )	0.10
		Contribution rate ( $\tau_p$ )	0.199

Most microeconomic estimates on the intertemporal elasticity of substitution fall between zero and one, see the discussion in Auerbach and Kotlikoff (1987) or İmrohoroğlu and Kitao (2009). We use in our benchmark  $\gamma = 0.5$ . The intratemoral elasticity of substitution between consumption of goods and leisure is set to  $\rho = 0.6$ . Then we calibrate the leisure preference parameter  $\alpha = 0.9$  in order to set average work hours in the economy at 0.26, which implies a 40 hours work week length. Finally, in order to calibrate a realistic capital to output ratio of 3.3, the discount factor  $\beta$  is set at 0.86 which implies an annual discount rate of about 3.1 percent.

On the production side we let the capital share in production be  $\epsilon = 0.35$  reflecting the average share of capital income in Germany. The annual depreciation rate for capital is set at 4.75 percent (i.e. the periodic depreciation rate is  $\delta = 0.26$ ) which yields a realistic investment share in output. Finally we specify the general factor productivity  $\theta = 1.53$  in order to normalize the initial wage rate to unity.

We chose an accrual rate  $\kappa$  such that the replacement rate of net income amounts to 50 percent, which yields a realistic pension contribution rate for Germany. Taxable labor income as well as capital income are taxed at a proportional rate of 10 percent. In the initial long-run equilibrium, we assume a debt-to-output ratio of 60 percent and fix the consumption tax rate at 20 percent in order to generate a realistic public consumption ratio  $G/Y$ .

<sup>4</sup> The SOEP data base is described in Wagner et al. (2007).

### 3.2 Estimation of productivity profiles and income uncertainty

We estimate productivity profiles for men and women of different skill classes using inflated hourly wages  $w_{ijt}$  of primary household earners from the German SOEP. Our unbalanced panel data covers full-time workers between ages 20 and 60 of the years 1984 to 2008 who were divided into secondary and tertiary educated subgroups according to the International Standard Classification of Education (ISCED) of the UNESCO of 1997. This approach leads us to a total of 130 693 observations with 61 798 low-skilled males, 49 438 low-skilled females, 10 636 high-skilled men and 8 821 high-skilled women.

With this data, we estimate a simplified version of the Storesletten et al. (2004) model. Specifically, we assume log wages to follow a gender, skill group and age dependent trend and let shocks to individual wages be of AR(1) type. In addition we estimate time fixed effects to rule out business cycle components and technical change. Consequently we estimate the equations

$$\log(w_{ijt}) = \log(e_j) + \text{time}_t + \log(\eta_{ij})$$

with

$$\log(\eta_{ij}) = \varrho \log(\eta_{ij-1}) + \epsilon_{ij} \quad , \quad \epsilon_{ij} \sim N(0, \sigma_\epsilon^2).$$

We specify the time trend to

$$\log(e_j) = \beta_0 + \beta_1 \cdot j + \beta_2 \cdot j^2 / 100$$

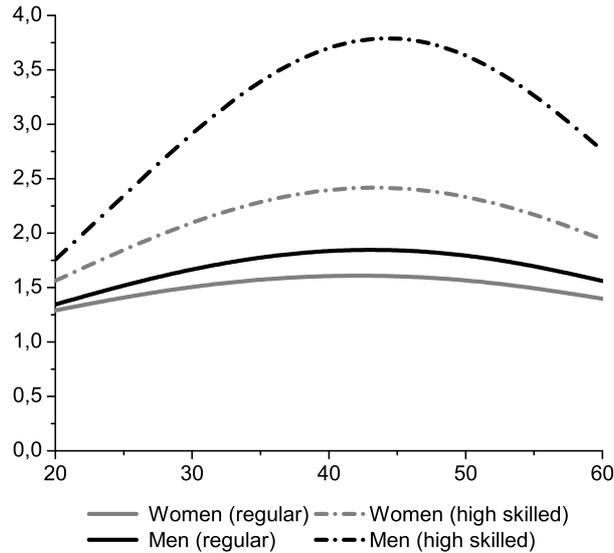
and estimate four separate equations, one for each gender and skill combination. Our parameter estimates are shown in Table 2 (standard errors are reported in parentheses).

Table 2: Parameter estimates for individual productivity

	<i>Men</i>		<i>Women</i>	
	<i>low-skilled</i>	<i>high-skilled</i>	<i>low-skilled</i>	<i>high-skilled</i>
Intercept $\beta_0$	1.4256 (0.0351)	06894 (0.1384)	1.7944 (0.0397)	1.4152 (0.1426)
age term $\beta_1$	0.0671 (0.0018)	0.1225 (0.0067)	0.0405 (0.0021)	0.0735 (0.0072)
age <sup>2</sup> term $\beta_2$	-0.0721 (0.0023)	-0.1324 (0.0079)	-0.0459 (0.0027)	-0.0810 (0.0088)
AR(1) correlation $\varrho$	0.8665 (0.0016)	0.9187 (0.0026)	0.8044 (0.0023)	0.8637 (0.0042)
transitory variance $\sigma_\epsilon^2$	0.0396 (0.0017)	0.0380 (0.0070)	0.0648 (0.0020)	0.0556 (0.0062)

Note that we find a strong AR(1) correlation of around 0.8 – 0.9 for the error term. Bayer and Juessen (2012) document similar values using SOEP data. The estimated wage profiles can be seen in Figure 1. While the gender productivity gap for high-skilled is quite drastic, the difference between low-skilled men and women is fairly small.

Figure 1: Estimated wage profiles



For computational reasons, we finally approximate the shock  $\eta$  by a first order discrete Markov process with three nodes using a discretization algorithm as described in Tauchen and Hussey (1991).<sup>5</sup>

### 3.3 The initial equilibrium

Table 3 reports the calibrated initial equilibrium of the base model and the respective figures for Germany in 2010. Since men have lower survival probabilities than women, their life expectancy (at age 20) is 76.8 years, while women on average become 4.3 years older. We consider a closed economy, so that the private consumption share in output also includes net exports. Pension benefits are slightly too high and tax revenues are slightly too low. Given the debt-to-output ratio, interest payments are roughly 3 percent of GDP, so that public consumption amounts to 19.2 percent of GDP which almost exactly matches the data. The fraction of bequest in GDP seems to be too low, but one has to keep in mind that our model only accounts for unintended bequest.

Overall, the base models' initial equilibrium reflects quite realistically the current macroeconomic situation in Germany and the distribution of household income. The distribution of wealth is more equal in the model as in reality. This is not surprising given the aggregation level of the model and the fact that it does not account for entrepreneurial income etc. About 50 percent of households younger than 30 years have no savings. The number of liquidity constrained households decreases for older cohorts towards zero but increases after retirement again steadily.

<sup>5</sup> We have also used a Markov process with five nodes. This approximation yields almost the identical equilibrium but increases computational time dramatically.

Table 3: The initial equilibrium

	<i>Germany 2010<sup>a</sup></i>	<i>base model</i>	<i>+ home production</i>	<i>+ families</i>	<i>+ marital risk</i>
<i>Calibration targets</i>					
Life expectancy (women) (in years)	81.9			81.1	
Life expectancy (men) (in years)	76.8			76.8	
Private consumption (% of GDP)	58.0	64.4	64.4	64.4	64.4
Investment (% of GDP)	17.5	17.4	17.4	17.4	17.4
Pension benefits (% of GDP)	11.6	12.4	11.7	11.8	11.7
Tax revenues (in % of GDP)	22.2	21.4	21.4	21.4	21.4
Capital-output ratio	3.5	3.3	3.3	3.3	3.3
<i>Other benchmark coefficients</i>					
Interest rate p.a. (in %)	–	4.8	4.8	4.8	4.8
Bequest (in % of GDP)	7.0 <sup>b</sup>	5.4	4.3	5.3	6.2
...intragenerational		–	–	2.2	2.9
Gini-coefficient for net income	29.0 <sup>c</sup>	28.0	40.9	37.9	39.2
Gini-coefficient for wealth	79.9 <sup>c</sup>	61.4	64.2	64.2	65.1

Source: <sup>a</sup>IdW (2012), <sup>b</sup>Schinke (2012, p. 41), <sup>c</sup>SVR (2009).

## 4 Simulation results

The remainder of this paper will focusses on the macroeconomic, welfare and efficiency consequences of a complete phase-out of the public pension system. This is accomplished by simply setting  $\kappa = 0$  in equation (5) so that individuals keep their pension claims, but do not accumulate additional ones in the future. We finance existing pension claims by time-invariant payroll and consumption tax rates computed from the respective intertemporal budgets of the pension system and the government in order to smooth the burden across current and future generations. As a consequence, the periodical budgets (8) and (9) are balanced by endogenous public and pension debt.

In the first subsection we explain how welfare and efficiency effects are computed. The next part considers the policy reform in the base version of the model discussed above. The following three subsections extend the base version of the model by successively introducing home production, two-earner households as well as divorce risk and remarriage.

### 4.1 Computation of welfare and efficiency effects

The concept we apply to quantify welfare effects is compensating variation à la Hicks. Due to the homogeneity of our utility function,

$$u[(1 + \phi)x_j, (1 + \phi)\ell_j] = (1 + \phi)^{1 - \frac{1}{\gamma}} u[x_j, \ell_j]$$

holds for any  $x_j, \ell_j$  and  $\phi$ . In consequence, since utility is additively separable with respect to time, if consumption and leisure were simultaneously increased by the factor  $1 + \phi$  at any age, life-time utility would increase by the same factor. With this considerations lets again turn to our simulation model. Assume an individual at state  $z_j$  had utility  $V^b(z_j)$  in the initial long-run equilibrium path and  $V^r(z_j)$  after the policy reform. The compensating variation between the baseline and the reform scenario for the individual characterized by  $z_j$  is then given as

$$\phi = \left( \frac{V^r(z_j)}{V^b(z_j)} \right)^{\frac{1}{1-\frac{1}{\gamma}}} - 1.$$

$\phi$  then indicates the percentage change in both consumption and leisure individual  $z_j$  would require in the initial equilibrium in order to be as well off as after the policy reform. We may also say that an individual is  $\phi$  better (or worse) off in terms of resources after the reform. If  $\phi > 0$ , the reform is therefore welfare improving for this individual and vice versa.

A special rule applies to individuals not having entered their economically relevant phase of life in the year before we conduct our pension reforms (the so-called future generations). We evaluate their utility behind the Rawlsian veil of ignorance, i.e. from an ex-ante perspective where their gender but neither their skill level nor any labor market shock has been revealed. The concept of compensating variation thereby applies likewise.

In order to isolate the pure efficiency effects of the reform, we apply the hypothetical concept of a Lump-Sum Redistribution Authority (LSRA) in a separate simulation.<sup>6</sup> The LSRA thereby proceeds as follows: to all generations already being economically active before the reform it pays lump-sum transfers or levies lump-sum taxes in order to make them as well off after the reform as in the initial equilibrium. Consequently their compensating variation amounts to zero. Having done that, the LSRA may have run into debt or build up some assets. It now redistributes this debt or assets across all future generations in a way that they all face the same compensating variation. This variation can be interpreted as a measure of efficiency. Consequently, if the variation is greater than zero, the reform is Pareto improving after compensation and vice versa. With this concept in hand, we can now proceed to our simulation results.

## 4.2 Pension funding in the base version of the model

The base model and the reform experiment in this subsection are similar to the simulation exercise with rational consumers in Fehr et al. (2008). However, we now consider a closed economy with a proportional tax system instead of a small open economy with a progressive income tax. In addition, we now neglect population growth, apply a different leisure preference parameter and assume a more realistic income process over the life cycle.

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<sup>6</sup> The LSRA was introduced by Auerbach and Kotlikoff (1987, 62f.) and has recently been applied by Nishiyama and Smetters (2007) as well as Fehr et al. (2008) in similar stochastic frameworks.

Consequently, the economic effects presented here are qualitatively the same but there are significant differences in absolute figures.

*Macroeconomic implications* The left part of Table 4 reports the macroeconomic effects of our privatization reform. In order to finance all existing pension claims accumulated in the pre-reform years, a permanent payroll tax of  $(19.9 - 5.6 =)$  14.3 percent is required. The reported reduction of the contribution rate roughly quantifies former implicit savings that can be invested explicitly on the capital market after the reform. Higher private savings trigger rising bequests, which increase from 5.4 percent to almost 12 percent of GDP in the long-run. While most of the savings increase is absorbed by rising public debt, the long-run capital stock still increases by 8.5 percent. The reduced contribution rate mirrors the previous implicit tax of the paygo system. However, while the implicit tax rate was falling with age<sup>7</sup>, the new payroll tax rate is constant over the life cycle. Consequently, the contribution rate adjustment reduces tax burdens for young and increases tax burdens for middle-aged agents. As a result labor supply changes only very slightly initially so that employment only falls by 0.2 percent in the short run. Since future cohorts receive higher bequest, the positive income effect reduces labor supply steadily during the transition. Overall, the long-run output increase is only modest at 0.6 percent while wages rise quite significantly by 4.1 percent. Higher income tax revenues allow to reduce the consumption tax rate by 0.5 percentage points. Given the constant consumption tax rate, public debt decreases from 60 to 53.7 percent of GDP in the long run while the existing implicit debt of the pension system amounts to roughly 200 percent of GDP.

*Welfare and efficiency* With the above discussion in mind, we can now turn to the welfare effects of our reform. Table 5 summarizes welfare consequences measured in compensating variation for different cohorts. For agents already taking economic decisions in the reform year, we report average welfare changes grouped by their gender and skill level. For future generations, we apply the concept of ex ante welfare and therefore only report aggregate numbers for each gender of the cohort. The first column indicates the age of the respective cohort in the reform year.

Without LSRA compensations, welfare of the already retired generations increases slightly. This reflects the reduction of the consumption tax rate and the compensating reduction of the interest rate. Not surprisingly, welfare gains increase with age of retirees since the (negative) interest rate effect hits elderly less due to lower assets. For the working cohorts in the reform year, welfare effects are also clear-cut. First, older workers lose significantly more than younger cohorts which is mainly due to the adverse change in the effective payroll tax rate explained above. In addition, high-skilled lose less than regular-skilled individuals which is due to the contribution ceiling. Finally, since women have a higher life expectancy they lose more than men. Future generations benefit from pension funding since they receive rising

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<sup>7</sup> This is due to the fact that accumulated pension claims of the paygo-system earn no interest during the working period, see the discussion in Fenge et al. (2006).

Table 4: Macroeconomic effects in the base model without and with home production<sup>a</sup>

Period	without home production				with home production			
	1	3	5	$\infty$	1	3	5	$\infty$
Private Assets	0.0	11.0	24.5	55.6	0.0	9.3	21.1	47.4
Capital Stock	0.0	1.3	3.0	8.5	0.0	0.2	1.1	4.7
Gov. Debt/GDP (in %)	60.1	60.2	59.1	53.7	60.7	59.8	58.4	54.6
Pens. Debt/GDP (in %)	0.0	38.5	86.5	194.8	0.0	37.1	82.5	178.0
Labor Supply	-0.2	-0.7	-1.2	-3.4	-1.7	-2.0	-2.5	-3.5
Home Production	-	-	-	-	0.9	1.3	1.7	2.6
GDP	-0.1	0.0	0.2	0.6	-1.1	-1.2	-1.3	-0.7
Bequests	0.0	0.7	9.0	117.4	-0.5	-0.3	8.4	106.9
Wage rate	0.1	0.7	1.4	4.1	0.6	0.8	1.3	2.9
Interest rate <sup>b</sup>	0.0	-0.1	-0.2	-0.6	-0.1	-0.1	-0.2	-0.5
Consumption tax rate <sup>b</sup>	-0.5	-0.5	-0.5	-0.5	0.0	0.0	0.0	0.0
Social Security tax rate <sup>b</sup>	-5.6	-5.6	-5.6	-5.6	-5.5	-5.5	-5.5	-5.5

<sup>a</sup>Changes in percent over value in initial equilibrium.

<sup>b</sup>Changes in percentage points.

bequest and significant wage increases. This dominates the cost from the elimination of the longevity insurance. Note that in the long run there is hardly any difference between the two genders.

Table 5: Welfare effects in the base model without and with home production\*

Gender	without home production					with home production				
	without LSRA				with LSRA	without LSRA				with LSRA
	Male	Female		Male		Female				
Skill level	regular	high	regular	high	regular	high	regular	high		
Retirees										
85-89	0.15	0.13	0.15	0.13	0.00	0.00	0.00	-0.01	-0.01	0.00
65-69	0.07	0.05	0.04	0.03	0.00	-0.08	-0.13	-0.13	-0.15	0.00
Workers										
45-49	-1.17	-0.90	-1.24	-1.09	0.00	-1.10	-1.25	-1.13	-1.28	0.00
25-29	-0.43	-0.32	-0.54	-0.50	0.00	-0.53	-0.56	-0.63	-0.71	0.00
Future Generations										
15-19	0.34		0.25		-0.20	0.07		0.00		-0.80
5-9	0.97		0.93		-0.20	0.62		0.58		-0.80
$\infty$	1.87		1.90		-0.20	1.30		1.28		-0.80

\*In percent of initial resources.

Finally, let's turn to welfare effects after LSRA compensation payments. As mentioned above, the LSRA makes all existing cohorts as well off as in the initial equilibrium and redistributes resources across future generations to make them all face the same welfare changes. The efficiency effects of the policy reform are depicted in the column "with LSRA" of Table 5. We find that the reform induces losses for any future generation of 0.2 percent of initial resources. The elimination of paygo pensions comes along with three major efficiency consequences. First, insurance provision against longevity risk is completely eliminated. Second, due to the previous Bismarkian system the reform mainly turns implicit taxes into explicit payroll taxes. Consequently, labor market distortions hardly change at all. Third, the lower contribution rate relaxes liquidity constraints for younger cohorts. Therefore, we find in this reform scenario that the first effect dominates the latter ones and therefore the move towards funded pensions is Pareto inferior.<sup>8</sup>

### 4.3 Home production

Following Rogerson (2009), we now introduce home production. The agent at age  $j$  solves the problem

$$V(z_j) = \max_{x_j, h_j, \ell_j} u(c_j, \ell_j) + \beta \psi_{j+1}^g E[V(z_{j+1})]. \quad (10)$$

Individual consumption

$$c_j(x_j, h_j) = \left\{ v x_j^{1-\frac{1}{\chi}} + (1-v) h_j^{1-\frac{1}{\chi}} \right\}^{\frac{1}{1-\frac{1}{\chi}}}$$

is now produced within the household by means of market goods  $x_j$  and home labor  $h_j$ . The production of the consumption good within the household follows a CES home production technology where  $v$  is a share parameter for market goods  $x_j$  and  $\chi$  defines the substitution elasticity between market goods  $x_j$  and effective working time in home production. Of course, market labor is now defined by  $l_j = 1 - h_j - \ell_j$ .

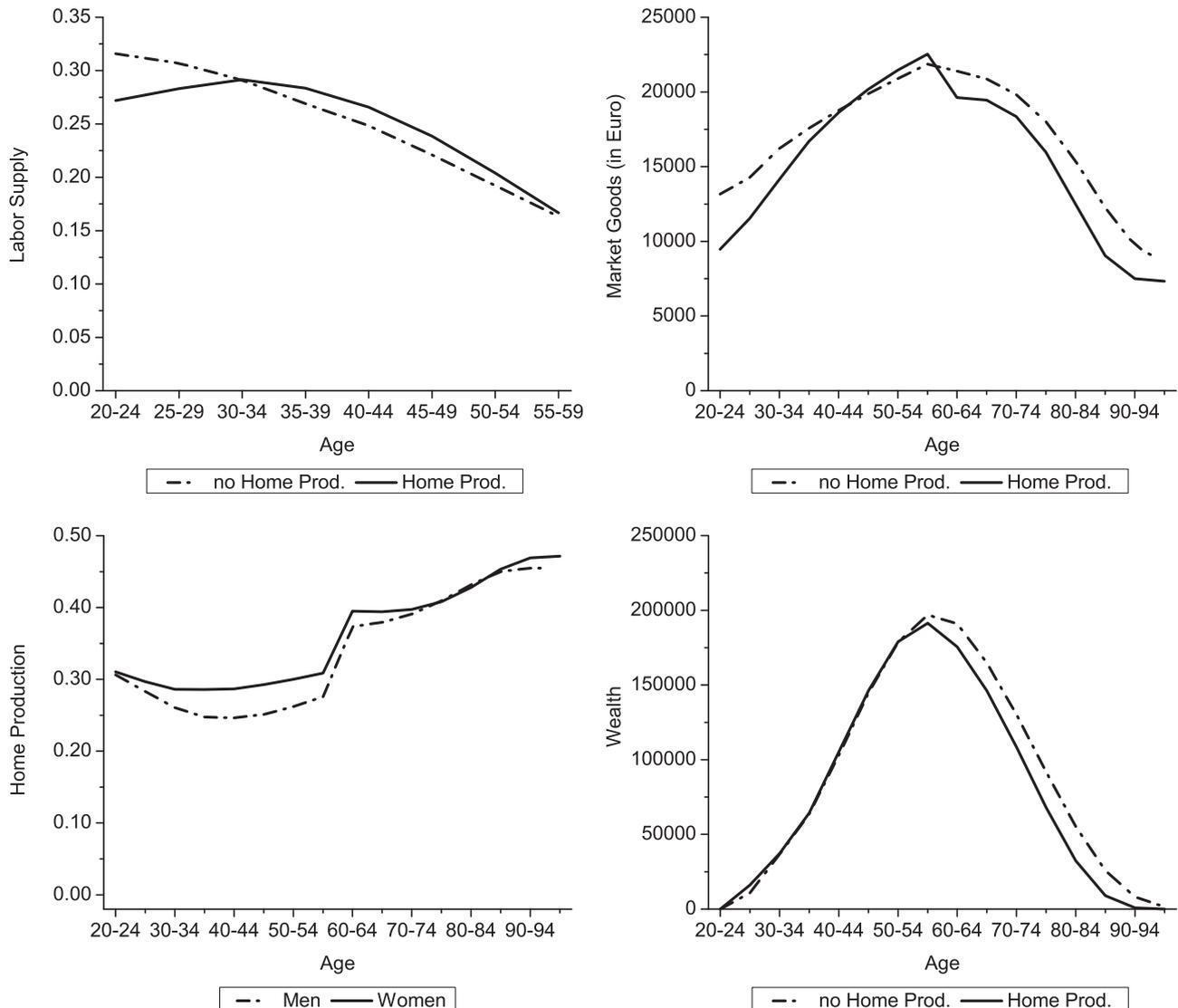
In order to calibrate the split-up of time use, we assume  $\chi = 0.5$ . Rogerson (2009, p. 596) surveys the literature and concludes that typical estimates of the substitution elasticity between market goods and home work lie between 0.4 and 0.6. We recalibrate the leisure preference  $\alpha = 1.13$  and chose the share parameter for market goods  $v = 0.48$  in order to match the time use from Burda et al. (2008) who report that on average men and women spend about 43, 26 and 31 percent of their time endowment as leisure time, market work and home work, respectively. Finally, in order to realize the same capital-output ratio as reported in Table 3 we adjust the discount rate which is now set at  $\beta = 0.856$ .

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<sup>8</sup> In Fehr et al. (2008) the intergenerational redistribution pattern is quantitatively stronger. Existing pensioners gain roughly 2 percent, medium-age workers lose up to 3 percent and future cohorts gain more than 3 percent of remaining lifetime resources. After compensation, the aggregate efficiency loss amounts to 0.57 percent of initial resources.

Since agents can substitute working at home and in the market, they now work more later in life when their productivity is high, see the upper left part of Figure 2. They also consume less initially, so that the fraction of liquidity constrained agents of the two youngest cohorts falls from 53.7 and 50.5 percent in the base model to 30.2 and 24.4 percent, respectively. The upper right part of Figure 2 also shows that consumption at retirement falls much stronger now since households substitute towards home production when they retire. The importance of home production in explaining the so-called "retirement consumption puzzle" is pointed out by Aguiar and Hurst (2005). Schwerdt (2005) presents some evidence for Germany. The

Figure 2: Life cycle behavior without and with home production



lower left part of Figure 2 also shows that – due to their lower productivity – women work more at home than men especially during the employment phase. Finally, since agents can now better self-insure against longevity risk, they drive down their assets more rapidly after retirement, see the lower right part of Figure 2.

Table 3 documents that the macroeconomic structure of the resulting initial equilibrium is

quite similar as in the base model. Since people with low (high) productivity substitute to home (market) work, the resulting income distribution in the initial equilibrium becomes more unequal and the Gini-coefficient increases from 28 to 40. The more unequal income distribution also reduces aggregate pension benefits, since more households receive an income above the contribution ceiling. Finally, since assets are run down faster after retirement, bequest fall from 5.4 to 4.3 percent of GDP.

How does the consideration of home production change the results of the pension funding experiment? The right part of Tables 4 and 5 report the resulting macroeconomic and welfare effects which can be directly compared with the respective left part.

*Macroeconomic implications* Comparing the right part and the left part of Table 4 shows that the fall in the payroll tax is almost the same without and with home production. The accumulation of pension debt is significantly reduced, since the benefit share in GDP was lower initially, see Table 3. Now the short-run labor supply reaction is much stronger than before. On the one side, agents now work more in the years before retirement where the effective payroll tax increases due to the reform. On the other side, agents can now immediately substitute towards home production so that labor supply becomes more elastic. Due to their lower productivity, especially women increase home production after the reform. Since people work less on the market than before, asset, bequest and capital accumulation are dampened significantly compared to the left part of Table 4. As a result, pension funding now even reduces output in the long run by 0.7 percent. The lower capital stock dampens the long-run wage rate increase which in turn decreases income tax revenues so that the consumption tax could not be reduced as before.

*Welfare and efficiency* The right part of Table 5 shows the impact of home production on intergenerational welfare and efficiency. Since consumption taxes remain constant, pensioners are only hurt by the reduction of the interest rates which mainly affects younger pensioners with savings. Especially high-skilled workers of both genders lose more since they worked more in the initial equilibrium. After the reform, their labor supply falls significantly stronger. In addition, workers are also hurt more since the consumption tax rate remains constant. Finally, future cohorts gain much less than before, since future wages and bequest increase less. After compensation, the efficiency loss from pension funding increases from 0.2 percent to 0.8 percent. There are two main reasons for this result: On the one side, labor supply distortions rise with home production, on the other side the gains from higher liquidity are much smaller now.

Note that our results are in sharp contrast to the findings from Dotsey, Li and Yang (2012) who argue that the consideration of home production increases the welfare gains from pension funding. The positive impact of home production is explained by the fact that home production provides some self-insurance which dampens the cost from the elimination of the longevity insurance. Dotsey et al. (2012) only compare the long-run welfare gains derived from a model without and with home production without considering the transition

path and aggregate efficiency changes. In our model home production has a negative impact on long-run welfare gains and aggregate efficiency effects of pension funding. Of course, there are also positive self-insurance effects but they are dominated by negative labor supply effects.

#### 4.4 Singles and married households

Next we distinguish between single and married households with two earners similar as in Kaygusuz (2011), Sanchez-Martin and Sanchez Marcos (2010) or Nishiyama (2010). Conditional on getting married, individuals of a gender  $g$  and skill level  $s$  are assigned to a  $s^*$  spouse with probabilities  $\pi_{g,s}^{s^*}$  before they enter the life cycle.<sup>9</sup> This marriage remains stable afterwards, i.e. at this stage we abstract from divorce and remarriage. The individual state vector of an age- $j$  agent now changes to

$$z_j = (g, s, \eta_j, m_j, \eta_j^*, a_j, p_j), \quad (11)$$

In case of a married couple,  $m_j$  and  $\eta_j^*$  denote the labor market status of the respective partner, i.e.  $m_j \in \mathcal{M} = 0, 1, \dots, S$ . In case the individual is a single we set  $m_j = \eta_j^* = 0$ .

Future utility of married households is computed over the distribution of future states of productivity  $\eta_{j+1}$  and  $\eta_{j+1}^*$ . We assume a collective model of household decision making. Consequently, married couples of skill groups  $s$  and  $s^*$  at age  $j$  maximize a joint welfare function with equal weights in order to obtain efficient outcomes

$$\max_{x_j, h_j, h_j^*, \ell_j, \ell_j^*} \left\{ u(c_j, \ell_j) + \beta \psi_{j+1}^g E[V(z_{j+1})] \right\} + \left\{ u(c_j, \ell_j^*) + \beta \psi_{j+1}^{g^*} E[V(z_{j+1}^*)] \right\}. \quad (12)$$

Consumption for each family member is now computed from

$$c_j(x_j, h_j, h_j^*) = \frac{1}{1 + \omega} \left\{ v x_j^{1 - \frac{1}{\lambda}} + (1 - v) \Phi \left[ (h_j)^{1 - \frac{1}{\sigma}} + (h_j^*)^{1 - \frac{1}{\sigma}} \right]^{\frac{1 - \frac{1}{\lambda}}{1 - \frac{1}{\sigma}}} \right\}^{\frac{1}{1 - \frac{1}{\lambda}}}. \quad (13)$$

Aggregate home labor is derived using a CES production function, where  $\sigma$  measures the elasticity of substitution between time spent in home production by the two partners,  $\Phi$  is a scale parameter and  $\omega$  captures the degree of joint consumption in the household. If all household consumption is consumed jointly (separately) then we would set  $\omega = 0$  ( $\omega = 1$ ).

The respective household budget constraint reflects the fact that both assets and pension claims are pooled within a marriage.<sup>10</sup> Consequently, pension claims now evolve according to

$$p_{j+1} = p_j + \kappa \frac{y_j + y_j^*}{2}, \quad (14)$$

<sup>9</sup> Variables pertaining to a partner are denoted by an asterisk. In particular, if  $s = \text{male}$ , then  $s^* = \text{female}$  and vice versa.

<sup>10</sup> The pooling of pension claims approximates the German widow's pension benefit.

while the household budget constraint reads

$$2a_{j+1} = 2[1 + (1 - \tau_r)r] a_j + (1 - \tau_w)(1 - \tau_p) [y_j + y_j^*] + 2(1 - \tau_w)\tilde{p}_j + 2b_j - (1 + \tau_x)x_j. \quad (15)$$

Note that married couples in our benchmark are not altruistic and don't receive direct utility from being married. Consequently, they still value consumption and leisure according to the function (2).

Beneath the productivity processes for both partners, married agents take into account the possibility that one of the spouses dies. In this case the surviving partner, e.g. the partner of gender  $g$ , inherits the assets of the partner completely and his state turns into  $z_{j+1} = (g, s, \eta_{j+1}, 0, 0, 2a_{j+1}, p_{j+1})$ . Consequently, couples' assets are only passed on to younger cohorts if both partners die at the end of the same period.

Mating probabilities  $\pi_{g,s}^{s*}$  were again estimated from the SOEP data base. While 96.6 percent of low-skilled males marry a low-skilled female spouse, only 53.3 percent of high-skilled men marry a woman from the same skill level. This documents the well known fact that a couple with a low-skilled man and a high-skilled women is fairly rare (only 2.6 percent of couples) while the opposite combination is quite common (with 11.0 percent). Overall we assume that 58 percent of the population lives as couples, which reflects the average number of married individuals within a cohort in Germany, see below.

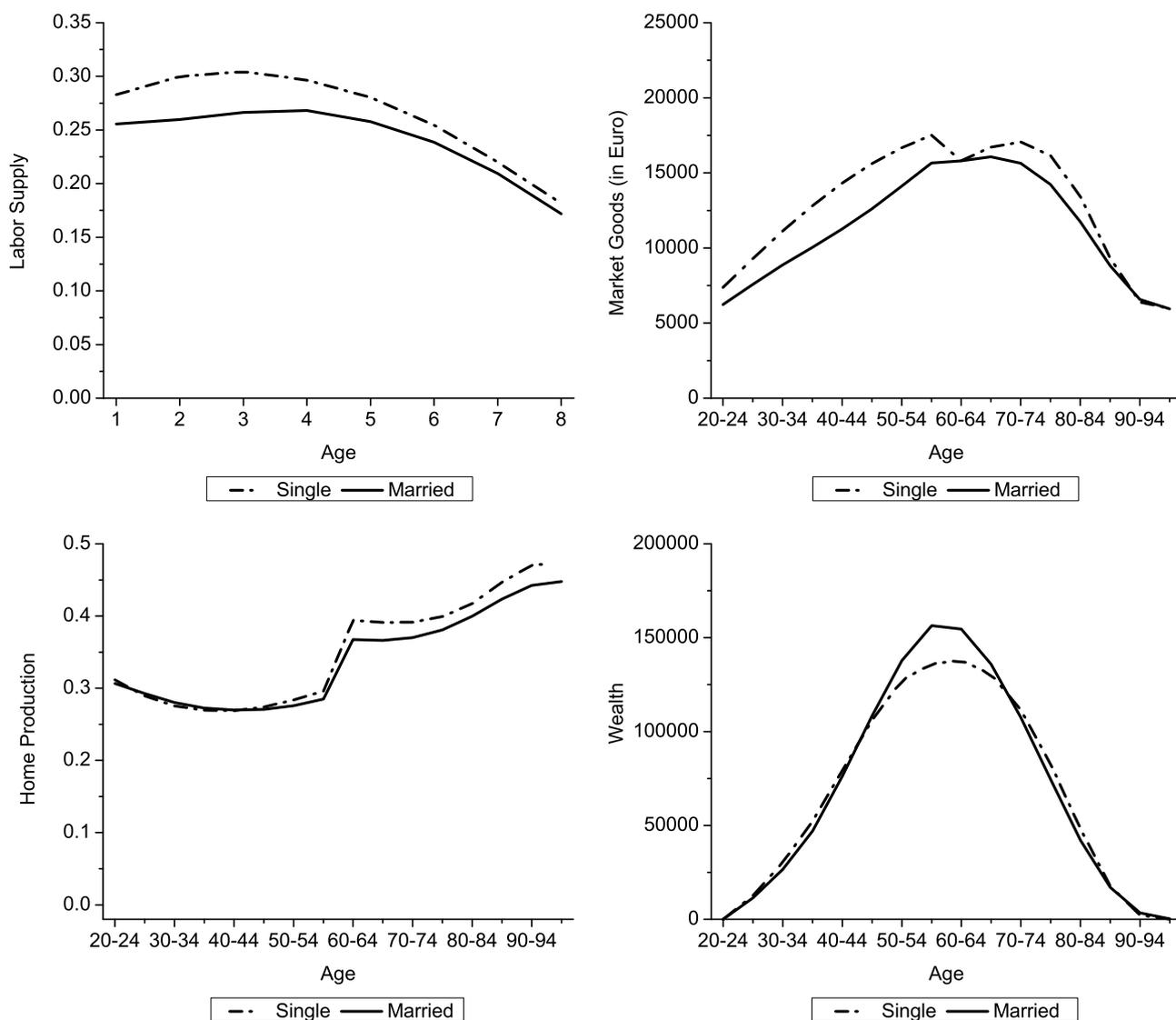
With respect to home production we calibrate a substitution elasticity between male and female home work of  $\sigma = 0.9$  so that we obtain a time difference in home labor for married men and women similar to the one reported in Burda et al. (2008). We chose a scaling factor  $\Phi$  in order to make sure that aggregate household home labor never exceeds two. The degree of joint consumption in the household is taken from Nishiyama (2010) and set at  $\omega = 0.6$ . Again we (slightly) recalibrate the leisure preference  $\alpha = 1.03$  in order to match time use data and adjust the discount rate  $\beta = 0.885$  to realize the previous capital-output ratio.

Figure 3 reports the behavioral differences of singles and married individuals. Since couples can save resources due to joint consumption, they work less on the market and at home compared to singles. At the same time they save more and can smooth their consumption better at retirement. Since widows and widowers receive the assets of the former spouse, consumption of singles again increases after retirement.

Table 3 shows that the resulting macroeconomic structure is quite similar as in the base model. Due to higher savings aggregate bequest rise to 5.3 percent of GDP. However, now younger cohorts only receive 3.1 percent of GDP as bequest, since 2.2 percent are absorbed by the surviving spouse. Since couples can better self-insure against wage risk, they save less at the beginning of the employment phase so that more couples than singles are liquidity constrained initially.

Next we consider in the left part of Tables 6 and 7 the macroeconomic and welfare consequences of the pension funding experiment when we apply the model with singles and married couples.

Figure 3: Life cycle behavior of singles and families (without marital risk)



*Macroeconomic implications* Comparing the left part of Table 6 and the right part of Table 4 shows that the reduced fraction of *intergenerational* bequests slightly dampens the long-run increase in assets and the capital stock. Since younger cohorts receive less bequest, labor supply falls less and the substitution towards home production is dampened. As a result, the wage increase and the fall in output is smaller than before. Higher income tax revenues allow to reduce the consumption tax now slightly by 0.2 percent.

*Welfare and efficiency* The reduction in bequest is also the main driving force which explains the changes in welfare and aggregate efficiency reported in the left part of Table 7. Note that we now distinguish in each gender between singles and married households. As one would expect, existing single households experience almost the same welfare changes as in the previous subsection. At the same time, existing families are significantly better off than singles, since they can self-insure against labor income and longevity risk. Finally, long-run

Table 6: Macroeconomic effects of household structure without and with marital risk<sup>a</sup>

Period	without marital risk				with marital risk			
	1	3	5	$\infty$	1	3	5	$\infty$
Private Assets	0.0	9.2	20.7	45.9	0.0	9.0	20.5	44.5
Capital Stock	0.0	0.2	0.8	3.5	0.0	-0.2	0.3	3.3
Gov. Debt/GDP (in %)	60.5	59.8	58.6	54.6	60.4	59.7	58.4	55.0
Pens. Debt/GDP (in %)	0.0	36.8	81.7	174.4	0.0	37.2	82.3	169.5
Labor Supply	-1.2	-1.5	-1.8	-2.4	-1.0	-1.2	-1.5	-2.0
Home Production	0.6	1.0	1.2	1.8	0.5	0.8	1.0	1.4
GDP	-0.8	-0.9	-0.9	-0.4	-0.7	-0.9	-0.9	-0.2
Bequests	-0.3	-0.3	6.2	109.3	-0.3	-0.1	5.5	99.1
Wage rate	0.4	0.6	0.9	2.1	0.4	0.4	0.7	1.8
Interest rate <sup>b</sup>	-0.1	-0.1	-0.2	-0.3	-0.1	-0.1	-0.1	-0.3
Consumption tax rate <sup>b</sup>	-0.2	-0.2	-0.2	-0.2	-0.2	-0.2	-0.2	-0.2
Social Security tax rate <sup>b</sup>	-5.5	-5.5	-5.5	-5.5	-5.7	-5.7	-5.7	-5.7

<sup>a</sup>Changes in percent over value in initial equilibrium.

<sup>b</sup>Changes in percentage points.

welfare gains are smaller due to lower bequest and wages.

Table 7: Welfare effects of household structure without and with marital risk

Gender	without marital risk				with marital risk					
	without LSRA		with LSRA		without LSRA		with LSRA			
	Male	Female	Male	Female	Male	Female	Male	Female		
Family status	single	married	single	married	single	married	single	married		
Retirees										
85-89	0.02	0.02	0.01	0.01	0.00	0.03	0.03	0.02	0.02	0.00
65-69	-0.04	-0.07	-0.07	-0.09	0.00	-0.01	-0.04	-0.04	-0.06	0.00
Workers										
45-49	-1.14	-0.89	-1.12	-0.93	0.00	-1.10	-0.81	-1.10	-0.83	0.00
25-29	-0.49	-0.29	-0.56	-0.19	0.00	-0.23	-0.24	-0.24	-0.28	0.00
Future Generations										
15-19	0.13	0.15	-0.65	-0.65	0.21	0.18	-0.49	-0.49	-0.49	-0.49
5-9	0.52	0.54	-0.65	-0.65	0.55	0.54	-0.49	-0.49	-0.49	-0.49
$\infty$	0.97	0.99	-0.65	-0.65	0.94	0.91	-0.49	-0.49	-0.49	-0.49

\*In percent of initial resources.

Although long-run welfare gains are smaller, aggregate efficiency losses are reduced compared to the previous simulation in the right part of Table 5. This demonstrates that long-run welfare alone is not a good indicator for economic efficiency! The lower efficiency losses isolate the benefits from the (partial) provision of family insurance already discussed in Kotlikoff and Spivak (1981) as well as Brown and Poterba (2000). Quite surprisingly, the implicit annuity provision of the family is fairly small in our model compared to the impact of home production. While the latter increased aggregate efficiency losses from 0.2 to 0.8 percent, the former only decrease the losses from 0.8 to 0.65 percent of aggregate resources.<sup>11</sup>

## 4.5 Divorce risk and remarriage

Finally, we introduce the fully specified model with demographic dynamics similar to those of Domeij and Klein (2002) or Hong and Rios-Rull (2007). Singles now can get married at any age  $j$  with probability  $\pi_j^m$  and married couples can get divorced with probability  $\pi_j^d$ . We restrict (mainly for computational reasons) marriage, divorce and re-marriage to working periods. After retirement, single individuals remain single until death while married couples could only become widows/widowers.

Single agents still solve the problem (10), but now future utility is computed over the distribution of future states of own productivity  $\eta_{j+1}$ , marital status  $m_{j+1}$  and productivity of the partner  $\eta_{j+1}^*$ . If the agent stayed single with probability  $1 - \pi_{j+1}^m$ , his state would move to  $z_{j+1} = (g, s, \eta_{j+1}, 0, 0, a_{j+1}, p_{j+1})$ . However, if he was to get married to an agent of same age with probability  $\pi_{j+1}^m$ , his future state would change to

$$z_{j+1} = \left( g, s, \eta_{j+1}, s^*, \eta_{j+1}^*, \frac{a_{j+1} + a_{j+1}^*}{2}, \frac{p_{j+1} + p_{j+1}^*}{2} \right). \quad (16)$$

Single agents take into account the mating probabilities  $\pi_{g,s}^{s^*}$  and form expectations over future spouses' productivity  $\eta_{j+1}^*$ , assets  $a_{j+1}^*$  and pension claims  $p_{j+1}^*$  according to the distribution of singles of gender  $g^*$  and skill group  $s^*$  over the state space at age  $j$ . Note that, if two agents get married, their assets and earning points will be pooled, which highlights the risk sharing aspect of a marriage.

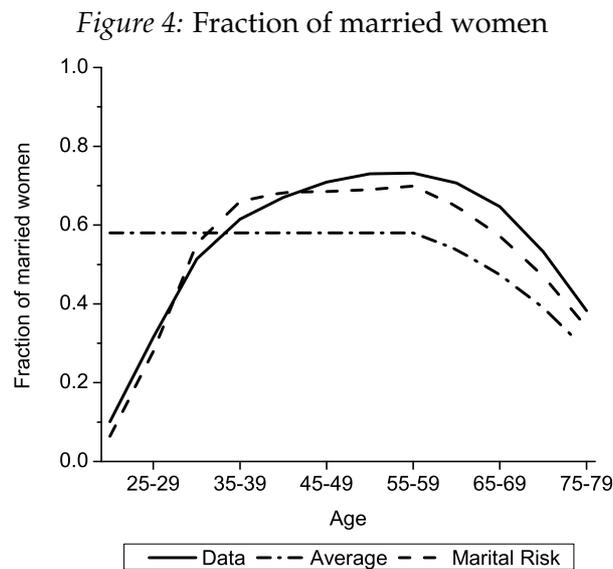
Married agents also maximize as before (12) but beneath the productivity processes for both partners, they take into account three different scenarios: The first of them reflects the situation when the marriage continues with probability  $1 - \pi_{j+1}^d$  in the next period and the spouse survives. The second case covers the situation when one of the spouses dies. Finally, the third case describes the situation when the marriage is divorced. Here, the individual status changes to  $z_{j+1} = (g, s, \eta_{j+1}, 0, 0, a_{j+1}, p_{j+1})$ .

Age-specific marriage and divorce probabilities  $\pi_j^m$  and  $\pi_j^d$  are derived from cohort data

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<sup>11</sup> Of course, this result depends on the fraction of married households in the population. When we simulate the same policy reform in a model without single households long-run welfare gains are further reduced and aggregate efficiency losses amount to 0.5 percent.

reported in the Statistical Yearbook of the Statistisches Bundesamt (2007). Specifically we adjust actual marriage and divorce rates in order to match existing cohort-specific fractions of married couples. Figure 4 shows the fraction of married women in each cohort we obtain when applying our estimated marriage and divorce probabilities to the model. We see an increase of married couples in the early years of life until age 35 due to high marital risk. Passing age 35, the number of married couples stays roughly constant. Finally, with survival probabilities being lower than one at retirement, the fraction of married women again declines as the number of widows/widowers increases. Figure 4 also shows the fraction of married women we obtain when applying the actual data on married couples in Germany computed from Statistisches Bundesamt (2007). Finally, we also report the respective fractions with the constant share from the previous section without marital risk.



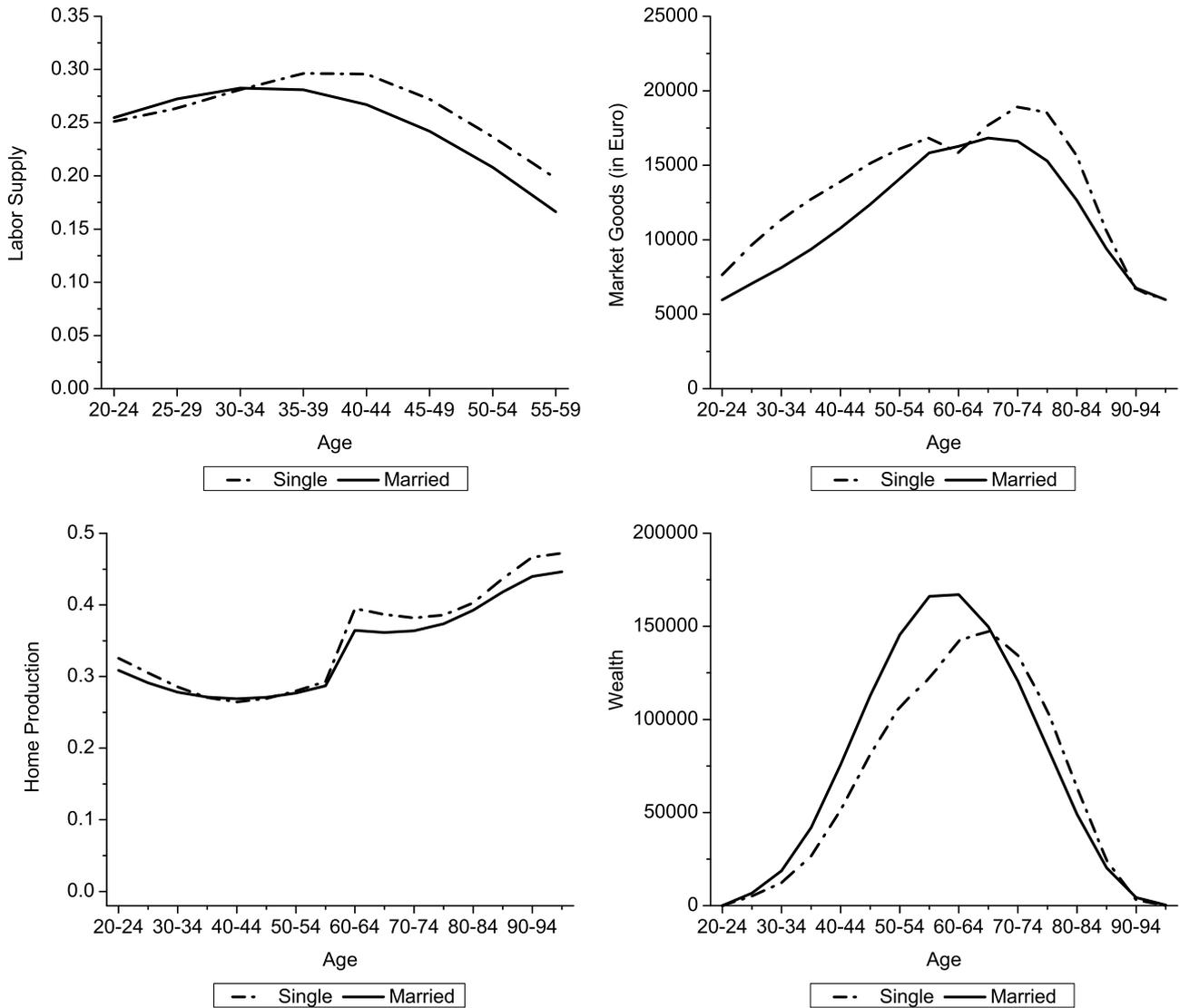
Again we (slightly) recalibrate the leisure preference  $\alpha = 1.05$  and adjust the discount rate  $\beta = 0.90$  to realize the capital-output ratio of 3.3. As shown in Figure 5, when singles still hope to get married and couples face the risk of divorce, labor supply differs significantly in the later employment phase. Singles also save less than couples since they might need to split their assets in case of marriage<sup>12</sup>, while couples have to close this savings gap. Note from Figure 4 that the fraction of couples is now higher at retirement than in the previous model. Consequently, more widowers receive bequest from the previous spouse so that average consumption of singles now peaks after retirement.

The last column of Table 3 documents that aggregate bequest now rise to 6.2 percent of GDP. Since 2.9 percent of GDP are transfers to the surviving spouse the redistribution towards younger cohorts is hardly changed compared to the previous model.

Table 8 also illustrates that while single men and women have quite similar labor supply elasticities, married women's labor supply is significantly more elastic than that of men.

<sup>12</sup> Glazer (2008) has analyzed a similar prisoner's dilemma in a non-cooperative family model.

Figure 5: Life cycle behavior of singles and families (with marital risk)



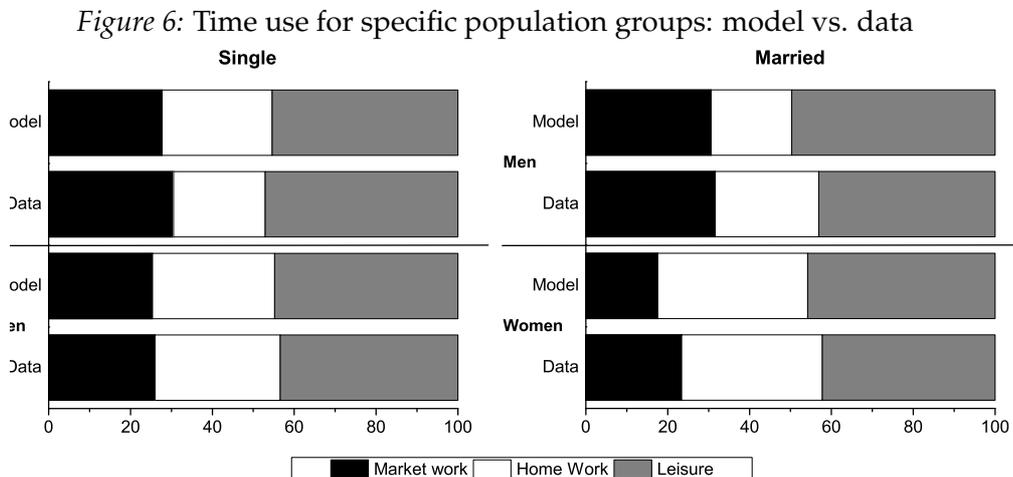
The latter reflects the fact that labor supply at the extensive margin is more flexible than at the intensive margin for married women.

Table 8: Labor supply elasticities with marital risk\*

	<i>Total</i>		<i>Single</i>		<i>Married</i>	
	<i>Men</i>	<i>Women</i>	<i>Men</i>	<i>Women</i>	<i>Men</i>	<i>Women</i>
uncompensated	0.155	0.334	0.281	0.344	0.079	0.324
compensated	0.606	0.885	0.851	0.917	0.458	0.855

Figure 6 compares the fractions of market work, home work and leisure for different marital statuses and genders generated by the model with those from the data. The right side re-

veals that single men and women spend roughly the same time on leisure consumption, but they are quite different with respect to their shares of market work and home work. Even single men work much less at home than women, which is mainly due to the gender wage gap described above. The left part of Figure 6 documents that specialization increases in a marriage.



Source: Burda et al. (2008) and Statistisches Bundesamt (2003).

*Macroeconomic implications* Comparing the right and the left part of Table 6 shows that although the individual behavior differs quite strongly, the consideration of marital risk does hardly change the macroeconomic effects of pension funding. Since more married households enter the retirement phase, pension funding induces a stronger increase in intragenerational bequests than before. Consequently, asset and capital accumulation are dampened compared to the previous model which in turn reduces the fall in output and wage growth.

*Welfare and efficiency* The reduction in bequest is also the main driving force which explains the reduced intergenerational redistribution reported in the right part of Table 7. Note that now the differences between younger single and married households have almost disappeared. Note that especially younger singles who still can get married experience lower welfare losses from pension funding than in the previous model. This reflects the positive impact of family insurance. Consequently, the aggregate efficiency loss is slightly lower than before. Overall, however, the aggregate efficiency loss from pension funding is still significant. The benefits from family insurance appear to be fairly small in our model.

## 5 Conclusion

Summing up the simulation results from the previous section, we can draw three major conclusions. First, our results demonstrate that long-run welfare changes are a bad indicator

for economic efficiency. While our approach replicates the long-run welfare gains of pension funding found in many previous studies, we also find significant aggregate efficiency losses arising mainly from the elimination of longevity insurance. Second, we show that home production reduces long-run welfare gains and increases the efficiency losses from pension funding since higher labor market distortions dominate self insurance effects of home production. Third, two-earner households positively affects the efficiency of pension funding due to the provision of family insurance. Quite surprisingly, the benefits of family insurance are rather small so that pension funding even generates an efficiency loss in the family model.

Of course, our simulation approach is based on various assumptions which might affect our qualitative results. First, the initial pension system is Bismarkian so that labor supply distortions are small and the insurance provision against income risk is limited. It could be that the effects of home production are quite different when a more progressive pension system is eliminated. Second, bequest motives may play an important role especially in a family context. But this is completely neglected here which might at least partly explain why we find a surprisingly small impact of family insurance provision. A third issue for further study concerns the preference structure of the family household. On the one side it might be interesting to change the weights assigned to husband and wife in the decision problem of families or even move from a cooperative to a non-cooperative family decision modeling. On the other side there is some evidence that women are more risk averse than men which indicates to modify the assumption that men and women have the same preferences. Finally, since social security may as well affect household formation, an obvious extension for future work will be to endogenize marriage and divorce probabilities along the lines of Chade and Ventura (2002).

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